TECHNICAL PAPER

Error control of non‑equal diameter machining of globoidal cam profle based on adaptive method

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Abstract

In order to efectively tackle the machining error of globoidal cam profle, an adaptive compensation method of tool position error was proposed. The actual profle equation of the non-equal diameter machining of globoidal cam was constructed through the space meshing principle and the rotation transformation tensor method. To minimize the maximum deviation of the actual tool axis was regarded as an objective, and then the adaptive compensation of the tool position was performed through searching the optimal tool ofset amount and direction during the machining process. When tool radius compensation amount Δ*r* was the value optimized by the adaptive method, the theoretical maximum value of the normal error of globoidal cam was closest to the simulation result. The validity of the error control technique of tool position of the adaptive method was verifed by the simulation results and calculation. Consequently, the machining accuracy of the globoidal cam in nonequal diameter machining is improved greatly.

Keywords Adaptive compensation method · Globoidal cam · Non-equal diameter machining · Error control

1 Introduction

As an efficient and simple transmission indexing mechanism, globoidal cam has broad application prospects. The globoidal cam mechanism is not only of high precision, compact structure, light weight and high yield, but also of strong carrying capacity, good dynamic performance and high reliability. Consequently globoidal cam mechanism plays a pivotal role in mechanical transmission, and widely served in automobile manufacturing, textile, packaging service, aerospace, automatic machine tools and other felds [\[1](#page-8-0)].

Globoidal cam is a precision mechanical element, and cam production is a complicated task [[2\]](#page-8-1). The NC machining methods of globoidal cam are generally divided into two

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categories, i.e., equal diameter machining and non-equal diameter machining [[3\]](#page-8-2). Most recently, the machining methods of globoidal cam are divided into two categories: equal diameter machining and non-equal diameter machining [\[4](#page-8-3)]. The tool with the same geometrical parameters as the roller surface is regularly devoted in equal diameter machining process, and then the tool and the workpiece moved in a conjugate motion by the mechanism meshing transmission so as to machine globoidal cam. Although equal diameter machining is implemented easily, its accuracy is poor. The nonequal diameter machining is that globoidal cam is machined with the smaller tool radius than the roller, which is generally superior to equal diameter machining. Compared with equal diameter machining, the machining efficiency can be improved with concurrently low production cost by using non-equal diameter machining.

The machining error cannot be disregarded during the machining process of globoidal cam. The machining accuracy is typically afected by the principal factors such as machining error, programming error, installation error and tool wear error. The machining method of globoidal cam was analyzed based upon the theoretical error. The original theoretical errors exist in resembling freeform surface machining and tool compensation method. For several decades, the focus of research has been on the tool position optimization of globoidal cam

