**TECHNICAL PAPER** 



# Generalized diffusion effects on Maxwell nanofluid stagnation point flow over a stretchable sheet with slip conditions and chemical reaction

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#### Abstract

The aim of this article is to investigate the heat and mass diffusion (Cattaneo–Christov model) of the upper convected Maxwell nanomaterials passed by a linear stretched surface (slip surface) near the stagnation point region. Convocational Fourier's and Fick's laws are employed to investigate heat and mass diffusion phenomena. Using the similarity transformations, the governing PDEs are rendered into ODEs along with boundary conditions. The boundary value problem is solved numerically using RK-4 method along with shooting technique (Cash and Karp). The effects of embedded parameters, namely fluid relaxation parameter, Hartmann number, Brownian moment, thermophoresis parameter, thermal relaxation parameter, Lewis number, chemical reactions concentration relaxation parameter, and slip parameter on velocity, temperature, and concentration distributions, are deliberated through the graphs and discussed numerically. The skin friction coefficient is deliberated numerically, and their numerical values are accessible through graphs and table. The comparison of current article is calculated in the last section, and a good agreement is clear with the existing literature.

**Keywords** Maxwell nanofluid  $\cdot$  Generalized Fourier's and Fick's laws  $\cdot$  Slip conditions  $\cdot$  Stagnation point  $\cdot$  Chemical reaction  $\cdot$  Shooting method

#### 1 Introduction

Nanotechnology has important interest in manufacturing, aerospace, and medical industries. The term nanofluid was generated by Choi [1] in 1995, designated as fluids that contain solid nanoparticles having 1–100 nm size dispersed in the base fluids, namely ethylene, water, toluene, oil, etc. Nanoparticles such as coppers, silicone, aluminum, and titanium tend to improve the thermal conductivity and convective heat assignment rate of liquids. Impact of variable viscosity on flow of non-Newtonian material with convective

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conditions over a porous medium is investigated by Rundora et al. [2]. Babu and Sandeep [3] discussed the numerical solution on MHD nanomaterials over a variable thickness of the surface along with thermophoresis and Brownian motion effects. Haiao [4] presented the numerical solution of magnetohydrodynamic micropolar fluid flow with the addition of nanomaterials toward a stretching sheet with viscous dissipation. Mahdavi et al. [5] illustrated the slip velocity along with multiphase approach of nanofluids. Xun et al. [6] obtained the numerical solution of bioconvection heat flow nanofluid flow over a rotating plate with temperaturebased viscosity. Khan et al. [7] numerically analyzed heat and mass diffusion in Jeffery nanofluid passed by inclined stretching surface. Lebon and Machrafi [8] analyzed the two-phase change in Maxwell nanofluid flow along with thermodynamic description. Ansari et al. [9] investigated the comprehensive analysis in order to calculate the relative viscosity of nanofluids. Khan et al. [10] considered the chemical reaction on Carreau-Yasuda nanomaterials over a nonlinear stretching surface.

Magnetohydrodynamic (MHD) flow of heat and mass transfer Maxwell fluid flow over a continuous stretching surface has

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great significance in several applications in engineering such as melts, aerodynamics extrusion of plastic sheet, geothermal extractions, and purification of molten metals. Numerous researchers have made great interest and evaluated the transport phenomena for magnetohydrodynamic. Zhao et al. [11] solve the differential equations labeling MHD Maxwell fluid in permeable sheet by considering Dufour and Soret impact. Hsiao [12] investigated the combined effects of thermal extraction on MHD Maxwell fluid over stretching surface with viscous dissipation and energy conversion. Ghasemi and Siavashi [13] demonstrated the Cu-water MHD nanofluid in square permeable surface with entropy generation. Nourazar et al. [14] illustrated the heat transfer in flow of single-phase nanofluid toward a stretching cylinder with magnetic field effect. Dogonchi and Ganji [15] addressed the unsteady squeezed MHD nanofluid flow over two parallel plates with solar radiation. Hayat et al. [16] investigated the heat and mass diffusion for stagnation point flow toward a linear stretching surface along with magnetic field. Sayved et al. [17] investigated the analytical solution of MHD Newtonian fluid flow over a wedge occupied in a permeable sheet. Representative analyses on MHD flow can be seen in Refs. [18–20].

