Journal of the Brazilian Society of Mechanical Sciences and Engineering (2020) 42:453
https://doi.org/10.1007/s40430-020-02513-0

TECHNICAL PAPER

Formulating the optimum parameters of modified hanger system in the cable-arch bridge to restrain force fluctuation and overstressing problems

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Received: 4 October 2019 / Accepted: 16 July 2020 / Published online: 7 August 2020

Abstract

Inclined hangers behave better than vertical ones under lateral loads (i.e., wind or earthquake) despite they are prone to fatigue phenomenon. In some instances, the slackness problem is seen in some inclined hangers, while others may become overstressed. By considering the modified hanger system, the disadvantages of vertical and inclined systems are resolved, while keeping their advantages. The objective of this study is to propose formulations for the optimum application of novel arrangement of hangers in a cable-arch bridge to suppress problems such as overstressing, force fluctuation, and decreasing the probability of fatigue phenomenon in hangers. To define optimum parameters, the modified hanger system was analyzed and compared with vertical and inclined ones considering nonlinear static analysis under dead load as an initial state plus seismic excitation and dynamic impact load of vehicles. Results indicate that the modified hanger system is improved remarkably in comparison with the inclined and vertical hanger systems.

Keywords Modified hanger system · Inclined hanger system · Cable arrangements · Cable-stayed arch bridge · Nonlinear dynamic analysis

1 Introduction

To improve the vibration characteristics induced by dynamic loads, the cable systems comprising hangers or main cables may increase the stiffness of the cable or arch bridges. Inclined or vertical hanger systems are the applicable forms in the arch or suspension bridges in which forces are transferred from the deck to the arch by these systems. Due to the damping role of the inclined hangers under dynamic loads, their performance is better than vertical hangers. Nevertheless, due to early fatigue or excessive tensile forces in hangers of several bridges, inclined hanger systems require a modification to achieve optimality.

The probability of the fatigue phenomena in inclined hangers is high. Due to loads on bridges with inclined hangers, two adjacent cables may be subjected to tension and compression, respectively. By changing the load direction, the compression in one cable changes into tension and vice versa. Therefore, fatigue phenomenon may happen under repeated load changing. A typical example can be mentioned for the Severn suspension bridge in which some inclined hangers were replaced within the first 10 years of its lifetime [1]. Also, remarkable signs of distress were seen in forces of several inclined hangers, while in some others, slackness problem was observed because of their position against loads. The expected types of dynamic load such as earthquake excitations and vehicles' impact load, and also the traffic density have significant influences on the dynamic performance of bridges. Accordingly, severe earthquakes or dense vehicle traffic may cause several problems such as fluctuation in force of hangers, overstressing, and undesirable dynamic vibrations. To restrict the structural vibrations to desirable limits, several modifications may be applied to the existing structures that can decrease the internal stresses and corresponding forces. In this way, the damping role is achieved, excessive tensile stresses and force fluctuation are decreased and probable slackness problem is removed [2]. For this reason, modifications on inclined hanger systems were proposed to achieve a system with better dynamic

Technical Editor: Thiago Ritto.

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performance. For the first time, Barghian and Moghadasi [3] proposed the modified hanger system just for a case study of a suspension pedestrian bridge named Soti Ghat. Likewise, Faridani and Barghian [4], and Farahmand-Tabar and Barghian [5] studied the dynamic behavior of bridges with the modified hanger system. Similar to pedestrian suspension bridges, the modification of inclined hanger systems in a highway bridge such as cable-stayed arch bridge indicated the improvement of the dynamic properties and reduced several aforementioned disadvantages. There may be several changes in frequencies and natural modes to reduce the probability of intense vibrations in the cable-stayed arch bridge under seismic or dynamic vehicle load.

To improve the dynamic behavior of hanger system in bridges, most investigations are related to cable-stayed and suspension bridges. These investigations are classified by some factors such as type of hangers, vibration characteristics and comfort criteria, slackness problem, and fatigue phenomenon. The Millennium bridge in London is an instance of a bridge suffered from large horizontal vibrations due to pedestrians' footfall that finally necessitated the closure of the bridge to the public [6–8]. Some bridges such as a steel pedestrian bridge in Forchheim, Germany, are sensitive to vertical vibrations due to vertical loads such as running pedestrians [9]. To investigate the vertical footfall forces, Kerr and Bishop [10] measured more than 1000 individual time-histories of footfall force from 40 subjects walking with the frequencies between 1 and 3 Hz on a surface. According to the observations, it was concluded that the comfortable walking pace was in the range of 1.7–2.2 Hz with an approximate mean value of 1.9 Hz. A realistic load model proposed by Figueiredo et al. [11] was developed to unify the pedestrian induced effects in the dynamic response of footbridges. The results indicated that this bridge type could achieve high levels of vibration that could accommodate safety and comfort for the pedestrian bridge users in particular. Several field measurements on the M-bridge in Japan showed that not only the pedestrians can excite the fundamental frequency, but also their distribution can excite the higher-order modes of vibration [12]. The highway steel arch bridge, King's Mindaugas—in Lithuania—was probable to experience severe loading of the crowd. Soon after completion, the bridge testing indicated its impermissible dynamic characteristics. The improvement was obtained through the addition of inclined suspenders. The higher vertical fundamental frequency of 2.5 Hz was achieved after the modification [13]. The defects and disadvantages of inclined hangers in suspension bridges have been investigated by some researchers, especially the Bosphorus Bridge in Turkey and the Humber and Severn bridges in England, which all are highway bridges with inclined hangers. The early fatigue and fracture of hangers were noticed in these bridges [1, 14]. In the case of inclined hangers' slackness problem, the influence of inclined cables slackness on nonlinear parametric vibrations, subjected to periodic stimulations of their support, was studied by Wu et al. [15, 16].

The hangers are especially susceptible to fatigue phenomena, but the axial force variation in hangers has not been investigated deeply, although it is a crucial parameter relating to the hangers' fatigue behavior. For this purpose, Pellegrino et al. [17] considered different arrangements of hangers in a tied arch bridge such as fan, network, and vertical arrangement with inclined hangers. Their work was focused on a slight modification of hangers and reducing the fatigue stressing of those hangers. The arch bridges are dealt with the stability problems [18, 19] which are sometimes caused by fatigue phenomenon in hangers, especially where the rise to span ratio of the arch is high and the arch anchored with cables for its stability.

Studničková investigated the influence of pedestrian load on the dynamic performance of footbridges in a report and presented the range of the frequencies in horizontal and vertical directions for the dynamic response from moving pedestrians that could be important [20]. The maximum accelerations of the bridge subjected to dynamic loading were proposed by Bachmann [21]. In a study, Huang et al. have suggested a cable model with reverse-profiled and pre-tensioned cables to accomplish theoretical research on the shallow suspension footbridges' vibration characteristics [22, 23]. Results from the numerical analysis indicate that for cable-stayed bridges with little sag, the lowest frequencies related to torsional and lateral modes of vibration are always mixed. Brownjohn et al. investigated the vibration characteristics of suspension footbridges with the superstructure composed of large-sagged suspending cables, a structural system of cross-bracing, and towers [24, 25]. The stiffness of the structure, in mentioned systems, depends on the cross-bracing structure and suspended cables. Hauksson [26] studied parametrically the dynamic performance of Millennium pedestrian bridges under vibrations induced by pedestrians. A comprehensive literature review about the vibration serviceability of pedestrian bridges was subjected to human-induced excitation and investigation on the extension of the model from a single person to multi-harmonic approach, and different load patterns were done in several researches [27–30].