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in non-equal diameter fank milling. An improved optimal deviation method to study the more reasonable allocation of globoidal cam mechanism tolerance was proposed by Yang Shiping [[5\]](#page-8-4). The measuring principle of equidistant model was analyzed preliminarily, and then cam profle error of the equidistant model was evaluated through measuring the feature line [[6\]](#page-8-5). A novel method was devised by Bu Fanhua [\[7](#page-8-6)] to refne the profle error, and then the minimum distance from a point on the actual profle machined to the desired one was defned as the profle error; ultimately, the mathematical model of the caused profle error was established through the rotational deviation of the workpiece. As claimed by the minimum deviation between the trajectory surface of tool axis and the equidistant surface of globoidal cam, subsequently an optimization algorithm was presented for calculating the tool position data of the single-sided NC machining of globoidal cam [[8](#page-8-7)]. In order to resolve the problems of the approximation error resulted from data point discretization and great fuctuation of the ruled surface of tool path, a tool path optimization algorithm of spatial cam fank milling was conducted by HU [\[9](#page-8-8)] based upon NURBS surface. The normal error calculation of tool position compensation was carried out by Ge Rongyu [\[10](#page-8-9)]; consequently, the tool position control was effectively optimized and the machining error was reduced simultaneously. The bow height error was calculated by Rong-Shean Lee through generating method and resembling freeform surface method [[11](#page-8-10)], and subsequently a tool path generating method to curb bow height error was discovered; thus, its error was reduced to large extent. Nevertheless, this method is just suitable for equal-step process; accordingly, the application range is much limited. The bow and chord height error during the machining process was investigated by $J N$ Lee $[12]$ $[12]$, and then a scheme to reduce the error was propounded. The large amount research achievements in profle error analysis and measurement of globoidal cam have been made by Xiangtan University [\[13](#page-8-12)]. The special machine tool to machine globoidal cam was analyzed, and then its error was evaluated by Zhang [\[14\]](#page-8-13). The relationship between the helical angle of globoidal cam and the tool radius compensation was established by Chaiqing [\[15](#page-8-14)]; subsequently, the *X*-axis and *Z*-axis were compensated, respectively; eventually, theoretical error of globoidal cam was eliminated to large extent. Based on the equidistant surface theory, the cam globoidal profle equation was derived and concurrently the mathematical model of the machining error was conducted by Ji Shuting [[16\]](#page-8-15), and the infuence of the center distance error on the transmission accuracy and the meshing performance of cam was manipulated obviously. In the aforementioned literature, the real-time local errors during the machining process of globoidal cam was just considered, whereas the generation rules of the whole tool path were not investigated deeply in NC machining. In order to establish the equidistant surface of the cam profle, the theoretical tool path surface needs to be discretized primarily and then reconstructed. However, the initial samples data required are large, and the acquisition process is awkward and inefficiency. With the advancement of manufacturing technology and diversifcation of market demands, the machining methods of globoidal cam are increasingly multipliable. Complicated and rawness as the theory of non-equal diameter machining is, non-equal diameter machining is still of great signifcance to globoidal cam and other complex surfaces. Therefore, a novel tool position control technique based upon the adaptive method with the tool radius smaller than the roller was devised in order to efectively control the machining error of cam profle.

2 The mathematical model of globoidal cam

The simplifed model of the globoidal cam mechanism is shown in Fig. [1](#page-2-0), where $o - xyz$ stands for a fixed coordinate system, *z* denotes the axis direction of the driven turntable, and *x*-axis direction is from the driven turntable center to the cam center. $o_1 - x_1y_1z_1$ denotes a dynamic coordinate system, which is connected to the driven turntable and rotates with it. x_1 axis is coincided with the roller axis and turns around the point o with the roller, and α_1 means the angular displacement of the driven turntable. $o_2 - x_2y_2z_2$ denotes the dynamic coordinate system fxed with the cam and rotates around y_2 axis with the cam. The α_2 means the angle between x_2 and x axis, c denotes the center distance from the driven turntable to cam center, β is the contact angle of the roller and cam at the contact point *D*, and the n_1 represents the unit vector of the common normal at the point *D* between the roller and cam.

The position vector of the contact point *D* on the roller in $o_1 - x_1y_1z_1$ can be expressed as follows:

$$
\boldsymbol{R}_1 = (l_0 + l, R\cos\beta, R\sin\beta)^{\mathrm{T}} \tag{1}
$$

where l_0 denotes the distance from the driven turntable center to the roller inner end, and the *l* is the meshing depth, i.e., the distance from the roller inner end to the contact point *D*.

In light of Ref. [[17\]](#page-8-16) and the meshing principle of conjugate surface, the relative velocity direction between the two surfaces is orthogonal to each other at the common normal. Then the unit normal vector of the contact point *D* on the roller of $o_1 - x_1y_1z_1$ is:

$$
\boldsymbol{n}_1 = \frac{\frac{\partial R_1}{\partial l} \times \frac{\partial R_1}{\partial \beta}}{\left| \frac{\partial R_1}{\partial l} \times \frac{\partial R_1}{\partial \beta} \right|} = \begin{pmatrix} 0\\ \cos \beta\\ \sin \beta \end{pmatrix} = (0, \cos \beta, \sin \beta)^T
$$
(2)

Suppose F_1 and F_2 are, respectively, the angular velocity vectors of the roller and cam of the fxed coordinate system *o* − *xyz*. Thus, the relative velocity at the contact point *D* of the meshing surface is drawn:

$$
v = F_1 \times R_1 - F_2 \times (R_1 - c) \tag{3}
$$

where $c = (c, 0, 0)^T$.