The Maxwell model is a subclass of rate-type fluids, which calculates stress relaxation so it has become popular. This model also eliminates the complicating behavior of shear-dependent viscosity and is thus useful for focusing exclusively on the impact of a fluid's elasticity on the characteristics of its boundary layer. Nadeem et al. [21] deliberated the numerical study on heat transfer of Maxwell nanofluid flow over a linear stretching sheet. Reddy et al. [22] studied the approximate solution of magnetohydrodynamic Maxwell nanofluid flow over exponentially stretching surface. Liu [23] indicated the 2D flow of frictional Maxwell fluid over a variable thickness. Solution of the differential equations was obtained numerically here by  $L_1$  technique. Yang et al. [24] considered the fractional Maxwell fluid through a rectangular microchannel.

Inspired by the above studies, the current study illustrates the MHD Maxwell nanofluid flow over a linearly stretched sheet near the stagnation point and slip boundary conditions. Fourier's and Fick's laws are presented in the constitutive relations. The nonlinear ODEs are deduced from the nonlinear PDEs by similarity transaction. The solutions are obtained via shooting method (Cash and Karp). The different involved physical parameters are examined for velocity, concentration, and temperature fields.

#### 2 Mathematical formulation

Let us consider two-dimensional laminar steady heat and mass transfer flow of an electrically conducting Maxwell nanofluid flow passed by a linear stretched surface placed along x-axis and y-axis vertical to the sheet with stagnation point at the origin (as illustrated in Fig. 1). The free stream velocity  $\mathbf{u} = \mathbf{u}_e(\mathbf{x}) = \mathbf{cx}$  and the velocity via which sheet is stretched are  $\mathbf{u} = \mathbf{U}_w(\mathbf{x}) = \mathbf{ax}$ , where *a* and *c* are positive constants. The temperature at the surface is conserved at  $T_w$  and  $T_\infty$  far away from the plate; in similar a manner, the nanoparticle volume fractions are  $C_w$  and  $C_\infty$ . An external magnetic field  $H_0$  is applied normal to the sheet.

Under the above assumptions, the required equations are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,\tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda \left[ u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + uv \frac{\partial^2 u}{\partial x \partial y} \right]$$
  
=  $-\frac{1}{\rho} \frac{\partial P}{\partial x} + v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma \mu_e^2 H_0^2}{\rho} \left( u + \lambda v \frac{\partial u}{\partial y} \right),$  (2)

where  $\rho$  is the density,  $\mu_e$  is the magnetic permeability velocity,  $\sigma$  is the electrical conductivity,  $\lambda$  is the Maxwell fluid parameter, and v is the kinematic viscosity. Due to hydrostatic and magnetic pressure gradient, the force will be in equilibrium as given by

$$-\frac{1}{\rho}\frac{\partial P}{\partial x} = u_e \frac{du_e}{dx} + \frac{\sigma \mu_e^2 H_0^2}{\rho} u_e \tag{3}$$

Hence, Eq. (2) becomes

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda \left[ u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + uv \frac{\partial^2 u}{\partial x \partial y} \right]$$
  
$$= u_e \frac{\mathrm{d} u_e}{\mathrm{d} x} + v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma \mu_e^2 H_0^2}{\rho} \left( u - u_e + \lambda v \frac{\partial u}{\partial y} \right).$$
(4)



Fig. 1 Geometry of the problem

The classical form of Fourier's and Fick's laws with the ray of Cattaneo–Christov equations takes the following form:

$$q + \lambda_T \left[ \frac{\partial q}{\partial t} + V.(\nabla q) - q.(\nabla V) + (\nabla V)q \right] = -k_f \nabla T, \quad (5)$$

$$J + \lambda_c \left[ \frac{\partial J}{\partial t} + V.(\nabla J) - J.(\nabla V) + (\nabla V)J \right] = -D_B \nabla C.$$
(6)