The construction of a relatively slender and long footbridge in Portugal, that was liable to lateral and vertical vibrations induced by pedestrians, motivated the complete evaluation of the respective dynamic performance with the aim of installing a control system [31]. Yau studied the vibration of the arch bridge under the combination of vertical ground excitations and train moving loads [32]. Cable-stayed bridge's dynamic responses under vehicle loads including the influences of the local vibration of cables were studied by Zhang and Xie [33]. Kong et al. offered a new procedure of sub-structure method to long-span hybrid cable-stayed model bridges subjected to the vibrations due to vehicles [34]. A numerical investigation on the nonlinear vibration of inclined cables coupled with the deck in cable-stayed bridges was done by Kang et al. [35]. It was indicated that the periodic variation of the cable tension induced by the deck vibration would cause the parametric resonance of the stay cable under certain tuning conditions. The estimation of the ambient vibration-based cable tension in a case study was investigated by Kangas et al. [36]. Some experiments were accomplished to determine the application viability of the traditional vibration techniques to evaluate the cable tension. An experimental study was done by Yang et al. regarding the vibration characteristics of FRP Cables for long-span bridges with stay cables [37]. A simplified method was offered by Cheng and Dong using an artificial neural network (ANN) for free vibration analysis of cable-stayed bridges [38]. The suggested method was especially beneficial for the preliminary design stage of cable-stayed bridges where the analysis of free vibration needed to be accomplished. Papadopoulos et al. [39] developed a simple computer program to investigate the hysteresis damping influence of cross-ties on the vibration of cables in cable-stayed bridges. Torra et al. [40] investigated a solution for the oscillation mitigation of stay cables in bridges using SMA dampers for cable vibration. Results from experiments establish the vibration amplitude reduction to one half or less than the one achieved in the un-damped case and the increase in the cable frequency.

In this study, after illustrating the advantages of utilizing the modified hanger system over inclined and vertical ones in a highway bridge with hybrid system of arch and stay cables, the parameters of modified hangers were analyzed and investigated to achieve the better performance for the system and the bridge. The desired criteria for the system are the reduction in force fluctuation and overstressing of hangers, while satisfying the permissible geometrical state of them. Finally, the modification of hangers was generalized and formulated to achieve optimum parameters.

2 Numerical model

The Liancheng Bridge model—which is a hybrid system of arch bridge and two-pylon cable-stayed bridge—is shown in Fig. 1a, b. The length of the main and side span of the bridge is 400 m and 120 m, respectively. The bridge has 27 m width. According to Fig. 1c, both arch ribs of the bridge have a rectangular cross section constructed by six concrete-filled steel tubes (CFST). The outer diameter of the steel tubes is 850 mm, while the thickness alters from 20 to 28 mm regarding the position of the arch ribs. Two parallel main arch ribs are stabilized by 11 wind bracings: two K-shaped wind bracings and two connecting beams under the deck; also, six K-shaped wind bracings and one Ж-shaped wind bracing above the deck. The connecting beams and other wind bracings are consisting of the steel rectangle girders and steel pipes, respectively. The floor system of the main span is composed of the deck, I-shaped transverse girders, and longitudinal stringers, and the whole system is supported by 39 hangers in two rows with 8 m intervals. The stay cables are anchored to the bridge deck and arch ribs with the intervals of 10 and 8 m, respectively.

According to Fig. 2, a 3D finite element model of the whole bridge was created by a finite element software. Beam-column elements were used to model the arch and towers, while the deck, bracings, and cables were modeled by beam, truss, and cable elements, respectively. The used cable element is the tension-only bar element without the flexural/torsional stiffness. The effects of the prestressing and cable sag have been applied in the modeling. In the model, both ends of the main arch ribs, main towers, and arch ribs of the side spans were fixed in all degrees of freedom. The translational movements of the ends of the two side spans were also fixed in all three directions, whereas the rotation along the lateral axis was allowed. The basic properties regarding the main structural components of the bridge, comprising the modulus of elasticity, E, and

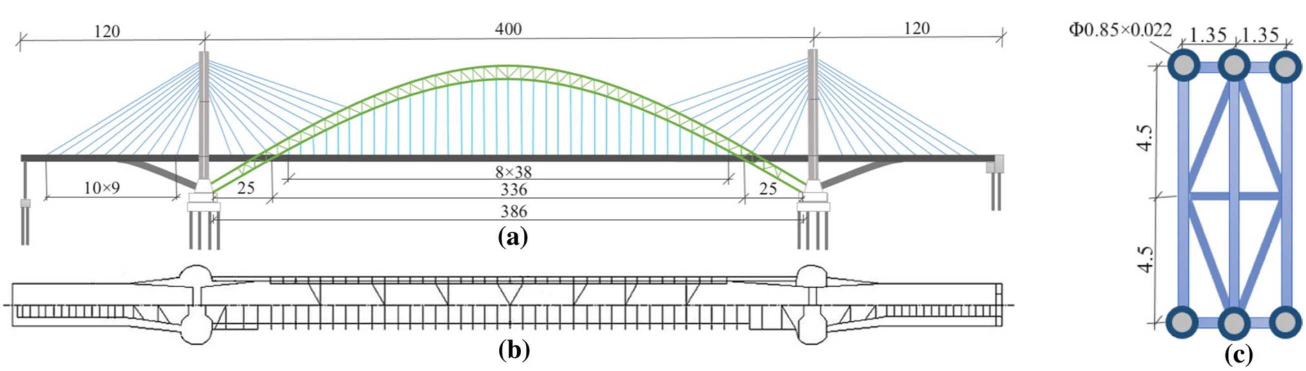

Fig. 1 General view of the LianCheng bridge (values in meter): **a** elevation **b** plan view **c** cross section of main arch rib

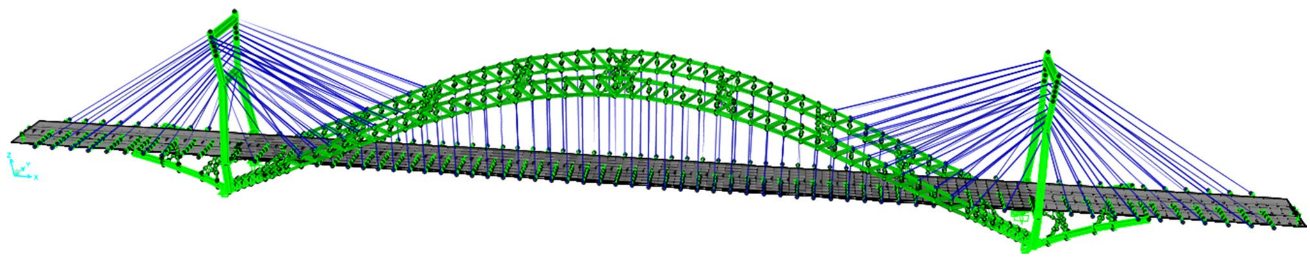

Fig. 2 FE model of the cable-stayed arch bridge

cross-sectional area, A, and the element types are presented in Table 1. Total mass of the structure is 33,313.37 kN by considering the mass per unit volume (m^3) of concrete and steel as 2.5 kN and 7.85 kN, respectively. The assumed damping is 5%, and the applied analysis is modal time-history analysis entitled fast nonlinear analysis (FNA). In FNA, the mass- and stiffness-orthogonal load-dependent Ritz vectors represent the structural equilibrium relationships according to Eq. 1. The uncoupled modal equations are solved exactly at each time increment, while using an iterative process, the forces within the nonlinear DOF ($R_{NL}(t)$) are resolved [41]. It should be mentioned that the time-history analysis is carried out considering the initial state of structural dead load.

$$\mathrm{M}\ddot{\mathrm{u}}(t) + \mathrm{C}\dot{\mathrm{u}}(t) + \mathrm{K}\mathrm{u}(t) + R_{\mathrm{NL}}(t) = R(t) \tag{1}$$

First ten mode shapes—achieved from the bridge model—as illustrated in Fig. 3 were the same as the available data given by Luo et al. [42] which demonstrate the reliability of the developed model. In this research, after considering vertical hangers, they are replaced by inclined ones in the investigated model to study the effect of hanger systems on dynamic performance of the cable-arch bridge subjected to seismic and vehicle loads (Figs. 4 and 5). The number and the cross-sectional diameter of inclined and vertical hangers are identical in both bridges.

