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Wake characteristics in high solidity horizontal axis hydrokinetic turbines: a comparative study between experimental techniques and numerical simulations

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Abstract

Hydrokinetic turbines utilizing renewable energy from water streams, such as tidal or river currents, ofer a sustainable solution for electricity generation in remote coastal or fuvial regions. While there has been extensive research on the wake characteristics of low-solidity turbines, studies focusing on high-solidity turbines are limited. This study aims to address this research gap by characterizing the turbulent wake of a high-solidity, four-bladed hydrokinetic turbine using a combination of experimental and numerical approaches. Experimental investigations are conducted in two wind tunnel facilities, employing particle image velocimetry and hot-wire anemometry to measure and identify fow structures in the near-wake region. Additionally, the applicability of the actuator line method for high-solidity turbines is validated, considering the challenges posed by the higher blockage efect associated with such turbines. Numerical simulations are performed using an unsteady Reynolds-averaged Navier–Stokes approach, incorporating a full-geometry model of the rotating rotor and employing the actuator line concept for simplifed modeling. The fndings from this study provide valuable insights into the wake characteristics of high-solidity hydrokinetic turbines, contributing to the understanding of their performance and environmental implications.

Keywords Hydrokinetic turbines · High solidity · Actuator line method · PIV · Hot-wire measurements · CFD

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1 Introduction

Nowadays, the renewable source of energy issued from water streams (tidal or river currents) is an alternative way to provide electricity for remote coastal or fuvial regions with local communitarian needs. Small- and medium-sized turbines can be used as a good solution for renewable and sustainable electricity production, employing hydrokinetic energy conversion devices with low environmental impacts [[1,](#page-15-0) [2\]](#page-15-1).

The characterization of the water flow, the understanding of the physical mechanisms of energy conversion, and hydrodynamics of horizontal-axis hydrokinetic turbines compose an important background to estimate the performance of the machines and also to verify how disturbed fow may impact the surroundings of the turbine installation and its environment. Therefore, the characterization of the turbulent wake is an important step for the entire evaluation of the operation of a hydrokinetic installation.

The wake is the downstream flow region formed by the interaction between the free fow and rotor blades. It exhibits high crossflow and spanwise vorticity, velocity deficit, and elevated turbulence levels [[3](#page-15-2)]. Vortex structures play a signifcant role, infuencing shape, hydrodynamics, and turbulent kinetic energy downstream. The near-wake region, within approximately 2.0–4.0 rotor diameters downstream, features strong hydrodynamic conversion, counter-rotating tip vortices, and hub vortices [\[4\]](#page-15-3). In the far-wake region, large structures dissipate, pressure recovers, and turbulence intensity decreases. Various numerical and experimental techniques are employed to study wake characteristics in horizontal-axis turbines [[5\]](#page-15-4).

In experimental studies, turbines are tested in water flumes [[6\]](#page-15-5) or wind tunnels [\[7](#page-15-6)] facilities to achieve a wide range of operational points. Tests are commonly equipped with a velocity-based measuring method, such as PIV [\[8](#page-15-7)], hot-wire [[9\]](#page-15-8), or laser Doppler anemometers [[10\]](#page-15-9), in order to defne the length and other parameters of turbulence inside the wake [[11\]](#page-15-10).

For estimation of the performance of the full hydrokinetic turbine, an experimental methodology based on smallscale tests in the wind tunnel facility was carried out. The approach developed by Macias et al. [[7\]](#page-15-6) is presented as an alternative to the experimental evaluation of hydrokinetic turbines, which is generally based on experiments in laboratory water fumes. The use of wind tunnel experiments was supplemented by a model-to-prototype extrapolation approach in which the performance results of the smallscale airfow experiments are transposed to the prototype real scale in water fow. This is achieved through a calibrated methodology using some arguments of blade element momentum theory. The works of Macias et al. [\[7\]](#page-15-6) and Nunes et al. [[12\]](#page-15-11) present a successful validation of the upscaling approach by numerical simulations (CFD) at both scaling levels and by experimental data.

On the other hand, numerical analysis plays a crucial role in describing wake structures, with CFD simulations being a primary tool. There are various methods to evaluate turbines in CFD, such as using the full rotor geometry [[13](#page-15-12)], or modeling the wake behavior using techniques such as the actuator disk $[14]$ $[14]$ $[14]$, actuator line method $[15]$ $[15]$, and actuator surface method $[16]$. The primary differences between these methods are related to their computational cost and the level of detail they can provide in the results. While the full-geometry method can produce more detailed flow characteristics, it requires much longer simulation times.

