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TECHNICAL PAPER

Optimal placement of damping devices in buildings

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Abstract

The appropriate use of energy dissipating devices improves the behavior of structures when subjected to external loads, defining the optimal location of the dampers; therefore, it is crucial to ensure their efficiency. In this work, the mathematical expressions of an efficient and systematic procedure proposed by Takewaki were adapted and detailed to find the optimum location of dampers when a structural damper is used. This procedure consists of minimizing the sum of the amplitudes of the transfer functions evaluated at the undamped fundamental frequency of a structural system subject to constraints on the sum of the damping coefficients of the added dampers. For instance, at the beginning and end of the calculation, the sum of the damping coefficients entered must be the same. A series of numerical examples on shear building models within a range of two to six stories are used to verify the efficiency of the systematic procedure. The results showed that the optimal placement method is efficient due to the amplitude reduction of the transfer function after the optimal distribution of the damping coefficients in the structure.

Keywords Structural damping · Incremental inverse problem · Optimal placement of dampers · Passive control · Transfer function

1 Introduction

Constructions are often subjected to external forces that may affect their stability, occupants and contents. Over time, many catastrophes have been recorded in large cities throughout the world, such as earthquakes in Mexico City (1985) and Loma Prieta (1989), where the devastating damage rate could have been mitigated using seismic control technology [1, 2]. Seismic control technology increases the structure capability for energy dissipation through devices that are incorporated into the structure. In this way, main structural elements will not receive all the applied energy, which means less likelihood of collapsing. The use of passive devices such as structural dampers represents an advantage compared to other kinds of devices, since they do not need an external source of energy for their operation. Moreover, once a natural phenomenon occurs and excitates the structure, these control devices can be removed and replaced by new ones.

Finding the optimal damper location is undoubtedly an important task for structural control design. Some studies proposed a procedure to establish the optimal position regarding the transfer function. The transfer function is defined as a mathematical model that expresses the relations between the output variables (system response) and the input variables (external excitation). Being an intrinsic property of the system, it does not depend on the nature and magnitude of the external loading. Transfer functions provide a full description of the system dynamic behavior without taking into account the physical structure of the system [3]. Takewaki [4–7] proposed an efficient procedure to minimize the transfer function amplitude evaluated at the undamped fundamental frequency of the structure and subject to a constraint on the sum of the added viscous dampers. Takewaki and Uetani [8] considering the amplification of the response due to the ground surface developed a method for optimal damper placement in building

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structures. Aydin et al. [9] studied different objective functions to determine the optimal placement of added dampers; instead of measuring the amplitude of the transfer function at the top of the building, the authors chose the evaluation of the amplitude of the transfer function of the base shear force as an objective function. Aydin and Boduroglu [10] presented a study for the optimal location of X steel diagonal braces (SDBs) using the amplitude of the transfer function as the objective function and the stiffness as the design variable. Aydin [11] proposed a method for finding the location and optimal size of viscous dampers in planar steel building frames; the author used the transfer function evaluated in the undamped frequency and defined the coefficients of the dampers as the design variable. The work aimed to minimize the amplitude of the transfer function, subjected to the restriction of the sum of the coefficients of the added dampers. Martinez et al. [12, 13] proposed a procedure to optimally define the damping coefficients of viscous and hysteretic dampers and comply with the structural design requirements established by seismic design codes. Sonmez et al. [14] used an artificial bee colony algorithm (ABCA) to calculate the size and location of viscous dampers and to reduce the damage caused by an earthquake. The efficiency of the method was evaluated through the amplitude of the transfer function.

The effective operation of the dampers depends not only on their mechanism and capacity of dissipate energy but also on their location in the structure. Other works that aimed the optimal location and the use of hysteretic dampers were published by Uetani et al. [15] and presented an innovative structure system design for buildings with hysteretic and viscous dampers. The program was based on the gradient projection algorithm and has the advantage of being easily manipulated by the structural designer to satisfy the design codes. Uz and Hadi [16] proposed a design strategy based on genetic algorithms (GA) applied in hysteretic dampers in order to reduce damage from pounding in adjacent structures. The authors concluded that a decrease in the number of dampers improves the dynamic response of the structure regarding to an unnecessary amount of anti-seismic devices. Murakami et al. [17] proposed an optimization method to find the size and location of the hysteretic dampers; this method includes a variable adaptive step length considering that hysteretic dampers have nonlinear restoring-force characteristics. Pu et al. [18] proposed a new methodology to reduce the seismic response of the structure using hysteretic dampers, thus improving the structural equivalent damping ratio and stiffness.

The optimal damper design in structures has been one of the predominant challenges in seismic control. There have been numerous studies on viscoelastic dampers. A simplified sequential search algorithm was published [19–21], which maximizes the effect of added dampers. Kim and Bang [22] worked with viscoelastic dampers and tried to reduce the torsional response of an asymmetric structure by an optimal damper distribution when subjected to an earthquake. Xu et al. [23, 24] used an optimization method called the simplex method, which finds the optimal parameters of the viscoelastic dampers as well as the location. The results showed that the seismic response of a structure will be reduced after applying the proposed methodology. Main and Krenk [25] developed a methodology that facilitates the setting of the location and optimal size of viscous dampers; the formulation was later applied to a 10-story building model with viscous dampers. Lang et al. [26] demonstrated the effectiveness of nonlinear viscous dampers over linear dampers as a method of vibration control through the use of a new technique for calculating the location and optimal design of dampers in multi-degree of freedom (MDOF) systems. Alibrandi and Falsone [27] presented a method to obtain the number and placement of the viscous or viscoelastic damping necessary to reduce the seismic response; the method is based on the minimization of the Expected value of the stochastic Dissipated Power (EDP). Kandemir and Mazanoglu [28] investigated the optimal parameters and number of viscous dampers needed to prevent pounding between two adjacent buildings when a seismic event occurs. The authors conducted parametric studies with varying story number as well as stiffness and capacity of viscous dampers. There are a number of solutions applied in practice and proposed in the related literature with optimal locations using different types of dampers [18, 29–34].

This work calculates the optimal placement of structural dampers based on the methodology proposed by Takewaki [4], which was developed and applied to a shear building model controlled by viscous dampers. Besides applying this methodology to another type of damper, this work also presents a step-by-step explanation of the Takewaki method, not found in the literature to the best of authors knowledge. In addition, some of the equations were simplified to a better understanding of the procedure. In other words, the mathematical expressions for a shear building model with structural damping are adapted and detailed. This methodology is about minimizing the amplitude of the transfer function evaluated at the fundamental frequency subject to a constraint on the sum of the damper coefficients. Through a numerical example using MATLAB software (The Mathworks, Inc.), the initial and optimal amplitudes of the transfer function responses are compared for shear buildings varying between two to six floors, keeping the stiffness unchanged on each floor. The initial amplitude is calculated with equal damping coefficients per floor, and the optimal amplitude is calculated with the optimal distribution of the damping coefficients after applying the methodology.

