

A boundary layer sliding mode control design for chatter reduction using uncertainty and disturbance estimator

Prasheel V. Suryawanshi · Pramod D. Shendge · Shrivijay B. Phadke

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Abstract This paper proposes a new boundary layer sliding mode control design for chatter reduction. The control scheme uses a discontinuous control outside the boundary layer and switches over to uncertainty and disturbance estimator (UDE) based control inside. The problem of large initial control underlying the method of UDE, is also addressed with a modified sliding surface. The overall stability of the system is proved and the results are verified on an illustrative example and application to flexible joint system. The results show that the proposed method exhibits much better control performance than the baseline SMC using ‘sat’ function, for reduced chattering.

Keywords Sliding mode control · Uncertainty and disturbance estimator · Chatter · Flexible joint

1 Introduction

Sliding mode control (SMC) is an effective strategy for controlling systems with significant uncertainties and unmeasurable disturbances [1–5]. The control however is discontinuous and requires the bounds of uncertainties and disturbances. In many situations, these bounds are hard to find,

which result in overestimation and consequently a large control. Additionally, the discontinuous control leads to chatter. The chatter is undesirable because it causes excessive wear and tear of components and can excite fast unmodelled dynamics. The chatter in control and states is a major concern in the practical implementation of SMC [6, 7].

The most popular method for chatter control is the boundary layer control [8, 9] in which the control is discontinuous outside a boundary layer, but is continuous inside. The method tries to strike a trade off between invariance of system trajectories and smoothness of control.

Lee and Utkin [10] use state-dependent or equivalent-control-dependent, magnitude of the discontinuous control for chatter suppression. A variety of other strategies like filtering of control signal [11], time varying feedback gain [12, 13], Euler time-discretization [14], integration of control signal [15, 16], fuzzy logic [17], neural network [18], quantitative feedback theory [19] have been proposed in literature to address the problem of chattering. The methods of chattering suppressions related to higher order sliding modes can be found in [20–22],

A combination of SMC with methods that give estimates of uncertainty and disturbance enables the reduction of discontinuous component of control, thereby suppressing the chatter significantly. The uncertainty and disturbance estimator (UDE) [23] is one such strategy for estimating slow varying uncertainties. This method has been applied to SMC [24] and in applications like load frequency controller [25], flexible joint [26], robotic control [27] to name a few. In [25–27] the focus is on estimation accuracy rather than chatter; and sliding surface used has the shortcoming of resulting in large initial control.

A strategy for chatter control based on UDE is proposed in this paper. The control scheme uses a discontinuous control outside the boundary layer and switches over to UDE

P. V. Suryawanshi (✉)
MIT Academy of Engineering, Alandi (D), Pune, India
e-mail: prasheels@gmail.com

Present Address:
P. V. Suryawanshi
Department of Instrumentation & Control Engineering,
College of Engineering, Pune, India

P. D. Shendge · S. B. Phadke
Department of Instrumentation & Control Engineering,
College of Engineering, Pune, India

based control inside. The method of UDE results in a large initial control for systems with non-zero initial conditions. The strategy proposed here will handle this drawback in a novel way. The stability of the system inside the boundary layer is proved.

In this paper, UDE is used to estimate the lumped uncertainty comprising of uncertainty in plant as well as input matrix and unknown disturbance. The main contributions of this paper are as follows:

- (i) The class of disturbances considered here is significantly larger.
- (ii) The sliding surface is modified to circumvent the problem of large initial control.
- (iii) No knowledge of bounds on uncertainties and disturbances is required.
- (iv) The ultimate boundedness of estimation error (\tilde{e}) and sliding variable (σ) is guaranteed inside the boundary layer.

The paper is organized as follows: Sect. 2 describes the discontinuous sliding mode control design. A control based on uncertainty and disturbance estimation using UDE is explained in Sect. 3. Section 4 gives the stability analysis. The performance is illustrated by a numerical example in Sect. 5 followed by application to flexible joint system in Sect. 6 and conclusion in Sect. 7.

2 Sliding mode control

Consider an uncertain single input single output (SISO) system,

$$\dot{x} = Ax + bu + \Delta Ax + \Delta bu + d(x, t) \tag{1}$$

where x is the state vector, u is the control input, A and b are known constant matrices, ΔA and Δb are uncertainties and $d(x, t)$ is the unknown, unmeasurable disturbance.

Assumption 1 *The uncertainties ΔA , Δb and disturbance $d(x, t)$ satisfy matching conditions given by,*

$$\Delta A = bD, \quad \Delta b = bE, \quad d(x, t) = bv(x, t) \tag{2}$$

where D and E are unknown matrices of appropriate dimensions and $v(x, t)$ is an unknown function.

The Eq. (2) is the well-known matching condition required to guarantee invariance and is an explicit statement of the structural constraint stated in [28]. The system (1) can now be written as,

$$\dot{x} = Ax + bu + be(x, t) \tag{3}$$

where $e(x, t) = Dx + Eu + v(x, t)$ is the lumped uncertainty comprising of uncertainties in A and b as well as external disturbance.

Assumption 2 *The lumped uncertainty $e(x, t)$ is bounded by a known function,*

$$|e(x, t)| = \rho(x, t) \tag{4}$$

where $\rho(x, t)$ is a known positive scalar function.

For a sliding surface,

$$\sigma = sx \tag{5}$$

the control that ensures sliding is given by,

$$u = u_n + u_{eq} \tag{6}$$

where

$$u_{eq} = -(sb)^{-1}(sAx + k\sigma) \quad k > 0 \tag{7}$$

and

$$u_n = -\rho(x, t) \operatorname{sgn}(\sigma sb) \tag{8}$$

The control given by (8) is discontinuous. Since discontinuous control is objectionable, a commonly used smooth approximation of u_n is given by,

$$u_n = \begin{cases} -\rho(x, t) \operatorname{sgn}(\sigma sb), & |\sigma sb| > \epsilon \\ \frac{-\rho(x, t)}{\epsilon}, & |\sigma sb| \leq \epsilon \end{cases} \tag{9}$$

where ϵ is a small positive number. The well known drawbacks of the smooth approximation are that a small ϵ retains invariance but may result in chatter while large ϵ suppresses chatter but results in significant loss of invariance.