Assume that  $\nabla .q = 0$ ,  $\nabla .J = 0$ , and for steady state  $\frac{\partial q}{\partial t} = 0$ ,  $\frac{\partial J}{\partial t} = 0$ , the new equations become:

$$q + \lambda_T \left[ V \cdot \nabla q - q \cdot \nabla V \right] = -k_f \nabla T, \tag{7}$$

$$J + \lambda_C \left[ V \cdot \nabla J - q \cdot \nabla J \right] = -D_B \nabla C.$$
(8)

Now in component form, energy and concentration Eqs. (7) and (8) take the following form:

$$\begin{aligned} u\partial_{x}T + v\partial_{y}T + \lambda_{T} \left[ \partial_{x}Tu(\partial_{x}u) + \partial_{x}Tv(\partial_{y}u) \right. \\ \left. + u(\partial_{x}v)\partial_{y}T + v(\partial_{y}v)\partial_{y}T \right] \\ \left. + \lambda_{T} \left[ 2uv\partial_{xy}T + v^{2}\partial_{yy}T + u^{2}\partial_{xx}T \right] \end{aligned} \tag{9} \\ = \alpha\partial_{yy}T + \tau \left\{ D_{B}(\partial_{y}T\partial_{y}C) + \frac{D_{T}}{T\infty}(\partial_{y}T)^{2} \right\}, \end{aligned}$$

$$u(\partial_{x}C) + v(\partial_{y}C) + \lambda_{c}[u\partial_{x}u(\partial_{x}C) + v\partial_{y}u(\partial_{x}C) + u(\partial_{x}v)\partial_{y}C + v(\partial_{y}v)\partial_{y}C]$$
  

$$\lambda_{c}[+2uv\partial_{xy}C + v^{2}\partial_{yy}C + u^{2}\partial_{xx}C]$$

$$= D_{x}\partial_{x}C + \frac{D_{T}}{2}\partial_{x}T - K_{c}(C - C_{x}).$$
(10)

 $= D_B \partial_{yy} C + \frac{1}{T\infty} \partial_{yy} T - K_1 (C)$ where  $\partial_x = \frac{\partial}{\partial x}$  and  $\partial_y = \frac{\partial}{\partial y}$ .

The specified boundary conditions of the current problem take the form

$$\mathbf{u}(\mathbf{x}, \mathbf{y}) = \mathbf{U}_{w}(\mathbf{x}) = \mathbf{a}\mathbf{x} + g\frac{\partial u}{\partial y}, \quad v(x, y) = 0,$$
  

$$T(x, y) = T_{w}(x) = T_{\infty} + bx, \quad C(x, y) = C_{w} = C_{\infty} + bx, \text{ at } y = 0,$$
  

$$\mathbf{u}(\mathbf{x}, \mathbf{y}) = \mathbf{u}_{e}(\mathbf{x}) = \mathbf{c}\mathbf{x}, \quad T(x, y) = T_{\infty}, \quad C(x, y) = C_{\infty} \text{ as } y \to \infty.$$
(11)

Here (u, v) are the velocity components along the (x, y) directions, q, J are the normal heat and mass flux, respectively,  $k_f$  represents the thermal conductivity,  $D_B$  is the Brownian motion,  $\lambda_T$ ,  $\lambda_C$  are the relaxation parameters for thermal and concentration,  $\alpha_f = \frac{(\rho c)_s}{(\rho c)_f}$  is the ratio of nanoparticle heat capacity to base fluid thermal capacity,  $\alpha_f = \frac{k_f}{(\rho C_p)_f}$  represents thermal diffusion,  $T_w(x, y)$  is known as temperature at the wall,  $C_w(x, y)$  is known as concentration at the wall, T and C are the temperature and concentration of the fluid, respectively,  $C_p$  is the specific heat, and

 $C_{\infty}$  and  $T_{\infty}$  are the concentration and temperature free streams. Temperature of the sheet is  $T_w = T_{\infty} + bx$ , for heated surface b > 0 so  $T_w > T_{\infty}$  and for cooled surface b < 0 and  $T_w < T$ , where b is a constant and  $D_T$  is known as thermophoresis diffusivity.