2 Optimal damper placement

Optimal damper placement (ODP) is an inverse problem. It is a challenge to find the optimal damper location that minimizes the sum of the transfer function amplitudes evaluated at the undamped fundamental frequency of a structural system subjected to a constraint. This constraint regards to the sum of the damping coefficients of the added dampers. It should also be noted that because the present formulation addresses a general dynamical property (i.e., the amplitude of the transfer function), the results are general and are not influenced by the characteristics of input motions [4].

Structural damping is attributed to the hysteresis phenomenon associated with cyclic stress in elastic materials, and the energy loss per stress cycle is equal to the area inside the hysteresis loop [35]. Systems with structural damping and subjected to harmonic excitation can be considered as having viscous damping with the equivalent coefficient $\left(C_{\text{eq}}\right)$.

$$C_{\text{eq}} = \frac{\alpha}{\pi\omega} \tag{1}$$

where α is a constant frequency independent of harmonic oscillation and ω denote a circular frequency. Thus, the differential equation of motion for a one degree of freedom (1 DOF) system can be written as:

$$m\ddot{u}(t) + \frac{\alpha}{\pi\omega}\dot{u}(t) + ku(t) = Ake^{i\omega t} \tag{2}$$

where m is the mass, k is the stiffness, and A is a constant of integration. Because $\dot{u} = i\,\omega\,u$, Eq. (2) can be rewritten as

$$m\ddot{u}(t) + k(1 + i\gamma)u(t) = Ake^{i\omega t} \tag{3}$$

where

$$\gamma = \frac{\alpha}{\pi k} = \frac{C_{\text{eq}} \cdot \omega}{k} \tag{4}$$

γ is the structural damping factor, which can be rewritten as

$$\gamma = \frac{2\zeta m\omega_n\omega}{k} = 2\zeta\frac{\omega}{\omega_n} = 2\beta \tag{5}$$

where ζ is the damping ratio of a viscous system, ω_n is a natural frequency, and β is the damping coefficient. The quantity $k(1 + 2i\beta)$ is the complex stiffness or complex damping. In this work, we assume that dampers connected to the main structure provide structural damping.

The mathematical formulations have been expressed for a three-story shear building model with structural dampers as shown in Fig. 1. Let $M = \{\bar{m}_1, \bar{m}_2, \bar{m}_3\}$ denote the masses of the story and $K = \left\{\bar{k}_1, \bar{k}_2, \bar{k}_3\right\}$ denote their stiffness. The design variables are the damping coefficient $\beta = \{\beta_1, \beta_2, \beta_3\}$ of the added structural dampers. It is also assumed here that the original structural damping is negligible compared with the damping of the added dampers. The equation of motion for the model shown in Fig. 1 when subjected to a base acceleration $\ddot{u}_g$ may be written as:

$$\begin{bmatrix} \bar{k}_1 + \bar{k}_2 & -\bar{k}_2 & 0 \\ -\bar{k}_2 & \bar{k}_2 + \bar{k}_3 & -\bar{k}_3 \\ 0 & -\bar{k}_3 & \bar{k}_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} + 2i \begin{bmatrix} \beta_1\bar{k}_1 + \beta_2\bar{k}_2 & -\beta_2\bar{k}_2 & 0 \\ -\beta_2\bar{k}_2 & \beta_2\bar{k}_2 + \beta_3\bar{k}_3 & -\beta_3\bar{k}_3 \\ 0 & -\beta_3\bar{k}_3 & -\beta_3\bar{k}_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} + \begin{bmatrix} \bar{m}_1 & 0 & 0 \\ 0 & \bar{m}_2 & 0 \\ 0 & 0 & \bar{m}_3 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{Bmatrix} = - \begin{bmatrix} \bar{m}_1 & 0 & 0 \\ 0 & \bar{m}_2 & 0 \\ 0 & 0 & \bar{m}_3 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \ddot{u}_g \tag{6}$$

where i is the imaginary unit, and the displacements of masses m_i are denoted by u_i.

The differential equation of motion, Eq. (6), is then transformed to the following form using the Fourier transform:

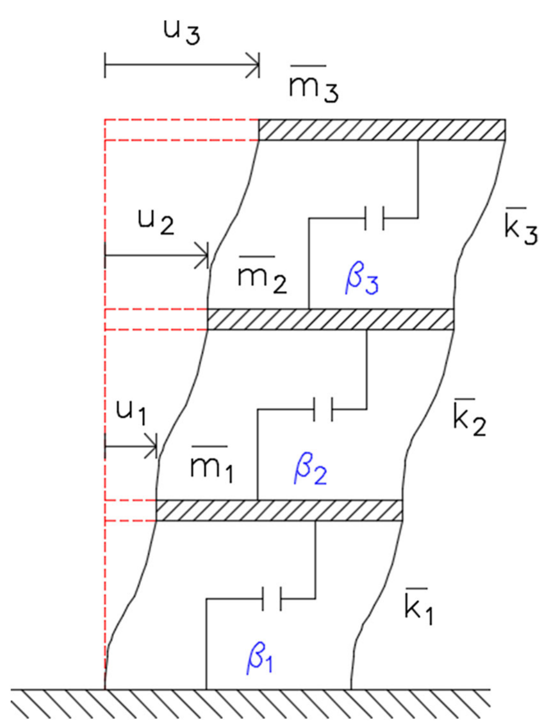

Fig. 1 Three-story shear building model with added structural damper

$$\left(\begin{bmatrix}\bar{k}_1+\bar{k}_2 & -\bar{k}_2 & 0\\ -\bar{k}_2 & \bar{k}_2+\bar{k}_3 & -\bar{k}_3\\ 0 & -\bar{k}_3 & \bar{k}_3\end{bmatrix}+ 2i\begin{bmatrix}\beta_1\bar{k}_1+\beta_2\bar{k}_2 & -\beta_2\bar{k}_2 & 0\\ -\beta_2\bar{k}_2 & \beta_2\bar{k}_2+\beta_3\bar{k}_3 & -\beta_3\bar{k}_3\\ 0 & -\beta_3\bar{k}_3 & \beta_3\bar{k}_3\end{bmatrix} -\omega^2\begin{bmatrix}\bar{m}_1 & 0 & 0\\ 0 & \bar{m}_2 & 0\\ 0 & 0 & \bar{m}_3\end{bmatrix}\right)\begin{Bmatrix}U_1(\omega)\\ U_2(\omega)\\ U_3(\omega)\end{Bmatrix} = -\begin{bmatrix}\bar{m}_1 & 0 & 0\\ 0 & \bar{m}_2 & 0\\ 0 & 0 & \bar{m}_3\end{bmatrix}\begin{Bmatrix}1\\ 1\\ 1\end{Bmatrix}\ddot{U}_g \quad (7)$$

where $U_1(\omega)$, $U_2(\omega)$, $U_3(\omega)$, and $\ddot{U}_g(\omega)$ denote the Fourier transforms of u_1, u_2, u_3, and $\ddot{u}_g$, respectively.