The meshing equation of the conjugate surface between the roller work surface and the globoidal cam profle is as follows:

$$
\tan \beta = \pm \frac{(l+l_0)}{[c - (l+l_0)\cos\alpha_1]} \left(\frac{\omega_1}{\omega_2}\right)
$$
 (4)

In Eq. [\(4](#page-2-1)), the "+" means a left-handed cam, "−" a righthanded one, ω_1 denotes the driven turntable angular velocity, and ω_2 represents globoidal cam angular velocity.

3 Tool position determination of the non‑equal machining of globoidal cam

3.1 Machinability analysis of contact line

The globoidal cam profle at scale division is a non-developable space surface. When the tool error occurs, the theoretical meshing line of tool and cam and the actual meshing line of the roller and cam are non-coincident. The contact line between cam and roller is a curve along the roller surface rather than a straight line; subsequently, the curve of the cam cannot be machined with the desired one no matter how tool radius is compensated; eventually, the globoidal cam profle error will exist. Therefore, globoidal cam groove cannot be machined by the same tool compensation mode as planar cam. If the tool radius is smaller than the roller radius during the machining process, there still is programming error itself no matter how to control the tool position. Fortunately, the machining error can be refned though an appropriate control method of tool position. The error size of globoidal cam

profle is determined by the position relationship between the theoretical contact line and the actual one. The two contact lines distance will closer and its coincidence degree is better concurrently through the proper tool position control technique; ultimately, the machining accuracy of the cam profle is more perfect.

The machining error is a significant index reflecting generally the machining accuracy of globoidal cam, and composed by tool error, programming error, theoretical error, actual error, etc. The machining error was also the other comprehensive result of all kinds of errors. Tool error was one of machining errors caused by tool position compensation, and the more diference between the tool and roller radius existed, the larger the machining error caused tool position compensation. Tool error was also the main infuence factors of globoidal cam researched in this paper, and it consisted of under-cut and over-cut states, as shown in Fig. [2](#page-2-2). An arbitrary contact line MN on the roller was taken and analyzed subsequently. When the generating surface of globoidal cam was non-developable, MN was a spatial curve on the roller surface. While a smaller cutter was adopted to machine cams, a certain offset compensation $\Delta_r = R - r$ was implemented; subsequently, the cutter and roller were tangent to the generatrix L. The generatrix L and MN intersected at point B, which denoted that the tool can reach B point on the theoretical contact line at most. Therefore, all

Fig. 2 The tool compensation error status

points except point B on MN were in over-cut or under-state, which resulted in tool position compensation error.

The tool compensation method is carried out by tool position offset through the generating machining of cam profile so as to minimize the machining error [[18](#page-8-17)]. The machining principle of the tool position compensation method is illustrated in Fig. [3](#page-3-0) where *R* denotes the roller radius, *r* is the tool radius, *A* and *D*, respectively, stand for the contact points of the tool and roller, the curve *MN* (i.e., the actual contact line) refers to the contact line of the roller and cam, *PQ* (i.e., the theoretical contact line) means the contact line between the tool and cam profle, *B* is the intersection point of the theoretical contact line and the actual one. When the tool radius is smaller than the roller, suppose it is offset by the compensation amount $\Delta_r = R - r$, then the roller and tool are tangent at L. Besides, when the actual contact line during the machining process is only *PQ*, the tool can reach to point *B*(i.e., zero machining error of tool radius compensation), while other points on the *MN* are in under-cut state, then under-cut error exists subsequently [\[19\]](#page-8-18). In this way, although the theoretical contact line and the actual one are on diferent cylinders, the two contact lines are intersected at a point *B* at half the length of the roller contact line; subsequently, the normal error of the profle of globoidal cam is 0; ultimately, the minimum error of the two contact line is realized.