3 A new smooth control inside the boundary layer

A smoothing approach using uncertainty and disturbance estimator (UDE) is used for chatter suppression. The key idea in UDE based control is to approximate and estimate the uncertainty using a filter of right bandwidth. The opposite of estimate is then used in control to negate the effect of the uncertainty [23, 24]. The following assumption is needed to ensure that \tilde{e} (i.e. $e - \hat{e}$) is bounded.

Assumption 3 *The lumped uncertainty $e(x, t)$ is continuous and satisfies,*

$$\left| \frac{d^{(j)}e(x, t)}{dt^{(j)}} \right| \leq \mu \quad \text{for } j = 0, 1, 2, \dots, r \tag{10}$$

where μ is a small positive number.

The lumped uncertainty $e(x, t)$ can be estimated as,

$$\hat{e} = e G_f(s) \quad (11)$$

where $G_f(s)$ is a strictly proper low-pass filter with unity steady state gain and sufficiently large bandwidth.

Using (3) and (5)

$$\dot{\sigma} = s A x + s b u + s b e \quad (12)$$

Using (6) and (7) in (12)

$$\dot{\sigma} = s b u_n + s b e - k \sigma \quad (13)$$

$$e = (sb)^{-1}(\dot{\sigma} + k\sigma) - u_n \quad (14)$$

The control strategy in UDE is to estimate lumped uncertainty e as \hat{e} (11) and use $-\hat{e}$ as a component in control; to cancel the effect of e . Let

$$u_n = -\hat{e} \quad (15)$$

Using (11), (14) and (15),

$$u_n = -(sb)^{-1}(\dot{\sigma} + k\sigma) \frac{G_f(s)}{1 - G_f(s)} \quad (16)$$

Specially for a choice of $G_f(s)$ given by,

$$G_f(s) = \frac{1}{1 + \tau s} \quad (\text{first order filter}) \quad (17)$$

where τ is a small positive constant,

$$u_n = -(sb)^{-1} \frac{1}{\tau} \left(\sigma + k \int \sigma \right) \quad (18)$$

From Eqs. (11) and (17)

$$\dot{\tilde{e}} = -\frac{1}{\tau} \tilde{e} + \dot{e} \quad (19)$$

From Eqs. (13) and (15)

$$\dot{\sigma} = -k \sigma + s b \tilde{e} \quad (20)$$

Remark 1 If $\dot{e} = 0$, \tilde{e} goes to zero asymptotically, otherwise it is ultimately bounded. As a consequence $\sigma \rightarrow 0$, if $k > 0$ and $\tau > 0$. If \dot{e} is not small, but \ddot{e} is small, i.e $j = 2$ in (10), then the accuracy of estimation can be improved by estimating e as well as \dot{e} .

The ultimate boundedness of \tilde{e} and calculation of bound is discussed in Sect. 4. The improvement obtained using an higher order filter is derived in Sect. 3.3 and shown in the results. The proposed estimator does not need any knowledge of the size of the uncertainty. It can estimate slow varying

lumped uncertainties accurately, if τ is chosen to be a small constant. However a small τ results in a large control u_n at $t = 0$, since $\sigma(0)$ may not be a small number in general. To circumvent this problem, a modified sliding surface is proposed.

3.1 Modified sliding surface

The conventional sliding surface is modified as,

$$\sigma^* = \sigma - \sigma(0) e^{-\alpha t} \quad (21)$$

where α is a user chosen positive constant.

It may be noted that at $t = 0$, the modified sliding variable $\sigma^*(0) = 0$ for any $\sigma(0)$ and $\sigma^* \rightarrow \sigma$ as $t \rightarrow \infty$.

3.2 Design of control

From (3) and (21),

$$\dot{\sigma}^* = s A x + s b u + s b e + \sigma(0) \alpha e^{-\alpha t} \quad (22)$$

selecting control,

$$u = u_{eq} + u_n \quad (23)$$

and

$$u_{eq} = -(sb)^{-1}(s A x + \sigma(0) \alpha e^{-\alpha t} + k \sigma^*) \quad (24)$$

Working on lines similar to Sect. 2, it is straightforward to obtain,

$$u_n = -(sb)^{-1} \frac{1}{\tau} \left[\sigma^* + k \int \sigma^* \right] \quad (25)$$

The advantage of (25) is that even for small τ , u_n is not large, since σ^* is small for all $t \geq 0$. It may be noted that u is continuous for all $t \geq 0$ and therefore the control is chatter free.

Remark 2 The control developed in this section is different from the one developed in [25]. The definition of the sliding variable is different and while the control in [25] requires real time integration to find the values of the sliding variable, the proposed control does not require such an integration.

3.3 Control using second order UDE

With reference to Remark 1, the accuracy of estimation can be improved by using a second order UDE. The uncertainty (e) and its derivative (\dot{e}) can be estimated using a second order filter of the form,

$$G_f(s) = \frac{1 + 2\tau s}{\tau^2 s^2 + 2\tau s + 1} \quad (26)$$

where τ is a small positive constant.

Using (22), (23) and (24)

$$\dot{\sigma}^* = s b u_n + s b e - k \sigma \tag{27}$$

$$e = (sb)^{-1}(\dot{\sigma}^* + k\sigma^*) - u_n \tag{28}$$

Using (11), (15) and (28),

$$u_n = -(sb)^{-1}(\dot{\sigma}^* + k\sigma^*) \frac{G_f(s)}{1 - G_f(s)} \tag{29}$$

With $G_f(s)$ as in (26),

$$u_n = -(sb)^{-1} \left(\frac{2}{\tau} \sigma^* + \frac{2\tau k + 1}{\tau^2} \int_0^t \sigma^* + \frac{k}{\tau^2} \int_0^t \int_0^t \sigma^* \right) \tag{30}$$