The Fourier transforms of the relative nodal displacements $d_1 = u_1$, $d_2 = u_2 - u_1$, and $d_3 = u_3 - u_2$ can be expressed in terms of $U_1(\omega)$, $U_2(\omega)$, and $U_3(\omega)$, respectively, by

$$\begin{Bmatrix}\delta_1(\omega)\\ \delta_2(\omega)\\ \delta_3(\omega)\end{Bmatrix} = \begin{bmatrix}1 & 0 & 0\\ -1 & 1 & 0\\ 0 & -1 & 1\end{bmatrix}\begin{Bmatrix}U_1(\omega)\\ U_2(\omega)\\ U_3(\omega)\end{Bmatrix} \quad (8)$$

From Eq. (8), the transfer function of interstory drifts can be calculated.

New complex-value quantities defined by Takewaki [4] as $\hat{U}_1$, $\hat{U}_2$, and $\hat{U}_3$ can be written as:

$$\hat{U}_1 = \frac{U_1(\omega_1)}{\ddot{U}_g(\omega_1)} \quad \hat{U}_2 = \frac{U_2(\omega_1)}{\ddot{U}_g(\omega_1)} \quad \hat{U}_3 = \frac{U_3(\omega_1)}{\ddot{U}_g(\omega_1)} \quad (9)$$

In Eq. (9), $\hat{U}_i$ indicates the resonant amplitude of the floor displacement at the fundamental circular frequency and represents the value such that ω_1 is substituted in the frequency response function obtained as $U_i(\omega)$ after substituting $\ddot{U}_g(\omega) = 1$ in Eq. (7). New complex-value quantities are defined by $\hat{\delta}_1 = \hat{U}_1$, $\hat{\delta}_2 = \hat{U}_2 - \hat{U}_1$ and $\hat{\delta}_3 = \hat{U}_3 - \hat{U}_2$, and the substitution of Eq. (9) into (7) with $\omega = \omega_1$ provides

$$A_H\begin{Bmatrix}\hat{U}_1\\ \hat{U}_2\\ \hat{U}_3\end{Bmatrix} = -\begin{Bmatrix}\bar{m}_1\\ \bar{m}_2\\ \bar{m}_3\end{Bmatrix}, \quad \rightarrow \quad A_H\hat{U}(\omega_1) = -Mr \quad (10)$$

where M is the mass vector, r is the influence vector in terms of the direction of the base acceleration, and A_H is the transfer function matrix that also includes the design variables $(\bar{\beta}_i)$ expressed as

$$A_H = \begin{bmatrix}k_1(1+2\bar{\beta}_1 i)+k_2(1+2\bar{\beta}_2 i)-\omega_1^2\bar{m}_1 & -k_2(1+2\bar{\beta}_2 i) & 0\\ -k_2(1+2\bar{\beta}_2 i) & k_2(1+2\bar{\beta}_2 i)+k_3(1+2\bar{\beta}_3 i)-\omega_1^2\bar{m}_2 & -k_3(1+2\bar{\beta}_3 i)\\ 0 & -k_3(1+2\bar{\beta}_3 i) & k_3(1+2\bar{\beta}_3 i)-\omega_1^2\bar{m}_3\end{bmatrix} \quad (11)$$

From Eq. (10), the transfer function vector of the displacements evaluated at the fundamental frequency of the structure can be written as follows:

$$\hat{U}(\omega_1) = -A_H^{-1}Mr \quad (12)$$

2.1 Optimal damper placement criteria

The methodology applied in this work aims to minimize the amplitude of the transfer function after optimal distribution of the damping coefficients, which means to decrease the sum of the relative floor displacement. The presented method in this paper proposed by Takewaki [4] automatically finds the optimal placement of the added dampers. For this model, the problem of optimal damper placement may be described as

$$V = \sum_{i=1}^{3}\left|\hat{\delta}_i(\beta)\right| \quad (13)$$

where V represents the global flexibility, and its minimization is preferred from the view of performance-based design. This methodology is subject to a constraint on the sum of the damping coefficients of the added dampers by

$$\sum_{i=1}^{3} \beta_i = \overline{W} \quad (\bar{W}: \text{ specified value}) \tag{14}$$

The optimality criteria for the optimal damper problem can be derived using the Lagrange multipliers method. The Lagrangian L for this problem may be written as

$$L(\beta, \lambda) = \sum_{i=1}^{3} \left|\hat{\delta}_i(\beta)\right| + \lambda\left(\sum_{i=1}^{3} \beta_i - \overline{W}\right) \tag{15}$$

The optimality criteria for the global flexibility V may be derived from the stationarity conditions of $L(\beta, \lambda)$ with respect to β and λ as

$$\left(\sum_{i=1}^{3} \left|\hat{\delta}_i\right|\right)_{,j} + \lambda = 0 \quad (j = 1,\ 2,\ 3) \tag{16}$$

$$\sum_{i=1}^{3} \beta_i - \overline{W} = 0 \tag{17}$$

The partial differentiation with respect to β_j is indicated by $(\cdot)_{,j}$. If $\beta_j = 0$ or $\beta_j = \bar{W}$, then Eq. (16) must be rewritten as follows:

$$\begin{aligned} &\left(\sum_{i=1}^{3} \left|\hat{\delta}_i\right|\right)_{,j} + \lambda \geq 0 \quad \text{for} \quad \beta_j = 0 \\ &\left(\sum_{i=1}^{3} \left|\hat{\delta}_i\right|\right)_{,j} + \lambda \leq 0 \quad \text{for} \quad \beta_j = \bar{W} \end{aligned} \tag{18}$$

In the construction of the optimality criteria, the term λ is used in Eqs. (15) and (18), and it may be convenient in the creation of a systematic solution algorithm to express it without that parameter. Thus, another expression to define the optimality criteria is given by the following:

$$\gamma_1 = \frac{\left(\sum_{i=1}^{3} \left|\hat{\delta}_i\right|\right)_{,2}}{\left(\sum_{i=1}^{3} \left|\hat{\delta}_i\right|\right)_{,1}} \quad \gamma_2 = \frac{\left(\sum_{i=1}^{3} \left|\hat{\delta}_i\right|\right)_{,3}}{\left(\sum_{i=1}^{3} \left|\hat{\delta}_i\right|\right)_{,1}} \tag{19}$$

where γ is an optimality index for each floor j. An optimal solution is achieved when the distribution of the dampers results in all values γ_j tending to the unit [36].