3.2 The determination of tool position ofset

In order to polish the adverse efects like tool wear, complicated cam groove widths, and the machining allowance, etc.,

Fig. 3 The tool compensation diagram

consequently the non-equal diameter machining was adopted. Suppose the tool center is at a meshing angle β_2 of half the width of roller, and its distance from the center of the roller is Δ_r . As shown in Fig. [4](#page-3-1), β_1 denotes the meshing angle at half of the tool axis between the tool and cam (also called the contact angle of equal diameter machining), where β_2 refers to contact angle of the non-equal diameter machining, *K* represents the contact point between roller and the cam; *H* stands for the contact point between the non-equal diameter machining tool and the actual machining profile Σ_1 , *G* means the contact point between non-equal diameter machining tool and the theoretical machining profile Σ_2 , n_2 is the normal at the contact point *K*, n_f is the normal of the contact point of the actual work profile; n_d is the normal of the contact point of the theoretical work profile, $\Delta_r \sin\beta_1$ denotes the offset amount of the tool in the *y* -axis direction in the dynamic coordinate $o_1 - x_1y_1z_1$, $\Delta_r cos\beta_1$ is the offset amount of the tool in the *z*-axis direction.

In the coordinate system $o_1 - x_1y_1z_1$, the *D* vector coordinates at the meshing points of tool and cam in any crosssection $(l + l_0)$ can be derived:

$$
(\boldsymbol{R}_H)_1 = \begin{bmatrix} -(l + l_0) \\ \Delta_r \cos \beta_1 + R \cos \beta_2 \\ \Delta_r \sin \beta_1 + R \sin \beta_2 \end{bmatrix}
$$
 (5)

The coordinate transformation matrix of the meshing point *D* from the dynamic coordinate system $o_1 - x_1y_1z_1$ into the fxed coordinate system *o* − *xyz* is:

$$
\boldsymbol{M}_{10} = \begin{bmatrix} \cos \alpha_1 & -\sin \alpha_1 & 0 \\ 0 & 0 & 1 \\ \sin \alpha_1 & \cos \alpha_1 & 0 \end{bmatrix}
$$
 (6)

In the dynamic coordinate system o_1 - $x_1y_1z_1$, the unit normal vector over the meshing point *D* is:

Fig. 4 The tool center position of the non-equal diameter machining

$$
\boldsymbol{n}_{H1} = \begin{vmatrix} i & j & k \\ \partial a/\partial l & \partial b/\partial l & \partial c/\partial l \\ \partial a/\partial \beta_2 & \partial b/\partial \beta_2 & \partial c/\partial \beta_2 \end{vmatrix} = \cos \beta_j \boldsymbol{j} + \sin \beta_1 \boldsymbol{k} \quad (7)
$$

where $a = -(l + l_0), b = \Delta_r cos\beta_1 + Rcos\beta, c = \Delta_r sin\beta_1 + Rsin\beta_2$. Then the actual machining working profle equation of the non-equal diameter machining of globoidal cam was drawn.

$$
\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} [c - \cos \alpha_1 (l + l_0) - (R \cos \beta_0 + \Delta_r \cos \beta_1) \\ \sin \alpha_1] \cos \alpha_2 - (R \sin \beta_0 + \Delta_r \sin \beta_1) \sin \alpha_2 \\ [c - \cos \alpha_1 (l + l_0) - (R \cos \beta_0 + \Delta_r \cos \beta_1) \\ \sin \alpha_1] + (R \sin \beta_0 + \Delta_r \sin \beta_1) \sin \alpha_2 \\ - \sin \alpha_1 (l + l_0) + (R \cos \beta_0 + \Delta_r \cos \beta_1) \sin \alpha_2 \end{pmatrix}
$$
(8)

