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TECHNICAL PAPER

A methodology based on empirical mode decomposition and synchrosqueezed wavelet transform for modal properties identification and damage detection

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Abstract
Structural health monitoring (SHM) and damage detection using vibration-based methods continue to be of interest in fields such as civil and mechanical engineering, among others. Early identification of damage can save human lives and facilitate low-cost recovery of existing infrastructure. The methods used in SHM can have a local or global approach. A local approach checks the components of the structure in detail, while a global approach detects general changes in the characteristics of the structure. In this article, a methodology with a global approach is proposed, which combines the properties of empirical mode decomposition (EMD), synchronized wavelet transform (SWT), and spline interpolation, with the aim of identifying modal properties and damage. In this methodology, it is possible to identify the frequency content of a signal over time; if there are changes in the modal properties of the structure, it is known that there were changes in the physical properties; therefore, there was damage in the structure. To validate the effectiveness of the proposed methodology, the instant of damage is identified, as well as the natural frequencies of the Benchmark Phase I, considering the structure without damage and with damage due to loss of stiffness in the first and third floors. The analyzed signal is created considering the undamaged and damaged states of the structure. First, the instant of damage and the change in the natural frequencies of the structure due to stiffness loss damage were identified using the SWT. Subsequently, the proposed methodology was validated by comparing the values obtained in the identification of the natural frequencies with the values reported by other authors. The minimum and maximum errors were 0.0% and 2.31%, respectively, compared to the results reported by the AISCE-ASCE group. The proposed methodology proved to be robust as a SHM method; it identifies frequencies with closely spaced modes and does not require a priori knowledge of the structure.

Keywords Empirical mode decomposition · Synchrosqueezed wavelet transform · IASC-ASCE Benchmark Phase I · Structural health monitoring

1 Introduction

Many civil structures around the world are in some state of deficiency, requiring intervention for their recovery. The cause of deterioration of a structure is diverse; it can be due to corrosion, service, destructive forces, or other abnormal events [1]. In 2021, the American Society of Civil Engineers (ASCE) issued a report on infrastructure in the USA [2]. The ASCE estimates that an investment of $ 5.94 trillion will be necessary to recover existing infrastructure between 2020 and 2029. Early damage detection techniques emerge as a challenge for structure health monitoring (SHM) that seeks to provide a solution to the recovery of an existing structure. Therefore, it will be necessary to develop new technologies

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and low-cost methods that allow the maintenance of civil structures.

Recently, several researchers have focused their interest on damage determination. Pereira, et al. [3] applied a bio-inspired algorithm known as Lichtenberg optimization algorithm (LA) in combination with finite element programs. The authors highlighted that the LA is able to detect damage in noisy conditions and low damage severity. Similarly, Pereira et al. [4] developed a study for crack identification through the implementation of LA. The results obtained by the authors were promising in damage identification and decision making, such as replacing parts of the structure. Several studies were reported by Gomes et al. [5] in their review on vibration-based inverse methods aimed at damage identification.

The methods used for damage detection can be classified into local and global approaches [6]. Methods with a local approach aim to detect damage or changes in structural components or materials, for example: the ultrasonic pulse velocity method [7], the radiograph method [8], the infrared thermographic method [8], among others. Methods with a global approach are based on the premise that the dynamic characteristics are a function of the physical properties of the structure. Therefore, if there are changes in the physical properties, it will be possible to detect changes in the dynamic characteristics [9]. Some methods with a global approach are: Wavelet transform [10]; Hilbert–Huang transform [11]; Empirical mode decomposition method (EMD) [12], among others. In this work, a methodology with a global approach based on the combination of three techniques is proposed: (1) empirical mode decomposition (EMD), (2) synchrosqueezed wavelet transform (SWT), and (3) spline interpolation.

Studies that have taken a global approach and used only SWT as a method for damage detection have been reported by Sanchez et al. [13]. The authors used SWT to identify the modal properties of Benchmark and reported six natural frequencies found in an undamaged signal, three in each direction (x, y). Babajanian Bisheh et al. [14] proposed a methodology based on vibration analysis to identify damage by pattern recognition. Five techniques were considered; one of them was SWT. Benchmark was used as the analysis model. To assess the structural health of bridges Liu et al. [15] implemented SWT using bridge-vehicle iteration models. The authors demonstrated that the proposed methodology is robust for identifying and quantifying damage, obtaining good results in five of the six simulated experimental tests.

Other works composed of two or more techniques including SWT were presented by Rafiei and Adeli [16]. The authors proposed a new methodology based on two signal processing methods, SWT and FFT. In addition, an unsupervised machine learning technique, the neural dynamics classification (NDC) algorithm, and the restricted Boltzmann machine were implemented. The authors evaluated the overall health status of a 38-story concrete building constructed on a 1:20 scale. Using the same experimental model, Amezquita-Sanchez and Adeli [17] validated a methodology to detect and quantify damage. This new methodology consisted of three steps: (1) SWT, (2) fractality dimension, and (3) Condition assessment of the structure. Similarly, Rafiei and Adeli [18] proposed a methodology to assess the state of health globally and locally through structural response. The methodology was composed of SWT, FFT, and an unsupervised deep Boltzmann machine. On the other hand, Z. Li et al. [19] implemented SWT in conjunction with HT and linear least square fitting. The authors implemented the methodology on a 123-story building and compared the values obtained with those found by EMD.

Works whose methodologies were validated through the Benchmark structure and implemented the SWT as a signal processing technique were performed by C. Perez-Ramirez et al. [20], who proposed a computational strategy for the accurate identification of modal parameters. A methodology that combines three techniques: (1) natural excitation technique, (2) synchrosqueezed wavelet transform, and (3) genetic algorithm. In the same line, C. Perez-Ramirez et al. [21] defined a new methodology based on SWT for the identification of the modal parameters of a structure subjected to environmental vibrations. The proposed methodology consisted of four techniques: (1) random decrement technique, (2) synchrosqueezed wavelet transform, (3) Hilbert transform, and (4) the Kalman filter. The values obtained were compared with other signal processing techniques.

This article proposed a methodology with a global approach for the identification of modal properties and damage detection, that is, based on the dynamic characteristics of the structure. To validate the effectiveness of this methodology, the Benchmark Phase I developed by the IASCE-ASCE group is used. The Benchmark structure consists of a 4-story, 2×2 bay steel structure built to scale 1:4. It has the following dimensions 3.6 m in height and a surface area of 2.5×2.5 m. IASCE-ASCE offers a finite element program known as Datagen. The program was developed in Matlab and allows simulating different damage patterns. In this work, the scenarios of the structure without damage and with damage due to loss of stiffness in the first and third floors are considered, as shown in Fig. 3. The aims of the numerical simulation in this article are: (1) Identification of the instant of damage and of the changes in the natural frequencies and (2) Validation of the proposed methodology through the accuracy in the calculation of the natural frequencies. The values obtained were compared with those found in other works, including Johnson et al. [22]. The proposed methodology proved to be robust in accurately identifying the frequencies of a damaged, non-stationary, and noise-embedded signal.

2 Methodology

The proposed methodology was developed entirely in Matlab programming language. This methodology can be applied to any type of civil structure, being required as input data for the acceleration response of the structure. According to Rytter [23], there are four levels of damage detection classification. This paper focuses on level 1, which corresponds to the determination if there is damage in the structure and not in determining the location of the damage. Figure 1 shows the methodology based on EMD–SWT methods for damage identification from vibration data. First, the intrinsic mode functions (IMFs) of the signal are extracted with the help of the EMD method [24]. Subsequently, each IMF is processed by the SWT method to obtain the natural frequencies. Finally, using spline interpolation, the graph containing the natural frequencies is smoothed.

The main advantage of this method lies in the use of EMD to decompose the signal into the IMF and thus improve the accuracy in the identification of the natural frequencies by the SWT. On the other hand, the main disadvantage of using EMD lies in the end effect of each IMF, which is partially distorted [25].