Alternatively, the actuator line method can be a useful solution to describe the wake with accurate results and less computation effort, as demonstrated in recent studies [\[17](#page-15-16)[–19](#page-15-17)]. The actuator line method ALM combines the blade element momentum (BEM) methodology with the Navier-Stokes equations computing the forces on the turbine and the fow around the rotor. The crucial issue is that the method does not require surface meshing or the resolution of the detailed fow in the boundary layer because the blades are discretized in actuator lines formed by points. In the literature, there are studies using unsteady Reynolds-averaged Navier–Stokes (URANS) approaches [[20,](#page-15-18) [21](#page-15-19)] and largeeddy simulations (LES) [\[22](#page-15-20), [23\]](#page-15-21), as well as works dedicated to the study of hydrokinetic arrangements with the objective of characterizing the wake and optimizing the positioning of an array of machines [[24,](#page-15-22) [25\]](#page-15-23), and studies using turbines with diffuser [\[26](#page-15-24)].

The identification of physical features in the wake flow of horizontal-axis turbines has been extensively studied and documented through numerous previous research efforts, concerning the fuid fow through based on wind turbines or large hydrokinetic turbines with small solidity (e.g., [[27–](#page-15-25)[32](#page-15-26)]), where the blockage efect is smaller, due to the geometry of the rotor with three sparse blades and often with a low rotational speed. Recently, several studies have focused on characterizing the wake of high-solidity turbines, particularly in the context of ducted turbines (e.g., [\[33](#page-15-27)[–35](#page-15-28)]). However, research specifcally investigating the wake characteristics of high-solidity turbines without ducts remains limited, which are typically found in small hydrokinetic turbine cases. Therefore, it is crucial to consider the diferences that high solidity can cause, such as a higher blockage efect, in the analysis of wake behavior.

This paper aims to characterize the wake flow from a high-solidity, four-bladed hydrokinetic turbine, as shown in Fig. [1,](#page-2-0) employing diferent experimental and numerical approaches. Experiments utilize a small-scale turbine in two wind tunnel facilities to measure and identify the flow structures at the near-wake region with two different approaches, i.e., particle image velocimetry (PIV) and hotwire anemometry. Additionally, another objective of this study is to validate the applicability of the actuator line method for high-solidity turbines, as this method is commonly employed for low-solidity rotors. Numerical simulations are conducted using an unsteady Reynolds-averaged Navier–Stokes (RANS) approach, encompassing the complete 3D rotating rotor formulation (full geometry), along with the implementation of the actuator line concept for simplifed modeling.

The organization of this paper is as follows. Section [2](#page-2-1) presents the high-solidity hydrokinetic turbine employed in this study providing an overview of the main non-dimensional parameters involved in the investigation. Section [3](#page-3-0) outlines the proposed experimental methodology, including the small-scale turbine description, the wind tunnel facilities, and the conducted experimental tests. Section [4](#page-5-0) includes details on the CFD simulations used for numerical studies. In Sect. [5](#page-7-0), the results obtained from the experimental tests and numerical simulations are compared and discussed.

Fig. 1 Main fow structures at high-solidity hydrokinetic turbine HK10

Table 1 HK10 turbine design

Finally, Sect. [6](#page-11-0) summarizes the conclusions drawn from this study.

Table 2 HK10 rotor geometry information along the blade: radial distribution, chord, and twist angle

2 High‑solidity hydrokinetic turbine

In the present study, the high-solidity, four-bladed HK10 turbine is considered as shown in Figs. [1](#page-2-0) and [2](#page-3-1). The rotor was developed in R&D Brazilian cooperation involving the University of Brasilia and the AES-Tiete company. With 2.2 m diameter and NACA4415 hydrodynamic profle to blade design, this machine has a rated power of 10 kW and operates at a water speed of 2.5 m/s (see [[36\]](#page-15-29), for instance). It was designed to convert the kinetic energy of water streams, close to the free surface, by means of a foating integrated electromechanical device. The description of the wake fow is important to establish an array confguration of several machines working together and also to evaluate the region afected by the turbine that can impact the surrounding environment.

Table [1](#page-2-2) summarizes generic technical information about the turbine employed in this study. Besides, Table [2](#page-2-3) presents blade profle descriptions in detail: axial distance, chord, and torsion angle.

In this study, a comparison between experimental and numerical methodologies is provided to investigate the highsolidity hydrokinetic turbine HK10 wake. Besides, the power conversion efficiency C_p is computed for various operating regimes defned by the tip speed ratio TSR. The non-dimensional parameters are described by

$$
C_p = \frac{\omega T}{\frac{1}{2}\rho A U_0^3},\tag{1}
$$

$$
TSR = \frac{\omega R}{U_0}.\tag{2}
$$

Fig. 2 HK10 turbine: **a** front and lateral views of conception model; **b** feld assembly

being the free flow velocity U_0 , the density ρ , the torque *T*, the rotation speed ω , and the radius *R*. The area swept by the rotor is defined by $A = \pi R^2$.

The rotor solidity σ refers to the ratio of the blade projections and the swept area of the turbine, where N_b is the number of blades, H_r is the hub radius, and c_r is the chord at radial distance *r*.

$$
\sigma = \frac{\int_{H_r}^R N_b c_r dr}{\pi R^2}.
$$
\n(3)

Rotors with high solidity carry a lot of material, and the fow hydrodynamic of the blades can interfere with neighboring blades. Table [3](#page-3-2) lists the main parameters to estimate the solidity rotor for HK10 and other low-solidity horizontalaxial turbines in the literature.