2.1 Cable-stayed arch bridge with modified hanger system

In a cable-stayed arch bridge with inclined hangers, several hangers may be subjected to overstressing, and force fluctuation (induced by applied dynamic loads). Nonetheless, to eliminate the mentioned problems of inclined hangers, a modified model is considered to resolve the defects of inclined hangers. The parameters of this modification are studied to achieve the optimality of the hanger system. The model consists of inclined hangers so that the load distribution, among two adjacent hangers, is carried out through a member parallel to the deck. The tensile load is transferred from the overstressed hanger to the adjacent hanger with a lower force by added horizontal member. Moreover, the modification of hangers improves the dynamic properties of the cable-stayed arch bridge. The modified hanger system and its application in the bridge are illustrated in Figs. 6 and 7.

3 Load models

Generally, bridges have four main kinds of vibration modes: vertical, lateral, longitudinal, and torsional modes. Nonetheless, numerical results indicate that the torsional and lateral modes do not always appear as pure torsional or lateral modes of vibration. Most vertical modes of

Table 1 Structural properties of the bridge components [43]

Content	A (m^2)	E (N/m^2)	Material	Element type
Main arch rib (steel tube only)	1.430×10^{-3} (upper rib)	2.1×10^{11}	Steel	Beam-column
	1.304×10^{-3} (lower rib)	2.1×10^{11}	Steel	
Bracing	4.128×10^{-3}	2.1×10^{11}	Steel	Bar
Hanger	2.117×10^{-3}	2.05×10^{11}	Steel	Cable
Cross-girder	8.000×10^{-1}	2.1×10^{11}	Steel	Bar
Stay cable	4.340×10^{-3}	1.95×10^{11}	Steel	Cable
Main tower	1.143×10^{1} (upper tower)	3.5×10^{10}	Concrete	Beam-column
	1.074×10^{1} (middle tower)	3.5×10^{10}	Concrete	
	1.304×10^{1} (lower tower)	3.5×10^{10}	Concrete	
Arch rib of side span	8.800×10^{0}	3.5×10^{10}	Concrete	Beam-column

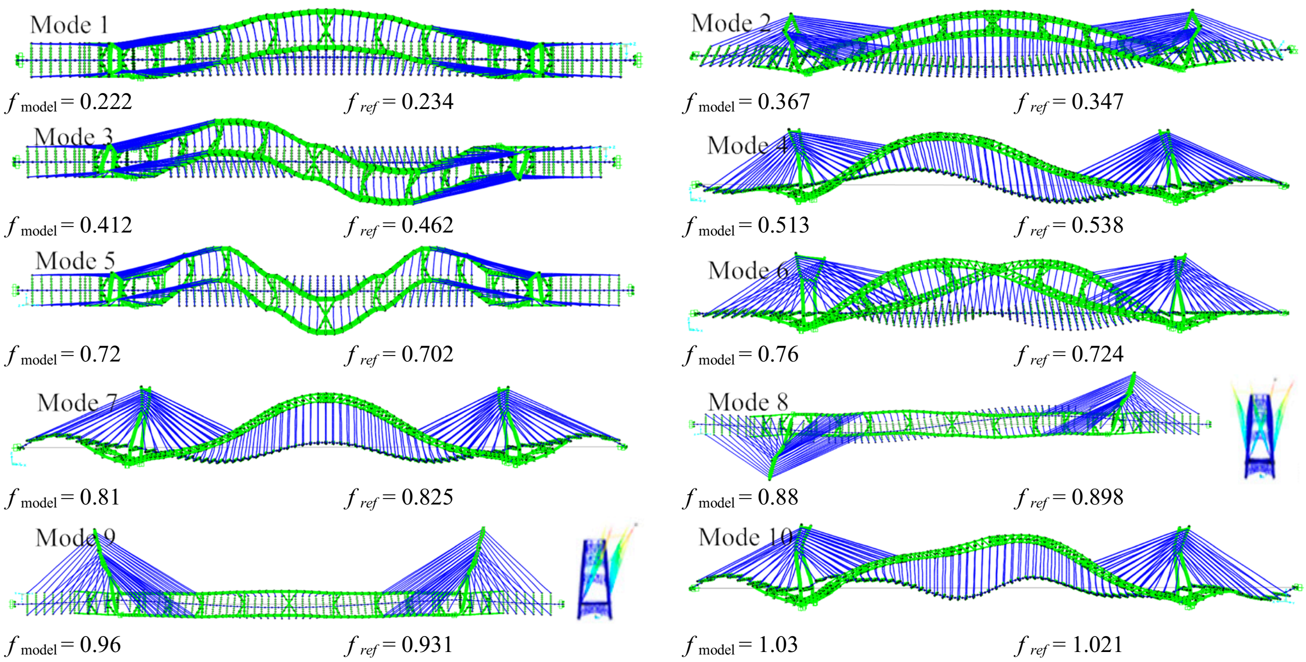

Fig. 3 First 10 mode shapes of the cable-stayed arch bridge

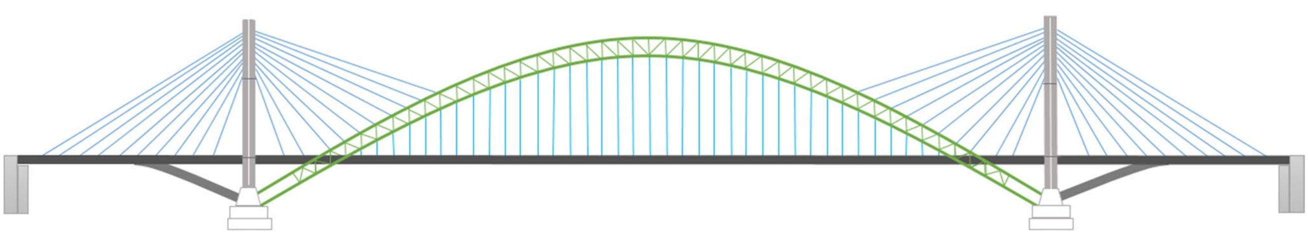

Fig. 4 The cable-stayed-arch bridge with vertical hangers

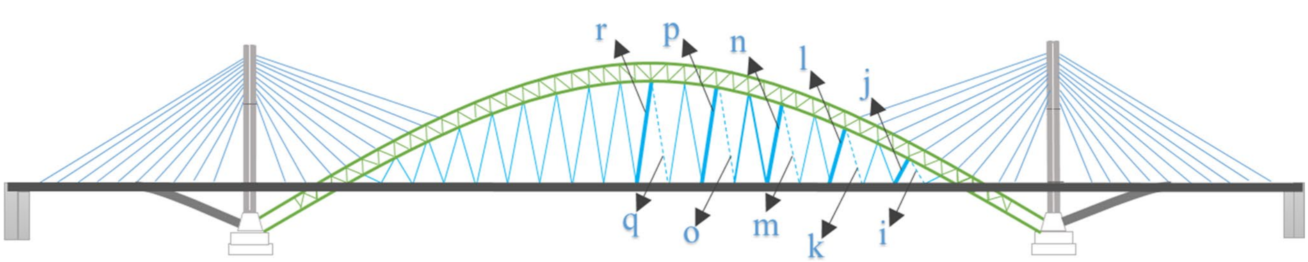

Fig. 5 The cable-stayed-arch bridge with inclined hangers with typical locations of overstressed hangers

vibration appear as pure vertical modes, without correspondence to torsional or lateral ones. Nevertheless, it is possible to consider coupled torsional-lateral or lateral-torsional modes for harmonic loads. Accordingly, to apply loads in the case of coupled modes, the torsional and lateral loads have been positioned concurrently on the surface of bridges to the shape of torsional and lateral signs of coupled modes [23].

