Journal of the Brazilian Society of Mechanical Sciences and Engineering (2020) 42:239
https://doi.org/10.1007/s40430-020-02324-3

TECHNICAL PAPER

Numerical investigation of material properties and operating parameters effects in generating motorcycle break squeal using the finite element method

Matheus Henrique Pires Miranda[1] · Rômulo do Nascimento Rodrigues[1] · Roberto de Araújo Bezerra[1] · Pierre Maurice Christophe Lamary[1] · Raimundo Arrais de Oliveira Neto[2]

Received: 10 June 2019 / Accepted: 31 March 2020 / Published online: 18 April 2020

Abstract

One of the factors of influence on driver comfort is undoubtedly the noise. The noise generated by the brake, in addition to causing discomfort, can cause uncertainty as to the existence of mechanical failure in the brake system. Among the types of noise related to disc brakes, what has been generating a greater interest of researchers is squeal noise. This research paper is concerned with the disc brake squeal problem for motorcycles. The aim of the present research is developing a finite element model of the motorcycle disc brake in order to improve the understanding of the influence of material and operational parameters on squeal generation. Stability analysis of the disc brake assembly was accomplished to find unstable frequencies. A parametric study was carried out to investigate the effect of changing Young's modulus of the disc, back plate, friction material and operational parameters, as rotational velocity of the disc, pressure and temperature, on squeal generation. The results of simulation indicated that material and operational parameters play a substantial role in generating the squeal noise.

Keywords Motorcycle brake squeal · FEM · Young's modulus · Complex eigenvalue analysis

List of symbols

F_y	Normal forces (N)
F_x	Friction forces (N)
μ	Coefficient of friction
k_c	Stiffness of the contact elements (N/m)
E	Elasticity module (GPa)
y	Vertical displacement (m)
x	Horizontal displacement (m)
$\ddot{x}$	Horizontal acceleration (m/s^2)
$\dot{x}$	Horizontal speed (m/s)
u	Generic displacement
y_i	Modal coordinate
λ_i	Complex eigenvalues
σ_i	Real part of eigenvalue
ω_i	Imaginary part of eigenvalue
j	Imaginary part $\left(\sqrt{-1}\right)$
$[K_f]$	Stiffness matrices of the contact interface elements
$[k]$	Stiffness matrices
$[C]$	Damping matrices
$\{u\}$	Coordinate transformation
$\{\phi_i\}$	Eigenvector normalized
$[\Lambda^2]$	Diagonal matrix containing the natural frequencies of the system
$[I]$	Moment of inertia matrices
$[\psi]$	Complex eigenvalues and eigenvectors

Technical Editor: José Roberto de França Arruda.

✉ Rômulo do Nascimento Rodrigues
eng.romulorodrigues@gmail.com

Matheus Henrique Pires Miranda
matheushpmiranda@gmail.com

Roberto de Araújo Bezerra
roberto.bezerra@gmail.com

Pierre Maurice Christophe Lamary
pierre.lamary@ufc.br

Raimundo Arrais de Oliveira Neto
arrais.neto@hotmail.com

[1] Laboratório de Vibrações – LabVib, Departamento de Engenharia Mecânica, Centro de Tecnologia, Universidade Federal do Ceará, Fortaleza, CE, Brazil

[2] LMEst - Structural Mechanics Laboratory, School of Mechanical Engineering, Federal University of Uberlândia, Av. João Naves de Ávila, 2121, Uberlândia, MG 38408-196, Brazil

1 Introduction

The disc brake has been developed since about 1890, and the first patent came in 1902, registered by the British engineer Frederick William Lanchester (1868–1946) [1]. The basic disc brake assembly consists of several components such as a disc, a piston, a caliper, an anchor bracket, a wheel hub, a steering knuckle, piston and finger pads, two bolts and two guide pins. The operating principle of a disc brake is relatively simple. After the hydraulic pressure of the system is generated, the brake fluid passes through the hydraulic inlet of the caliper and through the piston transforms the hydraulic pressure into mechanical force, which induces contact of the brake pad against the brake disc. Through action and reaction, the clamp, through the pins, slides in the opposite direction and presses the friction material against the other face of the brake disc. Due to the friction generated between the pads in contact with the disc, the kinetic energy of the vehicle is converted into heat and the vehicle loses speed. Therefore, the brake disc must be able to withstand high temperatures and mechanical stresses.

Motorcycle has been one of the essential ground transportations for people to move from one place to another. The motorcycle braking system acts as one of the most fundamental safety–critical components in modern designs. Therefore, the braking system of a motorcycle is significant, especially in slowing down or stopping the vehicle. Due to the braking operation, the brake system generates an unwanted high-frequency noise called squeal. Since its development, brake systems have suffered major technological advancements by academic and research communities. However, brake noise is a challenging issue for researchers and developers up to nowadays, because of its highly complex nature. Most of the research about brake noise is conducted on cars', trucks' and trains' brake systems, while it can be found less work done about motorcycle disc brake systems. Although similar, these brake systems have particularities that must be observed. Motorcycle brake discs have considerably lower mass than a car disc brake, for example.

Many forms of classification can be found in the literature for types of brake generate noise. Akay [2], in his work, found twenty-five or so designations to describe noise in automotive brakes. Among the brake-related noises, the type that, certainly, raises more interest from researchers is squeal, which is a friction-induced noise, auto-excited, that has self-sustained vibration for a short period [3]. Squeal is a problem faced by researchers and engineers since 1930. It can be classified as low-frequency squeal, or high-frequency squeal. Noises in the spectrum band from 1 to 7 kHz are classified as low-frequency squeal, whereas high-frequency squeal manifests between 5 and 20 kHz [4, 5].

Two dominant theories have been studied for squeal prediction: stick–slip friction and modal coupling of the components, [1, 3, 6]. Stick–slip theory main characteristic is the consideration that, for the system to be auto-excited, friction coefficient must be a function of the relative slipping velocity between the contact surfaces. Mills, in 1938, examined various brake systems and friction materials combinations in which he observed that the friction coefficient decayed according to the relative slipping velocity [1].

In order to reduce the occurrence or even intensity of noise, after conceived and built, the brake system is tested experimentally to characterize its acoustics and vibrational behavior. Processes for that purpose are, aside from slow and laborious, expensive for the industry. In recent years, due to major computing advancements, researchers have been using the finite element method to deal with brake noise phenomenon as Jordan et al. [7] states:

> The need to predict noise and vibration generation in an early phase of these systems projects diverted great effort to development of numerical models using Finite Element Method (FEM).

According to Ouyang et al. [8], simulations and analysis methods for brake squeal can be divided into two large categories: complex eigenvalue analysis in frequency domain and transient analysis in time domain. In complex eigenvalue analysis, it is possible to find some or all eigenvalues at once, while transient analysis program must run several times, until a cycle-limit movement is found. Therefore, complex eigenvalue method produces lower computing costs [8]. The complex eigenvalue approach linearizes the brake squeal solution at static steady sliding states. Liles [9] pointed out that:

> knowing the unstable modes facilitates several courses of action; modal frequencies could be moved by altering components or damping could be added such that the mode in question becomes stable.

Currently, most production work in industry uses this method. The real parts of the eigenvalues dictate the stability (or lack of it) of the system. If the real part of an eigenvalue is positive, the corresponding imaginary part was thought to be a possible squeal frequency. Many researchers in their studies on the brake system tried to reduce squeal noise through changing the factors associated with the brake squeal. As for example, Liles [9] found that shorter pads, damping, softer disc and stiffer back plate could reduce squeal, while in contrast, higher friction coefficient and wear of the friction material were prone to squeal. Lee et al. [10] reported that reducing back plate thickness led to less uniformity of contact pressure distributions, consequently increasing the squeal propensity. Kung et al. [11] in their simulations presented that instability of the disc brake was dependent upon a range of disc Young's modulus. Liu et al. [12] commented that the squeal can be reduced by decreasing the friction coefficient. The objective of this work is to perform a complex eigenvalue analysis using FE software ANSYS. The parametric study as a guide is to evaluate the influence of the material parameters (disc Young's modulus, back plate Young's modulus, friction material Young's modulus and friction coefficient) and operative parameters (disc rotational velocity, brake pressure, temperature).

A better comprehension of the influence these parameters have on squeal generation allows developers to make less expensive and faster decisions in the early stages of brake design, in order to improve noise performance. Scientists should use this knowledge as a guide when investigating the physical mechanisms involved in this phenomenon, in pursuance of a unified analytical solution for the brake squeal problem. The implications of the understanding of this problem extend to other fields of knowledge where there is friction-induced noise, such as in musical instruments.

2 Parametric studies

In order to conduct the finite element analysis in ANSYS, various material properties and operative parameters were selected to be implemented. The detailed description of the chosen parameters and their respective characteristics on the brake system is described as follows.

2.1 Temperature

Preston and Forthofer [13] highlight that friction material performance is strongly influenced by thermal effects and that those are the most important factors to establish correlation between tests conducted in laboratory and tests conducted in vehicles. An aspect that should be considered is the amount of heat transferred to disc and pads, as a function of these materials' thermal conductivity. Silva [14] determined, through FE simulations, a specific system in which 5% of total heat generated in braking is transferred to the pad, 93% to the disc and 2% is dissipated to the medium. According to Tirovic and Todorovic [15], friction material properties are dependent on temperature. Friction materials can be considered as composite materials and due to the microstructural arrangement, these elements are very sensitive to temperature variations, in terms of their elastic properties. Therefore, the temperature has a strong influence on the properties of the friction materials, mainly on their stiffness.