3 Experimental approach

Two wind tunnel experiments were carried out in the present study. The characterization of the wake flow and the performance of the hydrokinetic turbine HK10 has been developed in a framework of an experimental international collaboration involving the University of Brasilia (UnB), in Brazil, and the Ecole Nationale d'Arts et Métiers (ENSAM-Paris), in France, to design and evaluate the small hydrokinetic turbines for remote communities.

Experimental tests were conducted on a small-scale turbine, analyzing the wake flow using particle image velocimetry (PIV) and computing the characteristic curves $C_p \times TSR$ through the torque and rotational velocity measurements. The small-scale curve performance was scaled up obtaining the characteristic curves of the full-scale hydrokinetic turbine employing the upscaling approach developed by Macias et al. [[7](#page-15-6)], as also reported by Nunes et al. [[12](#page-15-11)] to HK10 turbine. Subsequently, computational fluid dynamics (CFD) simulations were carried out on the full-size turbine to validate the flow

Table 3 Solidity rotor value estimation for horizontal-axial turbines

measurements obtained in the wind tunnel and to verify the upscaling method based on the experimental results.

3.1 Small‑scale turbine

The experimental study was conducted on a small-scale turbine model of 1:10th of real dimensions. The fourblade model is manufactured in an ABL 3D printer system and finished with epoxy paint (Fig. [3](#page-4-0)). The same blade profile (NACA 4415) was employed in small-scale and full-size turbines to keep the geometric similarity between both machines.

The kinematic similarity is ensured by the same velocity triangles in the inflow of the blade section, and consequently, the same values of *TSR* in both scales must be guaranteed. Finally, the dynamic similarity is not achieved for any regime operation because the Reynolds numbers are overly different due to differences in the complex phenomena over the turbine blades such as transition laminar-turbulent and boundary layer detachment. The Reynolds numbers based on the rotor diameter Re_D were computed such as 1.8×10^5 and 5.5×10^6 , to model and full size, respectively. To address the discrepancy between Reynolds numbers and achieve scaling of results from small- to full-scale turbines, an empirical power law from [[7](#page-15-6)] has been applied. This power law serves to correct the lack of dynamic similarity in C_p (power coefficient) results and enables a more accurate comparison between different scales. The power law was obtained relating the power coefficients and the Reynolds numbers for the model in the wind tunnel and full-scale hydrokinetic turbine by means of BEM theory equations. Table [4](#page-4-1) summarizes the main parameters of the full-scale and model turbines.

Table 4 Full-size and small-scale turbine parameters

3.2 Wind tunnel facilities

The first group of experiments was performed in the openloop wind tunnel of the Energy and Environment Laboratory of the University of Brasília. This tunnel has a test section of $1.2 \text{ m} \times 1.2 \text{ m}$ with a length of 2.0 m . With a contraction ratio of 8:1 and an axial fan of 10 *kW*, this wind tunnel can reach a controlled airspeed of up to 20 m/s, with a turbulence intensity of less than 0.2%. Figure [4](#page-5-1) a illustrates a general overview of the test facility. A Pitot tube is used to measure the free fow velocity, and the air temperature and relative humidity are monitored with calibrated electronic sensors. The experimental uncertainty of the velocity measurements is \pm 0.1 m/s.

The small-scale 1:10 turbine model is placed in the center of the test section as shown in Fig. [3.](#page-4-0) The experiments to obtain the performance curves are carried out using a torsional torque meter load cell with a small electrical generator as a break. The system maintains the operation of the turbine runner at a constant rotational speed, where an adjusted electronic resistive load through a PWM-PID control system dynamically dissipates the converted electricity power.

Fig. 3 a Small-scale turbine scheme (dimensions in mm): (1) runner; (2) tower; (3) permanent-magnet generator; (4) torsional load sensor. **b** The 1:10 small-scale turbine inside the wind tunnel test section

 (a)

 (b)

Fig. 4 Wind tunnel facilities: **a** University of Brasilia open-loop and **b** ENSAM-Paris closed-loop

A closed-loop PID control system achieves stable rotation for a given wind speed by using electronically controlled settings for the generator load levels, with the accuracy of torque measurements by a torsional load cell equal to 10[−]⁴ N m. The operation of the turbine model keeps the rotation, N, constant at a fixed value within ± 10 rpm fluctuations in the range of 300–2500 rpm (measured by an optical infrared rotation sensor). For a fxed wind tunnel velocity (around 12 m/s), a performance curve can be automatically obtained by the programmable control system, which covers the required range of rotational speed, measuring and registering the torque and the rotation by the data-logger.

In this facility, a 3D transverse system displaces the hot-wire probe for all positions in the test section (see Fig. [4a](#page-5-1)). A DANTEC system with MiniCTA 54T40 with probe 55P01 is employed. It allows the measurement of the wake profles of one component mean velocity and turbulence intensity.