3.4 Proposed control

The proposed boundary layer SMC with first order UDE, for chatter reduction can be written as,

$$u_{eq} = -(sb)^{-1}(s A x + \sigma(0) \alpha e^{-\alpha t} + k \sigma^*) \tag{31}$$

$$u_n = \begin{cases} -\rho \operatorname{sgn}(\sigma s b), & |\sigma s b| > \epsilon \\ -(sb)^{-1} \frac{1}{\tau} \left(\sigma^* + k \int \sigma^* \right), & |\sigma s b| \leq \epsilon \end{cases} \tag{32}$$

4 Stability

The estimation error \tilde{e} and sliding variable σ are ultimately bounded. Here the general case when $j = 1$ in (10) is considered. The bounds on \tilde{e} and σ are found by considering the Lyapunov function as,

$$V(\sigma, \tilde{e}) = \frac{1}{2} \sigma^2 + \frac{1}{2} \tilde{e}^2 \tag{33}$$

Taking derivative of $V(\sigma, \tilde{e})$ along (19) and (27)

$$\dot{V}(\sigma, \tilde{e}) = -k\sigma^2 + s b \sigma \tilde{e} - \frac{1}{\tau} \tilde{e}^2 + \tilde{e} \dot{\tilde{e}} \tag{34}$$

$$\leq -k\sigma^2 + |sb| |\sigma| \|\tilde{e}\| - \frac{1}{\tau} \|\tilde{e}\|^2 + \|\tilde{e}\| \mu \tag{35}$$

Using Young's inequality [29],

$$ab \leq \frac{a^2 + b^2}{2}$$

for any real a and b , one can obtain

$$\dot{V}(\sigma, \tilde{e}) \leq -k\sigma^2 + \frac{|sb|}{2} (|\sigma|^2 + \|\tilde{e}\|^2) - \frac{1}{\tau} \|\tilde{e}\|^2 + \|\tilde{e}\| \mu \tag{36}$$

$$\dot{V}(\sigma, \tilde{e}) \leq -|\sigma|^2 \left(k - \frac{|sb|}{2} \right) - \|\tilde{e}\|^2 \left(\frac{1}{\tau} - \frac{|sb|}{2} \right) + \|\tilde{e}\| \mu \tag{37}$$

$$\dot{V}(\sigma, \tilde{e}) \leq -|\sigma|^2 \left(k - \frac{|sb|}{2} \right) - \|\tilde{e}\| \left[\|\tilde{e}\| \left(\frac{1}{\tau} - \frac{|sb|}{2} \right) - \mu \right] \tag{38}$$

With $\left(k - \frac{|sb|}{2} \right) > 0$ and $\left(\frac{1}{\tau} - \frac{|sb|}{2} \right) > 0$, the system will be ultimately bounded. Using (38), the bound on \tilde{e} works out to be,

$$\|\tilde{e}\| \leq \frac{\mu}{\left(\frac{1}{\tau} - \frac{|sb|}{2} \right)} \tag{39}$$

Similarly the bound on σ works out to be,

$$|\sigma| \leq \frac{|sb| \mu}{k \left(\frac{1}{\tau} - \frac{|sb|}{2} \right)} \tag{40}$$

Thus $\|\tilde{e}\|$ and $|\sigma|$ are ultimately bounded and the bounds can be lowered using control parameters k and τ .

5 Numerical example

The following continuous system is considered to illustrate the results:

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix};$$

$$\Delta A = \begin{bmatrix} 0 & 0 \\ -1 & -2 \end{bmatrix}, \quad \Delta b = \begin{bmatrix} 0 \\ -0.4 \end{bmatrix};$$

ΔA and Δb are the uncertainties in the plant. The initial conditions for the plant are $x_0 = [1 \ 0]^T$ and control gain is $k = 5$. The disturbances considered are sinusoidal and sawtooth as shown in Fig. 1.

The discontinuous control is approximated with continuous approximation inside the boundary layer using *sat* function. The sliding plane used is $\sigma = 4x_1 + x_2$. The well known trade-off between smoothing and invariance is verified for varying values of ϵ ; for both sinusoidal as well as sawtooth disturbance. It is clearly evident that the chatter is dependent on the value of ϵ . The RMS values of σ for different values of ϵ are tabulated in Table 1.

The control performance is illustrated in Fig. 2 for a sinusoidal disturbance and boundary layer width of $\epsilon = 0.02$.

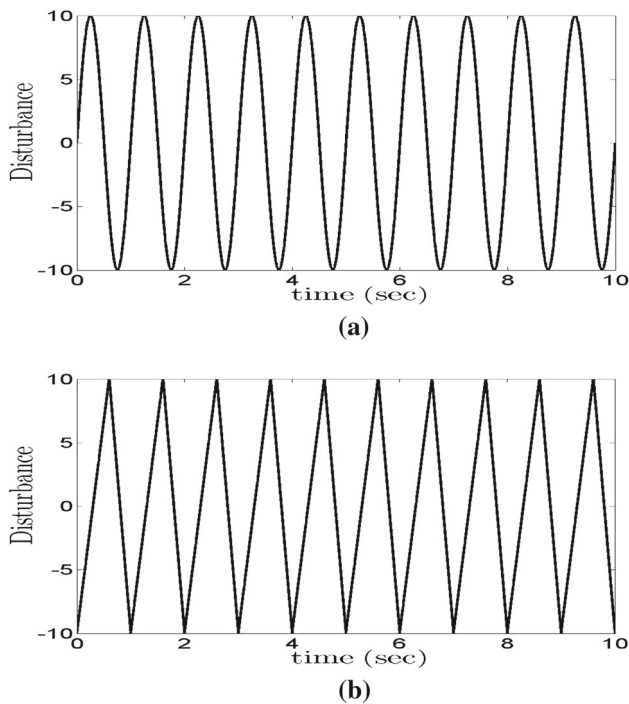


Fig. 1 Disturbances considered. **a** Sinusoidal, **b** triangular

Table 1 Variation in σ for different values of ϵ

Boundary layer width – ϵ	RMS value of σ	
	For sinusoidal	For sawtooth
0.02	0.0004524	0.02545
0.2	0.03646	0.1908
2	0.5244	0.4639

The states are shown in Fig. 2a, while a magnified view of the same graph is shown in Fig. 2b. It can be seen that for this case, the evolution of states inside the boundary layer depends on the disturbance acting. The sliding variable is shown in Fig. 2d. The chattering is evident in Fig. 2d.