Considering the similarity transformations, we have

$$\eta = y \sqrt{\frac{a}{\nu}}, \ \boldsymbol{\theta}(\eta) = \frac{\mathbf{T} - \mathbf{T}_{\infty}}{\mathbf{T}_{w} - \mathbf{T}_{\infty}}, \ \phi(\eta) = \frac{\mathbf{C} - \mathbf{C}_{\infty}}{\mathbf{C}_{w} - \mathbf{C}_{\infty}}, \ u = \frac{\partial \psi}{\partial y}, \ v = -\frac{\partial \psi}{\partial x}.$$
(12)

Substituting Eq. (12) into Eqs. (1), (4), (9), (10), the present problem boundary conditions (11) are as follows:

$$\begin{aligned} \frac{\mathrm{d}^{3}f}{\mathrm{d}\eta^{3}} + \frac{\mathrm{d}f}{\mathrm{d}\eta} \frac{\mathrm{d}^{2}f}{\mathrm{d}\eta^{2}} &- \left(\frac{\mathrm{d}f}{\mathrm{d}\eta}\right)^{2} + \lambda_{m} \left(2f(\eta)\frac{\mathrm{d}f}{\mathrm{d}\eta}\frac{\mathrm{d}^{2}f}{\mathrm{d}\eta^{2}} - (f(\eta))^{2}\frac{\mathrm{d}^{3}f}{\mathrm{d}\eta^{3}}\right) \\ &+ Ha^{2} \left(1 - \frac{\mathrm{d}f}{\mathrm{d}\eta} + \lambda_{m}\frac{\mathrm{d}f}{\mathrm{d}\eta}\frac{\mathrm{d}^{2}f}{\mathrm{d}\eta^{2}}\right) + 1 = 0, \end{aligned}$$

$$\begin{aligned} \frac{\mathrm{d}^{2}\theta}{\mathrm{d}\eta^{2}} + Pr\left[f(\eta)\frac{\mathrm{d}\theta}{\mathrm{d}\eta} + N_{b}\frac{\mathrm{d}\varphi}{\mathrm{d}\eta}\frac{\mathrm{d}\theta}{\mathrm{d}\eta} + N_{t}\left(\frac{\mathrm{d}\theta}{\mathrm{d}\eta}\right)^{2} \\ &+ \delta_{t} \left(f(\eta)\frac{\mathrm{d}f}{\mathrm{d}\eta}\frac{\mathrm{d}\theta}{\mathrm{d}\eta} - \frac{\mathrm{d}^{2}\theta}{\mathrm{d}\eta^{2}}(f(\eta))^{2}\right)\right] = 0, \end{aligned}$$

$$(14)$$

$$\frac{\mathrm{d}^{2}\phi}{\mathrm{d}\eta^{2}} + \frac{N_{t}}{N_{b}}\frac{\mathrm{d}^{2}\theta}{\mathrm{d}\eta^{2}} + PrLe\left[f(\eta)\frac{\mathrm{d}\phi}{\mathrm{d}\eta} - K\phi(\eta) + \delta_{c}\left(f(\eta)\frac{\mathrm{d}f}{\mathrm{d}\eta}\frac{\mathrm{d}\phi}{\mathrm{d}\eta} - \frac{\mathrm{d}^{2}\phi}{\mathrm{d}\eta^{2}}(f(\eta)^{2})\right)\right] = 0.$$
(15)

Using (11), the associated boundary conditions become

$$\frac{\mathrm{d}f}{\mathrm{d}\eta} = 1 + k \frac{\mathrm{d}^2 f}{\mathrm{d}\eta^2}, \ f(\eta) = 0, \theta(\eta) = 1, \ \phi(\eta) = 1, \ \text{at } \eta = 0,$$
$$\frac{\mathrm{d}f}{\mathrm{d}\eta} \to A, \ \theta \to 0, \ \phi \to 0, \ \text{as } \eta \to \infty.$$
(16)