2.1 Empirical mode decomposition (EMD)

Huang et al. [26] proposed the EMD method for time series analysis. EMD decomposes a signal that is generally non-stationary into a series of IMF functions that are quasi-stationary. Two conditions are necessary for an IMF to be recognized as a modal response function: (1) the number of zero crossings and the number of extrema must be of the same size or differ by 1; (2) the average (mean) between the upper and lower envelopes must be equal to zero [12].

For a signal $x(t)$, the following steps should be considered to obtain the IMFs.

Step 1 Locate the maximum and minimum points of $x(t)$.

Step 2 Interpolate by spline the local maxima and local minima to construct the upper and lower envelopes $\left(e_{\min}(t), e_{\max}(t)\right)$.

Step 3 Compute the mean by:

$$m(t) = \frac{e_{\min}(t) + e_{\max}(t)}{2} \tag{1}$$

Step 4 Extract the component

$$d(t) = x(t) - m(t) \tag{2}$$

Step 5 Iterate on the residual $r(t)$.

To obtain an IMF, steps 1 to 4 must be repeated on the component $d(t)$ until the two conditions mentioned are satisfied (stopping criterion). When step 4 is satisfied, consider $d(t)$ as the first IMF. Once an IMF is obtained, the residual is calculated, $r(t) = x(t) - d(t)$, and step 5 is applied. Thus, a finite number of IMFs and a residual are obtained.

2.2 Synchrosqueezed wavelet transform (SWT)

Synchrosqueezed transform is an analysis algorithm that follows the same philosophy as the EMD approach. SWT decomposes a signal into its building blocks functions, but with a different approach than EMD in the construction of the components [27, 28]. SWT is a time–frequency analysis method [29]. SWT combines wavelet analysis and a reallocation method that can improve the identification and extraction of oscillatory components as natural frequencies.

There are various types of the wavelet transform, including SWT. A wavelet transform is constructed from a function called the *mother wavelet,* ψ, and is defined as:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right), a, b \in \mathbb{R}, a \neq 0 \tag{3}$$

where a measures the degree of compression, known as the scaling parameter, and b determines the time location, called the translation parameter.

SWT uses two types of analytic wavelet: amor (analytic Morlet wavelet) [30] and bump [31]. The analytic bump wavelet, $\hat{\psi}_{bump}$, will be used as a *mother wavelet* [32] in SWT, defined by:

$$\hat{\psi}_{bump}(s\omega) = e^{\left(1-\frac{1}{1-(s\omega-\mu)^2/\sigma^2}\right)} 1_{[(\mu-\sigma)/s\,,(\mu+\sigma)/s\,]} \tag{4}$$

where $1_{[(\mu-\sigma)/s\,,(\mu+)/s\,]}$ is the indicator function for the interval $(\mu-\sigma)/s \ \leq \omega \leq (\mu+\sigma)/s$ the center frequency.

SWT algorithm is summarized in three steps [17]:

Step 1 Calculate the CWT coefficients, $W_x(a, b)$, using any mother wavelet to recover the amplitudes at the instantaneous frequencies.

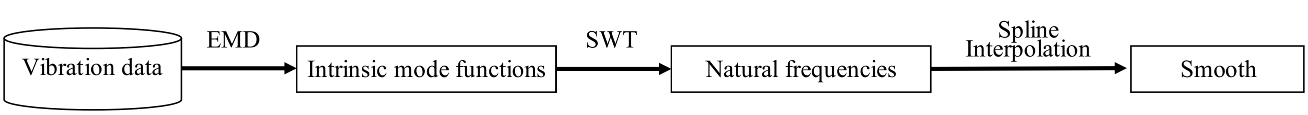


Fig. 1 Flowchart of methodology

$$W_x(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t)\psi^*\left(\frac{t-b}{a}\right)dt \tag{5}$$

where ψ is the mother wavelet, the symbol $*$ is the complex conjugated, a is the scale and b is the time shift.

Step 2 For the signal $x(t)$ is computed the instantaneous frequency, $\omega_x(a,b)$, differentiating at any point (a,b) the CWT coefficients, $W_x(a,b)$.

$$\omega_x(a,b) = \begin{cases} \frac{-j}{W_x(a,b)} \frac{\partial[W_x(a,b)]}{\partial b} & |W_x(a,b)| > 0 \\ \infty & |W_x(a,b)| = 0 \end{cases} \tag{6}$$

Step 3 Finally, the synchrosqueezing process is done by reassigning the CWT coefficients, $W_x(a,b)$, to the time–frequency domain considering the map $(a,b) \rightarrow (\omega_x(a,b), b)$. Only at the centers, ω_c, is determined the Synchrosqueezed Transform, $T_x(\omega_c, b)$, for the frequency range $[\omega_c - \Delta\omega/2, \omega_c + \Delta\omega/2\ ]$:

$$T_x(\omega_c, b) = \Delta\omega^{-1} \sum_{a_k : \omega_x(a,b) \in [\omega_c - \Delta\omega/2, \omega_c + \Delta\omega/2]} W_x(a_k, b) a_k^{-3/2} (\Delta a)_k \tag{7}$$

where $\Delta a = a_k - a_{k-1}$ and $\Delta\omega = \omega_c - \omega_{c-1}$.

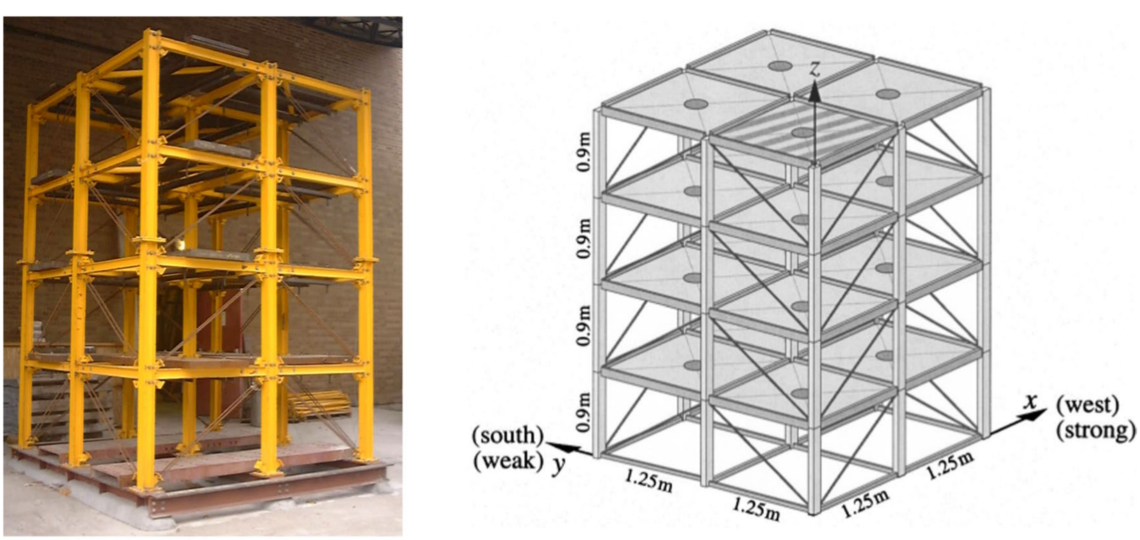
3 Numerical application

It is proposed to validate this methodology in the Benchmark Phase I. The Benchmark structure was built at the University of British Columbia (UBC—Canada), it is a 4-story 3D steel-frame quarter-scale model structure of 2×2 bay (see Fig. 2a). The structure is exposed to white noise in both the x-direction (strong side) and y-direction (weak side), as shown in Fig. 2b. The Benchmark model has six different damage patterns; in this article, the undamaged cases and damage pattern (ii) will be considered (see Fig. 3). The synthetic signal to be used is constructed considering case 3 (roof excitation), as shown in Table 1. The signal is obtained through sensors 15 and 16 (see Fig. 4). The choice of sensor location is very important to obtain a good dynamic response signal. Since one of the objectives of this work is to compare the results obtained with those of other authors, the same sensors used in other reference articles were used [21].

