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REVIEW

Nonlinear updating method: a review

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Abstract This paper is a survey of the literature about the nonlinear updating process. It is focused on the computation of the difference between the numerical model and the reference data as well as the algorithm uses to find the optimal parameters. In both parts of the nonlinear updating process, the popular approaches are presented. Special emphasis is given to methods based on Volterra series.

Keywords Model updating · Nonlinear model · Nonlinear identification · Sensitivity

1 Introduction

Numerical models are powerful tools to design a mechanical structure, to control its integrity or to predict its life's service. In the particular case of the linear behaviour of the mechanical systems, the methods for updating the numerical models are well known and are widely used for industrial applications (e.g. [36, 38, 70]). Typically, a linear model is a first approximation of a practical system. Indeed, nonlinear behaviour (e.g., due to large displacements, gaps, jumps, material behaviour, discontinuities) is very common in real mechanical structures (e.g. [52, 104]). The modelling of these nonlinear effects increases the reliability of the numerical model. However, the number of numerical parameters increases with the complexity of the model and it is not easy to identify the value of these parameters due to the uncertainties in the experimental system (e.g., mechanical properties, boundary conditions). Consequently, the first step is to define the numerical model. On one hand, this model could be complex enough to obtain a sufficiently reliable result. On the other hand, this model could be simple enough to minimise CPU time needed to solve it. The choice of the numerical model is still a balance between numerical cost and reliability.

1.1 The nonlinear model

An important part of the nonlinear numerical model study into the literature uses finite element method. For example, Astroza et al. [1] has dealt with the linear and nonlinear civil structural finite element models and has proposed a new updating method combining simulated annealing with the unscented Kalman filter. Nonlinear finite element model has also been updated by Ebrahimian et al. [31] using a batch Bayesian estimation approach. Many other examples can be found into the literature, as well as the finite element model with localized nonlinearity studied by da Silva et al. [95] and the nonlinear finite element model treated by Bussetta et al. [12]. Both examples use the Volterra series into the updating process.

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To optimise the computational time, an iterative process using meta-model can be used. First, a meta-model created with the solution of the finite element model is used to identify the parameters' value. Then, the updated finite element model is solved and the new value is used to correct the meta-model. This process continues until convergence is achieved (see Peeters et al. [80]).

A review of regression and Kriging meta-models is presented by Kleijnen [58]. The numerical model can use reduced-order modelling. For example, Claeys et al. [25] used the Craig–Bampton method to compute a simple model (i.e. with 1 or 2 degree of freedom) from a complex finite element one. Touzé et al. [98] proposed a reduced-order modelling using nonlinear normal modes. Čermeljand Boltežar [99] presented a reduced model in the frequency domain based on the sub-structuring of complex structures into linear and nonlinear parts. Lucia et al. [72] dealt with a large bibliography review and a comparison of order-reduction modelling techniques. They were focused on Volterra series representation, the proper orthogonal decomposition (POD) and harmonic balance (HB), particularly in the multi-disciplinary field of computational aeroelasticity.

Some investigations have used numerical models representative of complex physics. For example, the HB has been used by Detroux et al. [29] for the detection and tracking of bifurcations of nonlinear systems. Li and Billings [68] have dealt with the extension of the of Volterra analysis to weakly nonlinear Duffing systems at a much wider range of excitation amplitude. Restoring force surface (RFS) model has been studied by Kerschen et al. [49] into the framework of the identification of non-linear systems. The black-box model is another kind of these nonlinear models. Juditsky et al. [46] has discussed about several aspects of the mathematical foundations of the nonlinear black-box identification problem. The artificial neural network is a similar model. The properties of this model have been investigated by Billings et al. [6] through to the study of non-linear dynamical systems. Auto-regressive with exogenous inputs (ARX) models are widely utilised for describing dynamic data regimes for linear and non-linear systems. Chen et al. [23] have proposed a new fitness function to improve the quality of the (N)ARX model using a genetic algorithm.