where the fluid relaxation parameter is  $\lambda_m = \lambda a$ , Hartmann number is  $Ha^2 = \mu_e H_0 \sqrt{\frac{\sigma}{\rho a}}$ , concentration relaxation parameter is  $\delta_c = a\lambda_c$ , ratio parameter is  $A = \frac{c}{a}$ , Lewis number is  $Le = \frac{\alpha}{D_B}$ , thermal relaxation parameter is  $\delta_t = a\lambda_T$ , thermophoresis parameter is  $N_t = \frac{\tau D_t (T_w - T_0)}{T_w v}$ , Prandtl number is  $Pr = \frac{\mu c_p}{k}$ , Brownian motion parameter is  $N_b = \frac{\tau D_B (C_w - C_0)}{v}$ , slip parameter is  $\mathbf{k} = \mathbf{g} \sqrt{\frac{a}{v}}$ , and chemical reactive species is  $K = \frac{K_1}{a}$ . Friction factor coefficient  $(C_f)$  is defined as:

$$C_f = \frac{\tau_w}{\rho u_e^2} = R e_x^{-\frac{1}{2}} (f''(0) + \lambda_m (f''(0)f'(0) + f(0)f'''(0)).$$
(17)

Here  $\tau_w$  denotes the wall shear stress and is given by

$$\tau_w = \mu \frac{\partial}{\partial y} (u + \lambda v \frac{\partial u}{\partial y})_{y=0}.$$
(18)

where  $Re_x = u_e x \frac{x}{v}$  is local Reynolds number.

#### **3** Numerical procedure

Numerical solution of the nonlinear differential Eqs. (13)–(15) along with Neumann boundary conditions (16) is achieved by applying the shooting method with RK-4 integration technique for various values of parameters. Let  $y_1 = f$ ,  $y_2 = f'$ ,  $y_3 = f''$ ,  $y_4 = \theta$ ,  $y_5 = \theta'$ ,  $y_6 = \phi$  and  $y_7 = \phi'$ .

Hence, the leading equations become

$$y_1 = y'_2,$$
 (19)

$$y'_2 = y_3,$$
 (20)

$$y'_{3} = \frac{y_{2}^{2} - y_{2}y_{3} - \lambda_{m}(y_{1}y_{2}y_{3}) - Ha^{2}(1 - y_{2} + \lambda_{m}y_{2}y_{3}) - 1}{(1 - \lambda_{m}y_{1})},$$
(21)

$$y'_4 = y_5,$$
 (22)

$$y'_{5} = \frac{-Pr\left(y_{1}y_{5} + N_{b}y_{7}y_{5} + N_{t}\left(y_{5}\right)^{2} + \delta_{t}\left(y_{2}y_{1}y_{5}\right)\right)}{\left(1 - \delta_{t}\left(y_{1}\right)^{2}\right)},$$
 (23)

$$y'_6 = y_7,$$
 (24)

$$y_{7}' = \frac{-PrLe(y_{1}y_{6} - Ky_{6} + \delta_{c}(y_{2}y_{1}y_{6})) - \frac{N_{c}}{N_{b}}y_{5}'}{\left(1 - \delta_{c}(y_{1})^{2}\right)},$$
 (25)

and subsequent initial conditions are:

$$y_1(0) = 0, \ y_2(0) = 1 + ky_2, \ y_4(0) = 1, \ y_6(0) = 1,$$
  
$$y_2(\eta) \to A, \ y_4(\eta) \to 0, \ y_6(\eta) \to 0 \text{ when } \eta \to \infty.$$
 (26)

This technique is successfully used to solve the different problems related to boundary layer flows. The boundary conditions f'(0),  $\theta(0)$  and  $\phi(0)$  for  $\eta \to \infty$  are converted into finite interval length (here it is  $\eta = 5$ ). Insert three initial guesses to f''(0),  $\theta'(0)$  and  $\phi'(0)$  for approximate solution. Here the step size and convergence criteria are chosen to be 0.001 and  $10^{-6}$  (in all cases).