3.1 Earthquake excitation

To obtain the vibration properties of the hangers, a typical earthquake excitation was used. For this purpose, the El Centro ground motion with the epicentral distance of 15–45 km and the soil type of D was chosen. The horizontal and vertical components of the selected earthquake (Fig. 8) are matched to the design response spectrum according

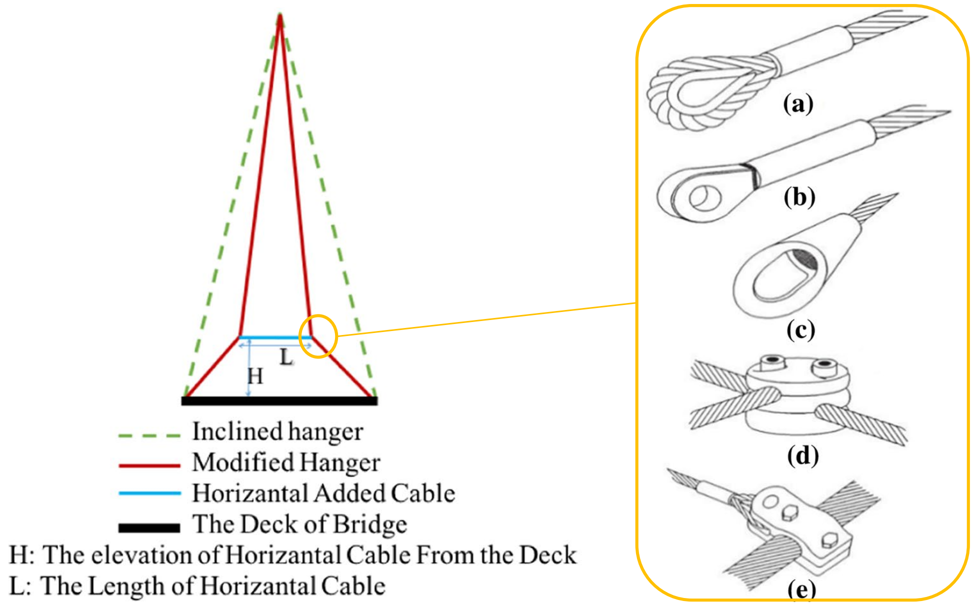

Fig. 6 The proposed model of the modified hanger (Detail A in Fig. 7) and the possible cable fittings for connections: **a** swaged eye, **b** swaged terminator, **c** hot-poured white metal eye, **d** cross clamp, **e** net and boundary connection

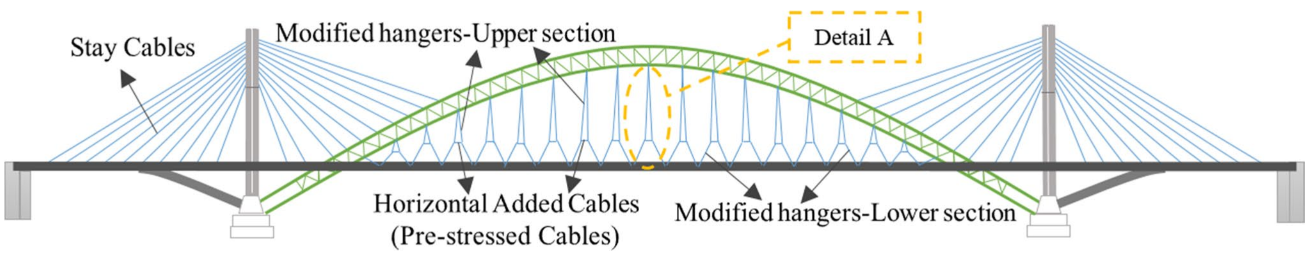

Fig. 7 The cable-arch bridge with modified hangers

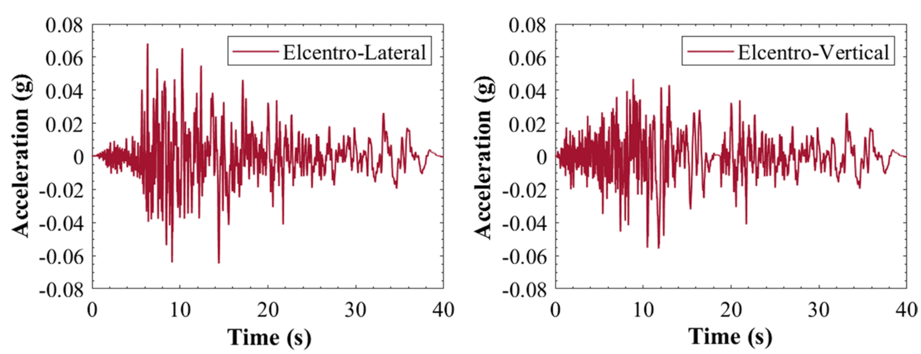

Fig. 8 Time-history of El Centro matched to the design spectrum

to Chinese code, GB50011-2010 [44] which is shown in Fig. 9. For further explanation, refer to the comprehensive seismic assessment of the cable-arch bridge investigated by Farahmand-Tabar and Bargian [45].

3.2 Vehicle dynamic load

The bridge excitation can be induced by the combination of high density of vehicles and low natural frequencies within the specified frequency range. Here, dense vehicle load was modeled as uniformly distributed lane load on the entire bridge deck. The load model shares a uniformly distributed harmonic load, $P_{(t)}$, that represents the design live load for each lane of the bridge. According to AASHTO [46], the amplitude of the HS20-44 vehicle load, $P_{(t)}$ as design live load, is equal to 9.52 kN/m. To apply the dynamic behavior of the vehicle live load (impact load on the deck induced by vehicles), a triangular time function (Fig. 10) was defined as an approximate and alternative method to exact solution [47]. Due to the sudden application of rectangular time

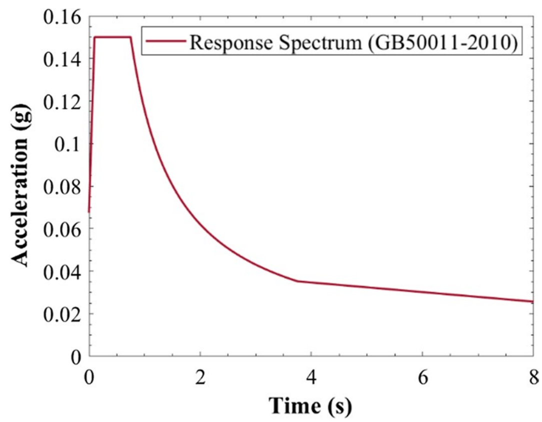

Fig. 9 Design response spectrum

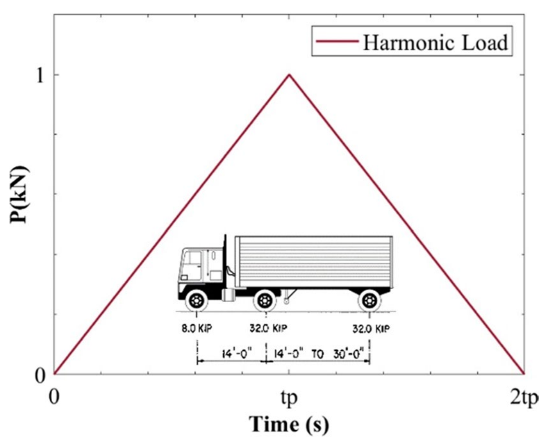

Fig. 10 Time history of the vehicle dynamic load

function, the results of the velocity and acceleration can be significantly inaccurate. However, the results of displacements and moments are satisfactory using either time functions. For this study, the time step t_p was assumed to be equally as 0.15.