The solution of the algorithm is achieved using the differentiation of Eq. (10) with respect to a design variable (β_j) as follows:

$$A_{H,j}\begin{Bmatrix} \hat{U}_1 \\ \hat{U}_2 \\ \hat{U}_3 \end{Bmatrix} + A_H \begin{Bmatrix} \hat{U}_{1,j} \\ \hat{U}_{2,j} \\ \hat{U}_{3,j} \end{Bmatrix} = 0 \tag{20}$$

where the matrix $A_{H,j}$ in Eq. (20) is expressed as

$$A_{H,1} = 2i\begin{bmatrix} \bar{k}_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad A_{H,2} = 2i\begin{bmatrix} \bar{k}_2 & -\bar{k}_2 & 0 \\ -\bar{k}_2 & \bar{k}_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$A_{H,3} = 2i\begin{bmatrix} 0 & 0 & 0 \\ 0 & \bar{k}_3 & -\bar{k}_3 \\ 0 & -\bar{k}_3 & \bar{k}_3 \end{bmatrix} \tag{21}$$

The first derivatives of $\hat{U}_1$, $\hat{U}_2$ and $\hat{U}_3$ from Eq. (20), since A_H is regular, can be written as

$$\begin{Bmatrix} \hat{U}_{1,j} \\ \hat{U}_{2,j} \\ \hat{U}_{3,j} \end{Bmatrix} = -A_H^{-1} A_{H,j} \begin{Bmatrix} \hat{U}_1 \\ \hat{U}_2 \\ \hat{U}_3 \end{Bmatrix} \tag{22}$$

where Eq. (22) represents the first-order sensitivity of the transfer function vector.

The first derivate of the interstory drift by substituting $\hat{\delta}_1 = \hat{U}_1$, $\hat{\delta}_2 = \hat{U}_2 - \hat{U}_1$ and $\hat{\delta}_3 = \hat{U}_3 - \hat{U}_2$, can be denoted as

$$\begin{aligned} &\begin{Bmatrix} \hat{\delta}_{1,j} \\ \hat{\delta}_{2,j} \\ \hat{\delta}_{3,j} \end{Bmatrix} = -TA_H^{-1} A_{H,j} T^{-1} \begin{Bmatrix} \hat{\delta}_1 \\ \hat{\delta}_2 \\ \hat{\delta}_3 \end{Bmatrix}, \\ &T = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \end{aligned} \tag{23}$$

The deformation-displacement transformation matrix is indicated by T, and $\hat{\delta}_i$ may be expressed as

$$\hat{\delta}_i = Re\left[\hat{\delta}_i\right] + iIm\left[\hat{\delta}_i\right] \tag{24}$$

The first derivative of $\hat{\delta}_i$ may be written as

$$\hat{\delta}_{i,j} = \left(Re\left[\hat{\delta}_i\right]\right)_{,j} + i\left(Im\left[\hat{\delta}_i\right]\right)_{,j} \tag{25}$$

The absolute value of $\hat{\delta}_i$ is defined by

$$\left|\hat{\delta}_i\right| = \sqrt{\left(Re\left[\hat{\delta}_i\right]\right)^2 + \left(Im\left[\hat{\delta}_i\right]\right)^2} \tag{26}$$

The first derivative of $\left|\hat{\delta}_i\right|$ can be written as

$$\left|\hat{\delta}_i\right|_{,j} = \frac{1}{\left|\hat{\delta}_i\right|}\left\{Re\left[\hat{\delta}_i\right]\left(Re\left[\hat{\delta}_i\right]\right)_{,j} + Im\left[\hat{\delta}_i\right]\left(Im\left[\hat{\delta}_i\right]\right)_{,j}\right\} \tag{27}$$

where $\left(Re\left[\hat{\delta}_i\right]\right)_{,j}$ and $\left(Im\left[\hat{\delta}_i\right]\right)_{,j}$ are calculated from Eq. (23).

According to the γ variation defined in Eq. (19), the distribution of the damping coefficients is based on an iterative process that considers the relation of relative displacements. The linear increments $\Delta\gamma_1$ and $\Delta\gamma_2$ may be expressed as

$$\Delta\gamma_1 = \left(\frac{1}{B_1}\frac{\partial B_2}{\partial \beta} - \frac{B_2}{B_1^2}\frac{\partial B_1}{\partial \beta}\right)\Delta\beta = \frac{1}{B_1}\left(\frac{\partial B_2}{\partial \beta} - \frac{\partial B_1}{\partial \beta}\gamma_1\right)\Delta\beta$$
$$\Delta\gamma_2 = \left(\frac{1}{B_1}\frac{\partial B_3}{\partial \beta} - \frac{B_3}{B_1^2}\frac{\partial B_1}{\partial \beta}\right)\Delta\beta = \frac{1}{B_1}\left(\frac{\partial B_3}{\partial \beta} - \frac{\partial B_1}{\partial \beta}\gamma_2\right)\Delta\beta \tag{28}$$

where B_1, B_2, and B_3 are defined as

$$B_1 = \left(\sum_{i=1}^{3}\left|\hat{\delta}_i\right|\right)_{,1} \quad B_2 = \left(\sum_{i=1}^{3}\left|\hat{\delta}_i\right|\right)_{,2} \quad B_3 = \left(\sum_{i=1}^{3}\left|\hat{\delta}_i\right|\right)_{,3} \tag{29}$$

Due to the constraint in Eq. (17), the increments $\Delta\beta$ must satisfy the following relation:

$$\sum_{i=1}^{3}\Delta\beta_i = 0 \tag{30}$$

The following set of simultaneous linear equations with respect to $\Delta\beta$ is arranged from Eqs. (28) and (30):

$$\begin{bmatrix} \frac{1}{B_1}\left(\frac{\partial B_2}{\partial \beta_1} - \frac{\partial B_1}{\partial \beta_1}\gamma_1\right) & \frac{1}{B_1}\left(\frac{\partial B_2}{\partial \beta_2} - \frac{\partial B_1}{\partial \beta_2}\gamma_1\right) & \frac{1}{B_1}\left(\frac{\partial B_2}{\partial \beta_3} - \frac{\partial B_1}{\partial \beta_3}\gamma_1\right) \\ \frac{1}{B_1}\left(\frac{\partial B_3}{\partial \beta_1} - \frac{\partial B_1}{\partial \beta_1}\gamma_2\right) & \frac{1}{B_1}\left(\frac{\partial B_3}{\partial \beta_2} - \frac{\partial B_1}{\partial \beta_2}\gamma_2\right) & \frac{1}{B_1}\left(\frac{\partial B_3}{\partial \beta_3} - \frac{\partial B_1}{\partial \beta_3}\gamma_2\right) \\ 1 & 1 & 1 \end{bmatrix} \begin{Bmatrix} \Delta\beta_1 \\ \Delta\beta_2 \\ \Delta\beta_3 \end{Bmatrix} = \begin{Bmatrix} \Delta\gamma_1 \\ \Delta\gamma_2 \\ 0 \end{Bmatrix} \tag{31}$$

Equation (31) indicates that the optimal distribution of damping coefficient $(\Delta\beta_i)$ can be found once $\Delta\gamma_i$ is given and the left-hand side of the equation is evaluated. The increments $\Delta\gamma_i = (1 - \gamma_{0i})/N$ where N is the step number, and γ_{0i} denotes the initial value of γ_i obtained from Eq. (19). If one of the β_i values vanishes, it is necessary to check whether the ratio γ_{i-1} corresponding to $\beta_i = 0$ satisfies the condition $\gamma_{i-1} \leq 1$.