Consequently, the meshing equation of the non-equal diameter machining of globoidal cam was derived.

$$
\beta_0 = \arctg \frac{\frac{d\alpha_1}{d\alpha_2}(l + l_0) - \Delta_r \sin \alpha_1 \sin \beta_1}{c - \cos \alpha_1 (l + l_0) \Delta_r \cos \beta_1 \sin \alpha_2}
$$
(9)

where β_0 and $\pi + \beta_0$ correspond to the upper and lower profle of globoidal cam, respectively.

3.3 The normal error calculation of cam profle

As illuminated in Fig. [4,](#page-3-1) the theoretical machining profle Σ_2 and the actual machining profile Σ_1 both were equidistant curved surfaces. It is drawn that the normal n_f at the contact point of the actual machining profle is approximately parallel to the normal n_d at the contact point of theoretical one. Suppose the normal over any point *P* on the actual machining profile Σ_1 intersects with the theoretical one Σ_2 at point *Q*, and thus |*PQ*| is the normal error of globoidal cam at point *P*. Consequently, the normal equation of the *P* was derived.

$$
\frac{x - x_p}{p_x} = \frac{y - y_p}{p_y} = \frac{z - z_p}{p_z} \tag{10}
$$

$$
\begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} -\sin \alpha_1 \cos \alpha_2 \cos \beta - \sin \alpha_2 \sin \beta \\ \sin \alpha_1 \sin \alpha_2 \cos \beta - \cos \alpha_2 \sin \beta \\ \cos \alpha_1 \cos \beta \end{pmatrix}
$$
(11)

According to the meshing principle of the roller and cam profle, when the machining error exists, the point *H* on the actual profle surface is in meshing state, whereas the point *G* on the theoretical profle may not in meshing state or has already been entered meshing state. Therefore, Eq. [\(10](#page-4-0)) can be transformed to:

$$
\begin{cases}\n x = x_P + \frac{p_x}{p_y}(y - y_P) \\
 z = z_P + \frac{p_z}{p_z}(y - y_P)\n\end{cases}
$$
\n(12)

Equation (12) (12) was a set of binary nonlinear equations with respect to $\alpha(k)$, $l(k)$, it can be solved by the Newton iteration method, and the point *P* was the initial point of the iteration.

$$
\begin{cases}\nf_1 = x - x_P - \frac{p_x}{p_y}(y - y_P) \\
f_2 = z - z_P - \frac{p_x}{p_y}(y - y_P)\n\end{cases}
$$
\n(13)

Equation (14) (14) was derived through expanded Eq. (13) (13) by using upon Newton's iteration method.

$$
\begin{cases} \frac{\partial f_1}{\partial \alpha_2}(\alpha_{20}, l_0) \Delta \alpha_2 + \frac{\partial f_1}{\partial l}(\alpha_{20}, l_0) \Delta l = -f_1(\alpha_{20}, l_0) \\ \frac{\partial f_2}{\partial \alpha_2}(\alpha_{20}, l_0) \Delta \alpha_2 + \frac{\partial f_2}{\partial l}(\alpha_{20}, l_0) \Delta l = -f_2(\alpha_{20}, l_0) \end{cases}
$$
(14)

where

$$
\Delta = \begin{vmatrix} \frac{\partial f_1}{\partial \alpha_2} & \frac{\partial f_1}{\partial l} \\ \frac{\partial f_2}{\partial \alpha_2} & \frac{\partial f_2}{\partial l} \end{vmatrix}, \quad \xi = \begin{vmatrix} -f_1 & \frac{\partial f_1}{\partial l} \\ -f_2 & \frac{\partial f_2}{\partial l} \end{vmatrix}, \quad \psi = \begin{vmatrix} \frac{\partial f_1}{\partial \alpha_2} & -f_1 \\ \frac{\partial f_2}{\partial \alpha_2} & -f_2 \end{vmatrix}, \quad \begin{cases} \Delta \alpha_2 = \frac{\zeta}{\Delta} \\ \Delta l = \frac{\psi}{\Delta} \end{cases}.
$$