2.2 Disc rotational velocity

Disc rotational velocity plays an important role over brake system analysis. The most direct influence is on the system coefficient of friction, which generally decreases with the increase in rotational velocity, going against Amontons' law, as Tsang et al. [16] describe. Friction coefficient variation in friction materials, due to changes in velocity, is in the order of 0.05 at maximum [17]. Friction sensitivity to velocity is explained as a thermal effect, relating the temperatures on material contact interface to corresponding values of friction coefficient, and, therefore, torque variation.

2.3 Brake pressure

Pressure exerted on the brake pads is directly linked to the friction coefficient between disc and friction material. An increase in brake pressure often results in a decrease in coefficient of friction, as described by Tsang et al. [16]. Contrary to what may seem, contact pressure is not uniform on the brake pads. Tirovic and Day [18] performed a study on the pressure distribution in disc brake systems and concluded that pad deformation occurred from the deformation of the other components in the disc brake system. High actuating force, friction force and thermal effects may cause significant deformation on pads, causing uneven pressure distribution, which can result in reduction in braking torque, irregular pad wear and elevated temperatures in frictional contact surface.

2.4 Friction coefficient

According to Rabinovicz [19] and Duffour [20], friction is the resistance to movement that exists when a solid body slips tangentially to the surface of another body, while in contact, or when an attempt is made to produce such movement. Neuman and McNinch [21] affirm that the main variables which affect friction performance during braking are slipping velocity, contact pressure and temperature. Brake squeal is generally defined as a friction-induced instability phenomenon. Since friction is the main cause of instability, it must always be considered for brake noise analysis, being experimental or numerical.

2.5 Friction material Young's modulus

Friction material purpose is to control the movement by deceleration of the vehicle, through transformation of kinetic energy in heat, via friction, and dissipate the heat to the medium. Therefore, the main component for brake performance is the friction material, once a good friction material ensures stability in several conditions of usage [21]. Friction material stiffness is a critical property for its application given that it influences the behavior of the coefficient of friction and, ergo, vehicle stability. The material properties of the friction materials are diverse and, generally, dependent on temperature, highlighting Tirovic and Todorovic [15]. Accordingly, evaluating the performance characteristics of the friction material in relation to its elasticity properties is essential.

2.6 Disc Young's modulus

Disc and friction material form the frictional pair of the disc brake system. Selection of materials for the disc is of vast importance. There is a set of limitations to development of special materials for disc brake applications, cost being the primary. The disc, besides braking, is responsible for heat dissipation in the braking system. Most of the kinetic energy converted to heat must be absorbed and dissipated by the disc as quickly as possible. Beyond that, there are other properties that must be considered when selecting disc brake material, such as mechanical resistance, usinability, resistance to wear and damping capacity. Disc material selection is, therefore, critical to the project of a disc brake system.

According to Belhocine and Ghazaly [22], the discs for brake systems in wide use today are made of gray cast iron, because they have acceptable thermal properties, sufficient mechanical strength, satisfactory wear resistance and good damping properties, they are cheap, and they are also relatively easy to cast. Gray cast irons differ somewhat to steels and most other structural metals in that the Young's modulus can be varied significantly by changing carbon equivalent. Belhocine and Ghazaly [22] state that this allows the discs to be manufactured with Young's modulus that runs from below 100 GPa through to approximately 140 GPa.

2.7 Back plate Young's modulus

The disc brake pads consist of two parts, friction plates which are made of organic composite material and back plates, made of steel. The main function of the back plate is to absorb the vibration and consequently to reduce the noise. In this study, the baseline Young's modulus of the back plates is 210 GPa. Steel does not present much variation in Young's modulus, but an alternative case is executed with the back plate Young's modulus set to 200 and 220 GPa.

2.8 Contact area of the friction material

The contact area of the friction material is influential on the phenomenon of the brake noise, because it is directly connected to the friction coefficient between the friction material and the disc, which is, also, connected to the variation of the contact pressure distribution. Spurr [23] found that brake squeal depends upon the magnitude of the coefficient of friction and on the position of the contact areas between the friction material and surface of the disc. In his work, he concluded that squeal is therefore fugitive and occurs only at certain pressures and is very dependent upon how the friction material is bedded in. Therefore, to analyze the variation of the contact area is important, as this geometric parameter is intrinsically related to the phenomenon of noise in disc brakes.

3 Methodology

The aim of this section is to propose a methodology in order to examine the effects of materials properties and operative parameters of the disc brake components on disc brake squeal generation.

3.1 Material properties of the components

The first step in the methodology of the work was to define the components of the disc brake system to be studied in the simulations as well as the main mechanical properties of its materials. The basic disc brake assembly consists of several components such as a disc, a piston, a caliper, an anchor bracket, a wheel hub, a steering knuckle, piston and finger pads, two bolts and two guide pins. For this work, it was decided to carry out simulations considering three basic components of the disc brake system: the friction material, which is made of an organic composite material, the back plate, made of structural steel, and the disc, made of gray cast iron, based on a commercial motorcycle brake model (Fig. 1). In order to carry out the simulation, the mechanical properties of the materials with respect to Young's modulus, the density and Poisson coefficient are used in the calculations. The baseline material properties of the disc brake components used in the simulation are listed in Table 1.

3.2 Parameters of operation, material properties for the planned simulation

There are some system parameters that act on the performance of disc brake systems. According to the literature, these variables are slip velocity, contact pressure, temperature and relative humidity of the air. The way in which these variables affect the performance of the systems is related to the friction and wear mechanisms involved in the braking process. There is, therefore, an interrelationship between variables, mechanisms and performance. The literature shows several studies that observe relationships between these variables to each other in a way that facilitates for engineers and researchers to choose the design and study parameters for the disc brake system. Some important studies found in the literature demonstrate, by experimental means, correlations between some important variables related to the performance of disc brake systems, such as the relationship between temperature and coefficient of friction, temperature and friction material Young's modulus, disc rotational velocity and coefficient of friction and, finally, braking pressure and coefficient of friction.

The second step in the methodology of this work was to define relationships between the selected study parameters in order to obtain the maximum similarity of results in comparison with the real brake system. The first relationship worked was between the temperature and the coefficient of friction. Neis [24] demonstrated, through tests carried out under different disc temperature levels and under constant disc rotational velocity, a graphic display of disc temperature influence on the variation of the coefficient of friction between the disc and the friction material. Interpolating this graph, it was possible to elaborate an equation correlating these two variables. Considering the temperature with minimum values of 50 °C and maximum of 250 °C (Table 2) and using the relationship previously found, it was possible to obtain the value of the coefficient of friction associated with each temperature. The coefficient of friction stipulated has a minimum value of 0.33 and maximum of 0.49 (Table 2).

The next relationship studied was the relationship between temperature and friction material Young's modulus. According to Trichês et al. [25], friction materials can be considered as composite materials due to the high number of elements in the final composition. Due to the microstructural arrangement, these materials are very sensitive to temperature variations, in terms of their elastic properties. Trichês et al. [25], in their work, determine a relationship between temperature and friction material Young's modulus. Considering the same temperature range used to determine the coefficient of friction, it was possible to obtain the friction material Young's modulus associated with each temperature. The friction material Young's modulus stipulated has a minimum value of 7.65 GPa and maximum of 11.13 GPa (Table 2).

The third relationship studied was between coefficient of friction and braking pressure. Xiao et al. [26] defined, through experimental analysis, a graphical representation

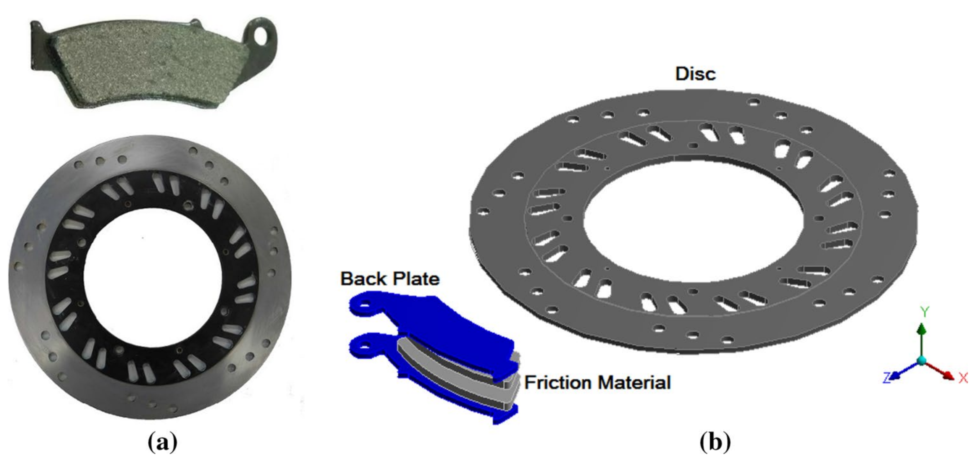


Fig. 1 **a** Commercial disc and pad of the motorcycle brake model used as study model, **b** detailed CAD model of disc brake assembly used in the simulation

Table 1 Material properties of disc brake components

	Material	Young's modulus (GPa)	Density (kg/m^3)	Poisson's ratio
Disc	Gray cast iron	125	7155	0.23
Back plate	Structural steel	210	7850	0.30
Friction material	Organic composite	11.13	2045	0.34

Table 2 Braking conditions determined for the sensitivity analysis

Operating and material parameters	Condition 1	Condition 2	Condition 3	Condition 4	Condition 5
Disc rotational velocity (rps)	11.85	9.96	8.06	6.17	4.28
Pressure (MPa)	2.22	2.02	1.82	1.62	1.42
Friction material Young's modulus (GPa)	11.13	10.26	9.39	8.52	7.65
Friction coefficient	0.33	0.37	0.41	0.45	0.49
Temperature (°C)	50	100	150	200	250

of the relationship between these two parameters. Using the same coefficient of friction, it was possible to find the associated braking pressures. The braking pressure stipulated has a minimum value of 1.42 MPa and maximum of 2.22 MPa (Table 2).