#### 4 Results and discussion

The main effort of this work is to examine the influence of magnetic field and stagnation point Maxwell nanofluid flow due to a linear stretching surface with slip conditions. The governing differential Eqs. (12)–(15) along with corresponding boundary conditions (16) are solved numerically by implying shooting procedure (Cash and Karp). Figure 2 represents the change in velocity profile for distinct values of Hartmann number Ha. From this figure, the enhancement in Ha results in decrease in velocity profile. Since the Hartmann number Ha represents the ratio of MHD force to viscous force, the enhancement in Ha leads to stronger the MHD force, which declares the velocity motion. Figure 3 depicts the variation of slip parameter k on velocity profile. The influence of slip parameter k significantly enhances the velocity profile. Figure 4 illustrates the variation of fluid relaxation parameter  $\lambda_m$  on velocity profile. It can be analyzed that the velocity of the fluid reduces by enhancing the fluid relaxation parameter  $\lambda_m$ . Figure 5 represents the change in temperature distribution for distinct values of  $N_t$ . It is found that by enhancing  $N_t$ , the temperature distribution also increases. Figure 6 depicts the variation of Brownian motion  $N_b$  on temperature distribution. It can be analyzed that by increasing  $N_b$ , the mass diffusivity trekked up which leads to enhancement in the temperature and



**Fig. 2** Outcome of *Ha* on  $f'(\eta)$ 

Pr = 1.1

Pr = 1.2

Pr = 1.3

 $k = 0.1, Le = 1.1, N_t = N_b = \lambda_m = 0.1,$ 

 $K = \delta_{c} = 0.1, \delta_{t} = Ha = A = 0.1$ 



**Fig. 3** Outcome of k on  $f'(\eta)$ 



0.9  $N_{i} = 0.5$  $N_t = 0.9$ 0.8  $k = 0.1, Pr = Le = 1.1, \lambda_m = N_b = 0.1,$ 0.7  $\mathbf{K} = \delta_c = \mathbf{0.1}, \, \delta_t = \mathbf{H}\mathbf{a} = \mathbf{A} = \mathbf{0.1}$ 0.6 θ(μ) 0.5 0.4 0.3 0.2 0.1 0 0 2 4 3 η

**Fig. 4** Outcome of  $\lambda_m$  on  $f'(\eta)$ 

the boundary layer thickness. Figure 7 shows the behavior of Prandtl number Pr on temperature profile. It is found that the temperature profile reduces with rising values of Prandtl Pr. Figure 8 presents the deviation of temperature profile for distinct values of  $\delta_t$ . It is seen that by enhancing thermal relaxation parameter  $\delta_t$ , fluid

**Fig. 6** Outcome of  $N_b$  on  $\theta(\eta)$ 

0.9

0.8

0.7

0.6

0.5

θ

particles require more time to heat the boundary layer region, and as a result temperature profile reduces. Figure 9 displays the effect of relaxation parameter  $\delta_c$  on concentration distribution. From this figure, it is observed that by increasing the relaxation parameter  $\delta_t$ , the concentration profile reduces. Figure 10 represents the variation





**Fig. 7** Outcome of  $N_t$  on  $\theta(\eta)$ 





**Fig. 9** Outcome of  $\delta_t$  on  $\phi(\eta)$ 



**Fig. 8** Outcome of  $\delta_t$  on  $\theta(\eta)$ 

**Fig. 10** Outcome of K on  $\phi(\eta)$ 

on concentration profile for different values of chemical reaction K. It is found that for large values of chemical reaction K reduces the concentration profile. Figure 11 represents the influence of Lewis number Le on nanoconcentration profile. It is found that the higher values of Lewis number Le lead to reduction in the mass diffusivity, so the concentration profile reduces. Figures 12 and 13 validate the distribution of skin friction coefficient  $C_f Re_x^2$  with respect to Hartmann number Ha and

for several values of slip parameter k and fluid relaxation parameter  $\lambda_m$ . It is very important to see that the skin friction coefficient  $C_f Re_x^2$  enhances by enhancing slip parameter k but decreases by increasing the fluid relaxation parameter  $\lambda_m$ . Table 1 shows that the fraction factor rises due to an increase in Hartmann number Ha and fluid relaxation parameter  $\lambda_m$  and opposite behavior is noticed for slip parameter k. The achieved results are in