The numerical example consists of two analysis stages: (1) identification of changes in natural frequencies and damage instant by SWT, and (2) the proposed methodology is validated through the accuracy in the identification of the natural frequencies. The signal used in this exercise is constructed from two signals, a first undamaged signal that goes from second 0 to second 20 and another signal with damage pattern (ii) that goes from second 20 to second 40 (see Fig. 5). This signal approximates the characteristics of a real signal, that is, a signal read in situ from a real structure.

According to Johnson et al., [22] the Benchmark structure allows to recreate six damage patterns between major and minor (see Fig. 3). The major damages patterns are represented by the numbers (i) to (iv) and the minor ones by (v) and (vi). The signal used in damage identification is affected by damage pattern (ii).

The Benchmark structure has a total of sixteen sensors, four on each floor, as shown in Fig. 4. The signals used are obtained from sensors 15 and 16 (x and y-direction, respectively). These two sensors were chosen to compare



(a) (b)

Fig. 2 Benchmark structure: **a** real structure on 1:4 scale model and **b** analytical model [22]

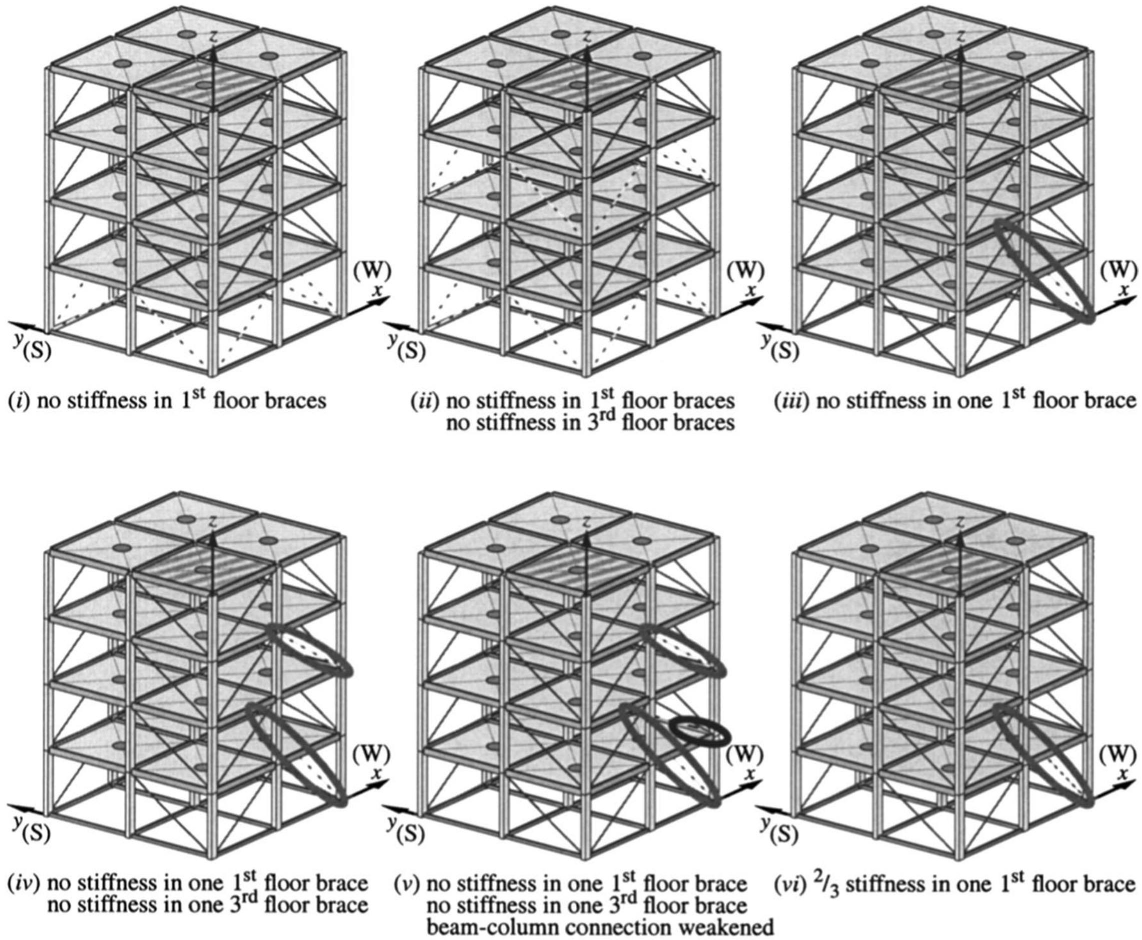


Fig. 3 Damage patterns for Benchmark structure [22]

the values obtained with those of other authors who used the same monitoring points.

The signal presented in Fig. 5 was constructed using a FEA code in MATLAB developed by the IASC-ASCE [22]. This code generates the dynamic response of the structure when exposed to a white noise type excitation, Case 3 (roof excitation). The signal obtained from sensor 15 (x-direction) is presented in Fig. 5a, and that from sensor 16 (y-direction) in Fig. 5b. Both signals were constructed by joining an undamaged signal with a damage pattern signal (ii), i.e., from 0 to 20 s an undamaged signal, from 20 to 40 s a damaged signal. The parameters used in the construction of both signals are a sampling frequency of 1000 Hz, 10% white noise, and a modal damping ratio of 1%. Sampling frequency and signal duration have an influence on obtaining robust results. In this paper, for reasons of comparison, the same data from the reference article will be used [22].

SWT was applied to the signals in Fig. 5, using the analytic bump wavelet as the mother wavelet with 42 voices per octave. The values obtained are presented in Fig. 6 for sensor 15 (x-direction) and in Fig. 7 for sensor 16 (y-direction). It can be observed in Fig. 6a a discontinuity of the value of the frequencies at second 20, it is precisely at that point where the transition from the undamaged signal to the signal with damage occurs; therefore, the SWT manages to identify the instant of damage accurately. Figure 6b shows the frequencies identified for the complete signal (40 s). In contrast, Fig. 6c shows the frequencies identified for each segment of the undamaged signal and the damage pattern (ii). The same result was found for sensor 16 (y-direction) presented in Fig. 7.

From Figs. 6c and 7c, the natural frequencies of the undamaged signal, that is, the frequency content of the signal between 0 and 20 s, are extracted (see Fig. 5). The values obtained are shown in Table 2 together with those of the reference model (FEA [22]) and Sanchez et al.[13] for comparison purposes. Sanchez et al.[13] implemented the SWT, using the analytic Morlet wavelet as the *mother wavelet* with 32 voices per octave, being able to identify the first three frequencies in both directions. The minimum error was of 0.12% and the maximum error was 3.06% with respect to the reference model [22]. In this work, SWT was

Table 1 Simulation cases of Benchmark structure [22]

Description	Case 3 (roof excitation)	Case 4 (3D)
Data generation model:		
1. Floors rigid (USC 12DOF)	x	x
2. Floors rigid in-plane (HKUST 120DOF)		
Mass distribution:		
1. Symmetric (four 400 kg masses on roof)	x	
2. Asymmetric (three 400 kg, one 550 kg)		x
Excitation:		
1. "Ambient"		
2. Shaker diagonal on roof	x	x
ID model: Linear 12DOF shear building	x	x
ID data: four sensors/floor with 10% RMS noise		
1. Known input		
2. Unknown input	x	x
3. Unknown input; sensors on second, fourth floors		
Damage patterns: remove the following		
i. All braces in first story	x	x
ii. All braces in first and third stories	x	x
iii. One brace in first story		x
iv. One brace in each of first and third stories		x
v. As iv, and loosen floor beam at first level		
vi. 2/3 stiffness in one brace in at first story		x

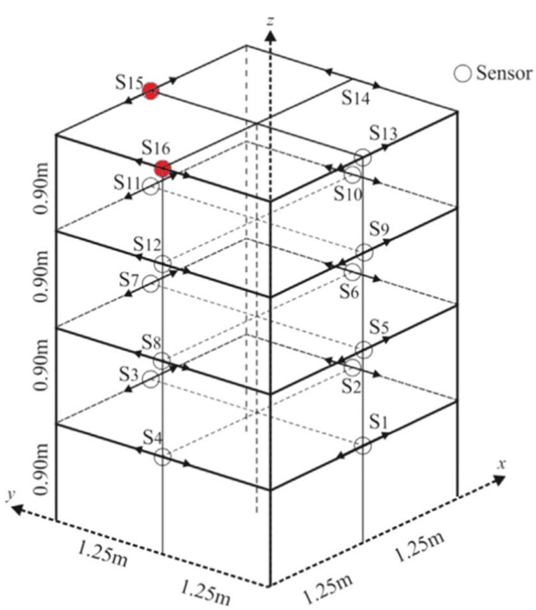


Fig. 4 Sensor location on Benchmark structure [21]

also implemented, using the analytic bump wavelet as the *mother wavelet* with 42 voices per octave. All frequencies (x and y-direction) were identified and a minimum error of 0.0% and a maximum of 2.31% were obtained compared to the FEA [22]. It can be stated that the analytical bump wavelet has a better performance compared to the values obtained when using the analytic Morlet wavelet.