The braking conditions selected in Table 2 are designed to emulate a progression of sequenced real brakes, in which the temperature of the disc and pads increases, what causes changes in the coefficient of friction and stiffness of the components, being the friction material Young's modulus variation, the most significant. The coefficient of friction is also altered by changes in braking pressure and rotational velocity. Using the relationships established above, the values of pressure and rotational velocity were determined to maintain the coefficient of friction as a function only of temperature.

The last relationship studied was between the disc rotational velocity and the coefficient of friction. Dezi et al. [27], in their work, through experimental analysis, produced a graphical display, in which can be interpolated an equation that correlates these two parameters. Using the same coefficient of friction obtained previously and using the relationship obtained by Dezi et al. [27], it was possible to find the associated disc's rotational velocity. The disc rotational velocity stipulated has a minimum value of 4.28 rps and maximum of 11.85 rps (Table 2).

Starting from the temperature range of 50–250 °C, and using each of the relationships defined above, the values for the operational parameters are calculated for five distinct braking conditions (Table 2). Thermal effects on disc and back plate material properties are negligible in this range of 50–250 °C [25]. Preston and Forthofer [13] highlight that friction material performance is strongly influenced by thermal effects and that those are the most important factors to establish correlation between tests conducted in laboratory and tests conducted in vehicles. According to Triches et al. [6], the effect of temperature is more important for the pad, where the increase in temperature reduces the brake pad stiffness, altering the coupling mechanisms between the rotor and pad.

The third and final step of the methodology was to define some parameters that were called project parameters. Three project parameters were selected to be evaluated with respect to their influences regarding disc brake noise. The parameters analyzed were the disc material Young's modulus (GPa), back plate material Young's modulus (GPa) and contact area of the friction material (mm^2). The values of each project parameter that were used in the simulations are disposed in Table 3. The variation of the contact area of the friction material was established by varying the geometry of the friction material of the brake disc assembly used in the simulation. The standard geometry was obtained by the commercial brake pad model that already has its contact area of the defined friction material (Fig. 2a). To perform the variation of the contact area, two new geometries of friction material were proposed to be analyzed (Fig. 2b, c), while the standard commercial brake pad model was maintained (Fig. 2a).

4 Finite element model

A motorcycle disc brake system consists of a disc coupled to the front wheel, a caliper–piston assembly where the piston slides inside the caliper, which is mounted to the vehicle suspension system, and a pair of brake pads. When hydraulic pressure is applied, the piston is pushed forward to press the inner pad against the disc and simultaneously the outer pad is pressed by the caliper against the disc. Numerical simulations using ANSYS Workbench® were carried out considering

Table 3 Values of project parameters analyzed in the simulation

Project parameters			
Disc material Young's modulus (GPa)	115 (D1)	125 (D2)	135 (D3)
Back plate material Young's modulus (GPa)	200 (BP1)	210 (BP2)	220 (BP3)

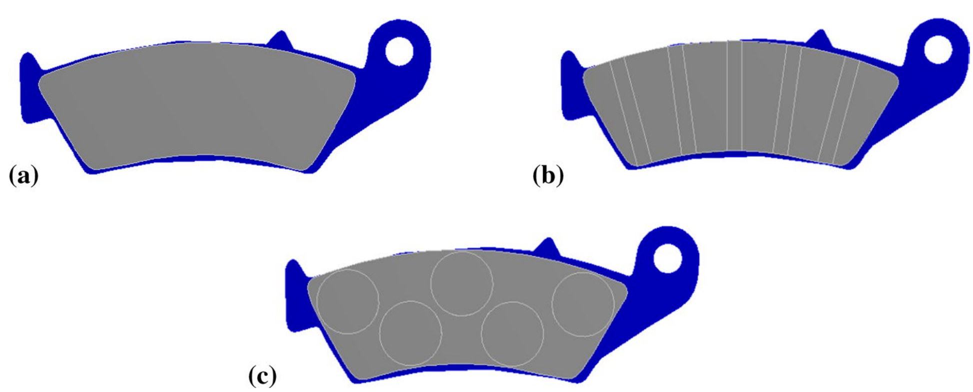


Fig. 2 Brake pad models for contact area variation (Values observed by selecting the contact area in ANSYS Workbench); **a** contact area of 2038,892 mm^2, **b** contact area of 1491,832 mm^2, **c** contact area of 1005,310 mm^2

a simplified model of a commercial disc brake system. The model considers the main elements contributing to the squeal noise, which are the disc and the brake pads. The pads contain both the contact plates and back plates (Fig. 1). The 3D design of the system is developed in CAD via Autodesk Fusion 360® and saved in IGES format, which communicates with ANSYS Workbench®.

When modeling a disc brake noise problem, one of the critical points is related to the contact interface, where there is friction between the disc and the pads, causing the vehicle to stop. One of the manners to incorporate friction in a system for squeal study, the most common nowadays, is modeling friction through geometric coupling, in which a spring is used to link the pair of corresponding nodes on the contact surfaces of the disc and the pads [8]. Thus, a friction element between the corresponding nodes on the disc and pads becomes necessary (Fig. 3), considering both the forces and the coefficient of friction of the contact. Dynamic instability generation in systems with constant coefficient of friction shall be proven through complex eigenvalue analysis, in which the inconstant friction force comes from an inconstant normal force. The interface element stiffness matrix, considering normal and friction forces, can be written as:

$$\begin{bmatrix} F_{ax} \\ F_{ay} \\ F_{bx} \\ F_{by} \end{bmatrix} = \begin{bmatrix} 0 & -\mu k_c & 0 & \mu k_c \\ 0 & k_c & 0 & -k_c \\ 0 & \mu k_c & 0 & -\mu k_c \\ 0 & -k_c & 0 & k_c \end{bmatrix} \begin{Bmatrix} x_a \\ y_a \\ x_b \\ y_b \end{Bmatrix} \tag{1}$$

In the context of a finite element-based model, the frequencies most likely to become unstable can be verified through the solution of the movement equation for an undamped system, by extracting its eigenvectors and eigenvalues. The complex eigenvalues reveal the unstable modes. Thus, a range of changes in the project might be made to avoid noise, as for example, changing the mechanical properties of the disc and the friction material, geometric variation of the components, increasing damping in a specific region, among others. The movement equation for a FE model with multiple degrees of freedom is given by the following equation:

$$[M]\ddot{x} + [C]\dot{x} + \left[K + K_f\right]x = 0 \tag{2}$$

where K_f is made up of the stiffness matrices of the contact interface element. An eigenvalue extraction, from Eq. (2), in which the matrices $[M]$, $[C]$ e $\left[K + K_f\right]$ might be real or complex, symmetric or asymmetric, should generate complex eigenvalues, given in Eq. (3), as follows:

$$\lambda_i = \sigma_i + j\omega_i \tag{3}$$

where σ_i is the real part of ith eigenvalue, representing damping, and ω_i, is the imaginary part, which represents the system natural frequency of this mode. When the real part of the eigenvalue (σ_i) is negative, the system is stable. However, if the real part of the eigenvalue (σ_i) is positive, the system amplitude grows exponentially, so the mode is considered unstable [28–30].

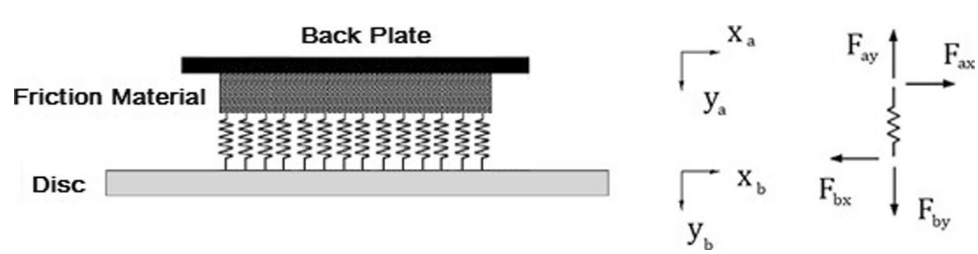
The method of solving the eigenvalues and eigenvectors problem in ANSYS software is the QRDAMP that allows the use of non-symmetric matrices and the extraction of complex eigenvalues. This method is a procedure for determining complex eigenvalues and corresponding eigenvectors of linear systems, allowing non-symmetric stiffness $[K]$ and damping matrices $[C]$. Basically, the method symmetrizes the stiffness matrix by rearranging the non-symmetric contribution, i.e., the original stiffness matrix is divided into a symmetrical and a non-symmetrical part. QRDAMP applies the orthogonal modal coordinate transformation to the system matrices, resulting in a quadratic eigenvalue problem and finds these eigenvalues.