The second wind tunnel facility is in the Laboratory of Fluid Engineering and Energy Systems of ENSAM Paris (Fig. [4](#page-5-1)b). It is a closed-loop installation where the test section is open to atmospheric pressure (Prandtl-like tunnel). The test section has dimensions of 1.35 $m \times 1.65$ m with a 2 m length. The contraction of 1:12 ensures homogeneous and low turbulence with an airfow velocity of up to 40 m/s. The velocity measurement with the PIV system can be carried out properly inside the open test section. All wind tunnel controls and airfow conditions are registered with accuracy for the velocity of \pm 0.1 m/s.

The PIV characterization of the turbine wake is obtained by 2D measurements using a synchronization triggered by the angular position of the blade. In this approach, each set of particle image pairs (olive oil microdrops) registers the flow field at one angular position, allowing scanning of the mean wake flow for the entire rotor (see [\[38](#page-16-1)], for instance).

Finally, in the present experimental campaign, the turbine model was tested in the UnB facility, obtaining the performance curve $(C_p \times \text{TSR})$ and horizontal profiles of mean axial velocity and turbulence intensity. After that, a PIV characterization of the wake was obtained in the facility of ENSAM, identifying the main vortice structures in the nearwake flow. All these experimental data are compared to a detailed simulation of the fuid fow (Fig. [5](#page-6-0)).

4 Numerical setup

Simulation of turbulent fow through the turbine rotor was performed using the platform ANSYS-CFX. The unsteady Reynolds-averaged approach (URANS) was used to describe the fow with time-dependent felds of velocity, pressure, and turbulence variables. This approach enables the analysis of the dynamic fow behavior, especially in the wake region, which is essential for characterization, particularly in wind turbine farms where the wake velocities and turbulence levels from upwind turbines serve as the incoming fow for downstream turbines [\[39,](#page-16-2) [40\]](#page-16-3). The turbulence model k - ω /*SST* with transition version $\gamma - Re_{\theta} - SST$, also known as transition SST [[41](#page-16-4)], was employed in two diferent subdomains: The frst is a parallelepiped domain defning the large fluid region affected by the machine [[42](#page-16-5), [43\]](#page-16-6). The second is the submerged cylindrical domains containing the turbine rotor. This surrounding cylindrical volume rotates at the same speed as the turbine blades. Figure [6](#page-6-1) shows both subdomains in a simplifed schematic form, and this fgure illustrates the boundary conditions employed in the simulations.

A 3D mesh with 5,572,162 nodes was generated by ANSYS-CFX/MESH, where tetrahedral elements in an

Fig. 5 Velocity measurements setup: **a** PIV and **b** hot-wire anemometer system, where (1) probe and (2), (3), and (4) step motors to move the probe in the Cartesian plane

unstructured mesh were used with refnement in the wake and in the near-wall regions using the infation approach, as shown in Figs. [7](#page-7-1)a and [8.](#page-7-2) The refnement near the wall guarantees small values for the average variable y_{ave}^+ = 1.6,

Fig. 6 Scheme of CFD domain defnitions and boundary conditions for full-geometry simulations

which is an important condition for better results with the turbulence model.

Complementary, another group of CFD simulations was carried out using the actuator line method (ALM). It was performed using the OpenFOAM platform where the model of source terms in the Navier–Stokes equations has been changed to take into account the hydrodynamical infuence of each blade section in the fuid fow. Hence, the four rotating actuator lines defne the source terms formulated by the hydrodynamical forces (drag and lift) computed after the geometry of the blade (airfoil section, chord length, and assembly angle), taking into account the distance of each node of the mesh to the set of points, which represents the blade. The airfoil hydrodynamical coefficients are computed using XFOIL tool. Besides, in ALM simulations the hub/ nacelle was modeled as an actuator cylinder. More details about the ALM process are given in Appendix.

For the ALM approach, the mesh is smaller than the Full 3D because it does not need the discretization of the blade surface and its near-wall gradients. The fnal mesh discretizes properly the wake region and has 4,000,000 nodes, as shown in Fig. [7](#page-7-1)b.

A previous mesh assessment was performed considering diferent dimensions and levels of refnement of the grids. Table [5](#page-7-3) shows the mesh convergence study to full 3D simulations at normal setting rotor condition ($\omega = 35$ *rpm* and $U_0 = 2.5 \, \text{m/s}$, and consequently, *TSR* = 1.6). All the tested mesh showed a similar C_p , varying less than %3 between coarse and fne mesh, where this value was validated in the experimental approach. Also, the refnement study proceeds until reaching an ideal y^+ for applying the SST turbulence model [[41](#page-16-4)]. The fnal meshes illustrated in Fig. [7](#page-7-1) were selected for the condition where the C_p and y_{ave}^+ T values had attained the proper condition in the nominal setting for the rotor. Then, the ALM mesh was reproduced to keep the same element density in the near-wake region. In Table [6](#page-7-4) is established a comparison between the simulation time

Fig. 8 Mesh details for CFD-Full 3D simulations: near-wake refnement region, mesh on the blade surface, and prismatic elements modeling the boundary layer region

Table 5 Mesh convergence study of the hydrokinetic rotor operating under the design conditions (TSR=1.6) in Full 3D simulations in Ansys-CFX

Grid levels	N° Nodes	y_{ave}^+	C_p
Coarse	4.2×10^{5}	323.62	0.381
Medium	1.2×10^{6}	117.77	0.386
Fine	5.5×10^{6}	1.57	0.390

Table 6 Simulation time comparison between the CFD-ALM and CFD-Full 3D methodologies

between the CFD-Full 3D and CFD-ALM methodologies. The residuals for all equations fall below 1×10^{-5} .