The *sat* function is replaced by a first order UDE with filter time constant τ . The sliding plane is modified as $\sigma^* = \sigma - \sigma(0)e^{-\alpha t}$ so that initial control is within limits. The Fig. 3 shows that the modified sliding variable ($\sigma^* = \sigma - \sigma(0)e^{-\alpha t}$) significantly reduces the initial control as compared to the conventional sliding plane ($\sigma = sx$.)

The state tracking, control and sliding variable for a first order UDE with $\tau = 0.001\text{ s}$ is shown in Fig. 4a–d.

The results clearly demonstrate that, use of first order UDE in comparison to *sat* function reduces the chatter by a substantial amount.

The RMS values of σ with modified sliding plane and first order UDE, for $\epsilon = 0.02$ in Table 2 validates the same.

The results can be improved further by using a second order filter. The results with such a filter for $\tau = 0.001\text{ s}$ are

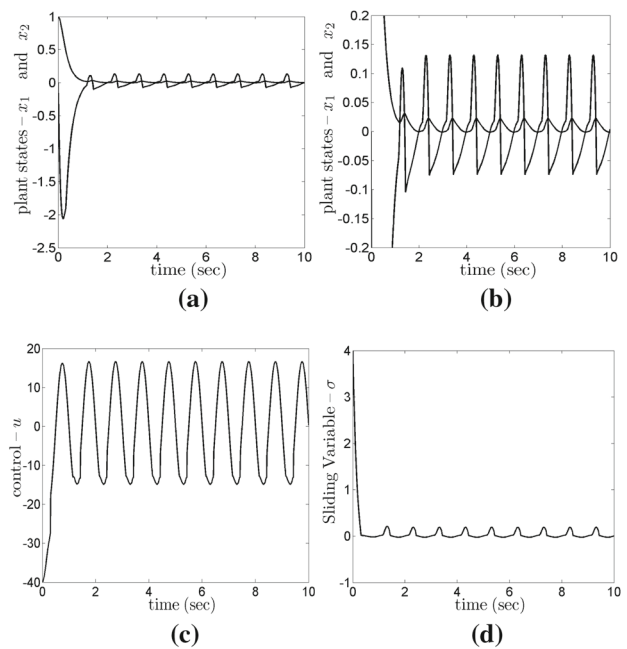


Fig. 2 States, control and sliding variable—*sat* function. **a** Plant states x_{plant} , **b** magnified plant states x_{plant} , **c** control u , **d** sliding variable σ

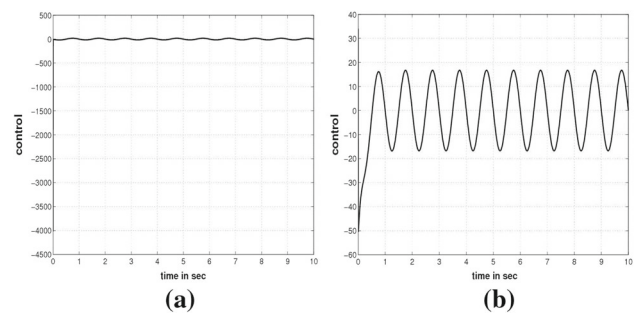


Fig. 3 Comparison of control. **a** Conventional sliding plane, **b** modified sliding plane

shown in Fig. 5, which shows that the improvement is indeed significant.

It is evident from Figs. 4d, 5d, that chattering is significantly reduced. The disturbance considered in these results is sinusoidal. The same results are observed for constant as well as sawtooth disturbance.

The value of τ has a direct effect on chatter mitigation, for a given boundary layer width ϵ . The value of τ and order of filter directly affect the accuracy of uncertainty estimation and consequently affect the achievable chatter reduction. The same can be seen in Fig. 6.

The RMS values of σ with $\epsilon = 0.02$ clearly demonstrates that UDE scores in comparison with *sat* function. The chatter is reduced as the order of UDE is increased. The value of $\tau = 0.001\text{ s}$ and sinusoidal disturbance is considered (Table 3).

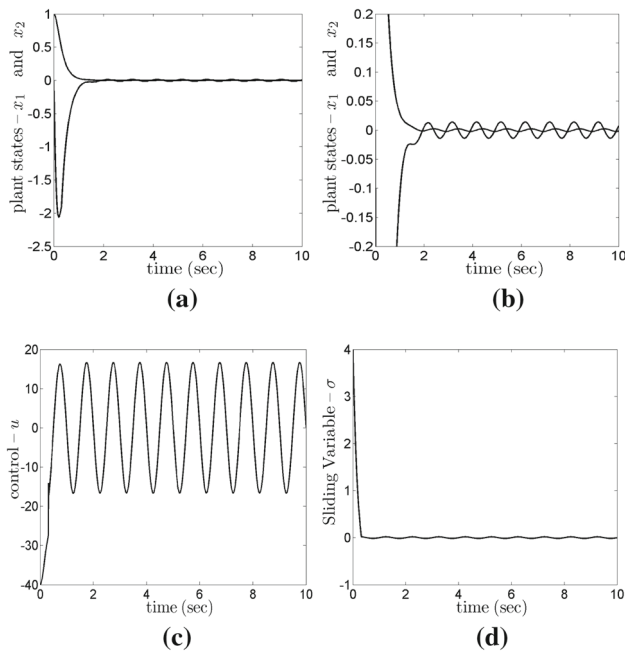


Fig. 4 States, control and sliding variable—first order UDE. **a** Plant states x_{plant} , **b** magnified plant states x_{plant} , **c** control u , **d** sliding variable σ