The modal superposition method is known to have greater computational efficiency than the complete eigenvalue method. The stiffness matrix can be symmetrized by rearranging the non-symmetric contribution, i.e., the global matrix can be divided into two parts, one symmetrical and the other non-symmetrical. Reducing the stiffness matrix, the eigenvalue problem is formulated and solved by the block Lanczos algorithm. The coordinate transformation used to transform the eigenvalue problem into modal overlay can be seen as follows:

$$\{u\} = \sum_{i=1}^{n} \left\{\phi_i\right\} y_i \tag{4}$$

where $\left\{\phi_i\right\}$ is the ith eigenvector normalized by the mass matrix [M] and y_i is the modal coordinate. By replacing

Fig. 3 Contact interface elements graphical representation



Eq. 4 in Eq. 2, we can write the equation of motion of the problem as follows:

$$[I]\{\ddot{y}\} + [\phi]^{\mathrm{T}}[C][\phi]\{\dot{y}\} + \left(\left[\Lambda^2\right] + [\phi]^{\mathrm{T}}\left[K_{\text{non-symmetric}}\right][\phi]\right)\{y\} = \{0\} \tag{5}$$

where $\left[\Lambda^2\right]$, is the diagonal matrix containing the first "n" natural frequencies of the system. The non-symmetric stiffness matrix associated with the global stiffness matrix is designed in modal superposition so that the reduced non-symmetric modal stiffness matrix is calculated $[\phi]^{\mathrm{T}}\left[K_{\text{non-symmetric}}\right][\phi]$. Introducing the state formulation, the equation can be rewritten as:

$$[I]\{\dot{z}\} = [D]\{z\} \tag{6}$$

where

$$\{z\} = \left\{ \begin{matrix} \{y\} \\ \{\dot{y}\} \end{matrix} \right\} \tag{7}$$

$$[D] = \begin{bmatrix} [0] & [I] \\ -\left[\Lambda^2\right] - [\phi]^{\mathrm{T}}\left[K_{\text{non-symmetric}}\right][\phi] & -[\phi]^{\mathrm{T}}[C][\phi] \end{bmatrix} \tag{8}$$

In ANSYS, the eigenvalue problem $2-n$ is calculated using the QR algorithm. To conclude, the original system is rewritten using Eq. 9. Thus, we have the extraction of the eigenvalues and complex eigenvectors so that the system instability can be analyzed.

$$[\psi] = [\phi]\{z\} \tag{9}$$

The brake squeal prediction model addressed in this paper is partially disturbed modal analysis. Figure 4 is a flowchart of the process adopted in order to investigate the influence of the input parameters on squeal generation (sensitivity analysis). A variation of the input parameters should have some effects on the output parameters. This workflow was performed for 135 different combinations of input parameters. For each iteration, the model was processed, and the output parameters were registered for postprocessing.

Simulation process starts with a static structural analysis, in which the disc is considered immobile and the brake pressure is applied, imposing the contact between the pads and the disc. This situation is represented by the following boundary conditions: The internal faces of the bolt holes in the disc were considered as fixed supports (Fig. 5a), the outer faces of the back plates were constrained in all degrees of freedom, except for translation in the direction normal to the contact surface (Fig. 5b, c), and a distributed pressure is also applied to the surface of the outer faces of the back plates (Fig. 5d, e). The key result of this process is the pressure distribution in the contact surfaces (Fig. 6), which, then, is used as a boundary condition in the modal analysis, as the friction force is determined in the contact surface.

Then, modal analysis is performed to extract the complex eigenvalues for the vibrating modes of the disc brake system up to 21 kHz, since brake squeal occurs when the system is excited within the frequency range of 1 kHz–20 Hz [4, 5]. Generally, the real part of the complex eigenvalues is plotted against the associated natural frequencies, in order

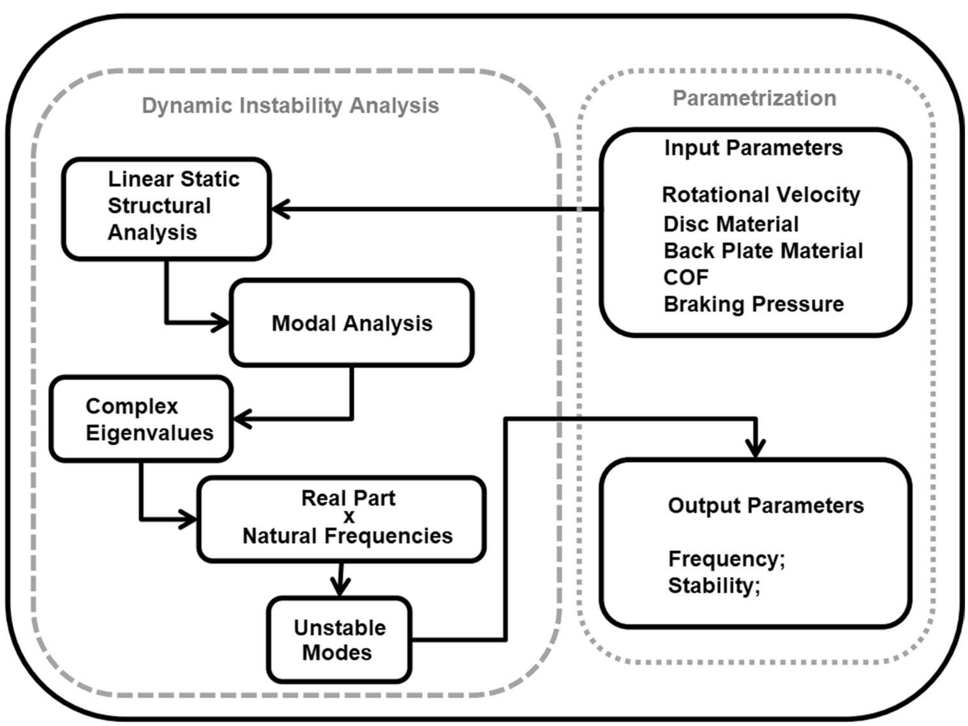
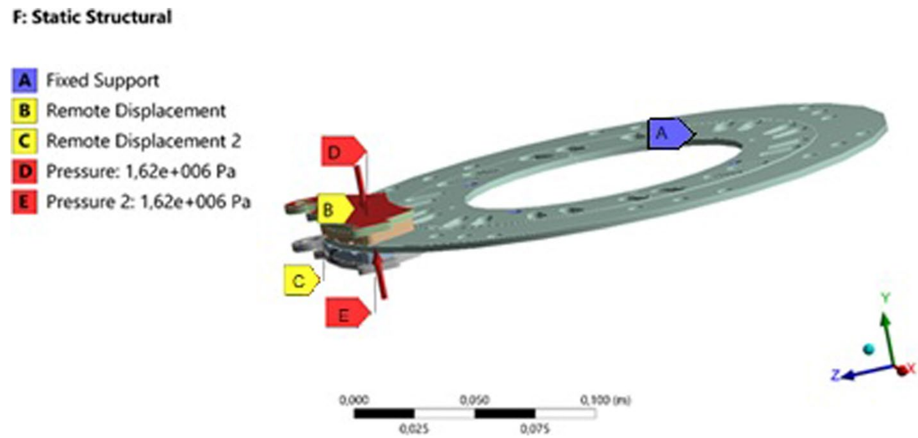


Fig. 4 Flowchart of the sensitivity analysis model

Fig. 5 Boundary conditions setup for the static structural analysis in ANSYS Workbench