5 Results and discussion

5.1 Performance results

In Fig. [9,](#page-8-0) the performance results as the $C_p \times TSR$ curve are presented. The wind tunnel experiment results for converted torsional power, corrected to the prototype scale,

Fig. 9 CFD simulations results. a Power coefficient for ALM approach, full-geometry CFD simulations, and experimental points from [[36](#page-15-29)]. The maximum values of C_p are represented by the vertical dashed line

are superposed to the results of the CFD-Full 3D and the CFD-ALM approach simulations obtained for the water flow.

The results for the CFD-Full 3D simulations and experiments have a good agreement in the TSR range between 0.5 and 2.0. For higher values, the results of simulations using the ALM approach are not too precise compared to the experimental observations and full CFD. In that situation, the results of the actuator line, based on the integration of the aerodynamic forces through the lines (like in the BEM approach), have an inherent uncertainty related to the lift and drag coefficients computed by XFOIL. Hence, the result for the stalled fow on the blade surface, equivalent to TSR higher than 2.0, does not present a good quality for the C_p estimates. On the other hand, for the nominal value in the maximum position of the power coefficient, the performance is properly predicted.

It is observed that the designed propeller machine has a good performance with a maximum value around $C_p = 0.39$,

occurring for $TSR = 1.7$. Values close to the maximum value of C_p occur, with a variation of $\pm 6\%$, in the range of *TSR* between 1.5 and 2.5. It is very useful to operate the machine close to its nominal condition, maintaining the conversion efficacy closest to the best value.

Finally, the performance results suggest that the parameters used in the CFD methodology are correct and the method can reproduce good results, especially close to the operating point. For this nominal condition, the analysis of the wake fow will be carried out.

5.2 Wake fow visualization

In Figs. [10,](#page-8-1) [11,](#page-9-0) and [12](#page-9-1), the wake visualization from the PIV measurements and CFD simulations are shown by timeaveraged velocity and vorticity during a turbine cycle. Figure [10](#page-8-1)b and c presents the near-wake fow visualization with a longitudinal mean velocity component. The dotted square in CFD-Full 3D and CFD-ALM fgures highlights the tip vortex region to be compared with the PIV data details in Fig. [10](#page-8-1)a. The PIV data identify properly the tip vortices spaced to the distance equivalent to the U_0 fblade where fblade denotes the blade passage frequency. The presence of the three vortex cores for the PIV case and the CFD simulations can be recognized in the fgures.

The near-wake vortex structures are also visualized from the numerical results for Full 3D and ALM (Fig. [11\)](#page-9-0). The large structures associated with the tip vortices and the hub core shadowing are correctly described. In Fig. [11a](#page-9-0), the blade tip vortex is observed as a helicoidal structure that appears in red color, located just after the rotor, resulting in the high angular velocities in this region. On the other hand, the hub/nacelle vortex with a cylindrical shape is located in the central zone of the near wake and appears in green/ blue color, representing the low-speed zone infuenced by the rotor nacelle. The two numerical methods present very similar results regarding the formation of the coherent structures of the wake.

Fig. 10 Near-wake fow visualization with longitudinal mean velocity component for **a** PIV data and for **b** CFD with Full 3D and **c** ALM

Fig. 12 Mean velocity feld U/U_0 in the rotor transverse plane: **a** CFD-Full 3D; and **b** CFD-ALM; **c** Rotor downstream transverse planes at distance 1D, 2D, 3D, and 4D

CFX

ALM

Figure [11](#page-9-0)b shows the dimensionless vorticity felds, also in the longitudinal midplane, for both numerical methods. It is observed how the results obtained by the ALM method also recover those extracted from the CFD-Full 3D, showing similar vorticity felds in morphology and intensity. The divergence between the fgures is related to the presence of the hub/nacelle, which induces a zone of greater vorticity in the central line, where the root vortex occurs. The fow visualization suggests that for this turbine the near-wake region remains close to the x/D<2.0−3.0, which is slightly diferent from the three-blade machines, for instance.

As mentioned, for the ALM simulation, the core region is smaller than the Full 3D simulation. It is difficult to describe the wake region near the center line by an effect of the blockage of the hub only considering the Navier–Stokes source terms. On the other hand, the main helicoidal structure of the vortices generated by the blade elements is properly obtained in computations, and both are compatible with the PIV data. The two modeling approaches can capture the destruction of this structure and as a consequence the end of the near-wake region. It is observed in Fig. [11](#page-9-0), in particular in the levels of the vorticity in the rotor plane.