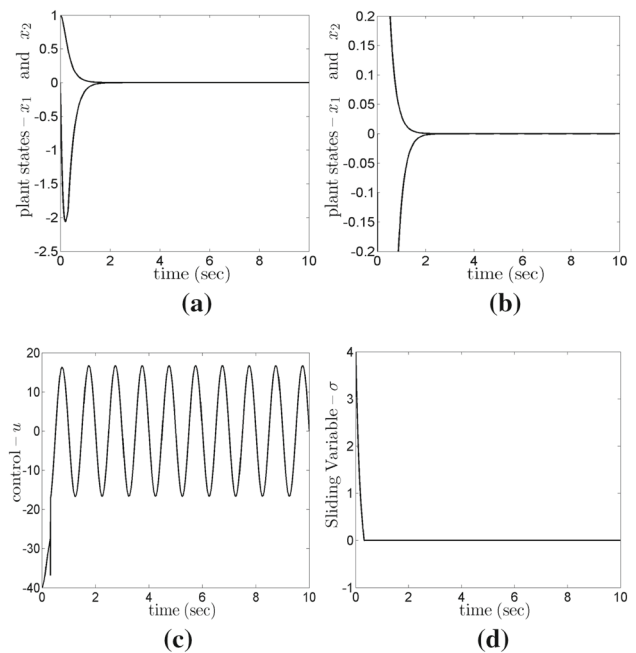


Fig. 5 States, control and sliding variable—second order UDE. **a** Plant states x_{plant} , **b** magnified plant states x_{plant} , **c** control u , **d** sliding variable σ

Table 2 Variation in σ with first order UDE

Time constant τ	RMS value of σ	
	For sinusoidal	For sawtooth
0.001	0.0001926	0.01641
0.01	0.01697	0.1434
0.1	0.4947	0.5625

6 Application to flexible joint system

The problem of joint flexibility has received considerable attention as the major source of compliance in most present day manipulator designs. This joint flexibility typically arises due to gear elasticity, shaft windup, etc., and is important in the derivation of control law. Unwanted oscillations due to joint flexibility, imposes bandwidth limitations on all algorithm designs; based on rigid robots and may create stability problems for feedback controls that neglect joint flexibility. A feedback linearization (FL) based control law made implementable using extended state observer (ESO) is proposed for the trajectory tracking control of a flexible joint robotic system in [30]. Controller design based on the integral manifold formulation [31], adaptive control [32], adaptive sliding mode [33] and back-stepping approach [34] are some other approaches reported in the literature. In this work, a model following SMC is proposed to control flexible joint manipulator with uncertainty and disturbance. A nonlinear disturbance is considered here

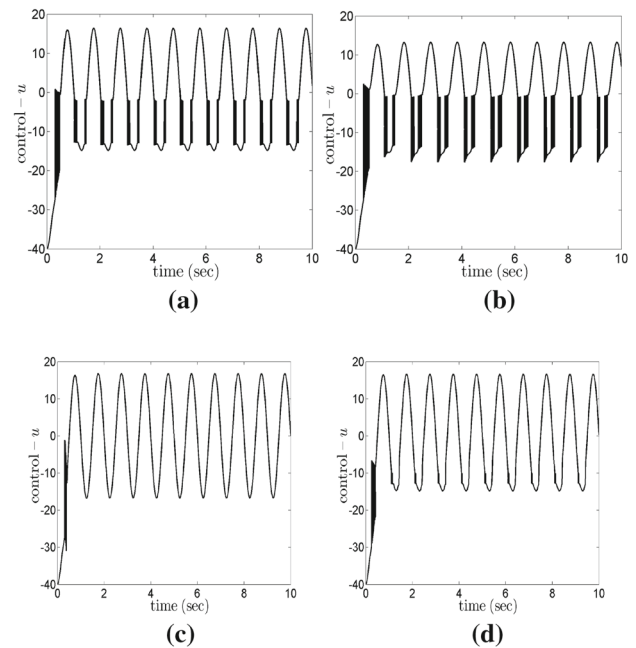


Fig. 6 Effect of τ and filter order on chatter mitigation. **a** First order UDE ($\tau = 0.01$ s), **b** first order UDE ($\tau = 0.1$ s), **c** second order UDE ($\tau = 0.01$ s), **d** second order UDE ($\tau = 0.1$ s)

and the plant model is controlled to follow the desired states and the uncertainty and disturbance is estimated with UDE.

Table 3 Variation in σ with different approximations

Approximation	RMS value of σ
Sat function	4.524×10^{-4}
First order UDE	1.926×10^{-4}
Second order UDE	6.475×10^{-5}

The equations of motion for the Quanser’s Flexible Joint module as given in [35] are,

$$\begin{aligned} \ddot{\theta} + F_1 \dot{\theta} - \frac{K_{stiff}}{J_{eq}} \alpha &= F_2 V_m \\ \ddot{\theta} - F_1 \dot{\theta} + \frac{K_{stiff}(J_{eq} + J_{arm})}{J_{eq} J_{arm}} \alpha &= -F_2 V_m \end{aligned} \tag{41}$$

where,

$$\begin{aligned} F_1 &\triangleq \frac{\eta_m \eta_g K_t K_m K_g^2 + B_{eq} R_m}{J_{eq} R_m} \quad \text{and} \\ F_2 &\triangleq \frac{\eta_m \eta_g K_t K_m}{J_{eq} R_m} \end{aligned}$$

The parameters are : θ is motor load angle, α is link joint deflection, η_m is the motor efficiency, η_g is the gearbox efficiency, K_t is the motor torque constant, K_m is the back EMF constant, K_g is the gearbox ratio, B_{eq} is the viscous damping coefficient, R_m is the armature resistance, J_{eq} is the gear inertia, K_{stiff} is the spring stiffness, J_{arm} is the link inertia, and V_m is the motor control voltage.

Considering the output of the system as $y = \theta + \alpha$, the dynamics (41) in terms of y and θ is re-written as,

$$\ddot{y} = \frac{K_{stiff}}{J_{eq}} F_3 y - \frac{K_{stiff}}{J_{eq}} F_3 \theta \tag{42}$$

$$\ddot{\theta} = \frac{K_{stiff}}{J_{eq}} y - \frac{K_{stiff}}{J_{eq}} \theta - F_1 \dot{\theta} + F_2 V_m \tag{43}$$

where $F_3 \triangleq \left(1 - \frac{J_{eq} + J_{arm}}{J_{arm}}\right)$.