Regular repetitive application of Eq. (31) in the range $(\gamma_{0i}, 1)$ will provide a definite optimal distribution of damping coefficient $\beta = \beta_i + \Delta\beta_i$.

The values of $\partial B_1/\partial \beta_i$, $\partial B_2/\partial \beta_i$ and $\partial B_3/\partial \beta_i$ in Eq. (31) can be obtained from partial differentiation of Eq. (27) with respect to β_k as

Then, $\partial B_1/\partial \beta_i$, $\partial B_2/\partial \beta_i$ and $\partial B_3/\partial \beta_i$ may be expressed as follows:

$$\frac{\partial B_1}{\partial \beta_1} = \left\{\left|\hat{\delta}_1\right|_{,11} + \left|\hat{\delta}_2\right|_{,11} + \left|\hat{\delta}_3\right|_{,11}\right\}$$
$$\frac{\partial B_1}{\partial \beta_2} = \left\{\left|\hat{\delta}_1\right|_{,12} + \left|\hat{\delta}_2\right|_{,12} + \left|\hat{\delta}_3\right|_{,12}\right\} \tag{33}$$
$$\frac{\partial B_1}{\partial \beta_3} = \left\{\left|\hat{\delta}_1\right|_{,13} + \left|\hat{\delta}_2\right|_{,13} + \left|\hat{\delta}_3\right|_{,13}\right\}$$

$$\frac{\partial B_2}{\partial \beta_1} = \left\{\left|\hat{\delta}_1\right|_{,21} + \left|\hat{\delta}_2\right|_{,21} + \left|\hat{\delta}_3\right|_{,21}\right\}$$
$$\frac{\partial B_2}{\partial \beta_2} = \left\{\left|\hat{\delta}_1\right|_{,22} + \left|\hat{\delta}_2\right|_{,22} + \left|\hat{\delta}_3\right|_{,22}\right\} \tag{34}$$
$$\frac{\partial B_2}{\partial \beta_3} = \left\{\left|\hat{\delta}_1\right|_{,23} + \left|\hat{\delta}_2\right|_{,23} + \left|\hat{\delta}_3\right|_{,23}\right\}$$

$$\frac{\partial B_3}{\partial \beta_1} = \left\{\left|\hat{\delta}_1\right|_{,31} + \left|\hat{\delta}_2\right|_{,31} + \left|\hat{\delta}_3\right|_{,31}\right\}$$
$$\frac{\partial B_3}{\partial \beta_2} = \left\{\left|\hat{\delta}_1\right|_{,32} + \left|\hat{\delta}_2\right|_{,32} + \left|\hat{\delta}_3\right|_{,32}\right\} \tag{35}$$
$$\frac{\partial B_3}{\partial \beta_3} = \left\{\left|\hat{\delta}_1\right|_{,33} + \left|\hat{\delta}_2\right|_{,33} + \left|\hat{\delta}_3\right|_{,33}\right\}$$

where $\left(\mathrm{Re}\left[\hat{\delta}_i\right]\right)_{,jk}$ and $\left(\mathrm{Im}\left[\hat{\delta}_i\right]\right)_{,jk}$ in Eq. (32) can be obtained from

$$\begin{Bmatrix} \hat{\delta}_{1,jk} \\ \hat{\delta}_{2,jk} \\ \hat{\delta}_{3,jk} \end{Bmatrix} = TA_H^{-1}A_{H,k}A_H^{-1}A_{H,j}T^{-1}\begin{Bmatrix} \hat{\delta}_1 \\ \hat{\delta}_2 \\ \hat{\delta}_3 \end{Bmatrix} - TA_H^{-1}A_{H,j}T^{-1}\begin{Bmatrix} \hat{\delta}_{1,k} \\ \hat{\delta}_{2,k} \\ \hat{\delta}_{3,k} \end{Bmatrix} \tag{36}$$

which is derived by differentiating Eq. (23) with respect to β_k and using the relation $A_{H,k}^{-1} = -A_H^{-1}A_{H,k}A_H^{-1}$. Since A_H is a linear function of β, $A_{H,jk}$ becomes a null matrix for all j and k.

3 Numerical example

In this work it is defined the optimal placement of structural dampers to minimize the sum of amplitudes of the transfer functions evaluated at the undamped fundamental

$$\left|\hat{\delta}_i\right|_{,jk} = \frac{1}{\left|\hat{\delta}_i\right|^2}\left(\left|\hat{\delta}_i\right|\left\{\begin{array}{c}\left(\mathrm{Re}\left[\hat{\delta}_i\right]\right)_{,k}\left(\mathrm{Re}\left[\hat{\delta}_i\right]\right)_{,j} + \mathrm{Re}\left[\hat{\delta}_i\right]\left(\mathrm{Re}\left[\hat{\delta}_i\right]\right)_{,jk} \\ + \left(\mathrm{Im}\left[\hat{\delta}_i\right]\right)_{,k}\left(\mathrm{Im}\left[\hat{\delta}_i\right]\right)_{,j} + \mathrm{Im}\left[\hat{\delta}_i\right]\left(\mathrm{Im}\left[\hat{\delta}_i\right]\right)_{,jk}\end{array}\right\} - \left|\hat{\delta}_i\right|_{,k}\left\{\begin{array}{c}\mathrm{Re}\left[\hat{\delta}_i\right]\left(\mathrm{Re}\left[\hat{\delta}_i\right]\right)_{,j} \\ + \mathrm{Im}\left[\hat{\delta}_i\right]\left(\mathrm{Im}\left[\hat{\delta}_i\right]\right)_{,j}\end{array}\right\}\right) \tag{32}$$

Table 1 Model properties

Property	Value	Units
$m_1 \ldots m_6$	8.00E+04	(kg)
$\beta_1 \ldots \beta_6$	0.20	
$k_1 \ldots k_6$	4.00E+07	(N/m)

Table 2 Undamped fundamental circular frequency

	2 DOF	3 DOF	4 DOF	5 DOF	6 DOF
ω_1 (rad/s)	13.820	9.951	7.766	6.365	5.391

frequency of a shear frame subjected to a constraint on the sum of the damping coefficients of added dampers. For each floor, a damping coefficient (β_i) of the added dampers is defined as a design variable; thus, the number of floors represents the number of design variables.

A total of five shear frames with structural damping between two to six floors are considered, and the optimal placement is obtained based on the method developed by Takewaki [4]. Through the amplitude of the transfer function and global flexibility concept (the sum of the amplitude of the transfer function of the model $V = \sum_{i=1}^{n} \left|\hat{\delta}_i(\beta)\right|$), the efficiency of the method can be observed, comparing the initial flexibility (V_I) to the optimal flexibility (V_O). The model properties are given in Table 1. $\{m_1 \ldots m_6\}$ are the masses, $\{k_1 \ldots k_6\}$ are the spring stiffness, and $\{\beta_1 \ldots \beta_6\}$ are the damping coefficients. The values of the masses and spring stiffness are equally distributed among floors and were taken from Takewaki [4]. The initial value damping coefficient $\beta = 0.20$ is suggested in this work. It also assumed here that the system intrinsic structural damping is negligible compared with the damping of the additional dampers.