Bringing more information about the wake computed in the simulations, Fig. [12](#page-9-1) presents cross-sectional planes of the rotor showing the velocity feld from another perspective. In Fig. [12a](#page-9-1) and b, the mean velocity distribution in the rotor plane is observed, for both numerical simulations, with similar behavior. CFD-ALM simulation presents a lower velocity in the blade region due to the absence of blade geometry in the simplifed method. Figure [12c](#page-9-1) displays the transverse planes of mean velocity at various distances downstream of the rotor. As mentioned previously, the main discrepancy between both methods is the lower velocity level in the nearwake central region, which appears more accentuated in the CFD-Full 3D case. Finally, at a 4D distance, it is observed that the maximum velocity defcit has been overcome.

5.3 Transversal profles and rear mapping

In Fig. [13,](#page-11-1) the experimental data profles of mean axial velocity and turbulence intensity are presented and compared to the CFD simulations in the region of near wake. Figure [13a](#page-11-1) presents the dimensionless mean velocity (U/U_0) in different position rotor downstream $(x/D = 1;2;3;4)$ providing information about the wake evolution. The velocity profles computed by CFD simulations show a good agreement in qualitative and quantitative terms. The main diference between the CFD results appears in the central region of the machine comprised by the interval $|z/D|$ < 0.2, referring to the hub/nacelle of the turbine, due to the velocity deficit. That characteristic is accentuated in the closest rotor profile $(x/D = 1)$ and decreases along the wake. In addition, Fig. [13](#page-11-1) presents the velocity profiles $(x/D = 2,3)$ extracted from the wind tunnel tests from hot-wire anemometer measurement for the HK10 machine on a 1:10 scale. Due to the kinematic similarity between both fows, it is possible to compare the velocity profles for operating conditions characterized by the dimensionless *TSR* = 1.6. See how the experimental and CFD results are very close in these profles. Furthermore, it is also observed that in the wind tunnel, the efect of the velocity drop in the hub/nacelle region is not as pronounced as in the CFD-Full 3D.

On the other hand, the efect on the blade tip velocity ($|z/D| \approx 0.5$) can be observed in the closest profiles $(x/D = 1;2)$, where the rotor geometry has a greater influence on the flow.

The profile $X/D = 1.5$ presents two peaks coinciding with the blade tip regions, identifying the position of the blade tip vortex. Furthermore, it is observed how the values of turbulence intensity *TI* decrease as the fow develops and also how the disturbed area increases. In the position $X/D = 1$, this was limited to the rotor area, and in the case $X/D = 4$ it goes even further. Both behaviors, also observed in the velocity profles, are due to the phenomena of difusion and mixing of the fow in the zones furthest from the wake due to the evolution of the fow.

In conclusion, the results between the simulations and the experiments have good adherence, reinforcing the quality of the integrated methodology using the diferent approaches to identify the main fow features in the wake. The velocity deficit in the wake is properly estimated, as well as the turbulence level. Therefore, the results presented for the HK10 rotor real machine operating in water fow show values and trends similar to those extracted from the wind tunnel tests for the scaled machine.

Finally, the mapping of the mean velocity and turbulence intensity for the plane $x/D = 1.5$ is presented in Fig. [14](#page-12-0) using the experimental data. In Fig. [14](#page-12-0)a, note that the flow, after crossing the rotor, presents a circular region, central to the rotor, with a large velocity deficit, as already observed in the profles previously introduced. In addition, for the foreground, the infuence of the tower on the fow is observed, delimited by the region of lower velocity.

In terms of the turbulence intensity, Fig[.14b](#page-12-0) shows the color map and Fig [14](#page-12-0)c the dimensionless energy density spectrum as a function of the dimensionless frequency by the optimal rotational speed, f/ω , for the points indicated in the image, located in the center of the rotor and the blade tip of each plane. The high-intensity regions represent locations with large velocity fuctuations, which is eventually attributed to fow recirculation zones or even the presence of coherent structures. In the case described in Fig. [14](#page-12-0)b, the sectors of high turbulence intensity correspond to the blade tip and root vortices regions. It can be observed the angular distribution of small points with high intensity of turbulence around the turbine center, identifying the position of the **Fig. 13** Transversal profles. **a** Mean axial velocity and **b** turbulence intensity. (—) CFD-Full 3D (- -) CFD-ALM and (**) Hot-wire measurements

blade tip vortex. Also, the hub/nacelle vortex's presence induces another great intensity region.

The signal spectra of the hot-wire measurements are shown in Fig. [14](#page-12-0)c, characterizing the typical bluf-body frequency distribution in the core region (2) and larger energy distribution, close to the blade passage frequency (1). The tip vortex allows in the near-wake region a supplementary supplement of turbulence energy due to the size of the vortices in the large-scale range of the spectrum.