Defining the state variables as, $x_1 = y$, $x_2 = \dot{y} = \dot{x}_1$, $x_3 = \theta$, $x_4 = \dot{\theta} = \dot{x}_3$, the dynamics (42)–(43) become,

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{K_{stiff}}{J_{eq}} F_3 (x_1 - x_3) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{K_{stiff}}{J_{eq}} (x_1 - x_3) - F_1 x_4 + F_2 V_m \end{aligned} \tag{44}$$

The state space form for (44) can be written as,

$$\dot{x} = A x + B V_m \tag{45}$$

where, $\dot{x} = [\dot{x}_1 \quad \dot{x}_2 \quad \dot{x}_3 \quad \dot{x}_4]^T$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{K_{stiff}}{J_{eq}} F_3 & 0 & -\frac{K_{stiff}}{J_{eq}} F_3 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K_{stiff}}{J_{eq}} F_3 & 0 & -\frac{K_{stiff}}{J_{eq}} F_3 & -F_1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ F_2 \end{bmatrix}$$

For the desired output, the relative output y can be differentiated in proper manner. In order to satisfy the model following conditions, the above system (45) is converted to phase variable form by using the transformation,

$$Z = T x$$

Then the Eq. (45) can be written as in [30],

$$\dot{z} = A z + B V_m \tag{46}$$

where, $\dot{z} = [\dot{z}_1 \quad \dot{z}_2 \quad \dot{z}_3 \quad \dot{z}_4]^T$

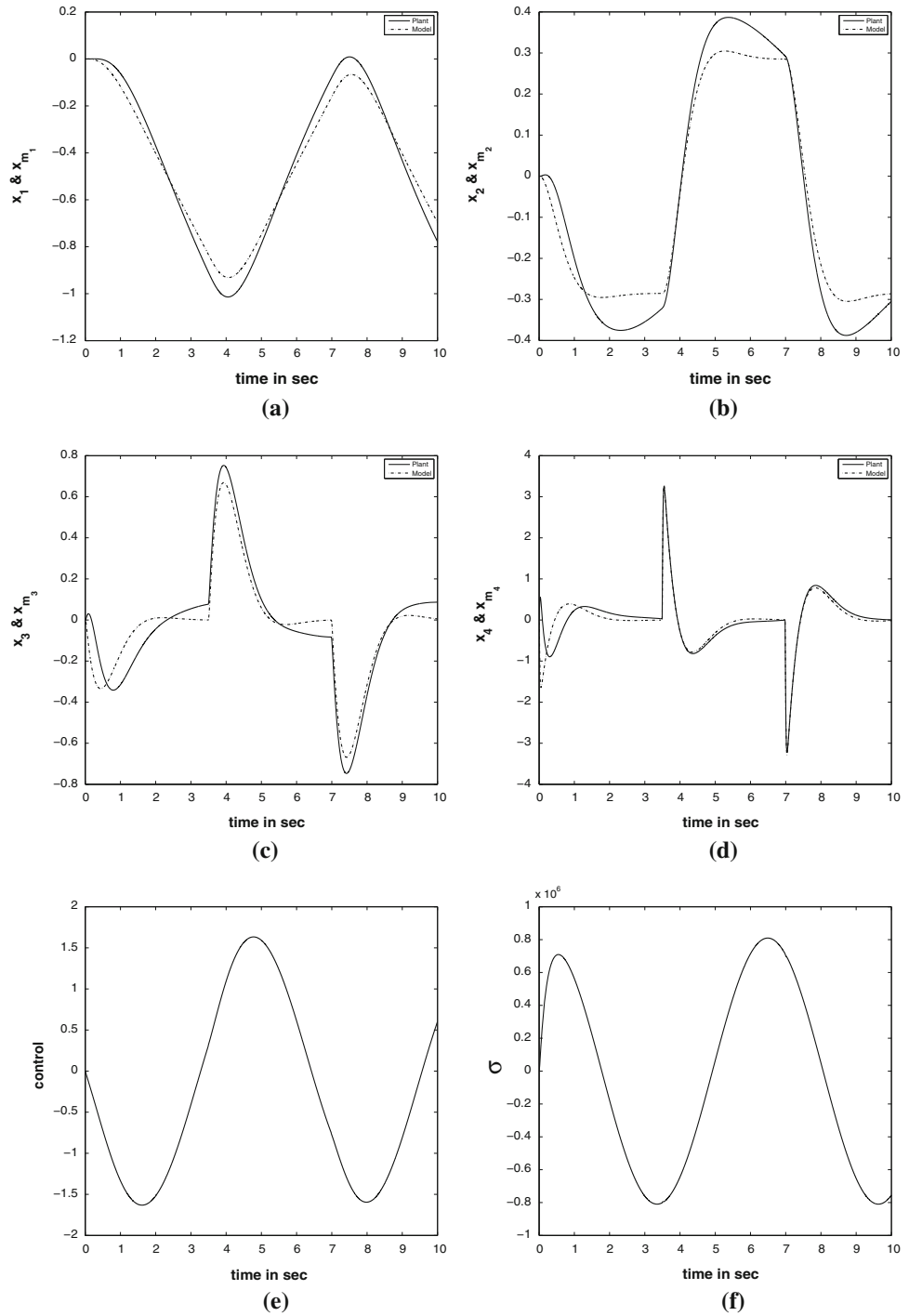
$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{K_{stiff} F_1}{J_{arm}} & -\frac{K_{stiff}(J_{eq} + J_{arm})}{J_{eq} J_{arm}} & -F_1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K_{stiff} F_2}{J_{arm}} \end{bmatrix}$$

The nominal values of the various flexible joint parameters are from [35]: $K_{stiff}=1.248$ Nm/rad, $\eta_m = 0.69$, $\eta_g = 0.9$, $K_t = 0.00767$ Nm, $K_g = 70$, $J_{eq} = 0.00258$ kgm², $J_{arm} = 0.00352$ kgm², $R_m = 2.6$ Ω . The initial conditions are $x(0) = [0 \ 0 \ 0 \ 0]$.