After demonstrating the capabilities of SWT to identify natural frequencies and instant damage, the methodology proposed in this work will now be validated (see Fig. 1). The signals presented in Fig. 5 are first decomposed into intrinsic mode functions (IMFs), as shown in Fig. 8 for sensor 15 (x-direction) and in Fig. 9 for sensor 16 (y-direction). Six IMFs were sufficient for the extraction of all frequencies. Subsequently, SWT and spline interpolation are applied to each of the calculated IMFs; the values obtained are presented in Figs. 10 and 11. The complete signal was analyzed: the signal composed of an undamaged and a damaged part. Figures 10 and 11 contain the group of frequencies before damage and after damage. The same procedure is applied for the analysis of the undamaged signal segment and the damaged signal segment.

Table 3 shows the frequencies of the undamaged signal identified from different procedures. First, the frequencies of the complete signal ($t=0$–40 s) were identified by applying only the SWT. The results showed a minimum error of 0.17% and a maximum of 2.54% compared to the FEA reference model [22]. Afterward, the proposed methodology was implemented and a minimum error of 0.09% and a maximum of 2.18% were obtained compared to the FEA [22]. Subsequently, the same procedure is carried out, only this time on the undamaged signal segment ($t=0$–20 s). The values obtained using SWT yielded a minimum error of 0.13% and a maximum of 1.20%, while with the proposed methodology, the minimum error value was 0.0% and the maximum was 1.05% compared to FEA [22]. It has been shown that the proposed methodology improves the values obtained from the SWT and is closer to the FEA reference values [22].

In the same way that the natural frequencies of the undamaged signal were identified, the natural frequencies present in the signal with damage pattern (ii) were identified (see Table 4). For the complete signal ($t=0$–40 s), the first three natural frequencies were obtained in both directions (x, y), both for SWT (see Figs. 6b and 7b) and for the proposed methodology (see Figs. 10 and 11). After applying SWT, a minimum error was 0.0% and the maximum error was 1.27%, whereas, with the proposed methodology, the minimum error was 0.04% and the maximum error was 1.44%. The signal segment with damage pattern (ii) ($t=20$–40 s) of the signal presented in Fig. 5 allowed the identification of all frequencies in both directions. Using SWT, a minimum error value of 0.13% and a maximum of 1.64% were obtained in comparison with FEA [22]. In the case of the proposed

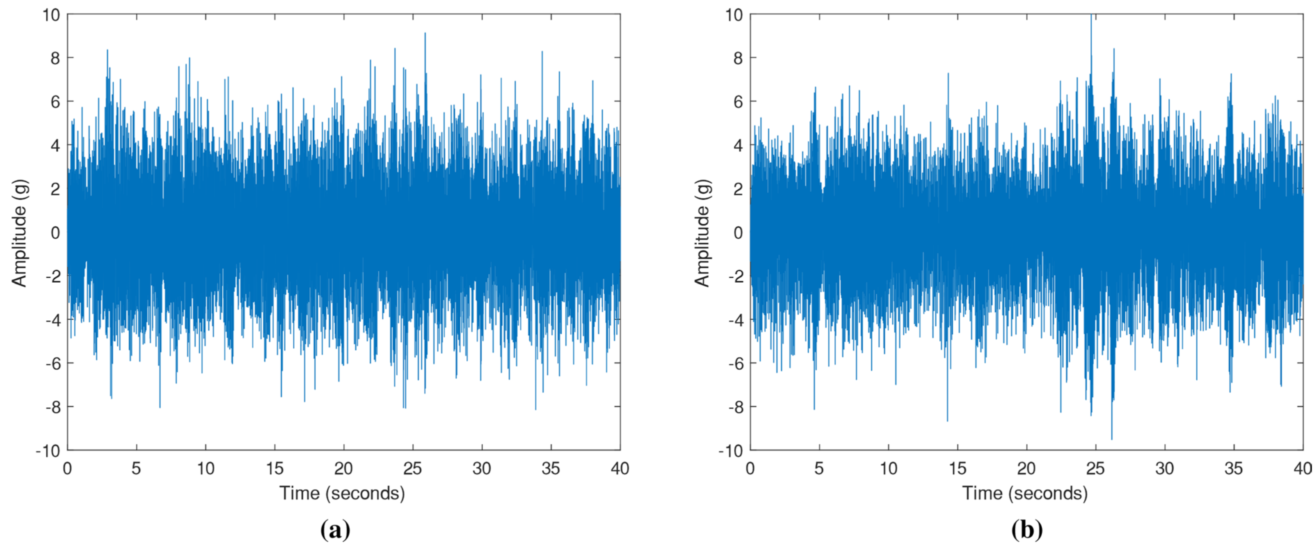


Fig. 5 Synthetic signals in: **a** *x*-direction and **b** *y*-direction

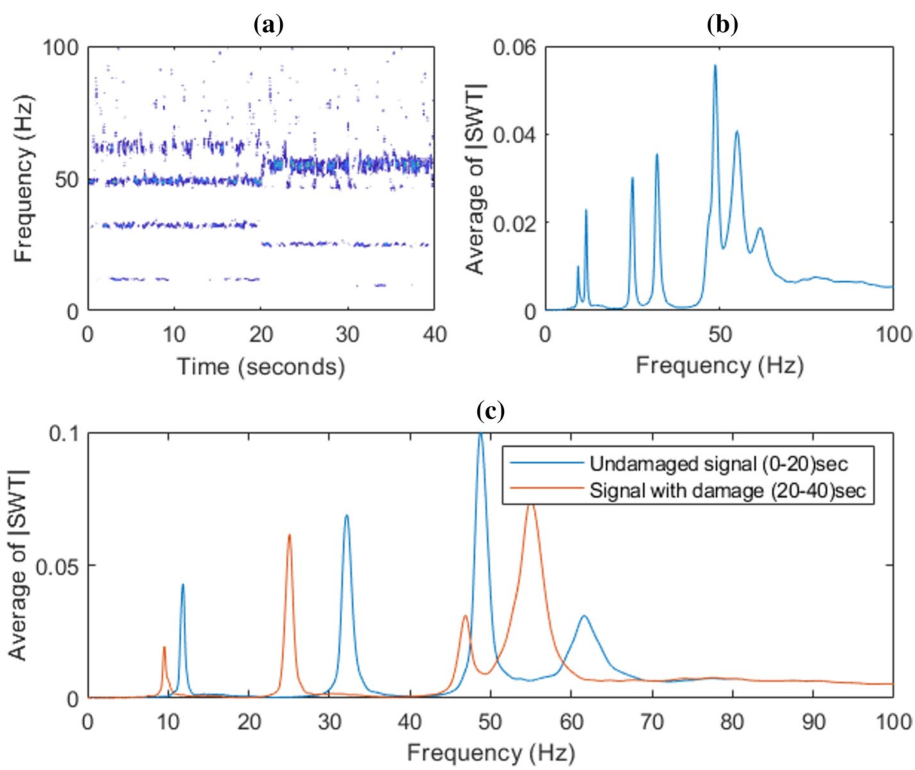


Fig. 6 Natural frequencies of the Benchmark using SWT—Sensor 15 (*x*-direction): **a** damage time instants, **b** $\left|\overline{\mathrm{SWT}}\right|$ for complete signal, and (c) $\left|\overline{\mathrm{SWT}}\right|$ for the signal segment

methodology, the minimum error was 0.13% and the maximum error was 3.13%. The values obtained after applying the proposed methodology were acceptable. In some ways, the natural frequencies were closer to the reference values. The authors did not find in the literature consulted studies presenting methodologies implemented in the identification of modal properties in the Benchmark structure with damage pattern (ii) for comparison purposes.