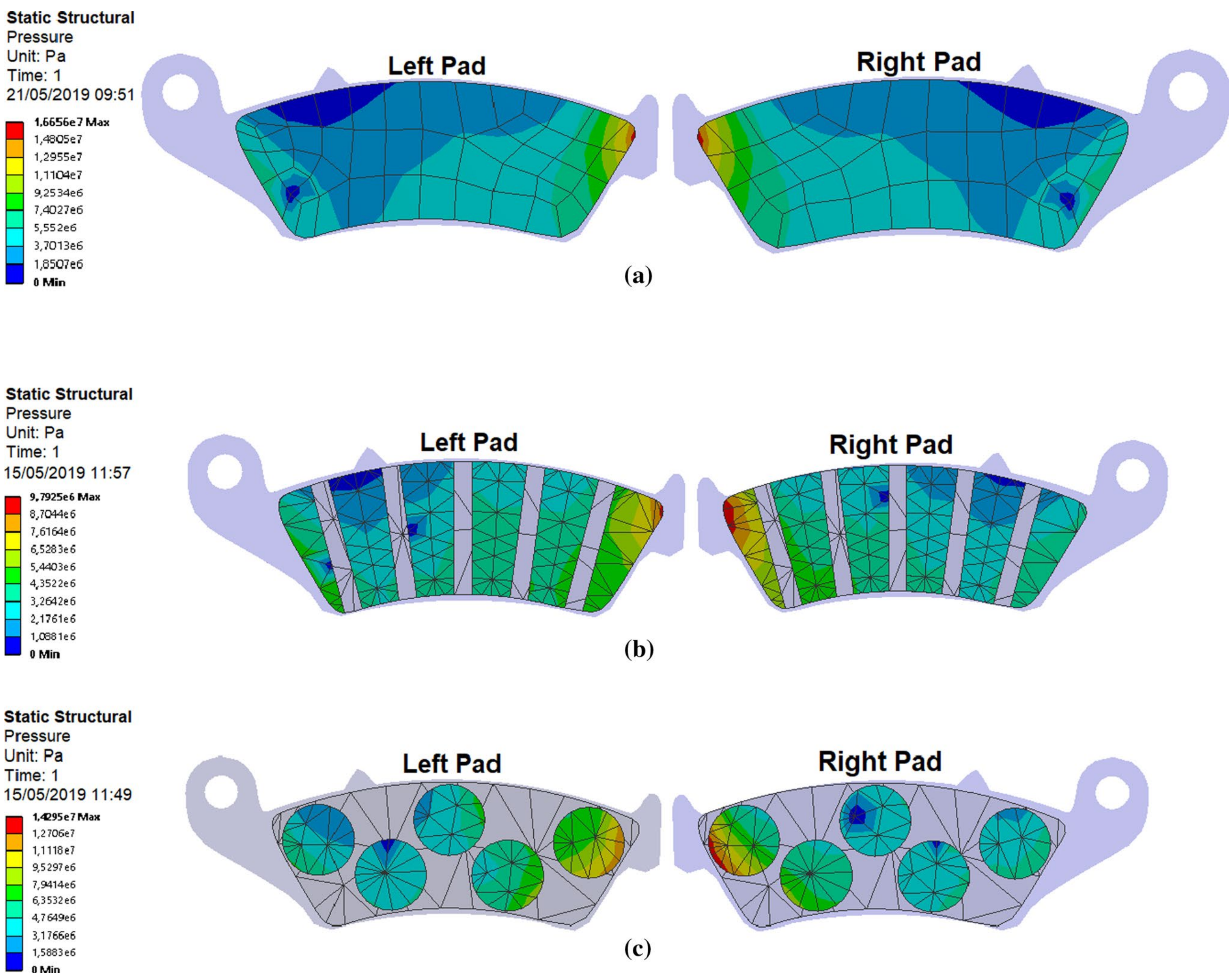


Fig. 6 Contact pressure distribution for each pad model; **a** Friction material with contact area of 2038,892 mm^2, **b** friction material with contact area of 1491,832 mm^2, **c** friction material with contact area of 1005,310 mm^2

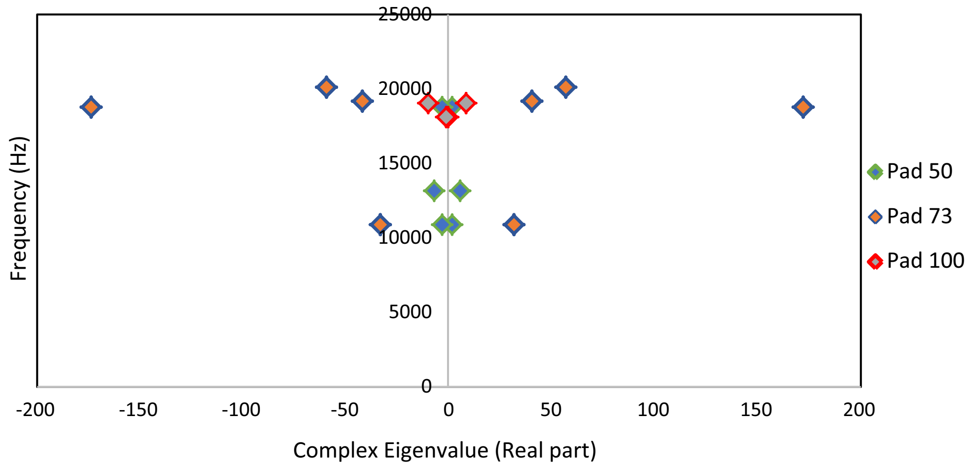


Fig. 7 Real part of complex eigenvalue for each friction material geometry, as a function of the natural frequency of the associated mode

to visualize the general occurrence of instability. A preliminary implementation of the model, for braking condition 3 in Table 2 and the material properties of the disc and the back plate from Table 1, was carried out to evaluate the consistency of the model (Fig. 7).

When there are modes coupled at the same frequency, one of them becomes unstable. The unstable modes can be identified during complex eigenvalue analysis, because the real parts of the complex eigenvalues are positive. Those modes are prone to squealing [31]. The FE model was validated, comparing the results with Ouyang et al. [8], Kinkaid et al. [1] and Belhocine and Ghazaly [22].

A first finite element mesh is generated automatically by ANSYS mesh assistant. Element sizing, element center and spam angle, mainly in the pads contact surface are employed as strategies for mesh refinement, and the results of the complex eigenvalues simulation are confronted to determine the best meshing strategy for the problem in hand, considering results convergence and computing time, as displayed in Fig. 8 and Table 4.

None of the strategies encountered convergence problems or presented incoherent behavior; therefore, the results can be considered as equivalent. Proven that the model works for the central conditions in Tables 2 and 3, the simulations were carried out for all the conditions in Table 2 and for each combination of the project parameters of Table 3. Therefore, the automatic mesh was chosen for these simulation tasks, given that it takes significant less time to solve.

5 Results

The simulations were conducted for each friction material geometry (Fig. 2), all the combinations in Table 3, in all the braking conditions in Table 2, producing 135 different configurations. Figures 7 and 8 summarize the results of the complex eigenvalue analysis for all configurations. Each simulation will deliver the real part of the eigenvalues for the natural frequencies (modes) of the brake system. If the real part of such eigenvalues is negative or zero, the system is stable, but if the real part is positive, the system becomes unstable and, therefore, prone to squealing. Thereby, it is important to know how many unstable modes the brake system will present (Fig. 10) and the magnitude of the eigenvalue (Fig. 9). In both the figures, the horizontal axis represents the braking conditions in Table 2, and each data series represents a combination of disc and back plate materials.

It can be observed from Figs. 9 and 10 that temperature plays an important role in the occurrence of unstable modes within the squeal zone. The higher the temperature, the coefficient of friction is increased, leading to more propensity to instability (both in intensity and in total number of unstable modes) and, therefore, brake squeal too.

Analyzing the variations of rotational velocity and pressure, it can be observed that a decrease in either one prompts a higher total of unstable modes, as well as the maximum eigenvalue increases. Both velocity and pressure are inversely proportional to the coefficient of friction, so a decrease in either one causes an increase in the coefficient of friction, thereby making the system more likely to squeal. In order to better understand the influence of the components material properties, it was necessary to conduct statistical analysis on the data from the simulations results. Pad geometry and coefficient of friction were confronted with Young's modulus of the components, for all braking conditions and associated with a total number of unstable modes within the squeal spectrum.

Statistical analysis was conducted on Statistica, software that provides data analysis, data management, statistics, data mining, machine learning, text analytics and data visualization procedures. Data analysis was performed through a factorial experimental design 2P with a central point. Such an experiment allows the investigator to study the effect of each factor on the response variable, as well as the effects