4 Results and discussion

4.1 Comparing the results of the analysis for different hanger systems

In this study, the cable-arch bridge with various parameters was analyzed. Dead load plus earthquake and vehicle dynamic loads were considered as load cases. Based on results, it was noticed that none of vertical hangers were overstressed under the dynamic load patterns, while the inclined hangers had force fluctuation along the layout line under the load patterns of vehicle and earthquake loads, so that, one of the two adjacent hangers became overstressed alternatively under the dynamic load pattern, as seen in Fig. 11. For the dynamic load pattern of vehicles, by moving the vehicle load from one end to the other end of the bridge deck, several inclined hangers became overstressed. After that, the direction of the movement was changed, and the overstressed hangers in the previous stage were subjected to the lower amount of forces. These actions in repetition can result in fatigue phenomenon, which causes hangers to be ruptured. In Fig. 11, the side inclined hangers are affected by axial force fluctuation more than central ones. The reason may be due to increasing of the hangers' length and decreasing their angle relative to the vertical axis when approaching the mid span. In this case, the hangers' performance approximately resembles the vertical hanger. To obtain the least force fluctuation in two adjacent hangers along the span of the bridge, different values were utilized for parameters of the modified hanger system (i.e., *H* and *L* as illustrated in Fig. 6) by try and error. First, various *H* parameters were used as the height of the horizontal member, while the *L* parameter was kept constant. Thus, the *H* value was begun from 0.1 m and then it was increased incrementally by 0.1 m in each phase of the analysis. Having determined the proper value for *H*, the height of 0.05 m below and above the considered height was investigated, and it was realized that the height equal to the 0.75 of the height of each hanger provided better results (i.e., no overstressing in hangers and minimum force fluctuation). The results of the analysis indicated that to achieve the minimum force oscillation in lower and upper sections of the two adjacent modified hangers, H equal to the 0.75 of the height of each hanger could be selected. Figure 12 illustrates the forces of lower and upper sections in the modified hanger induced by seismic and live vehicle load patterns at different levels of the added members. The amount of hangers' weight in bridges with vertical, inclined, and modified hanger systems were 384.96 kN,

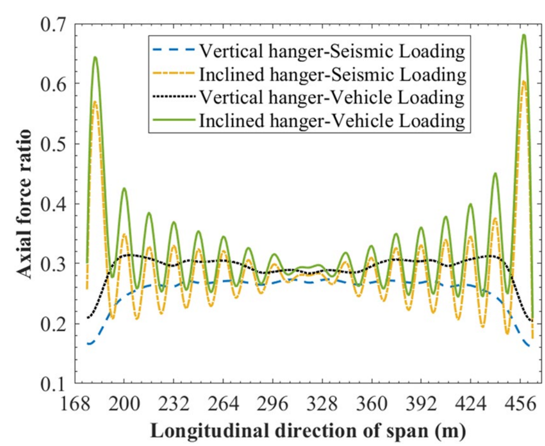

Fig. 11 Demand to capacity ratio of axial forces for vertical and inclined hangers due to dynamic loads

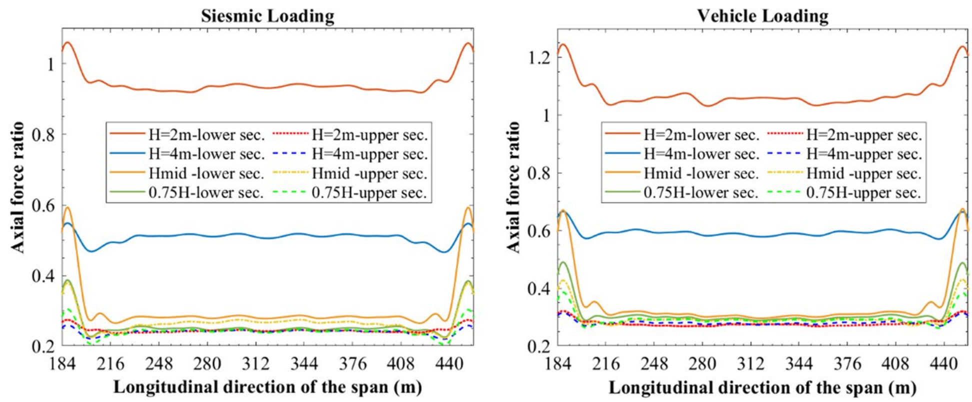

Fig. 12 Hanger's force ratio in the lower and upper sections of modified hangers under seismic and vehicle loading

401.72 kN, and 416.89 kN, respectively. It was found that the modified hangers' weight had been increased by about 3.78 percent in comparison with the inclined ones. After that, various lengths were applied for the added member as the parameter of L while $H = 0.75$ of the height of each hanger (the optimal height for the added member) was kept unchanged. First, the parameter of L was taken as 2 m, and then it was reduced up to the value of 0.25 m. It was found that by reducing the added member's length (L), the values of force oscillations in two adjacent hangers in the modified system were reduced. When the length of the added member became zero (in extreme condition), the modified hanger's upper section tended toward the position of the vertical hangers, so that the amount of forces in the upper sections of the modified hangers would be equal to the amount of the vertical ones. Based on the analysis results, it was found that for lengths less than 0.25 m, the reduction in the forces fluctuations was very slight and their oscillation was very close to the values obtained from $L = 0.25$ m. So, $L = 0.25$ m was chosen as the minimum length for the added member. Figure 13 illustrates the modified hanger's force for the constant height and different lengths of the added member.

As shown in Figs. 14 and 15, it was found that, by increasing the value of H, the forces of modified hangers in the lower and upper sections were getting very close to each other. If the height of the added member becomes equal to the hanger's vertical height as an extreme case—it means that there is no added member—the forces in the lower and upper sections of modified hangers will be equal. Figure 16 illustrates the modified and inclined hangers force induced by seismic and vehicle loadings.

In summary, the obtained results of the analysis for the three types of hanger systems are presented in Table 2. According to Table 2, the modified models indicate that of the seismic and vehicle load patterns, minimum force fluctuation exists in the adjacent cables. Furthermore, internal forces and their oscillations were reduced remarkably in the modified hangers compared with the inclined hangers (Fig. 17).