To validate the proposed methodology, the values obtained are compared with other studies that implemented SWT within their methodologies for the identification of modal properties in the Benchmark structure (see Table 5). Perez-Ramirez et al. [21] proposed a methodology composed of three processes: RDT, SWT, and HT. The minimum error was 0.03% and the maximum error was 0.33% compared to FEA [22]. Later, Perez-Ramirez et al. [20] proposed a new

Fig. 7 Natural frequencies of the Benchmark using SWT—Sensor 16 (*y*-direction): **a** damage time instants, **b** $\left|\overline{\mathrm{SWT}}\right|$ for complete signal, and **c** $\left|\overline{\mathrm{SWT}}\right|$ for the signal segment

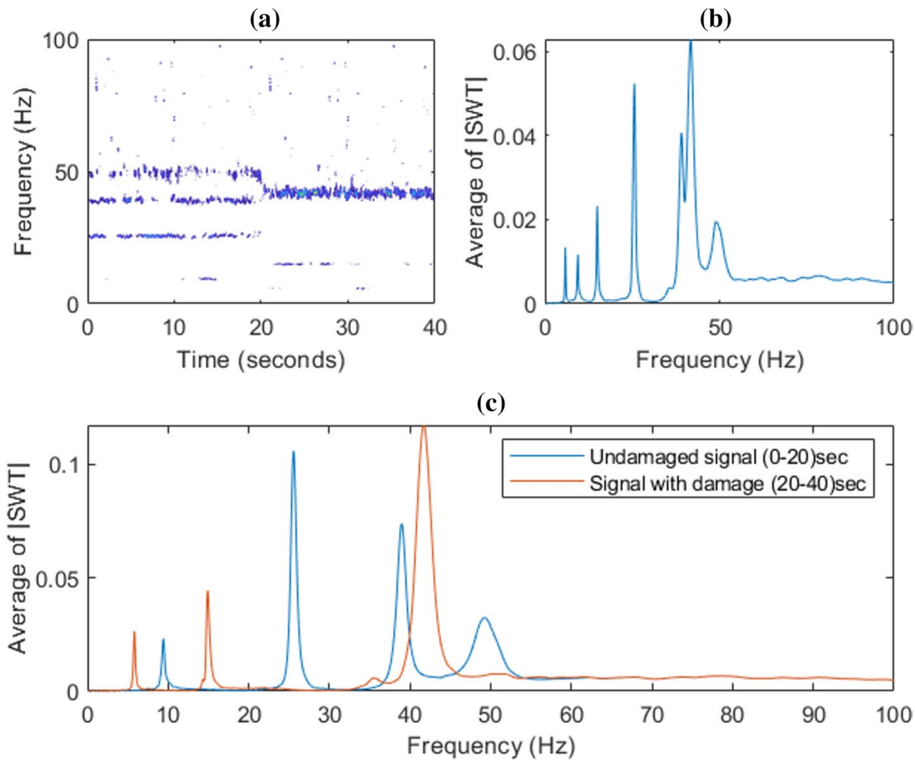


Table 2 Undamaged Benchmark structure (Case 3, 12 DOF, lumped mass)—identification of natural frequencies using only SWT

Mode	Natural frequencies (Hz) (% Error)		
	FEA [22]	Sanchez et al. [13]	SWT
1	9.41 (*y*)	9.48 (0.74)	9.46 (0.53)
2	11.79 (*x*)	11.83 (0.34)	11.79 (0.00)
3	25.54 (*y*)	25.68 (0.55)	25.54 (0.00)
4	32.01 (*x*)	32.05 (0.12)	31.93 (0.25)
5	38.66 (*y*)	39.12 (1.19)	38.98 (0.83)
6	48.01 (*y*)	- (-)	48.41 (0.83)
7	48.44 (*x*)	49.92 (3.06)	48.49 (0.10)
8	60.15 (*x*)	- (-)	61.54 (2.31)

methodology with three techniques: NExT, SWT, and GA. In this case, the minimum error was of 0.02% and the maximum error was 0.59% compared to FEA [22]. The methodology proposed in this article uses (1) EMD, (2) SWT, and (3) Spline interpolation. The minimum error was 0.0% and the maximum error was 2.31% compared to FEA [22]. Only one frequency exceeded 1% error, and two frequencies exactly reached the reference value. It can be concluded that the proposed methodology is efficient in identifying modal properties.

It was considered relevant to compare the proposed methodology with other methodologies that did not use SWT in the identification of the modal properties of the Benchmark structure. The studies used for comparison are shown in Table 6 [22, 33–35]. Amini and Hedayati [33] implemented the SCA technique, and the authors identified only the frequencies of sensor 16 (*y*-direction). The minimum error found was 0.08% and the maximum error was 12.86% compared to FEA [22]. Li, Xu, and Zhang [34] used Markov Chain Monte Carlo together with a probability-based method. The authors found a minimum error of 3.86% and a maximum error of 23.41% compared to FEA [22]. Cara et al. [35] used stochastic subspace identification method (SSI) and identified six of the eight frequencies. The minimum error found was 0.04% and the maximum was 3.81% compared to FEA [22]. Applying the proposed methodology, a minimum value of 0.0% and a maximum of 2.31% was obtained compared to FEA [22]. Therefore, it is demonstrated that the proposed methodology is more efficient than the other three methodologies presented, which did not include the SWT in the identification of the modal properties as the natural frequencies of the Benchmark structure.

Das & Saha [36] used frequency domain decomposition (FDD) in the identification of natural frequencies for Case 4, Pattern 3. Because it is a different pattern than the one addressed in this work, it was not possible to compare the results. The authors concluded that FDD has as a disadvantage the need to use a total of four sensors, one per floor. In addition, the authors reported that it was possible to use a single sensor when FDD is complemented with wavelet-based EMD.

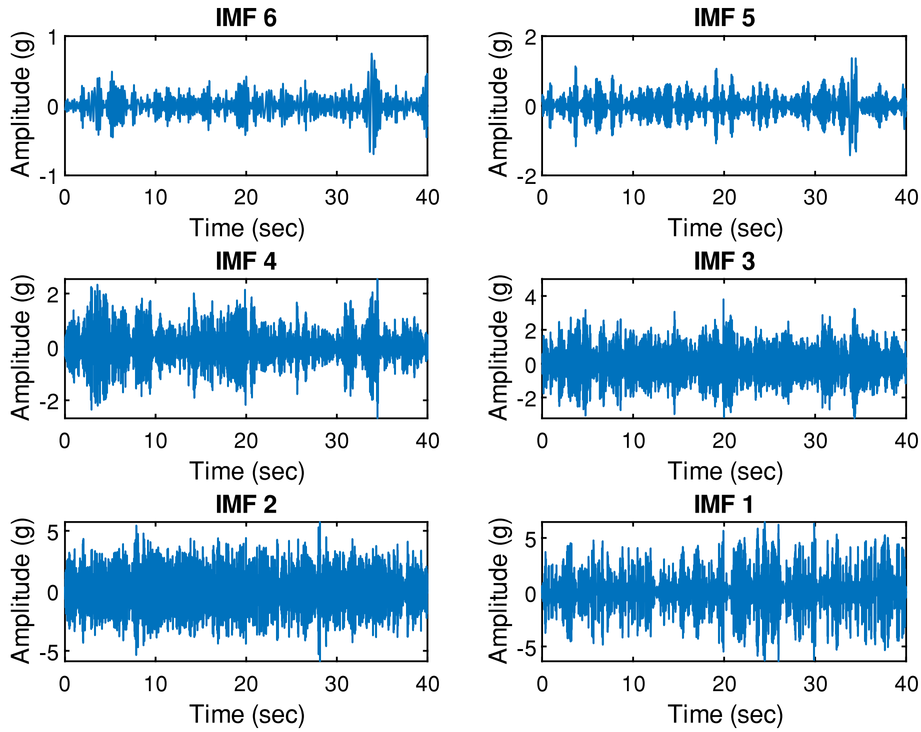


Fig. 8 IMFs from EMD in sensor 15 (*x*-direction)