6 Conclusion

Two numerical approaches have predicted accurate results to describe the wake from the high-solidity turbine rotor in the nominal operating conditions. The profles of mean and turbulent felds measured in the wind tunnel agree with the numerical simulation results in the near-wake positions. The

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CFD-ALM approach was able to predict good results of the wake flow in this turbine, where the solidity is higher than in conventional three-blade wind turbines (the situation where the method has been developed). The simulation using the URANS approach with ALM is close to the CFD-Full 3D and to the experimental observations, predicting the dynamics of the vortex's structures, velocity and vorticity felds, velocity deficits in the wake, turbulence intensity, and its spectra. Also, the wake morphology was compared by means of the recognition of the tip/root vortices.

All main fow structures were predicted and confrmed by experiments using hot-wire measurements and PIV. The tip vortex allows an organized contribution in terms of the remaining energy after the hydrodynamical conversion in the rotating blades, with a large-scale characteristic. It is observed in the PIV experiments and in both numerical simulations, with equivalent hydrodynamical characteristics of its advected behavior. The hot-wire measurements identifed

shadowing vortices

the two diferent characteristic spectra for the hub and tip vortex efect. In the last condition, the energy amount close to the characteristic frequency of blade passage contributes mostly to the total energy of the fuctuating feld. In the case of the core-shadowing region, the spectrum is equivalent to a conventional bluf-body fow. All these structures are alive downstream just at ∼ 3.0*D* and it is broken by shear and turbulence, with an interaction with the core region and the smaller eddies. After that, a far-wake difusion behavior is observed.

Results about the power coefficient C_p from CFD simulations and the experiments fit properly. The ALM C_p results show values close to experimental data around the nominal condition. The BEM in wind turbine aerodynamics has limitations that can lead to overestimation of aerodynamic performance. At high velocities, BEM assumes an attached boundary layer and neglects three-dimensional effects. This results in a simplified calculation of lift and drag coefficients based solely on the projected twisted angle, overlooking flow separation and non-uniformities in the boundary layer. Additionally, the neglect of three-dimensional efects disregards the radial fow gradients and spanwise distribution of aerodynamic forces. To improve accuracy, advanced models like computational fuid dynamics (CFD) should be employed to capture the complexities of the boundary layer and threedimensional fow, especially at high wind speeds and tip speed ratios. Despite this, as mentioned, the numerical methods employed in this work can describe properly the wake flow in hydrokinetic rotors. In conclusion, the ALM technique can accurately depict the fow morphology in high-solidity horizontal-axis turbines, including structures like tip and root vortices. However, it tends to yield inaccurate estimates of the power coefficient. Conversely, the CFD-Full 3D approach, when fully utilized, can also capture the correct flow morphology and provide accurate C_p results. Nevertheless, it requires a higher number of mesh nodes and longer simulation times.

Appendix

In this appendix, we describe the actuator line method $[15]$ $[15]$ based on an iterative process, combining the blade element momentum (BEM) and CFD equations. The method solves the Navier–Stokes equations with a font term counting the hydrodynamic forces on the blades. Firstly, it computes the 2*D* hydrodynamic forces on the blades (f_{2d}) . Later, it projects these forces in the feld fow using a regularization Kernel function (η_e) and incorporates the three-dimensional forces $f_i(\mathbf{x})$ in the Navier–Stokes equations.

In the ALM method, the rotor is a set of 'points' or 'elements' along the axis of the blade, as in the scheme in Fig. [15;](#page-13-0) it is no longer used as a surface, bringing advantages

Fig. 15 Full rotor geometry and blade discretization scheme to ALM. The hydrodynamic drag and lift forces, *D* and *L*, referred to any section of the blade are illustrated

such as lower computational costs or the possibility of structured mesh, suitable for large-eddy simulations.

The steps and main parameters to implement the ALM are described as:

1. *Rotor geometry defnition:* Rotor is constituted of 4 blades and 2.2 *m* in diameter, and the hydrodynamic profle is a NACA 4415. Other geometric data for the design of the blades (radial distribution, chord, and torsion angle) were extracted from the Hydrok project and are summarized in Table [2](#page-2-3).

> For the ALM, the blades were discretized in 38 points, an estimated value based on the relation-ship proposed by [\[44](#page-16-7)] such that $\Delta x = R/n$, which relates the rotor radius, $R = 1.1$ m, and the mesh element size, $\Delta x = 0.03$ m, for the mesh presented in this work. In this way, it guarantees an appropriate relation between mesh size and blade discretization to prevent more than one point reside in the same mesh element.