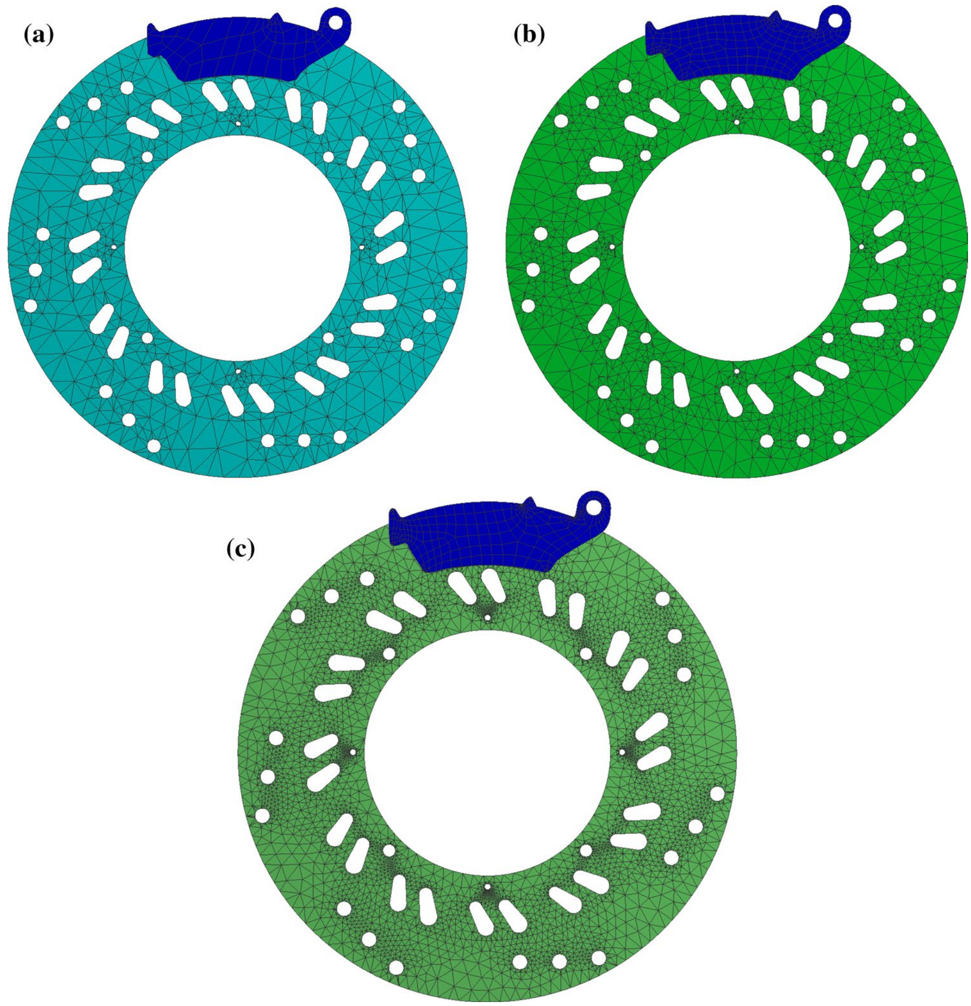


Fig. 8 Meshing strategies for FE analysis of the disc brake system; **a** Automatic mesh generated by ANSYS, **b** Intermediary refinement strategy, **c** refined mesh

Table 4 Mesh statistics and simulation results for comparison of the strategies

	Automatic (a)	Refinement 1 (b)	Refinement 2 (c)
Meshing strategies			
Nodes	15,369	34,553	103,195
Elements	6492	15,339	51,403
Computing time	7 min 50 s	32 min 4 s	2 h 56 min
Max. eigenvalue	9.12	1.57	9.92
Unstable modes	2	1	1

of interactions between factors on the response variable. The response variable considered was: total unstable points (modes), while the input variables are Young's modulus of the components (disc, friction material and back plate), coefficient of friction and the contact area of the friction material.

It is noticeable that both the coefficient of friction and the contact area of the friction material influence the number of unstable modes. An increase in the coefficient of friction causes a higher number of unstable modes, while there is a value for the contact area of the pad that produces the most unstable modes, while increasing or reducing the area leads to lesser instability (Fig. 11). The influence of the contact area on the number of unstable modes was experimentally proven by Bergman et al. [32], where the authors found experimentally that the contact area variation directly influences the pressure distribution variation, which induces a variation in the unstable modes. The influence of the friction coefficient was already expected for the simulation results, considering that friction is the most important factor for the occurrence of brake squeal phenomenon. Chen et al. [33], Fieldhouse et al. [34] and Lee et al. [35] demonstrated in their work the influence of friction on brake system instability, where the increase in friction value generated a significant increase in the number of brake system instabilities.

The influence of the Young's modulus of the components (disc, friction material and back plate) coupled with

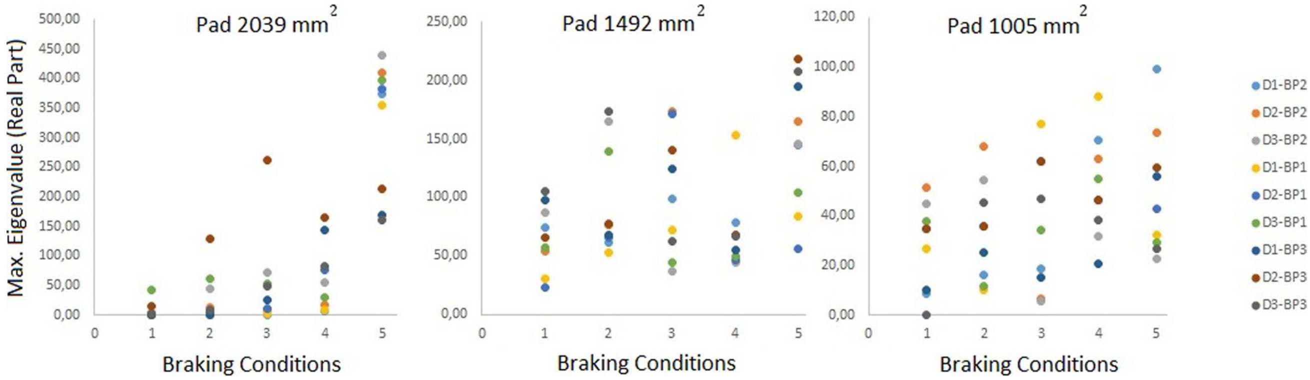


Fig. 9 Maximum Eigenvalue for the braking conditions in Table 2, for each contact area of the friction material in Fig. 2

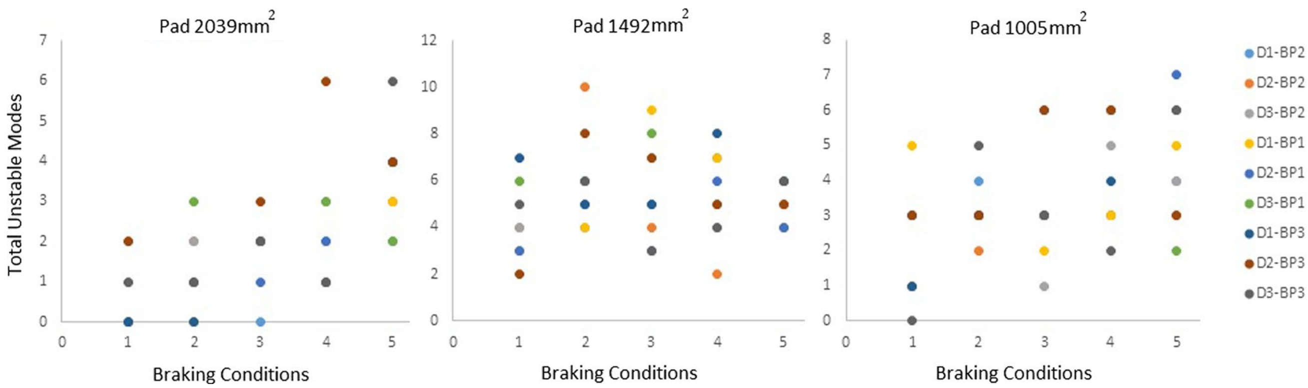


Fig. 10 Total unstable modes for the braking conditions in Table 2, for each contact area of the friction material in Fig. 2

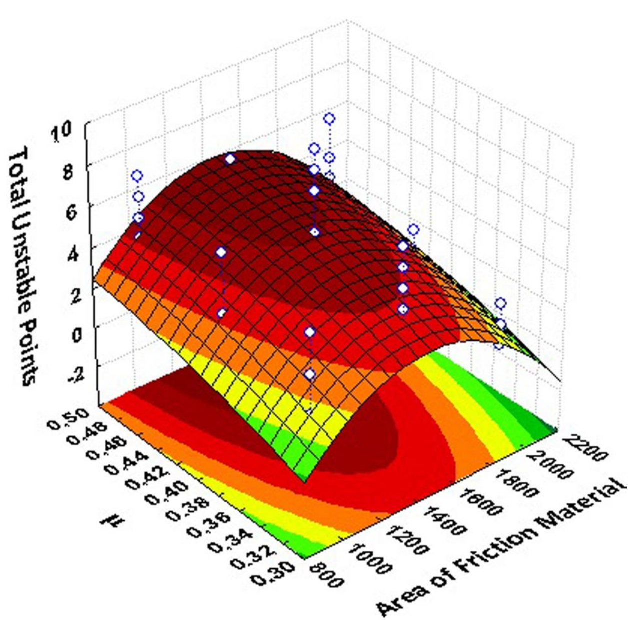


Fig. 11 Influence of the contact area of the friction material and the coefficient of friction (μ) on occurrence of unstable modes

the influence of the contact area of the friction material on the generation of unstable modes can be observed in Fig. 12. The contact area of the friction material presents the same behavior as in Fig. 11. Back plate Young's modulus has a central value that produces less instability, whereas a reduction or an increase in its value tends to produce more unstable modes in the coupled system. Disc Young's modulus variation, within the values of this study, bears no influence over occurrence of instability. Friction material Young's modulus, when decreased, will produce a higher number of unstable modes.