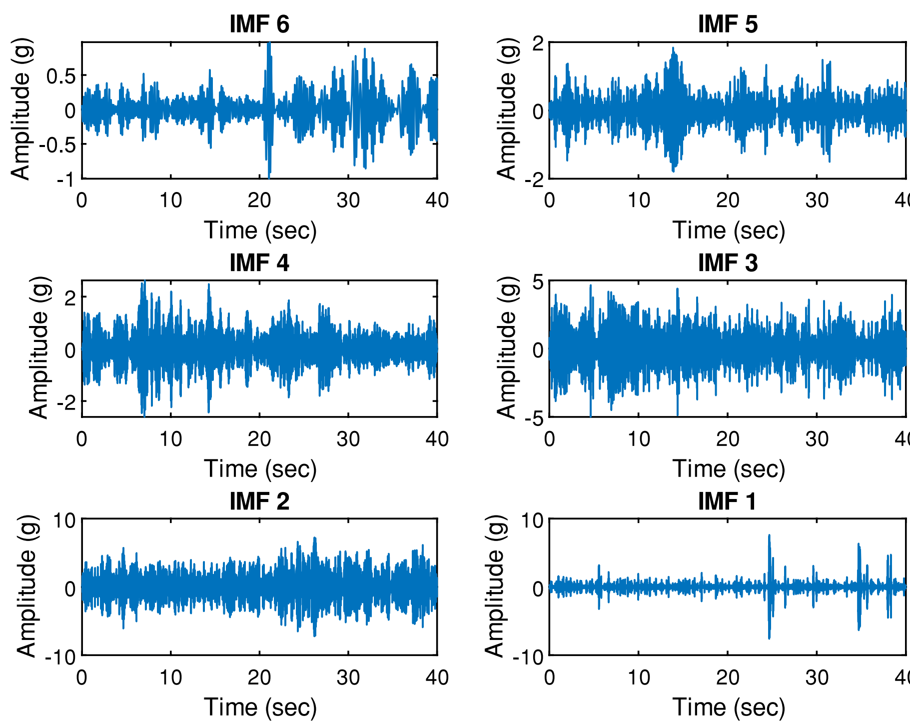


Fig. 9 IMFs from EMD in sensor 16 (*y*-direction)

4 Conclusions

In this work, the effectiveness of synchronized wavelet transform in identifying natural frequencies and damage of the Benchmark structure was evaluated. In addition, a methodology combining the properties of empirical mode decomposition, synchronized wavelet transform, and spline interpolation was proposed. The obtained values were compared with the reference model, finite element [22], and other methodologies applied to the identification of the modal properties of the Benchmark structure.

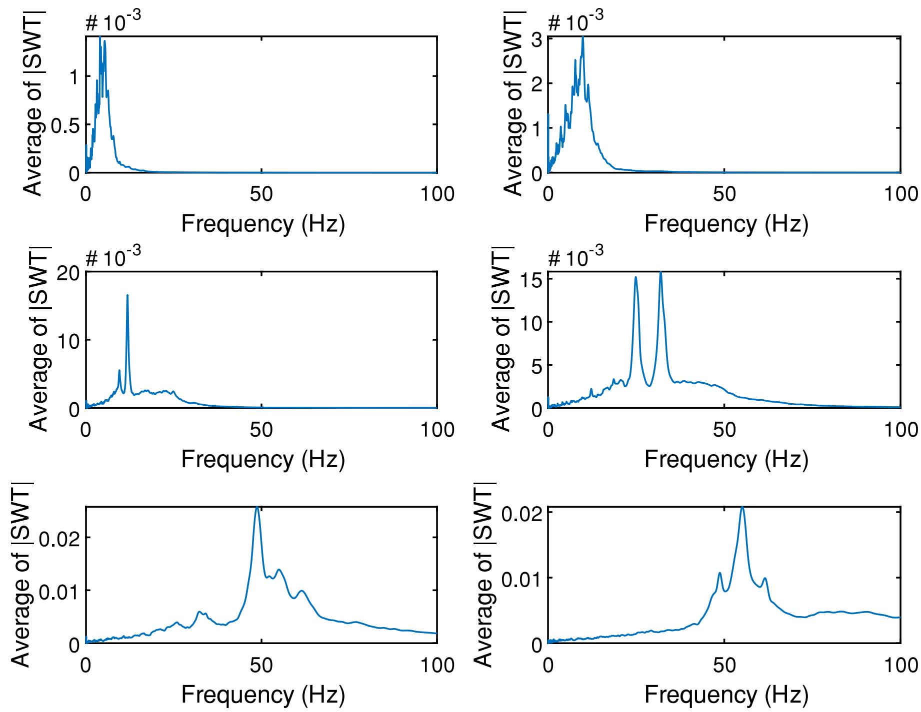


Fig. 10 Identification of frequencies in the sensor 15 (x-direction)

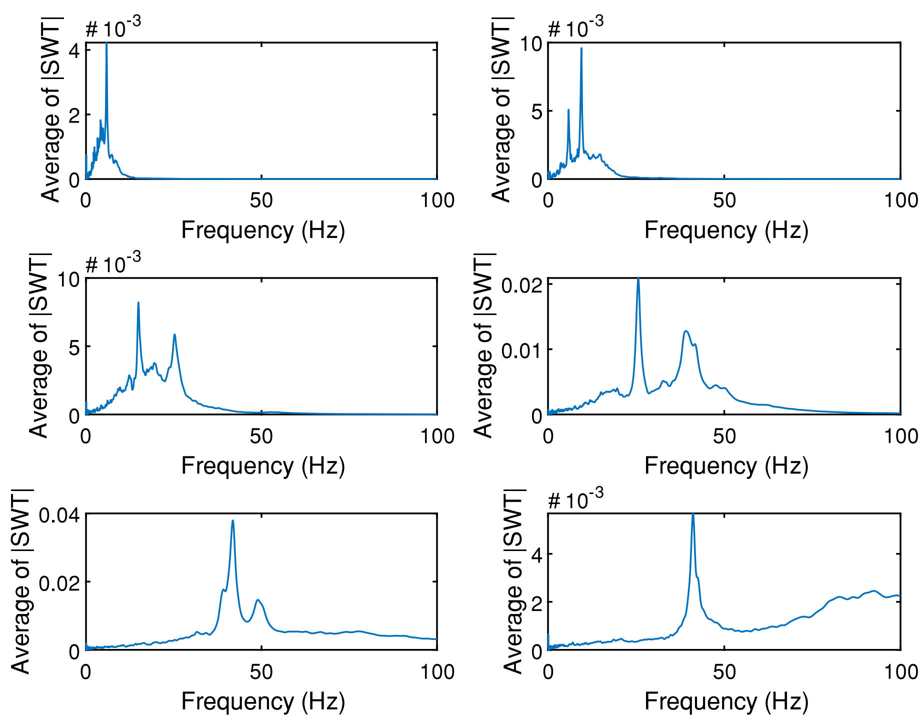


Fig. 11 Identification of frequencies in the sensor 16 (y-direction)

Synchronized wavelet transform proved to be a robust methodology in damage identification, given its property of working in the time–frequency domain. It was possible to identify the exact time of damage, that is, where there were changes in modal properties (natural frequencies) due to damage to the physical properties of the structure, for example, loss of stiffness. The authors of this work recently implemented synchronized wavelet transform using the analytic Morlet wavelet as the mother wavelet and identified three frequencies in both directions (x, y) [13]. Moreover, the minimum error was 0.12% and the maximum error was 3.06% compared to the finite element [22]. The analytic bump wavelet was implemented as the mother wavelet, and all frequencies of the structure were identified. The