The responses of the bridge under the considered loads are presented in Table 3. Accordingly, in the case of the earthquake load, the maximum deflection and acceleration of the model with modified hangers were obtained 0.36 m and 1.73 m/s^2, vertically and 0.0744 m and 0.388 m/s^2, laterally. Also, for the vehicle load case, the maximum acceleration and deflection were 2.2 m/s^2 and 0.37 m, respectively, in the vertical direction, and 0.011 m/s^2 and 0.00015 m in the lateral direction. The maximum vertical deflection and acceleration of the bridge with inclined hangers achieved 0.37 m and 2.87 m/s^2 (> 2.5). Therefore, according to maximum

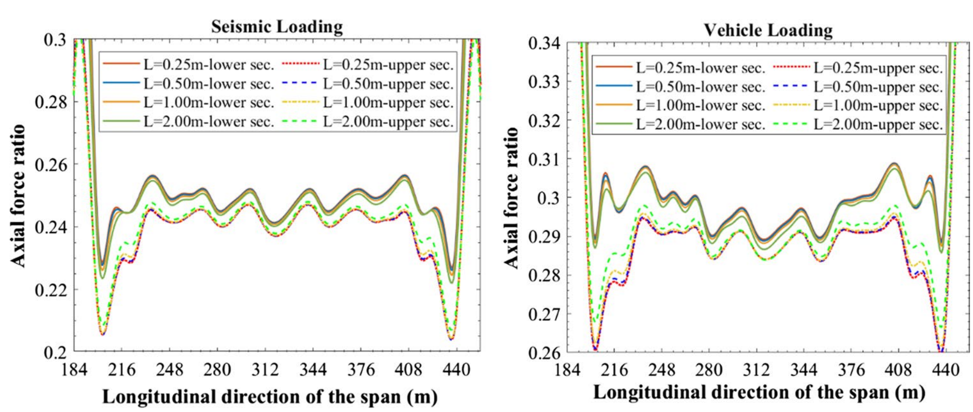

Fig. 13 Hanger's force ratio in the upper and lower sections of modified hangers with constant H ($= 0.75$ of the hangers' length) under seismic and vehicle loading

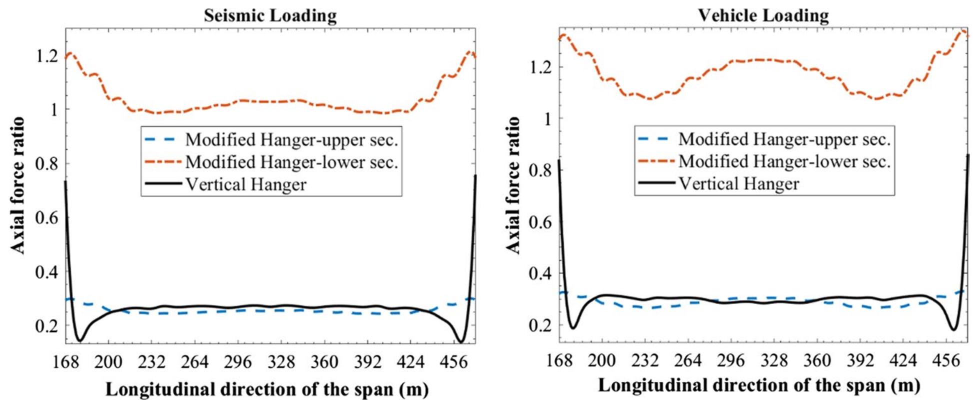

Fig. 14 Vertical and modified hangers' forces with $H=2$ m and $L=0.25$ m due to the dynamic load

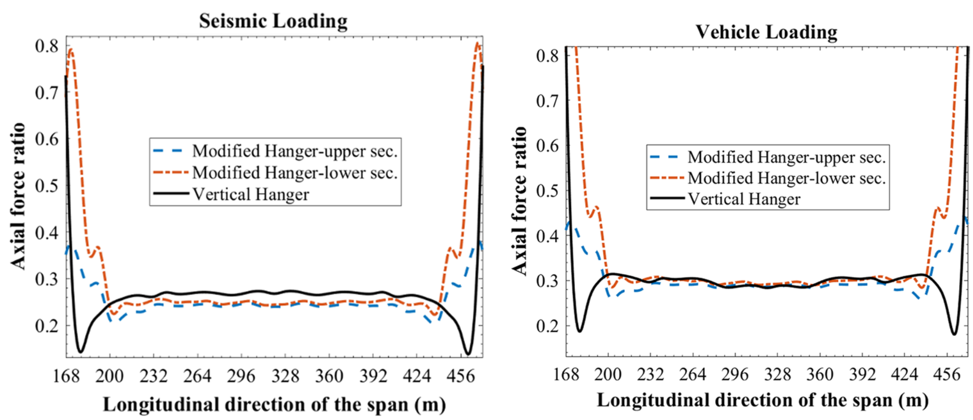

Fig. 15 Vertical and modified hangers forces with $H=0.75\ H_{hanger}$ and $L=0.25$ m due to the dynamic load

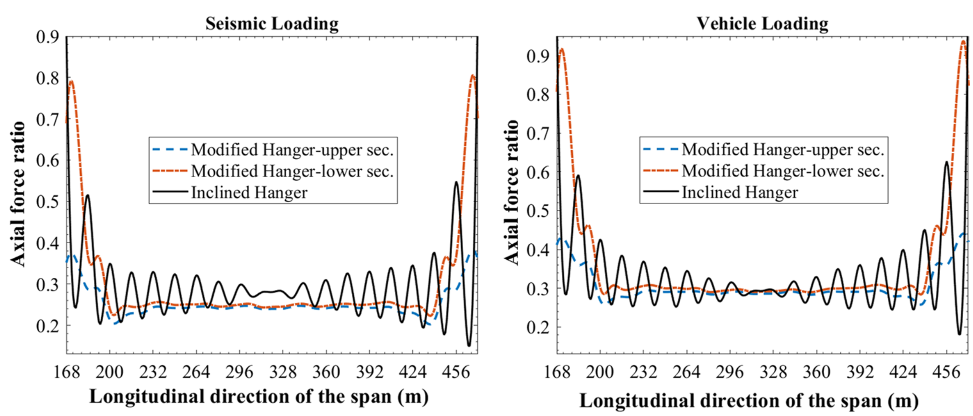

Fig. 16 Inclined and modified hangers' forces with $H=0.75\ H_{hanger}$ and $L=0.25$ m under the dynamic loading

values proposed by codes [48], the modified hanger systems can provide an acceptable comfort level and also it can reduce the resonance possibility.

4.2 Investigating the length and height of the added member in modified hangers for the cable-arch bridge

To achieve a mathematical relationship for the parameters of the added member in the cable-arch bridge, different values for parameters which influence on the structure have been investigated. The optimal height of the added member in a cable-arch bridge was achieved equal to the 3/4 of the cables' height. Also, it was found that the added members with smaller length (in a specific region) provide a better result. According to the results, the forces of the hangers with the added member at the 0.75 of the hangers' height have minimum oscillations than the other heights, and the minimum force oscillations of the modified hangers occur at $L=0.25$ m.

Table 2 Characteristics of the vertical, inclined, and modified hanger ($H = 0.75 \times H_{hanger}$) forces under the dynamic vehicle load and seismic excitation

The location of hangers	The type of hangers	Earthquake excitation (El-Centro)		Vehicle dynamic load	
		The maximum tensile forces (kN)	Forces fluctuation (%)	The maximum tensile forces (kN)	Forces fluctuation (%)
i	V^{α}	714.6	−20.01	933.84	−68.00
	I^{β}	3410	−99.97	2468.4	−26.93
	MU^{γ}	1010	−9.57	1289.5	−29.31
	ML^{δ}	1300	−9.60	1656.4	−29.31
j	V	873.9	−18.66	1150.2	−76.02
	I	1230	−95.28	982.4	−52.48
	MU	1013	−9.23	1288.8	−29.33
	ML	1300	−9.58	1654.9	−29.34
k	V	1040.8	−25.02	1213.9	−39.24
	I	2280	−99.9	1574.6	−30.14
	MU	925.6	−8.62	1120.1	−21.52
	ML	1008	−8.6	1220.0	−21.48
l	V	1043	−27.88	1188.9	−34.47
	I	1560	−92.02	960.8	−55.49
	MU	924.1	−8.65	1118.2	−21.49
	ML	1005	−8.62	1216.9	−21.47
m	V	1061.7	−33.86	1202.6	−37.83
	I	2370	−98.67	1420.2	−23.39
	MU	963.1	−8.99	1146.8	−20.15
	ML	999	−8.99	1189.6	−20.09
n	V	1062.7	−34.65	1207.1	−39.10
	I	1679	−95.10	972.8	−30.76
	MU	961.0	−8.99	1145.2	−20.29
	ML	994.6	−9.01	1185.6	−20.26
o	V	1063.9	−32.88	1171.5	−27.30
	I	2276	−99.84	1297.8	−18.29
	MU	963.5	−10.19	1120.8	−15.87
	ML	987.8	−10.18	1149.4	−15.79
	I	1885	−99.80	1034.4	−20.93
	MU	961.2	−10.24	1117.8	−15.87
	ML	982.8	−10.27	1143.5	−15.83
q	V	1081.4	−32.75	1141.4	−17.86
	I	2167	−99.82	1161.7	−16.25
	MU	959.2	−10.06	1125.2	−16.87
	ML	981.6	−10.07	1151.8	−16.89
r	V	1063.6	−33.53	1124.8	−19.24
	I	2125	−99.81	1149.8	−16.63
	MU	956.8	−10.17	1122.3	−16.88
	ML	976.6	−10.21	1146.1	−16.94