- 2. *Turbine operating condition:* characterized by free stream velocity $U_0 = 2.5$ m/s and rotational velocity $\Omega = 35$ rpm, and, consequently, defined by tip speed ratio $TSR = 1.6$. After geometric and kinematic definition, Reynolds number is computed through the mean chord, c_m , and the relative flow velocity (U_{rel}) at mean radial position R_m , to operation conditions, such as $U_{rel} = \sqrt{U_0^2 + (\Omega R_m)^2}$. Thus, the computed local Reynolds number is $Re_L = 2 \times 10^6$.
- 3. *Hydrofoil polar curves:* Using the free software XFOIL [[45](#page-16-8)], which employs panel methods with a boundary layer formulation, the lift and drag coefficient, $C_L(\alpha, Re_L)$ and $C_D(\alpha, Re_L)$ as a function of angle of attack (α) and local Reynolds number, are computed and presented in Fig. [16](#page-13-1). The input values to the software are the 2D hydrofoil geometry, the local Reynolds numbers, and the Mach number.
- 4. *Computing hydrodynamic forces by BEM method:* The two-dimensional hydrodynamic forces per unit length to each blade element are defned as

Fig. 16 Lift (C_L) and drag (C_D) coefficient values of NACA4415 hydrofoil at various angles of attack (α) from XFOIL software [\[45\]](#page-16-8)

$$
\mathbf{f}_{2d} = f_L \mathbf{e}_L + f_D \mathbf{e}_D,\tag{4}
$$

where f_L and f_D are calculated through equations [6](#page-14-0) and [5](#page-14-1), after to know the fow angle attack to each blade point, and to compute the relative fow velocity and the lift and drag coefficient.

$$
f_D = \frac{1}{2} C_D(\alpha) \rho U_{rel}^2 c,
$$
\n⁽⁵⁾

$$
f_L = \frac{1}{2} C_L(\alpha) \rho U_{rel}^2 c,
$$
\n⁽⁶⁾

where *c* is the chord length, ρ the flow density, U_{rel} the relative local velocity, and C_L and C_D , the lift and drag coefficients, respectively.

> In each radial position and angle of attack, there is a specifc value of the relative local velocity as well as C_L and C_D . So, the relative velocity is computed by

$$
U_{rel} = \sqrt{(U_z^2 + (\Omega r - U_\theta)^2)},
$$
\n(7)

being U_z and U_θ the axial and tangential velocities, Ω the angular velocity and *r* the radius varying along the blade. On the other hand, the angle of attack is expressed as

$$
\alpha = \Phi - \gamma,\tag{8}
$$

where Φ is the angle between the relative velocity and the rotor plane and, γ is the pitch angle, as observed in Fig. [17.](#page-14-2)

$$
\Phi = \tan^{-1}\left(\frac{U_z}{\Omega r - U_\theta}\right). \tag{9}
$$

Fig. 17 Hydrofoil cross section illustrating the lift and drag hydrodynamic forces and the velocities triangle

5. *Projection of hydrodynamic forces (2D) to the fow feld (3D):* computing the field flow, f_i , through the convolution integral of bi-dimensional forces and the kernel regularization function, η_e , such as

$$
f_i(\mathbf{x}) = \sum_{k=1}^{B} \int_0^R F \mathbf{f}_{2D}(r) \cdot \mathbf{e}_i \eta_c(|\mathbf{x} - r\mathbf{e}_k|) dr,
$$
 (10)

where e_k in the unit vector in the blade direction k , $|\mathbf{x} - r\mathbf{e}_k|$ is the distance between the mesh point and the actuator line point. The regularization function η_{ϵ} is defned as

$$
\eta_{\epsilon}(r) = \frac{1}{\epsilon^3 \pi^{3/2}} \exp[-(r/\epsilon)^2]. \tag{11}
$$

The parameter ϵ represents the reached kernel function and it is established as $\Delta x \geq 2\epsilon$ [[22,](#page-15-20) [46](#page-16-9), [47](#page-16-10)]. Based on the mesh generated, the ϵ value is defined as $\epsilon = 2\Delta x = 0.06$.

> Finally, to highlight that the function *F* in Eq. [10](#page-14-3) is the correction factor to tip blade effects developed by [[48\]](#page-16-11) such as

$$
F = \frac{2}{\pi} \cos^{-1} \left[\exp \left(-g \frac{B(R - r)}{2r \sin \Phi} \right) \right],\tag{12}
$$

the parameter *B* refers to number of blades and the *g* coefficient depends of blade numbers, *TSR* value, chord distribution, pitch angle, etc... For simplicity, it established *g* to be only dependent on the blade numbers and the *TSR* [[48\]](#page-16-11), in the form

$$
g = exp(-c1(B\Omega R/U_{\infty} - c2)), \qquad (13)
$$

being c_1 e c_2 coefficients estimated experimentally.

$$
g = exp(-0.125(B\Omega R/U_{\infty} - 21)) + 0.1.
$$
 (14)

6. *Navier–Stokes equations solving:* the fow is solved inserting the field force f_i in Navier–Stokes equations,

$$
\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + v \frac{\partial^2 u_i}{\partial x_j \partial x_j} + f_i.
$$
 (15)

After the velocity feld is computed in the new time step, new angles of attack and relative velocities are defned turning to step 4, in an iterative process, where forces are computed and projected in the fow feld again.

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Declarations

 Conflict of interest The authors declare that they have no confict of interest.

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