where α_{20} , l_0 mean the initial value of iteration, i.e., the parameters of the actual working profile of globoidal cam corresponding to point *P*. In the actual machining process, the machining error was generally small, which ensures the convergence of the nonlinear equations. When $|a_{i+1} - a_i|$ ≤ 10^{−6} and $|l_{i+1} - l_i|$ ≤ 10^{−6}, the iteration was
completed and the parameters α , and *l*, with regard to completed, and the parameters α_{2n} and l_{2n} with regard to intersection point *G* of the theoretical profile Σ_2 can be obtained. Consequently, $\alpha_{2n} = \alpha_{i+1} l_{2n} = l_{i+1}$, and the normal error of cam profle was calculated:

$$
\Delta n = |PQ| = \sqrt{(x_p - x_Q)^2 + (y_p - y_Q)^2 + (z_p - z_Q)^2} \quad (15)
$$

4 The optimization algorithm of adaptive compensation of tool position

An instantaneous α_2 of the cam was given, and thus the profle equation was expressed as a spatial curve *MN* (in Fig. [2\)](#page-2-2) along the roller axis through tool position compensation. Suppose that the tool position compensation was performed along the normal direction at point *B* on the theoretical contact line, besides the direction was O_1B , and the contact angle in this direction was β (*l*, α); consequently, O_1O_2 was the compensation amount. The compensation amount was set to Δ_r uniformly in order to establish the tool position optimization model. Eventually, the actual contact line *PQ* was drawn after tool position compensation. As for the determination of compensated tool center position, the tool center coordinates in cam coordinate system were obtained through coordinate transformation based upon the rotation transformation tensor

method and the diferential geometry principle. As shown in Fig. 5 , $o_1 - x_1y_1z_1$ denotes a dynamic coordinate system fixed to the driven turntable and rotates with it. The x_1 coincides with the roller axis and rotates around *O* point with the roller, and the α_1 means the angular displacement of the driven turntable. $o_2 - x_2y_2z_2$ represents a dynamic coordinate system fixed to the cam and rotates around the y_2 -axis with the cam, where α_2 stands for the angle between x_2 and x . *W'* means a point on the roller axis, also known as the tool center position before compensation, while the point *W* refers to the tool center position after compensation. The vertical distance from the tool center cross-section to the O_1 of the $o_1 - x_1y_1z_1$ *c*oordinate system was set *l*, and the tool feed rate in each step was set to a fxed value *l* ′ .

In roller coordinate system, the vector of tool center position W' before compensation was t_1 , and it was also a function with regard to the feed rate parameter *l'* in each machining step. Eventually, the tool center vector *W* after tool position compensation was derived.

$$
t'_{1}(\alpha_{2})\Big|_{l'} = t_{1}(l') + \Delta_{r} \cdot n(l, \alpha_{2})
$$
\n(16)

where $n(l, \alpha_2)$ denotes the compensation direction, Δ_r represents the tool radius compensation amount. During the error control process, the compensation direction of tool position was generally the normal vector direction at any point on the theoretical contact line, and thus $n(l, \alpha_2)$ was a binary function with regard to the parameter *l* along tool axis and the cam angle α_2 . In light of transformation matrix M_{10} from the dynamic coordinate system $o_1 - x_1y_1z_1$ to the fixed coordinate system $o - xyz$, consequently the tool center position in the cam machining process was derived.

$$
t_2(\alpha_2)|_{l'} = M_{10} \cdot t_1(l')|_{l'} \tag{17}
$$

The selection of tool position compensation direction and amount plays a pivotal role in non-equal diameter machining of globoidal cam. In the machining process, if