α: Vertical hanger β: Inclined hanger γ: Upper section of modified hanger δ: Lower section of modified hanger

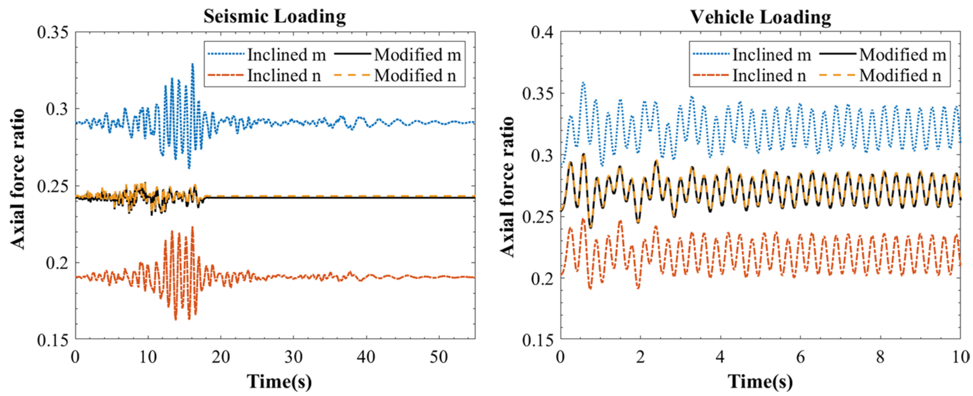

Fig. 17 Hanger's force ratio of typical adjacent inclined hangers m and n in comparison with respective modified hangers under earthquake and vehicle loading

Table 3 Deflections and accelerations of the bridge models under dynamic loads considering various hanger systems

Location	Hanger type	Load type	Deflection(m)				Acceleration(m/s^2)			
			Vertical		Lateral		Vertical		Lateral	
			Min	Max	Min	Max	Min	Max	Min	Max
Mid Span	Vertical	Earthquake	−0.2119	−0.2691	−0.1249	0.1113	−1.581	2.3989	−1.1930	1.0100
		Vehicle	−0.038	0.00	−0.0001	0.0001	−2.14	2.39	−0.034	0.035
	Inclined	Earthquake	−0.2274	−0.2869	−0.1066	0.1077	−1.193	1.855	−0.7563	0.8758
		Vehicle	−0.29	0.26	−0.0003	0.0003	−2.148	2.059	−0.1	0.105
	Modified	Earthquake	−0.283	−0.212	−0.074	0.056	−1.687	1.727	−0.388	0.382
		Vehicle	−0.284	−0.251	−0.00013	0.00015	−2.176	2.22	−0.007	0.008
¼ Span	Vertical	Earthquake	−0.2939	−0.3499	−0.0854	0.0801	−1.443	1.4690	−0.7679	0.7749
		Vehicle	0.04	0.001	−0.0004	0.0004	−3.27	3.32	−0.014	0.014
	Inclined	Earthquake	−0.2989	−0.3539	−0.0728	0.0755	−1.072	1.366	−0.7605	0.6872
		Vehicle	−0.37	−0.32	−0.00001	0.00001	−2.77	2.87	−0.0033	0.0033
	Modified	Earthquake	−0.360	−0.308	−0.053	0.040	−1.029	0.9494	−0.278	0.274
		Vehicle	−0.369	−0.344	−0.0001	0.0001	−1.963	1.918	−0.010	0.011

4.3 Proposing the optimum length and height for the added member in modified hangers

4.3.1 Mathematical generic relationship for the length of the added member

The obtained results indicated that by decreasing the length of the added member, the hangers' oscillations were reduced, while their forces were increased. So, the added member's length should be reduced as far as the force of the added member should not exceed its admissible amount.

In some cases, especially in the long and slender suspension bridges, hanger's slackness happens at the middle of the bridge span. The reason is that—for an added horizontal member with a constant length—after a specific height, the added member has no effect on eliminating slackness. This phenomenon begins for the hangers with the least height. Thus, for the added member, the shortest hanger plays a restrictive role. These explanations are

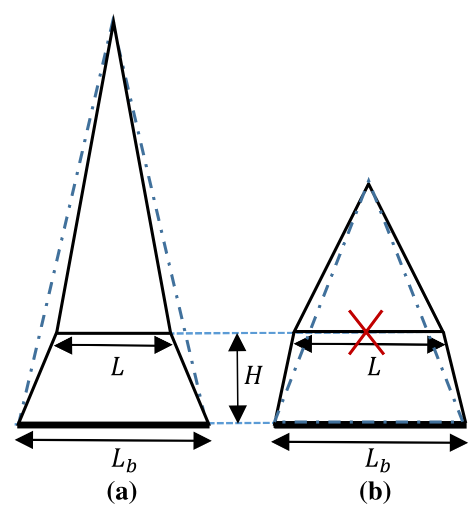

Fig. 18 **a** Permissible modified hanger. **b** Impermissible modified hanger

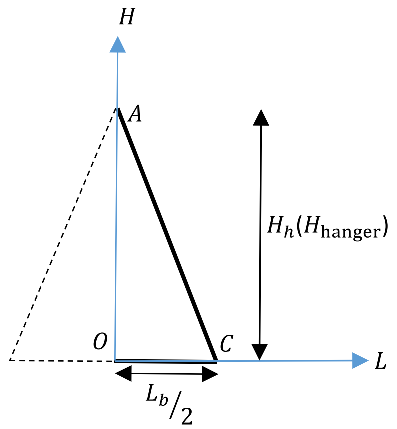

Fig. 19 Two adjacent hangers without adding the member

illustrated in Fig. 18. According to Fig. 12, adding the horizontal member to the specific constant height caused restrictions for the height ($H=5$ m is the maximum available constant height) and did not satisfy the optimum modification in the hanger system. Thus, the height of the added member should be related to the height of each hanger. Figure 19 represents typical adjacent hangers and based on this figure; the linear equation can be written for the line of AC as follows:

$$\frac{L - L_A}{L_c - L_A} = \frac{H - H_A}{H_c - L_A} \tag{2}$$

Considering the values from Fig. 19 and substituting them into Eq. (2) gives:

$$\frac{L}{L_b/2} = \frac{H - H_h}{-H_h} \tag{3}$$

where H, H_h, L_b, and L are the height of the added member, the hanger's height, the beam length between two adjacent hangers and the length of the added member, respectively. Eq. (3) indicates the half-length of the added member, and the complete length will be calculated in the following form:

$$L = \frac{L_b(H_h - H)}{H_h} \tag{4}$$

Thus, the following condition should be utilized:

$$L \leq \frac{L_b(\mathrm{H_h} - H)}{\mathrm{H_h}} \tag{5}$$

Equation (5) indicates the added member's length.