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -10007 & -837 & -28 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10007 \end{bmatrix} V_m \end{aligned} \tag{47}$$

Fig. 7 States, control and sliding variable ($\tau = 10$ ms). **a** Plant and model state x_1 , **b** plant and model state x_2 , **c** plant and model state x_3 , **d** plant and model state x_4 , **e** control u , **f** sliding variable σ



The model to be followed is assumed as;

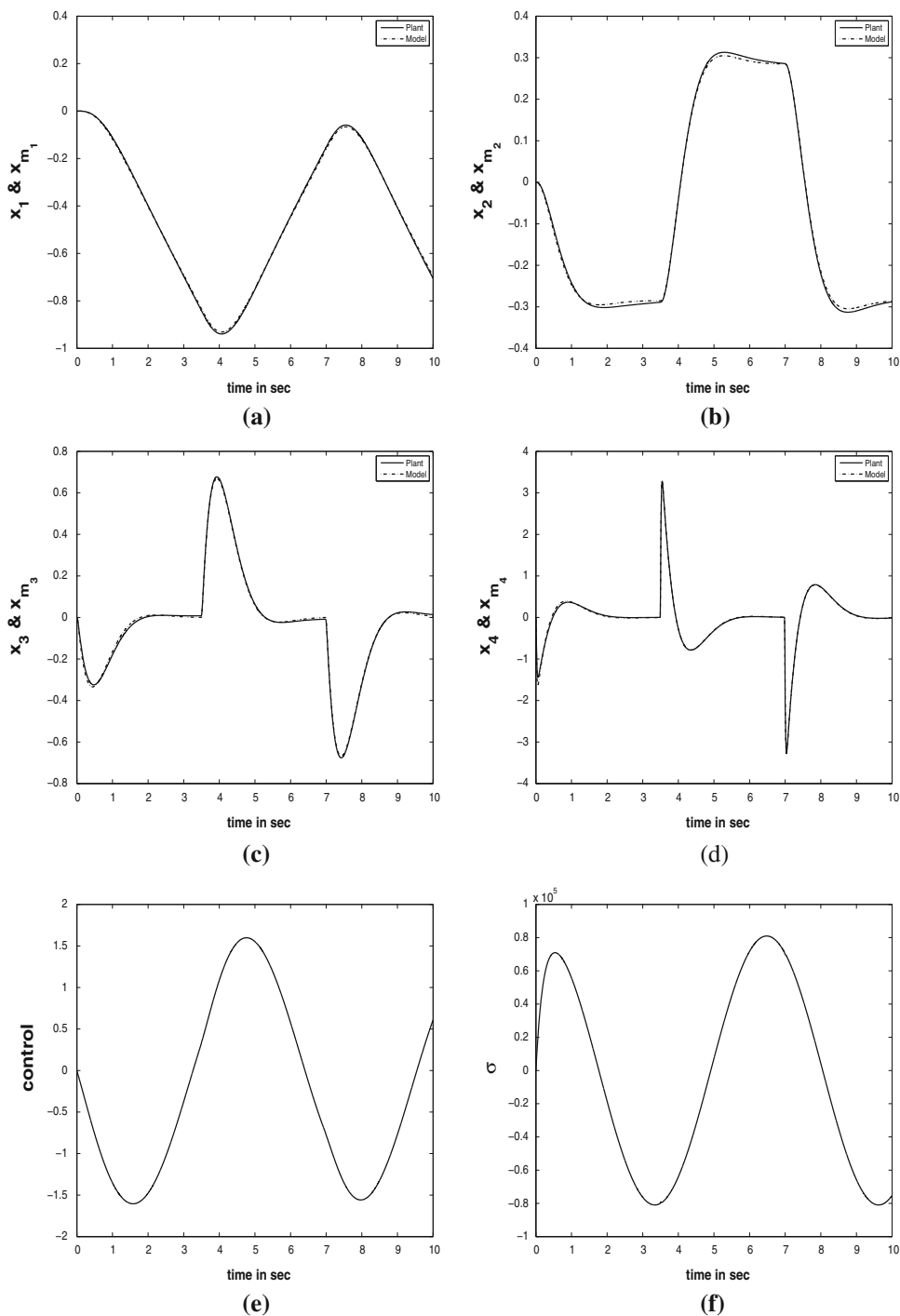
$$\begin{aligned}
 \begin{bmatrix} \dot{x}_{m1} \\ \dot{x}_{m2} \\ \dot{x}_{m3} \\ \dot{x}_{m4} \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -560 & -320 & -85 \end{bmatrix} \begin{bmatrix} x_{m1} \\ x_{m2} \\ x_{m3} \\ x_{m4} \end{bmatrix} \\
 &+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 160 \end{bmatrix} V_m \tag{48}
 \end{aligned}$$

with initial conditions are $x_m(0) = [0 \ 0 \ 0 \ 0]$. The disturbance $d(t) = 2 \sin(t)$, is sinusoidal with amplitude 2 and frequency 1 rad/s and uncertainty in the plant is 40%.

The simulation results are shown in Figs. 7, 8 for $\tau = 10$ ms and $\tau = 1$ ms respectively. The value of $k = 5$ is considered.

Figures 7a–d and 8a–d are the plant and model states i.e. displacement, velocity, acceleration and jerk. The control torque required and the sliding variable (σ) are also

Fig. 8 States, control and sliding variable ($\tau = 1$ ms). **a** Plant and model state x_1 , **b** plant and model state x_2 , **c** plant and model state x_3 , **d** plant and model state x_4 , **e** control u , **f** sliding variable σ



shown. The figure reveals the ability of the controller, to drive the system to follow the reference model. It is easily observed that system is robust even in presence of parameter variations and external disturbance. The tracking and robustness is improved as τ is decreased from 10 to 1 ms.