Table 3 Undamaged Benchmark structure (Case 3, 12 DOF, lumped mass)

Mode	Natural frequencies (Hz) (% Error)				
	FEA [22]	Complete signal (t=0–40 s)		Signal segment (t=0–20 s)	
		SWT	Methodology	SWT	Methodology
1	9.41 (y)	9.45 (0.43)	9.47 (0.64)	9.42 (0.11)	9.46 (0.53)
2	11.79 (x)	11.81 (0.17)	11.82(0.25)	11.85 (0.51)	11.79 (0.00)
3	25.54 (y)	25.61 (0.27)	25.61 (0.27)	25.55 (0.04)	25.54 (0.00)
4	32.01 (x)	32.09 (0.25)	31.98 (0.09)	32.14 (0.41)	31.93 (0.25)
5	38.66 (y)	39.08 (1.09)	39.15 (1.27)	38.96 (0.78)	38.98 (0.83)
6	48.01 (y)	49.04 (2.15)	48.92 (1.90)	49.19 (2.46)	48.41 (0.83)
7	48.44(x)	48.81 (0.76)	48.73 (0.60)	48.74 (0.62)	48.49 (0.10)
8	60.15 (x)	61.68 (2.54)	61.46 (2.18)	61.58 (2.38)	61.54 (2.31)

Table 4 Damage pattern (ii) on Benchmark structure (Case 3, 12 DOF, lumped mass)

Mode	Natural frequencies (Hz) (% Error)				
	FEA [22]	Complete signal (t=0–40 s)		Signal segment (t=20–40 s)	
		SWT	Methodology	SWT	Methodology
1	5.82 (y)	5.82 (0.00)	5.84 (0.34)	5.84 (0.34)	5.84 (0.34)
2	9.51(x)	9.51 (0.00)	9.50 (0.11)	9.55 (0.42)	9.53 (0.21)
3	14.89 (y)	14.97 (0.54)	14.95 (0.40)	14.91 (0.13)	14.91 (0.13)
4	24.91 (x)	25.08 (0.68)	24.90 (0.04)	25.06 (0.60)	25.10 (0.76)
5	36.06 (y)	- (-)	- (-)	35.47 (1.64)	34.93 (3.13)
6	41.35 (y)	41.74 (0.94)	41.14 (0.51)	41.69 (0.82)	41.22 (0.31)
7	46.79 (x)	- (-)	- (-)	46.88 (0.19)	46.22 (1.22)
8	54.34 (x)	55.03 (1.27)	55.12 (1.44)	54.99 (1.20)	54.91 (1.05)

minimum error was 0.0% and the maximum error was 2.31% compared to the finite element [22]. Thus, the effectiveness of synchronized wavelet transform as a structural health monitoring technique is ratified, and the superiority of analytic bump wavelet over Morlet in the identification of modal properties is demonstrated.

Table 5 Undamaged Benchmark structure (Case 3, 12 DOF, lumped mass)—Identification of natural frequencies using methodologies based on SWT

Mode	Natural frequencies (Hz) (% Error)			
	FEA [22]	Perez-Ramirez et al. [21]	Perez-Ramirez et al. [20]	Methodology
1	9.41 (y)	9.407 (0.03)	9.39 (0.21)	9.46 (0.53)
2	11.79 (x)	11.82 (0.20)	11.72 (0.59)	11.79 (0.00)
3	25.54 (y)	25.55 (0.03)	25.59 (0.20)	25.54 (0.00)
4	32.01 (x)	31.95 (0.20)	32.03 (0.06)	31.93 (0.25)
5	38.66 (y)	38.61 (0.13)	38.67 (0.03)	38.98 (0.83)
6	48.01 (y)	47.97 (0.08)	48.05 (0.08)	48.41 (0.83)
7	48.44 (x)	48.50 (0.10)	48.65 (0.44)	48.49 (0.10)
8	60.15 (x)	60.35 (0.33)	60.16 (0.02)	61.54 (2.31)

The proposed methodology also proved to be effective in identifying the frequency content of a signal affected by a damage pattern (ii) and immersed in noise (10%). It was possible to identify the frequency content of the signal, and found a minimum and maximum error of 0.13% and 3.13%, respectively, compared to the finite element [22]. According to the authors' knowledge, no studies have been found that have implemented their methodologies on the Benchmark structure for damage pattern (ii) and case 3 (roof excitation). Regarding the frequencies content of the undamaged and noisy signal, it is possible to compare the results with other authors. Perez-Ramirez et al. [21] found a minimum error of 0.03% and a maximum of 0.33%, while Perez-Ramirez et al. [20] reported a minimum and maximum error of 0.02% and 0.59%, respectively, compared to the finite element [22]. In this work, the minimum error was 0.0% and the maximum error was 2.31% compared to the finite element [22]. Although the values obtained by other authors are excellent, none reported a 100% accuracy in the identification of natural frequencies. Therefore, it can be concluded that the proposed methodology is robust as a structural health monitoring method.

Other methodologies implemented in the Benchmark structure, but which do not consider synchronized wavelet transform within their procedures were presented. These authors identified the frequencies contained in the undamaged signal. Amini and Hedayati [33] identified three of the eight frequencies contained in the signal, with a minimum error of 0.08% and a maximum of 12.86% compared to the finite element [22]. Li, Xu, and Zhang [34] identified all frequencies and reported a minimum error of 3.86% and maximum of 23.41%, compared to the finite element [22]. Cara et al. [35] identified six of the eight frequencies, with a minimum error of 0.04% and a maximum error of 3.81 compared to the finite element [22]. In this work, the values found presented a minimum and maximum error of 0.0% of

Table 6 Undamaged Benchmark structure (Case 3, 12 DOF, lumped mass)—Identification of natural frequencies using another SHM method

Mode	Natural frequencies (Hz) (% Error)				
	FEA [22]	Amini and Hedayati [33]	Li, Xu, and Zhang [34]	Cara et al. [35]	Methodology
1	9.41 (*y*)	10.62 (12.86)	8.48 (9.88)	9.40 (0.11)	9.46 (0.53)
2	11.79 (*x*)	–	9.03 (23.41)	11.78 (0.08)	11.79 (0.00)
3	25.54 (*y*)	25.39 (0.59)	23.07 (9.67)	–	25.54 (0.00)
4	32.01 (*x*)	–	25.45 (20.49)	–	31.93 (0.25)
5	38.66 (*y*)	–	36.32 (6.05)	37.95 (1.84)	38.98 (0.83)
6	48.01 (*y*)	47.97 (0.08)	41.81 (12.91)	49.84 (3.81)	48.41 (0.83)
7	48.44 (*x*)	–	46.57 (3.86)	48.42 (0.04)	48.49 (0.10)
8	60.15 (*x*)	–	56.09 (6.75)	59.86 (0.48)	2.31)

2.31%, respectively. Therefore, it can be said that the methodology based on the synchronized wavelet transform was superior in the identification of modal properties compared to the three methodologies presented in this work.

The objective of this research was to approach a real study through the Benchmark structure subjected to environmental vibration. A signal was constructed representing two reading phases of a structure, a first reading of the undamaged structure and then a second reading of the damaged structure. The proposed methodology proved to be robust in the identifying modal properties from non-stationary and noise-embedded signals. In addition, the proposed methodology can be applied to any signal since it does not require a priori knowledge of the structure.

Regarding the location and number of sensors used in this work, the following considerations must be taken into account: it is proposed for future work to perform other simulations using sensors in other positions to verify the robustness of the proposed methodology, and also to vary the number of sensors in order to locate the damage in the structure. It is important to emphasize that this work was focused on the determination if there is damage in the structure and not in determining the location of the damage. Therefore, a hybrid approach, a combination of methods, will be proposed for future work to locate and quantify damage in a structure.

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Declarations

Conflict of interest The authors declare that there is no conflict of interest between them.