**Fig. 13**  $C_f Re_x^{\frac{1}{2}}$  for various values of *Ha* and *k* 



**Table 1** Computational results of  $C_f Re_x^{\frac{1}{2}}$  for different values of Ha, k and  $\lambda_m$  when  $K = A = N_b = 0.1$ , Le = 1.0,  $\delta_c = \delta_t = 0.0$ ,  $N_t = 0.5$ , and Pr = 1.1

| На  | k   | $\lambda_m$ | $-C_f R e_x^{\frac{1}{2}}$ |
|-----|-----|-------------|----------------------------|
| 0.1 |     | 0.1         | 0.9097                     |
| 0.2 |     |             | 0.9559                     |
| 0.3 |     |             | 0.9999                     |
|     | 0.1 |             | 0.9097                     |
|     | 0.2 |             | 0.8042                     |
|     | 0.3 |             | 0.7225                     |
|     |     | 0.00        | 0.8918                     |
|     |     | 0.05        | 0.9008                     |
|     |     | 0.09        | 0.9097                     |

**Fig. 12**  $C_f Re_x^{\frac{1}{2}}$  for various values of *Ha* and  $\lambda_m$ 

**Fig. 11** Outcome of Le on  $\phi(\eta)$ 

good agreement with Afify and Elgazery [25] for different values Ha and  $\lambda_m$  as shown in Table 2. Table 3 is sketched for the comparative investigation between Hsiao [26] and present results.  $N_t N_b = 0.1$ , Le = Pr = 10 and  $\lambda_m = Ha = k = St = \delta t = \delta c = K = A = 0.0$ .

Ha

**Table 2** Comparisonof $(f''(0) + \lambda_m(f'(0)f''(0) + f(0)f'''(0)))$ withthepreviousliterature $N_b = k = St = 0.1$ , $\delta t = \delta c = K = A = 0.0$ , $N_t = 0.5$ andLe = Pr = 1.0andLe = Pr = 1.0and

| $\overline{\lambda_m}$ | На  | [25]      | Present results |
|------------------------|-----|-----------|-----------------|
| 0.5                    | 0.0 | - 1.68935 | -1.6752         |
| 0.5                    | 0.1 | - 1.69715 | -1.6821         |
| 0.5                    | 0.2 | - 1.72034 | -1.7000         |
| 0.5                    | 0.3 | - 1.75830 | -1.7321         |
| 0.0                    | 0.1 | - 1.00499 | -1.0001         |
| 0.1                    | 0.1 | -2.49440  | -2.1898         |

**Table 3** Comparison of  $-\theta'(0)$  and  $-\phi'(0)$  with the previous literature when  $\lambda_m = Ha = k = St = \delta t = \delta c = K = A = 0.0$ ,  $N_t = 0.1$  and Le = Pr = 10

| $N_t$ | Hsiao [26] | Hsiao [26] | Present outcome | Present outcome |
|-------|------------|------------|-----------------|-----------------|
| 0.1   | -0.9524    | -2.1294    | -0.9131         | -2.1526         |
| 0.2   | -0.6932    | -2.2740    | -0.6922         | -2.2945         |
| 0.3   | -0.5201    | -2.5287    | -0.5018         | -2.5002         |
| 0.4   | -0.4026    | -2.7952    | -0.4022         | -2.7590         |
| 0.5   | -0.3211    | -3.0352    | -0.3462         | -3.0617         |

## **5** Conclusions

The main results for heat and mass exchanger in MHD stagnation point Maxwell nanofluid flow past a linear stretching surface in the presence of slip effects are summarized below:

- Rising the Hartmann number *Ha* and slip parameter *k* leads to decline in velocity profile.
- Larger values of fluid relaxation parameter  $\lambda_m$ , increase the velocity profile.
- For increasing values of chemical reaction, *K*, Prandtl number *Pr*, thermal relaxation parameter  $\delta_t$ , concentration relaxation parameter  $\delta_c$  and Lewis number *Le* reduce the concentration and temperature profiles.
- Temperature profile increases for increasing values of thermophoresis parameter N<sub>t</sub> and Brownian motion N<sub>b</sub>
- Skin friction coefficient reduces  $C_f Re_x^2$  for large values of slip parameter k but opposing behavior is noticed for fluid relaxation parameter  $\lambda_m$ .

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### **Compliance with ethical standards**

**Conflict of interest** The authors declare that they have no conflict of interest.

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