Table 2 shows the undamped fundamental circular frequency (ω_1) for each model evaluated. The target values of γ are $\gamma_{F1} = \cdots = \gamma_{F6} = 1.0$, and the number of steps in the redesign process is $N = 75$, where N is used to calculate the increments $\Delta\gamma_i = (1 - \gamma_{0i})/N$ (see Eq. (31)).

The initial values of γ, the relationships between the relative displacements defined in Eq. (19), are presented in Table 3.

Table 3 Initial values of $\{\gamma_j\}$

DOF	γ_1	γ_2	γ_3	γ_4	γ_5
2	0.393	–	–	–	–
3	0.632	0.215	–	–	–
4	0.753	0.425	0.138	–	–
5	0.821	0.568	0.305	0.098	–
6	0.863	0.664	0.440	0.230	0.074

Table 4 Distribution of structural damping coefficients

DOF	β_1	β_2	β_3	β_4	β_5	β_6	$\sum\beta$
2	0.400	0.000	–	–	–	–	0.400
3	0.384	0.216	0.000	–	–	–	0.600
4	0.443	0.357	0.000	0.000	–	–	0.800
5	0.421	0.357	0.223	0.000	0.000	–	1.000
6	0.459	0.410	0.331	0.000	0.000	0.000	1.200

Table 5 Transfer function amplitude for the initial design and optimal design

Story (#)	Stage	2 DOF	3 DOF	4 DOF	5 DOF	6 DOF
1	Initial	0.0096	0.0139	0.0183	0.0225	0.0268
	Optimal	0.0074	0.0094	0.0115	0.0141	0.0162
2	Initial	0.0057	0.0108	0.0156	0.0202	0.0247
	Optimal	0.0054	0.0085	0.0107	0.0134	0.0156
3	Initial	–	0.0059	0.0114	0.0165	0.0214
	Optimal	–	0.0052	0.0100	0.0124	0.0147
4	Initial	–	–	0.0060	0.0117	0.0171
	Optimal	–	–	0.0054	0.0099	0.0144
5	Initial	–	–	–	0.0061	0.0119
	Optimal	–	–	–	0.0052	0.0102
6	Initial	–	–	–	–	0.0061
	Optimal	–	–	–	–	0.0053
	V_I	0.0153	0.0307	0.0513	0.0770	0.1079
	V_O	0.0128	0.0232	0.0375	0.0550	0.0764

4 Results

The obtained results, by the MATLAB code of the optimization procedure, are presented in Table 4. A total of five shear building models were evaluated, and the optimal distribution of the damping coefficients was calculated. Horizontally in Table 4, the amount of damping required for each floor is presented, and the last column verifies the constraint on the sum of the damping coefficients of added dampers as explained in detail in Eq. (14). The 2 DOF model result revealed lower amplitude of the transfer function (stage—optimal, see Table 5) and global flexibility (stage—optimal, see Table 5) of the structure, which occurs when all damping devices are placed on the first floor. Concerning the 6 DOF model, the smallest amplitude of the transfer function (stage—optimal, see Table 5) is obtained by damper location on the first three floors and none on the last three floors, as shown in Table 4.

The initial and optimal amplitudes of the transfer function evaluated at the undamped fundamental frequency of a structural system are presented in Table 5. It is important to note that for the calculation of the initial amplitude of the transfer function, a uniform damping was