Fig. 5 The tool center coordinates in compensation process

the compensation amount Δ_r and compensation direction $n(l, \alpha_2)$ were optimized and compensated reasonably, the normal error Δ*n* was a binary function with regard to the compensation amount and direction. Then the minimum value of the normal error was taken as the objective function; eventually, the optimized value can be obtained through the adaptive compensation optimization of tool position. The tool position selection and compensation of a certain point on the contact line were optimized properly based upon the adaptive method optimization. In order to efectively shorten the optimization compensation time, a small section of the middle position of the radial efective length of the roller was regarded as the domain of the compensation parameter $l \in (D/2, \sigma)$ during the selection process of compensation direction. As for compensation amount, a small section of radius difference Δ_r between the tool and roller was served as a domain, i.e., $\Delta \in C(\Delta_r, \sigma)$. Subsequently, the optimization algorithm of the tool position compensation was transformed into the optimization problem of two-dimension nonlinear constraint interval. Ultimately, the optimal solution can be solved by using the fmicon function in the optimization toolbox of MATLAB, i.e., the optimal tool compensation amount and direction were obtained.

5 The comparison between theoretical error and simulation of globoidal cam

In order to efectively verify the validity of adaptive tool position control method proposed for compensating cam profle machining error, the simulation experiment was performed subsequently. In view of the tool path surface of globoidal cam working profle, the diferent tool radius compensation was served to machine the cam profle. The accurate errors value in Table [1](#page-6-0) were acquired by the coordinate measuring machine (CMM), while the approximate errors were obtained through the adaptive method programming by MATLAB. The normal errors in Figs. [7](#page-7-0) and [8](#page-7-1) were calculated by Eq. [15](#page-4-4) in Chapter 3. Eventually, the normal error of cam profle was analyzed and compared concomitantly through the error distribution drawings of the reconstruction method and adaptive method, respectively.

The error calculation of a cam profile was analyzed through a specifc globoidal cam mechanism. The parameters are as follows: center distance $c = 180$ mm, 77 mm $\leq l_0 \leq 97$ mm, roller radius R = 20 mm, roller width 20 mm, cam rest angle 240°, and the number of driven turntable roller 8, and the modifed sine acceleration was considered as the periodic rule of turntable indexing. For diferent tool radius compensation amounts of Δ_r , the precision value of the normal error of cam profle caused by the tool wear error and the approximate value of the simulation experiment are, respectively, shown in Table [1.](#page-6-0) The rule can be

drawn easily from Table [2](#page-6-1) is that the profle error is apparently proportional to Δ_r .

According to the results in Table [2](#page-6-1) and the tool position determination through the adaptive optimization method, consequently the diference between the maximum normal error and the approximate value is very small, which is 10^{-5} mm order of magnitude; besides, the optimal compensation is gained when $\Delta_r = 3$ mm. The relationship between the optimal compensation position through the adaptive optimization method and the cam angle is illustrated in Fig. [6.](#page-6-2)

As shown in Table [2](#page-6-1), the profle error escalates gradually with the increase in tool radius compensation, and the maximum error is distributed linearly. So as to efectively reduce the normal error of the cam profle caused by the tool and maintain its accuracy, the tool radius compensation Δ*r*=3 mm was selected. The distribution of normal error along the roller axis cross-section is shown in Fig. [6.](#page-6-2)

In Fig. [7,](#page-7-0) when the tool radius compensation is different, the profle normal error curve is parabolic with the elevation of the roller cross-section position, and the intermediate error is nearly 0. The trajectory surface of the roller axis is concave, and thus the modifed drum roller can be served to carry out the meshing transmission. It can be drawn a striking conclusion from Fig. [7](#page-7-0) that when the tool radius compensation is $\Delta_r = 3$ mm, the normal error of the globoidal cam is signifcantly lower than when the tool radius compensation is $\Delta_r = 4$ mm and 5 mm. In addition, when tool radius compensation is $\Delta_r = 3$ mm and

Fig. 6 The relationship between optimal compensation position and cam angle

concomitantly the cam angle is 80°, the minimum of the normal error almost approaches 0. The maximum normal error Δn = 0.015154 mm of the cam is also calculated by the adaptive compensation method, and only occurs once.