References

1. Sánchez WED, Avila SM, de Brito JLV (2018) Optimal placement of damping devices in buildings. J Brazilian Soc Mech Sci Eng. https://doi.org/10.1007/s40430-018-1238-x
2. American Society of Civil Engineers (2021) A comprehensive assessment of America's Infrastructure. Asce 111
3. Pereira JLJ, Francisco MB, da Cunha SS, Gomes GF (2021) A powerful Lichtenberg Optimization Algorithm: a damage identification case study. Eng Appl Artif Intell. https://doi.org/10.1016/j.engappai.2020.104055
4. Pereira JLJ, Chuman M, Cunha SS, Gomes GF (2021) Lichtenberg optimization algorithm applied to crack tip identification in thin plate-like structures. Eng Comput 38:151–166. https://doi.org/10.1108/EC-12-2019-0564/FULL/HTML
5. Gomes GF, Mendez YAD, da Silva Lopes Alexandrino P, da Cunha SS, Ancelotti AC (2019) A review of vibration based inverse methods for damage detection and identification in mechanical structures using optimization algorithms and ANN. Arch Comput Methods Eng 26:883–897. https://doi.org/10.1007/s11831-018-9273-4
6. Doebling SW, Farrar CR, Prime MB, Shevitz DW (1996) Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: a literature review
7. Hwang E, Kim G, Choe G, Yoon M, Gucunski N, Nam J (2018) Evaluation of concrete degradation depending on heating conditions by ultrasonic pulse velocity. Constr Build Mater 171:511–520
8. Chang PC, Liu SC (2003) Recent research in nondestructive evaluation of civil infrastructures. J Mater Civ Eng 15:298–304
9. Xu YL, He J (2017) Smart civil structures. CRC Press, Taylor & Francis Group, 6000 Broken Sound Parkway NW, Suite 300, Boca Raton, FL 33487-2742
10. Daubechies I (1993) Ten Lectures on Wavelets. J Acoust Soc Am 93:1671–1671. https://doi.org/10.1121/1.406784
11. Yang JN, Lei Y, Lin S, Huang N (2004) Hilbert-Huang based approach for structural damage detection. J Eng Mech 130:85–95
12. Goncalves P, Rilling G, Flandrin P (2003) On empirical mode decomposition and its algorithms. IEEE-EURASIP Work Nonlinear Signal Image Process 3:8–11
13. Sanchez WD, De Brito JV, Avila SM (2020) Structural health monitoring using synchrosqueezed wavelet transform on

IASC-ASCE benchmark phase i. Int J Struct Stab Dyn. https://doi.org/10.1142/S0219455420501382
14. Babajanian Bisheh H, Ghodrati Amiri G, Darvishan E (2020) Ensemble classifiers and feature-based methods for structural damage assessment. Shock Vib. https://doi.org/10.1155/2020/8899487
15. Liu J, Chen B, Chen S, Berges M, Bielak J, Noh H (2020) Damage-sensitive and domain-invariant feature extraction for vehicle-vibration-based bridge health monitoring. In: ICASSP, IEEE international conference on acoustics, speech and signal processing - proceedings. pp 3007–3011
16. Rafiei MH, Adeli H (2017) A novel machine learning-based algorithm to detect damage in high-rise building structures. Struct Des Tall Spec Build 26:1400. https://doi.org/10.1002/tal.1400
17. Amezquita-Sanchez JP, Adeli H (2015) Synchrosqueezed wavelet transform-fractality model for locating, detecting, and quantifying damage in smart highrise building structures. Smart Mater Struct 24:065034. https://doi.org/10.1088/0964-1726/24/6/065034
18. Rafiei MH, Adeli H (2018) A novel unsupervised deep learning model for global and local health condition assessment of structures. Eng Struct 156:598–607. https://doi.org/10.1016/j.engstruct.2017.10.070
19. Li Z, Park HS, Adeli H (2017) New method for modal identification of super high-rise building structures using discretized synchrosqueezed wavelet and Hilbert transforms. Struct Des Tall Spec Build. https://doi.org/10.1002/tal.1312
20. Perez-Ramirez C, Jaen-Cuellar A, Valtierra-Rodriguez M, Dominguez-Gonzalez A, Osornio-Rios R, Romero-Troncoso R, Amezquita-Sanchez J (2017) A two-step strategy for system identification of civil structures for structural health monitoring using wavelet transform and genetic algorithms. Appl Sci 7:111. https://doi.org/10.3390/app7020111
21. Perez-Ramirez CA, Romero-Troncoso RJ, Valtierra-Rodriguez M, Camarena-Martinez D, Adeli H, Amezquita-Sanchez JP (2015) New methodology for modal parameters identification of smart civil structures using ambient vibrations and synchrosqueezed wavelet transform. Eng Appl Artif Intell 48:1–12. https://doi.org/10.1016/j.engappai.2015.10.005
22. Johnson EA, Lam HF, Katafygiotis LS, Beck JL (2004) Phase I IASC-ASCE structural health monitoring benchmark problem using simulated data. J Eng Mech 130:3–15. https://doi.org/10.1061/(ASCE)0733-9399(2004)130:1(3)
23. Rytter A (1993) Vibrational based inspection of civil engineering structures
24. Shrivastava Y, Singh B (2018) Estimation of stable cutting zone in turning based on empirical mode decomposition and statistical approach. J Brazilian Soc Mech Sci Eng. https://doi.org/10.1007/s40430-018-0989-8
25. Wu F, Qu L (2008) An improved method for restraining the end effect in empirical mode decomposition and its applications to the fault diagnosis of large rotating machinery. J Sound Vib 314:586–602. https://doi.org/10.1016/j.jsv.2008.01.020
26. Huang NE, Shen Z, Long SR, Wu MC, Shih HH, Zheng Q, Yen N-C, Tung CC, Liu HH (1998) The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. Proc R Soc London Ser A Math Phys Eng Sci 454:903–995. https://doi.org/10.1098/rspa.1998.0193
27. Thakur G, Brevdo E, Fučkar NS, Wu HT (2013) The Synchrosqueezing algorithm for time-varying spectral analysis: robustness properties and new paleoclimate applications. Signal Process 93:1079–1094. https://doi.org/10.1016/j.sigpro.2012.11.029
28. Daubechies I, Lu J, Wu HT (2011) Synchrosqueezed wavelet transforms: an empirical mode decomposition-like tool. Appl Comput Harmon Anal 30:243–261. https://doi.org/10.1016/j.acha.2010.08.002
29. Daubechies I, Maes SH (1996) A nonlinear squeezing of the continuous wavelet transform based on auditory nerve models
30. Sharma A, Amarnath M, Kankar PK (2017) Novel ensemble techniques for classification of rolling element bearing faults. J Brazilian Soc Mech Sci Eng 39:709–724. https://doi.org/10.1007/s40430-016-0540-8
31. Jiang Q, Suter BW (2017) Instantaneous frequency estimation based on synchrosqueezing wavelet transform. Signal Process 138:167–181. https://doi.org/10.1016/j.sigpro.2017.03.007
32. Meignen S, Oberlin T, McLaughlin S (2012) A new algorithm for multicomponent signals analysis based on synchrosqueezing: with an application to signal sampling and denoising. IEEE Trans Signal Process 60:5787–5798. https://doi.org/10.1109/TSP.2012.2212891
33. Amini F, Hedayati Y (2016) Underdetermined blind modal identification of structures by earthquake and ambient vibration measurements via sparse component analysis. J Sound Vib 366:117–132. https://doi.org/10.1016/j.jsv.2015.10.028
34. Li PJ, Xu DW, Zhang J (2016) Probability-based structural health monitoring through Markov chain Monte Carlo sampling. Int J Struct Stab Dyn 16:1550039. https://doi.org/10.1142/S021945541550039X
35. Cara FJ, Carpio J, Juan J, Alarcón E (2012) An approach to operational modal analysis using the expectation maximization algorithm. Mech Syst Signal Process 31:109–129. https://doi.org/10.1016/j.ymssp.2012.04.004
36. Das S, Saha P (2020) Performance of hybrid decomposition algorithm under heavy noise condition for health monitoring of structure. J Civ Struct Heal Monit 10:679–692. https://doi.org/10.1007/s13349-020-00412-5