7 Conclusion

In this paper, a boundary layer SMC design for chatter reduction is proposed. The control scheme uses a discontinuous control outside the boundary layer and switches over to uncertainty and disturbance estimator (UDE) based con-

trol inside the boundary layer. The use of UDE gives a better trade-off inside the boundary layer and this trade-off is improved by using a higher order UDE. The initial control is within limits, irrespective of the uncertainty. The results show improvement to the tune of 20–40% over the conventional chatter control method using saturation function inside the boundary layer. It is proved that the ultimate boundedness of uncertainty estimation error and sliding variable inside the boundary layer is guaranteed, and that the bounds can be lowered by appropriate choice of design parameters. The theoretically expected results are verified by computer simulation in MATLAB SIMULINK environment. The efficacy of the design is also confirmed on an application to flexible joint system in robotic control. The controller is able to force the plant to follow the given model inspite of parameter variations.

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References

- Utkin VI (1977) Variable structure systems with sliding modes. *IEEE Trans Autom Control* Acc 22(2):212–222
- Hung JY, Gao W, Hung JC (1993) Variable structure control: a survey. *IEEE Trans Ind Electron* 40(1):2–22
- Young KD, Utkin VI, Ümit Özgüner (1999) A control engineers guide to sliding mode control. *IEEE Trans Control Syst Technol* 7(3):328–342
- Fridman L, Moreno J, Iriarte R (2010) Sliding modes after the first decade of the 21st century. *Lecture notes in control and information sciences*. Springer, Berlin
- Sabanovic A (2011) Variable structure systems with sliding modes in motion control—a survey. *IEEE Trans Ind Inform* 7(2):212–223
- Utkin VI, Lee H (2006) The chattering analysis. In: *IEEE EPE-PEMC*, pp 2014–2019
- Boiko IM (2011) Analysis of chattering in sliding mode control systems with continuous boundary layer approximation of discontinuous control. In: *Proceedings of American control conference*, pp 757–762
- Slotine JJE, Sastry SS (1983) Tracking control of nonlinear systems using sliding surfaces with applications to robot manipulator. *Int J Control* 38(2):465–492
- Burton JA, Zinobar ASI (1986) Continuous approximation of variable structure control. *Int J Syst Sci* 17:875–885
- Lee H, Utkin VI (2007) Chattering suppression methods in sliding mode control systems. *Ann Rev Control* 31(2):179–188
- Lei M, Chen M-S (2010) Chattering reduction of sliding mode control by low-pass filtering the control signal. *Asian J Control* 12(3):392–398
- Xu Y (2008) Chattering free robust control for nonlinear systems. *IEEE Trans Control Syst Technol* 16(6):1352–1359
- Potluri R (2012) Comments on chattering free robust control for nonlinear systems. *IEEE Trans Control Syst Technol* 20(2):562
- Acary V, Brogliato B (2010) Implicit euler numerical scheme and chattering-free implementation of sliding mode systems. *Systems Control Lett* 59(5):284–293
- Chen MS, Chen CH, Yang FY (2007) An LTR-observer-based dynamic sliding mode control for chattering reduction. *Automatica* 43(6):1111–1116
- Bartiloni G, Pydynowski P (1996) An improved chattering free VSC scheme for uncertain dynamical systems. *IEEE Trans Autom Control* 41(8):1220–1226
- Liu JK, Sun FC (2006) Global SMC with adaptive fuzzy chattering free method for nonlinear system. In: *IMACS multiconference on CESA*, pp 541–546
- Fang Y, Chow TWS (1998) Chattering free sliding mode control based on recurrent neural network. In: *IEEE international conference man cybernetics*, pp 1726–1731
- Taha EZ, Happawana GS, Hurmuzlu Y (2001) Quantitative feedback theory (QFT) for chattering reduction and improved tracking in sliding mode control. In: *Proceedings of American control conference*, pp 5004–5009
- Bartiloni G, Ferrara A, Usai F (1998) Chattering avoidance by second order sliding mode control. *IEEE Trans Autom Control* 43(2):241–246
- Shtessel YB, Shkolnikov Brown J (2003) An asymptotic second-order smooth sliding mode control. *Asian J Control* 5(4):498–504
- Levant A (2003) Higher-order sliding modes, differentiation and output feedback control. *Int J Control* 76(9/10):924–941
- Zhong QC, Rees D (2004) Control of uncertain LTI systems based on an uncertainty and disturbance estimator. *ASME J Dyn Sys Meas Control* 126(4):905–910
- Talole SE, Phadke SB (2008) Model following sliding mode control based on uncertainty and disturbance estimator. *ASME J Dyn Sys Meas Control* 130(3):1–5
- Shendge PD, Patre BM (2007) Robust model following load frequency sliding mode controller based on UDE and error improvement with higher order filter. *IAENG Int J Appl Math* 37(1):216–221
- Shendge PD, Suryawanshi PV (2011) Sliding mode control for flexible joint using uncertainty and disturbance estimation. In: *Proceedings of world congress on engineering and computer Science*, pp 216–221
- Kolhe JP, Md Shaheed, Chandar TS, Talole SE (2011) Robust control of robot manipulator based on uncertainty and disturbance estimation. *Int J Robust Nonlinear Control* 23(1):104–122
- Drazenovic B (1969) The invariance conditions in variable structure systems. *Automatica* 5:287–297
- Trench WF (2003) *Introduction to real analysis*. Pearson, Upper Saddle River
- Talole SE, Kolhe JP, Phadke SB (2010) Extended state observer based control of flexible joint system with experimental validation. *IEEE Trans Ind Electron* 57(4):1411–1419
- Spong MW (1985) Modeling and control of elastic joint robots. *ASME J Dyn Sys Meas Control* 109:310–319
- Ghorbel F, Hung JY, Spong MW (1989) Adaptive control of flexible joint manipulators. *IEEE Control Syst Mag* 9(7):9–13
- Farooq M, Wang DB, Dar NU (2008) Adaptive sliding mode hybrid/force position controller for flexible joint robot. In: *Proceedings of IEEE international conference mechatronix and automation*, pp 724–731
- Oh JH, Lee JS (1997) Control of flexible joint robot system by backstepping design. In: *Proceedings of IEEE international conference robotics and automation*, pp 3435–3440
- Quanser (2008) *Rotary flexible joint: user manual*. Quanser Corp., Markham, Canada