In addition, in order to clearly verify the superiority of the adaptive compensation method, then globoidal cam is machined by the average contact angle compensation method, the roller midpoint compensation method and the adaptive compensation method, respectively; eventually, the maximum normal error is worked out. The variation of the maximum normal error with respect to cam angle of the three compensation methods is described in Fig. [8](#page-7-1). It is clearly observed that the adaptive compensation method is apparently superior to the roller midpoint compensation method and the average contact angle compensation method. The machining error of the average contact angle compensation method is 12.8% less than the roller midpoint compensation method, while the machining error of the adaptive compensation method is 9.73% less than the average contact angle compensation method. Consequently, the profle error of globoidal cam is signifcantly

Table 2 The error types of globoidal cam

The error types	The formed reasons	Relationships
The tool error	The different tool compensation radius	Related to Δ_r
The programming errors	The interpolation algorithm error	Belongs to system error
The theoretical error	The calculation error of mathematical model of theory	Verified by actual error
The actual error	The actual error during the machining process	Verify theoretical error
The normal error	The difference between actual machining profile and theoretical ones in normal direction	Related to tool compen- sation direction and amount

Fig. 7 The normal error distribution of the roller axis section

Fig. 8 $\Delta_r = 3$ mm, the comparison of normal errors

reduced by using adaptive tool position compensation method.

So as to further verify the validity of the adaptive method proposed and simulation results in this paper, the 3D model of globoidal cam mechanism was obtained by importing the model parameters and related data into UG8.5 parametric design module. Let the radius of driven turntable be 20 mm and the length be 180 mm, the movement cycle of the turntable be a curve of sines, and the indexing cycle of the globoidal cam be 120°, and thus a 3D model of globoidal cam mechanism was exported as shown in Fig. [8.](#page-7-1) Suppose the tool radius is 6 mm, then the data points of the profle were collected by discrete method, and thus the theoretical machining profle was obtained by surface reconstruction. The critical condition of machining error is set to 0.01 mm

Fig. 9 The 3D model of globiodal indexing cam mechanism

Fig. 10 The profle error distribution after reconstruction

in light of the requirement of machining accuracy (Fig. [9](#page-7-2)). The reconstructed profle error distribution map is shown in Fig. [10,](#page-7-3) while the profle error distribution map compensated by the adaptive compensation method is shown in Fig. [11.](#page-8-19) The dim part denotes that the ftting error is inferior to the critical value, while the yellow part is superior to the critical value. The revised tool path drawing is obtained through the machining data compensated imported into the machining module based on the adaptive method, such as illustrated in Fig. [11](#page-8-19). Comparing Fig. [10](#page-7-3) with Fig. [11,](#page-8-19) the remarkable conclusion is drawn that the machining error can be efectively controlled through the tool path compensated by the adaptive method (Fig. [12](#page-8-20)).

6 Conclusion

(1) The novel tool position control method is subtly adopted so as to perfectly machine globoidal cam, and then the machining error of cam profle caused by tool position compensation is reduced greatly; ultimately,

Fig. 11 The profle error distribution after compensation with the adaptive method

Fig. 12 The adaptive optimization tool path

the machining accuracy of cam profle is further elevated simultaneously.

- (2) The machined profle error of the globoidal cam can be analyzed and controlled concurrently within the allowable tolerance range more efectively in real time according to derived calculation of the normal error of globoidal cam.
- (3) The applicable range and service life are enlarged, respectively, by presented method; besides, the tool cost is saved to large extent concomitantly.

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Compliance with ethical standards

Conflict of interest The authors declare no confict of interests regarding the publication of this paper.

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