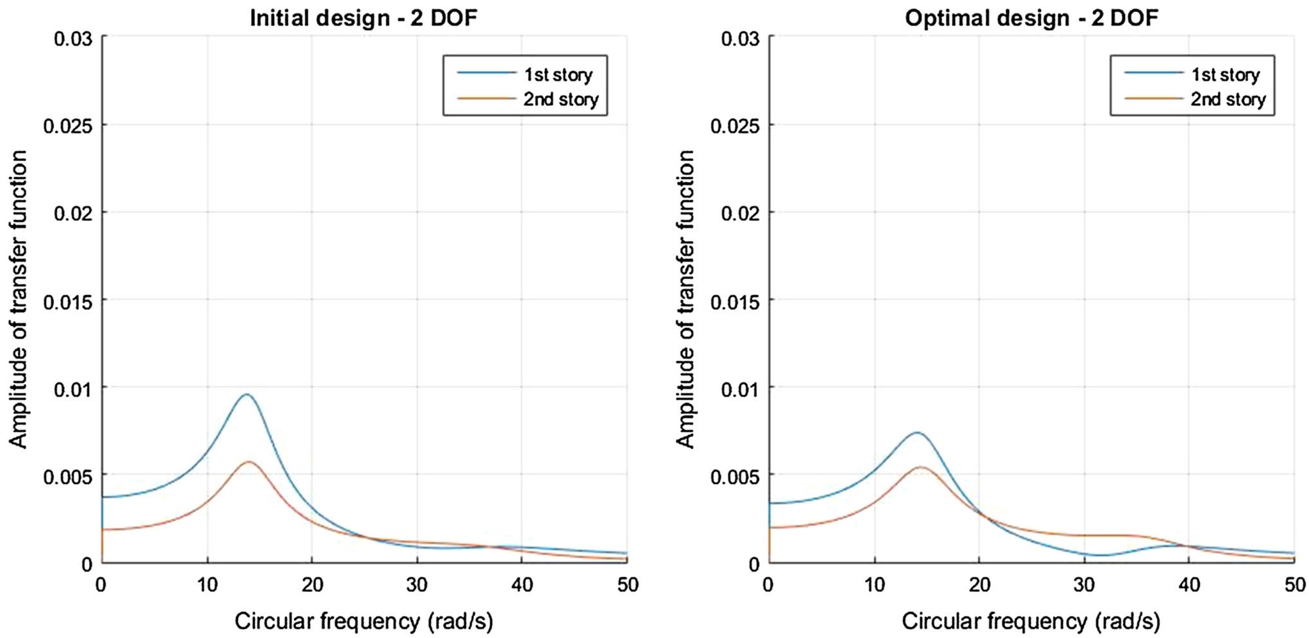

Fig. 2 Initial and optimal design—2 DOF

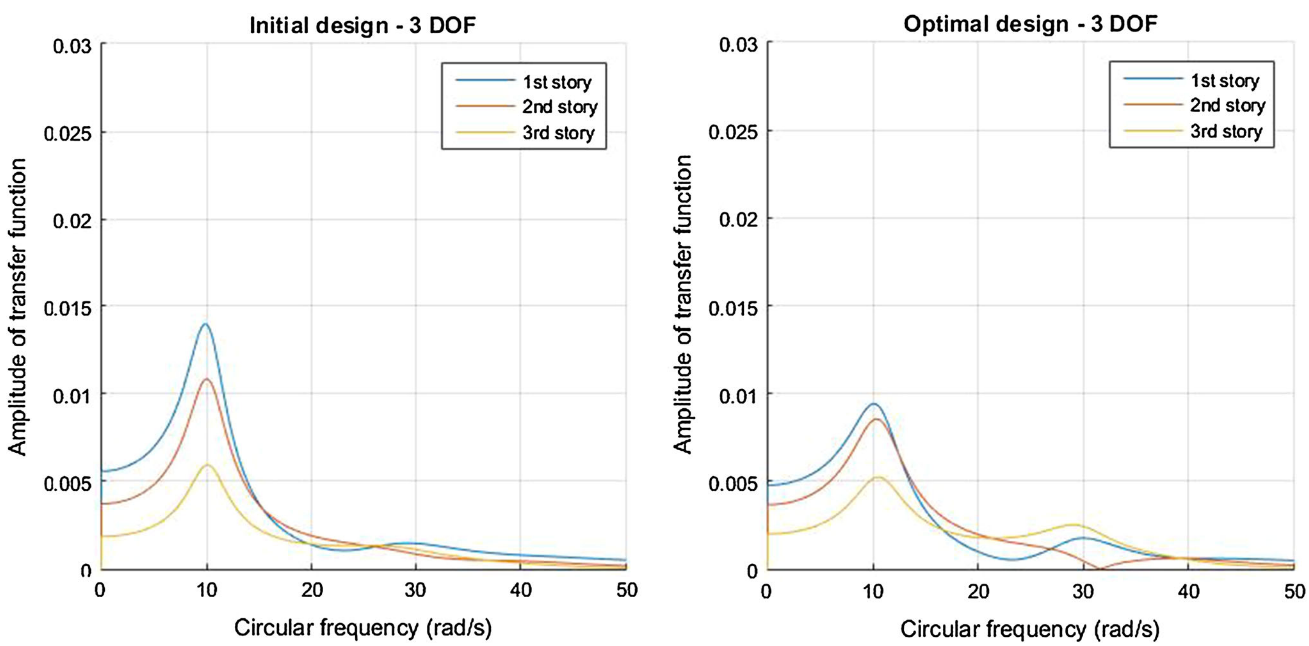

Fig. 3 Initial and optimal design—3 DOF

considered in each floor. The calculation of the optimal amplitude also used the distribution, presented in Table 4. In the last two lines of Table 5, it is shown the initial flexibility (V_I) and optimal flexibility (V_O); it was observed that there was a decrease in the transfer function amplitudes in all models when the damping coefficients were optimally distributed.

In Figs. 2, 3, 4, 5 and 6, the evolution of the initial and optimal amplitudes of the transfer function in the range of 0–50 rad/s is presented. There was a significant reduction in the transfer function amplitudes in all floors corresponding to the models evaluated, when the optimal distribution of the damping coefficients was considered.

The results demonstrated the importance of defining the optimal location of dampers in a building. It is not the best