To resolve the overstressing problem in the hangers, the analyses indicated that Eq. (5) should be improved. To obtain the state of hangers according to Figs. 7 and 18a, 20 cm (=0.2 m) less than the amount of Eq. (5) is proposed. Therefore, by considering the result achieved in previous sections (i.e., $L=0.25$ m), the proposed range of L is as follows:

$$0 \cdot 25 \leq L \leq \frac{L_b(H_h - H)}{H_h} - 0 \cdot 2 \tag{6}$$

Through the selection of the mentioned range, the force distribution in adjacent hangers becomes uniform and close to each other; thus, there is no overstressing or probable slackness in hangers and the probability of the fatigue or cable rupture induced by these problems is decreased.

4.3.2 General mathematical relationship for the height of the added member

The dimensions of the bridge were changed to achieve a general mathematical equation for the optimum height of the added member. It was again found that 0.75 of the hangers' height was proper for the height of the added members. For various bridges, it was found that optimum height can be obtained by altering the length of beams between two adjacent hangers (L_b) on the deck. To achieve a general equation, the performance of cable-arch bridges with different beam lengths (L_b) was investigated. It was realized that the height value of each added member was unique depending on the

Table 4 Achieved optimal height based on the length of the deck beams between two adjacent hangers

L_b (m)	H_i/H_{h_i} (m)
4	0.19
8	0.40
10	0.47
12	0.50
14	0.66
16	0.75

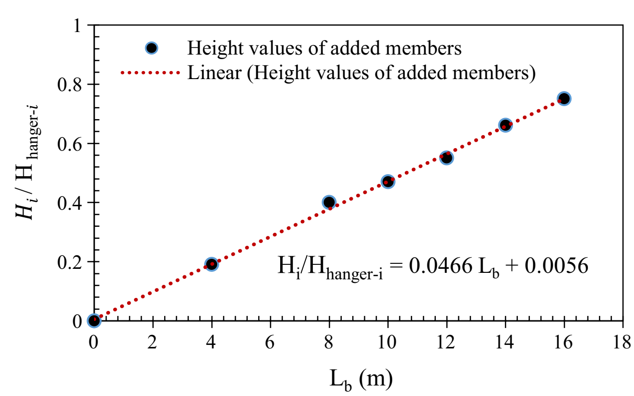

Fig. 20 Linear regression for the obtained optimum height of the added member

corresponding length of L_b. Several lengths and corresponding heights are presented in Table 4.

To achieve a mathematical relationship from the values of Table 4, they are plotted in Fig. 20 in which a linear regression was used. Basic equations of linear regression are given below [49]:

$$y = ax + b,, \tag{7}$$

$$a = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{2}}, \tag{8}$$

$$b = \frac{\sum y_i - a \sum x_i}{n}, \tag{9}$$

$$\text{corr} = \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sqrt{\left(\sum x_i^2 - \frac{(\sum x_i)^2}{2}\right)\left(\sum y_i^2 - \frac{(\sum y_i)^2}{2}\right)}}, \tag{10}$$

The extracted equation of the linear regression can be written as follows:

$$H_{\text{added member}} = \left(0 \cdot 0466 \times L_b + 0 \cdot 0056\right) \times H_{\text{hanger}}, \tag{11}$$

By ignoring the negligible amount, Eq. (11) becomes:

$$H_i = \left(\frac{233}{5000} L_b\right) \times H_{h_i}, \tag{12}$$

Equation (12) indicates the optimum amount and lower bound for the added member's height. Based on Fig. 19, the height of the added member (parameter H) should be limited to a specific amount. The H value's upper bound is determined by considering two important points: (1) the lower and upper bound of Eq. (6), (2) to know that the maximum height is achieved for the added member when L is considered to be minimum. The upper limit of Eq. (6)—which is equal to 25 cm—provides the upper limit for the parameter of H. The value of 25 cm is the optimum one, and it is the lower bound for the parameter of L. Therefore, the final equation for the height of the added member can be presented as follows:

$$\left(\frac{233}{5000} L_b\right) \times H_{h_i} \le H_i \le \frac{H_{h_i}\left(L_b - 0 \cdot 45\right)}{L_b}, \tag{13}$$

For example, the optimal length and height of the added member for the Liancheng cable-arch bridge, by considering the middle hanger with $H_h = 47.75$ m and $L_b = 16$ m using Eqs. (13) and (6), are equal to:

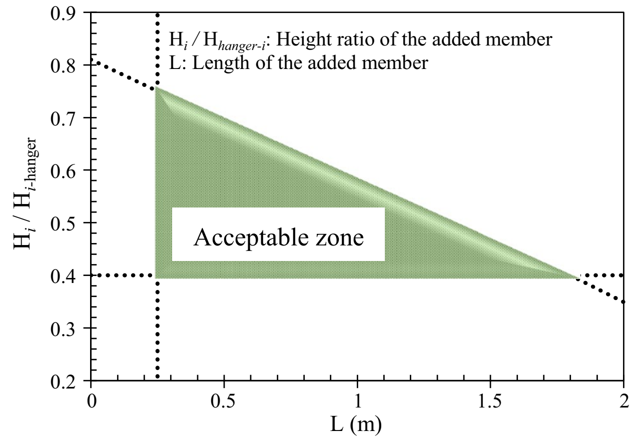

Fig. 21 Relationship diagram achieved from Lian Cheng cable-stayed arch bridge

$$35 \cdot 82 \le H_i \le 46 \cdot 41, \tag{14}$$

$$0 \cdot 25 \le L_i \le 15 \cdot 8 - 0 \cdot 335 H_i \tag{15}$$

The above equations can be plotted according to Fig. 21 to achieve the permissible zone. All points within the zone in Fig. 21 are permissible, but to get the minimum force oscillations in hangers, the optimal height (H) can be taken as equal to 75% of the hangers' height.

5 Conclusion

In this paper, the effects of various hanger systems (vertical, inclined, and modified hanger systems) were investigated on the cable-stayed arch bridge in order to modify the possible disadvantages of the systems while taking their advantages. However, the improvement in the modified hanger system is possible when the hanger system has a specific geometric and parametric limit that lead to generate the formula for the considered bridge. Proposing this formula for optimum length and height of the horizontal added member in the modified hanger system is the objective of this study which solves the problems of overstressing and force fluctuation in hangers properly.

Through the investigation, the results are summarized as follows:

The modification of hangers decreased the tensile forces and force fluctuations in inclined hangers remarkably by transferring the forces from overstressed hangers to the adjacent ones with lower amount of force using the added horizontal member between them. Also, it improves the acceptable comfort level laterally and vertically in the bridge under dynamic loads due to reduced vibration.

The modification of the hangers reduced oscillations in hanger forces considerably; therefore, it decreased the probability of the fracture in hangers induced by fatigue. It is possible for inclined hangers to be replaced after some years; on the contrary, the serviceability of modified hangers can last longer.

To achieve the improvements in the modified hanger system, formulations were offered to determine the height and length of the horizontal added member between two adjacent inclined hangers. For decreasing the force oscillation in two adjacent hangers, it would be better to use the added member which has the least length. But, by decreasing the length of the added member, its force is increased, while the oscillation is decreased. Therefore, by considering the parameters such as length and height of the added member, permissible shape of the modified hanger, intervals of two adjacent hangers and the total height of the cables, an acceptable range was obtained for the parameters of the modified hanger system.

According to the acceptable zone, the length of the added member was limited within 0.25–1.8 m, while the height was limited within 0.4–0.8 of the hangers' height. The ideal and optimal positions of horizontal members in modified hangers of the cable-stayed arch bridge model were obtained as the $^3/_4$ of the hangers' height so that they followed the curved shape of the arch. Also, the minimum force oscillations of the modified hangers occur when the length of the added member becomes 0.25 m.

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