The influence of the Young's modulus of the components (disc, friction material and back plate) coupled with the influence of the coefficient of friction between the contact surfaces of the disc and the pads can be observed in Fig. 13. The coefficient of friction will produce more unstable modes in the system, as it increases, being the parameter that most influences the occurrence of unstable modes in the squeal spectrum. Comparing the influences of the coefficient of friction with the back plate Young's modulus on the total number of unstable modes (Fig. 13a)

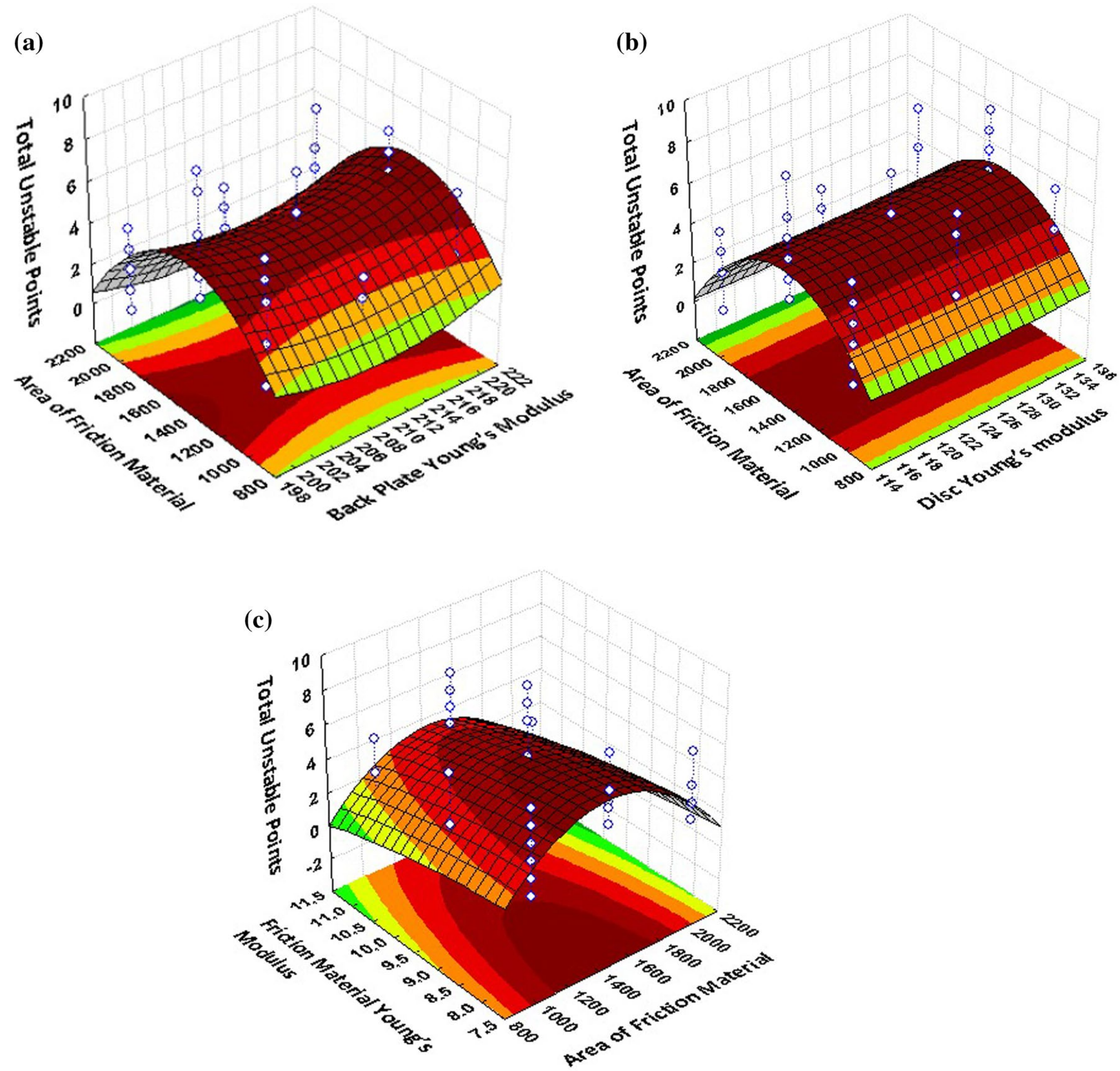


Fig. 12 Influence of the contact area of the friction material and Young's modulus of the components on occurrence of unstable modes; **a** back plate, **b** disc, **c** friction material

highlights a dominance by the coefficient of friction, but a slight influence of the back plate stiffness can be observed, just like in Fig. 12a, where a central value for the back plate Young's modulus produces less instability. Disc Young's modulus, in the range of values considered in this study, does not bear significant influence over the occurrence of instability, when confronted with the coefficient of friction (Fig. 13b). In Fig. 13c, it can be observed that the friction material Young's modulus bears little influence over the total unstable modes, when compared to the coefficient of friction. The results prove that friction material Young's modulus has greater influence on unstable mode variation than the Young's modulus of disc and back plate. The literature shows that friction material stiffness is a critical property for its application given that it influences the behavior of the coefficient of friction and, ergo, brake system stability [35, 36]. In the simulation, the increased stiffness of the motorcycle brake pad caused a reduction in the coefficient of friction (Table 2), causing a reduction in system instability. Back plate Young's modulus influence is well aligned with the observations in the literature. The main function of the back plate is to absorb the vibration and consequently to reduce the noise [37]. Therefore, the results demonstrate a worse damping condition for the back plate when varying the Young's modulus, which produces more unstable modes within the squeal zone. Disc Young's modulus did not show much interference in the variation of unstable modes given that the motorcycle

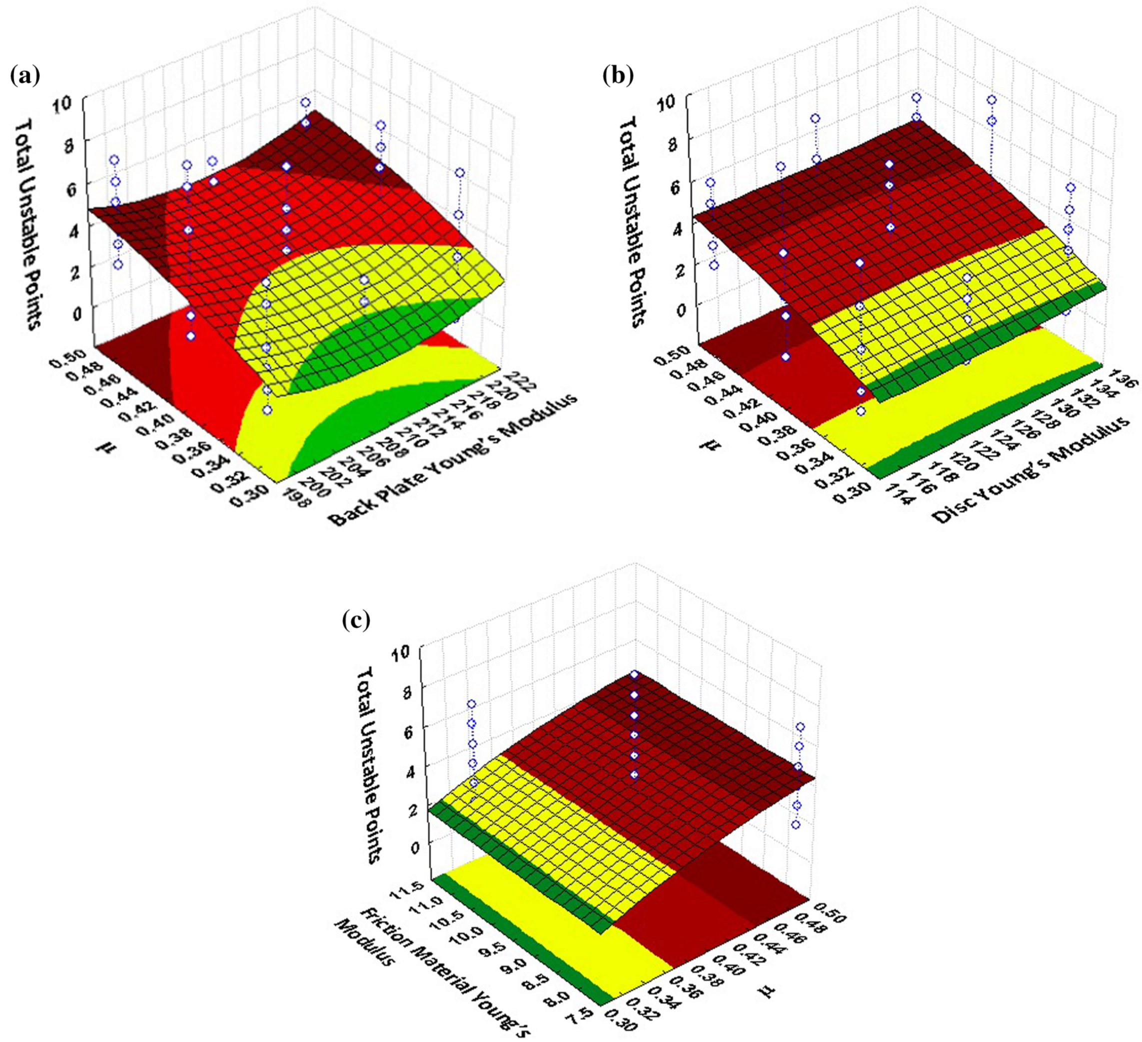


Fig. 13 Influence of the coefficient of friction and Young's modulus of the components on occurrence of unstable modes; **a** Back plate, **b** disc, **c** Friction material

brake disc has smaller size and mass than the car disc, which justifies its nonsignificant influence [12].

6 Conclusions

The effects of material properties and variations of the operating parameters of disc brake components on winch propensity were investigated using a simplified three-dimensional FE model. A reasonably good agreement is achieved between simulation results and the results of the literature. The results obtained in the simulations demonstrate that:

- The temperature plays an important role in squeal generation, because it influences, on direct proportionality, the coefficient of friction between the disc and the pads.
- Both velocity and pressure are inversely proportional to the coefficient of friction, so a decrease in either one causes an increase in the coefficient of friction, thereby making the system prone to squealing.
- The coefficient of friction has a relationship, almost linear, with instability occurrence within the squeal zone. A higher coefficient of friction means better braking performance, but also means poorer noise performance.
- It is noticeable that the contact area of the friction material influences the number of unstable modes. The contact area variation directly influences the pressure distribution variation, which, by consequence, induces a variation in the unstable modes. Pad geometry and contact area have a central value that produces more unstable modes, while smaller or larger contact areas generate less noise.