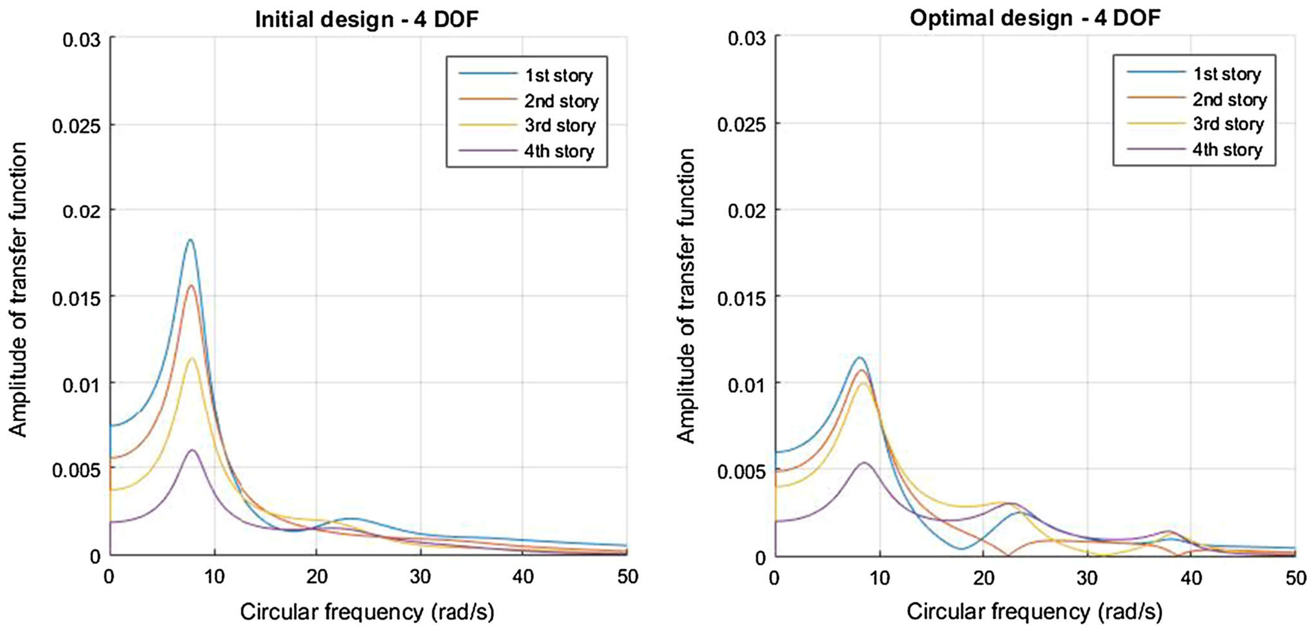

Fig. 4 Initial and optimal design—4 DOF

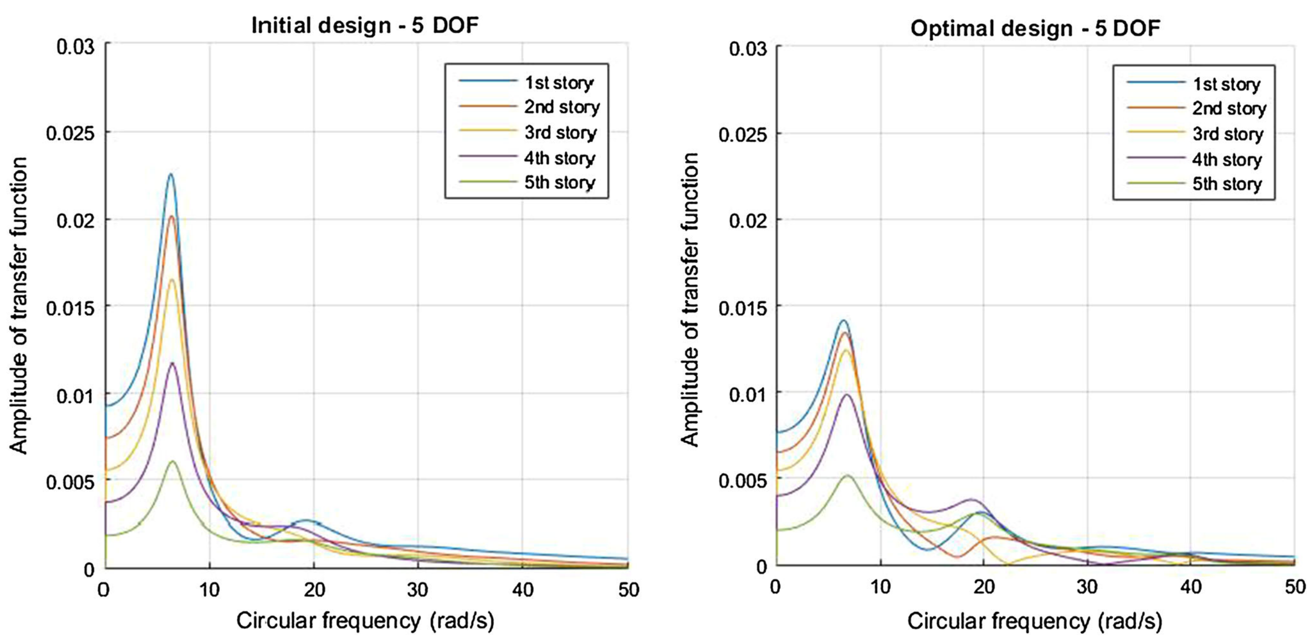

Fig. 5 Initial and optimal design—5 DOF

way to fill the building with damping devices; it is more indicated to smartly place the dampers. When the stiffness is equally distributed among floors, the results suggested to distribute the dampers mainly from half of the building, toward the first levels.

The efficiency of the Takewaki's method has been verified by Whittle et al. [36]. The authors conducted a review of five different damper placement methodologies. A comparative and detailed study was carried out in a ten-story, steel moment-resisting frame. Methods using simple empirical rules and advanced methods, including Takewaki's method, were performed. The results showed that the Takewaki and Lavan methods were the most efficient. An advantage of Takewaki's method is to require only minimal inputs and operate independent from ground motions. The results extracted from Wittle et al.'s paper are presented in Table 6.

Where DBE and MCE are seismic design parameters for building code design.

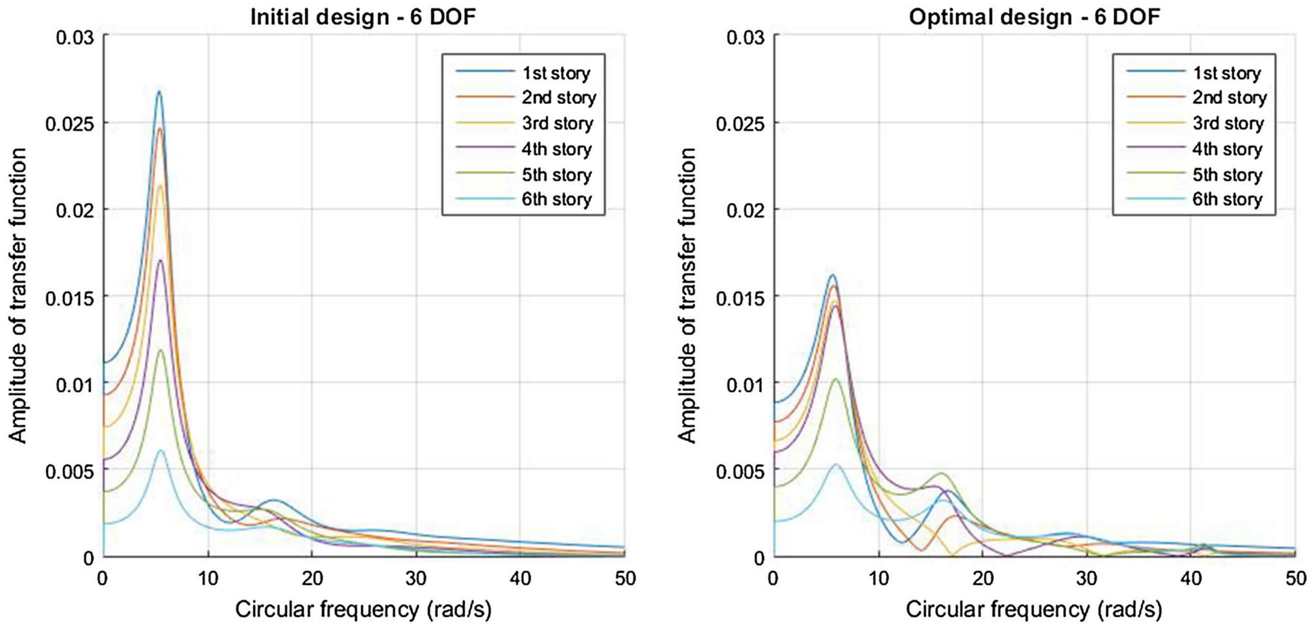

Fig. 6 Initial and optimal design—6 DOF

Table 6 Maximum of peak interstory drifts in regular building [36]

Method	DBE ground motion suite		MCE ground motion suite	
	%	mm	%	mm
No dampers	2.27	72.5	3.28	104.9
Uniform	1.08	34.7	1.64	52.6
Stiffness proportional	1.07	34.3	1.62	52.0
SSSA	0.94	30.1	1.41	45.2
Takewaki	0.90	28.7	1.34	43.0
Lavan	0.87	27.7	1.30	41.6

5 Conclusions

In this paper, a systematic procedure to find the optimal placement of structural dampers has been examined; shear building models were evaluated with device placement between two and six floors. The procedure was based on the method by Takewaki that proposed an optimal distribution to viscous devices. Mathematical equations were adapted and detailed for a system with structural damping devices.

Through this investigation, it has been shown that the random or equal distribution of dampers among the floors may result in a loss of performance. Table 4 shows the optimal distribution of the damping coefficients for the analyzed models, and it was evidenced that for a building with equal stiffness distribution among the floors, it is efficient to locate the dampers from the middle height of the building to the first level. This was also verified by comparing the initial and optimal amplitude of the transfer function presented in Table 5. Regarding to building and civil construction, cost is a relevant issue to be considered in the analysis of the feasibility of the proposed structural control system as well as materials and accessories. Therefore, it is very important and helpful to structural designers to be able to define, under high standard criteria, not only the quantity but also the location of dampers.

The initial and optimal amplitudes of the transfer function were also compared, as shown in Figs. 2, 3, 4, 5 and 6, and the smaller amplitude of the transfer function in a frequency range of 0–50 rad/s was obtained when the damping coefficients were distributed in the first levels. Analyzing the results for the 6 DOF model (Fig. 6), it is possible to observe that the initial and optimal amplitudes in the first three levels are similar. The effect of the optimal distribution of the damping coefficients is verified in the higher levels, where the difference between the amplitudes is higher. This investigation will help structural designers define a best location of structural damping devices to maximize its efficiency and protect structures from possible collapse in the case of occurrence excessive vibrations on the structure.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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