- The friction material Young's modulus has greater influence on unstable mode variation than the Young's modulus of disc and back plate. The increased stiffness of the motorcycle brake pad caused a reduction in the coefficient of friction, causing a reduction in system instability.
- The main function of the back plate is to absorb the vibration, and the results demonstrate a better damping condition for the back plate made of conventional steel (Young's modulus of 210GPa) which produces reductions in unstable modes.
- Disc Young's modulus did not show much interference in the variation of unstable modes given that the motorcycle brake disc has smaller size and mass than the car disc, which justifies its nonsignificant influence.

References

1. Kinkaid N, O'Reilly O, Papadopoulos P (2003) Automotive disc brake squeal: review. J Sound Vib 267:2003. https://doi.org/10.1016/s0022-460x(02)01573-0
2. Akay A (2002) Acoustics of friction. J Acoust Soc Am 111(4):1525–1548. https://doi.org/10.1121/1.1456514
3. Tan C, Chen F (2005) Disc brake squeal: mechanisms, analysis, evaluation, and reduction/prevention. Sae book monograph." Cap. 1 Mechanisms and Causes of Disc Brake Squeal, pp 1–25
4. Dunlap K, Riehle M, Longhouse R (1999) An investigative overview of automotive disc brake noise. In: International congress and exposition detroit, Michigan, USA: SAE. https://doi.org/10.4271/1999-01-0142
5. Matozo LT (2012) Estudo da Relação entre Propriedades de Compósitos de Fricção Aplicados a Sistemas de Freio a Disco e a Propensão à Ocorrência de Squeal Noise. Dissertation for the degree of Doctor of Philosophy. Federal University of Rio Grande do Sul, Brazil
6. Trichês JM, Gerges S, Jordan R (2004) Reduction of squeal noise from disc brake systems using constrained layer damping. J Braz Soc Mech Sci Eng 24:340–348. https://doi.org/10.1590/s1678-58782004000300011
7. Jordan R, Junior MT, Gerges S (2003). Determinação de propriedades mecânicas de materiais de fricção de pastilhas de freio através de ensaios dinâmicos. In: 6th international brake colloquium and engineering display, Caxias do Sul, RS: SAE, Brasil, pp 41–47
8. Ouyang H, Nack W, Yuan Y, Chen F (2005) Numerical analysis of automotive disc brake squeal: a review. Int J Veh Noise Vib 1:3–4. https://doi.org/10.1504/ijvnv.2005.007524
9. Liles GD (1989) Analysis of disc brake squeal using finite element methods. In: SAE paper 891150
10. Lee YS, Brooks PC, Barton DC, Crolla DA (1998) A study of disc brake squeal propensity using a parametric finite element model. In IMechE conference transaction, European conference on Noise and Vibration, pp 191–201
11. Kung S-W, Stelzer G, Belsky V, Bajer A (2003) Brake squeal analysis incorporating contact conditions and other nonlinear effects. In: SAE paper 2003-01-3343. https://doi.org/10.4271/2003-01-3343
12. Liu P, Zheng H, Cai C, Wang YY, Lu C, Ang KH, Liu GR (2007) Analysis of disc brake squeal using the complex eigenvalue method. Appl Acoust 68:603–615. https://doi.org/10.1016/j.apacoust.2006.03.012
13. Preston JD, Forthofer RJ (1971) Correlation of vehicle, dynamometer and other laboratory tests for brake friction materials. Society of Automotive Engineers, paper 710250. https://doi.org/10.4271/710250
14. Silva DT (2007) Determinação do Fluxo de Calor entre Pastilha e Disco de Freio Durante um Intervalo de Frenagem. Monografia, Escola de Engenharia - Universidade Federal do Rio Grande do Sul, Porto Alegre, RS, Brasil
15. Tirovic M, Todoronc J (1988) Flextural effects in commercial vehicle disc brake pads. Conference on Disc brakes for commercial vehicles, London, paper C455/88, pp. 47-52 (Institution of Mechanical Engineers, London). https://doi.org/10.1243/pime_proc_1978_192_002_02
16. Tsang PHS, Jacko MG, Rhee SK (1985) Comparison of chase and inertial dynamometer testing of automotive friction materials. In: Wear of materials: international conference on wear of materials, pp 129–137
17. Wilson AJ, Bowsher GT (1971) Machine testing for brake lining classification. Society of automotive engineers, paper 710249. https://doi.org/10.4271/710249
18. Tirovic M, Day AJ (1991) Disc brake interface pressure distributions. Proc Inst Mech Eng Part D J Autom Eng 205(2):137–146. https://doi.org/10.1243/pime_proc_1991_205_162_02
19. Rabinowicz E (1965) Friction and wear of materials, 2nd edn. Wiley
20. Duffour P (2002) Noise generation in vehicle brakes. Dissertation for the degree of Doctor of Philosophy. Engineering Department of Cambridge University
21. Neuman RF, McNinch JH (1983) Performance characterization of dry friction materials. In: IMechE conference publications (Institution of Mechanical Engineers). Published by Mechanical Engineering Publications for the Institution of Mechanical Engineers, London, p 233–238, ISSN 0144-0799
22. Belhocine A, Ghazaly NM (2015) Effects of material properties on generation of brake squeal noise using finite element method. Latin Am J Solids Struct 12:1432–1447. https://doi.org/10.1590/1679-78251520
23. Spurr RT (1961) A theory of brake squeal. Proc Autom Div Inst Mech Eng 1:33–40. https://doi.org/10.1243/pime_auto_1961_000_009_02
24. Neis PD (2012) Projeto e Construção de um Tribômetro com Controle Independente da Temperatura do Disco. Dissertation for the degree of Doctor of Philosophy. Federal University of Rio Grande do Sul, Brazil
25. Trichês Júnior M, Gerges SNY, Jordan R (2008) Analysis of brake squeal noise using the finite element method: A parametric study. Appl Acoust 69(2):147–162. https://doi.org/10.1016/j.apacoust.2007.10.003
26. Xiao X, Yin Y, Bao J, Lu L, Feng X (2016) Review on the friction and wear of brake materials. Adv Mech Eng 8(5):1–10. https://doi.org/10.1177/1687814016647300
27. Dezi M, Forte P, Frendo F (2013) Motorcycle brake squeal: experimental and numerical investigation on a case study. Meccanica 49(4):1011–1021. https://doi.org/10.1007/s11012-013-9848-y
28. Bajer A, Belsky V, Kung SW (2004) The influence of friction-induced damping and nonlinear effects on brake squeal analysis. In: SAE technical paper 2004-01-2794. https://doi.org/10.4271/2004-01-2794
29. Esgandari M, Taulbut R, Olatunbosun O (2013) Effect of damping in complex eigenvalue analysis of brake noise to control overprediction of instabilities: an experimental study. In: SAE technical paper 2013-01-2034. https://doi.org/10.4271/2013-01-2034
30. Silva JGP, Fulco ER, Varante PED, Nascimento V, Diesel FN, Boniatti DL (2013) Numerical and experimental evaluation of

brake squeal. In: SAE technical paper 2013-36-0030. https://doi.org/10.4271/2013-36-0030
31. Nouby M, Srinivasan K (2013) Parametric studies of disc brake squeal using finite element approach. J Mek 29:52–66
32. Bergman F, Eriksson M, Jacobson S (2000) The effect of reduced contact area on the occurrence of disc brake squeals for an automotive brake pad. Proc Inst Mech Eng Part D J Autom Eng 214(5):561–568. https://doi.org/10.1243/0954407001527844
33. Chen F, Tong H, Chen SE, Quaglia R (2003) On automotive disc brake squeal part IV reduction and prevention. SAE technical paper series. https://doi.org/10.4271/2003-01-3345
34. Fieldhouse JD, Steel WP (2003) A study of brake noise and the influence of the centre of pressure at the disc/pad interface, the coefficient of friction and calliper mounting geometry. Proc Inst Mech Eng Part D J Autom Eng 217(11):957–973. https://doi.org/10.1243/095440703770383866
35. Lee YS, Brooks PC, Barton DC, Crolla DA (2003) A predictive tool to evaluate disc brake squeal propensity. Part 1: the model philosophy and the contact problem. Int J Veh Des 31(3):289. https://doi.org/10.1504/ijvd.2003.003360
36. Joe Y-G, Cha B-G, Sim H-J, Lee H-J, Oh J-E (2008) Analysis of disc brake instability due to friction-induced vibration using a distributed parameter model. Int J Autom Technol 9(2):161–171. https://doi.org/10.1007/s12239-008-0021-x
37. Lee YS, Brooks PC, Barton DC, Crolla DA (2003) A predictive tool to evaluate disc brake squeal propensity Part 3: parametric design studies. Int J Veh Des 31(3):330. https://doi.org/10.1504/ijvd.2003.003364