Generally speaking, the updating methods are applied by assuming linear models of the structure with lumped non-linearities, often assuming some kind of nonlinear springs as being the source of the nonlinear behaviour. Although this kind of model is able to reproduce the nonlinear phenomena (e.g. Kerschen et al. [50]), it is actually a rough simplification of the structure. Kerschen et al. [52] presents an extensive bibliography review on the nonlinear system identification techniques. Recently, this review was updated by Noël and Kerschen [77].

1.2 Goal and outline

Despite of the important number of investigations, the nonlinear model updating techniques are not mature as the classical linear tools. The goal of this article is to give survey about the state-of-the-art of the nonlinear updating method. This bibliography review deals with the two main parts of the updating process: the computation of the distance between the reference and the model as well as the parameter identification algorithm. Both parts are independent and can be freely combined.

As the updating methods, the outline of this article is split into two critical parts. First, the Sect. 2 deal with the choice of the objective function (i.e. the difference between the reference and the model). Then, the algorithm used to find the optimal parameters of the numerical nonlinear model is discussed into the Sect. 3 (i.e. how to find the value of the parameters). Finally, some remarks about the nonlinear updating process are summarised in Sect. 4.

2 Error estimation: numerical model/reference data

The definition of the distance between the numerical model and the reference solution is the first difficult step on the way to increase the reliability of the model. This measure called error or objective function, is the main criterion uses to choose the value of the numerical parameters. Because of the uncertainty, the lack of knowledge and the specificities of each problems, a important number of error functions has been developed. These objective functions can use data in the frequency domain or in the time domain.

2.1 Harmonic balance (HB)

The harmonic balance (HB) method is the most widespread of the ones used in the frequency domain. This procedure represents the response of a nonlinear system to an harmonic input as a sum of sines with frequencies that are integer multiples of the input frequency. This consideration allows to analyse the frequency response of the system by substituting the input signal and the assumed output in the motion equation. To update the nonlinear model, the frequency response function (FRF) is linearised around the excitation frequency. Meyer and Link [74] used this method to identify the nonlinear two-degree-of-freedom elements into large linear finite element models. In Böswald and Link [8], a similar way was used to update the nonlinear stiffness and the damping of a joint. Moreover, a large number of investigation deals with the enhancement of the HB method. Hall et al. [42] developed an improved version of the HB method for the Navier-Stokes equations, and used it for modelling unsteady nonlinear flows in multi-stage turbomachinery. Liu et al. [71] used this kind of improved method for an aeroelastic airfoil with cubic restoring forces, and the results were compared with the results from the classical HB approach. The constrained optimization multi-dimensional harmonic balance method was proposed in Liao [69] to the uncertainty quantification problems in rotor dynamics with multiple frequency excitations. Another strategy to enhanced the HB method was proposed by Cochelin and Vergez [26], HB method was combined with a continuation method to follow the periodic solutions of dynamical systems. This method was extend to the case of non-polynomial non-linearities by Karkar et al. [48].

2.2 Constitutive relation error (CRE)

The concept of constitutive relation error (CRE) used to update linear models in the frequency domain (e.g., [2, 28]) can be extended to update nonlinear systems. CRE is an iterative method where each iteration is split in two steps. First, the most important error is localised in the model and then the parameters are updated to correct this error. Guchhait and Banerjee [40] used the CRE to update nonlinear hyperelastic material models. A modified constitutive relation error was proposed by Nguyen et al. [76] for nonlinear behaviour in general and for the delay damage model in particular. Isasa et al. [44] combined extended constitutive relation error with multi-harmonic balance to update linear model with local non-linearity.

2.3 Volterra series in the frequency domain

Another powerful method in the frequency domain uses the Volterra series to extend the concept of frequency response functions to nonlinear systems (the Fourier transform of the Volterra kernel is named higher-order frequency response functions). The Volterra series approach extends the input–output relationship for linear systems to nonlinear ones. With the Volterra series, the output of a nonlinear system is the sum of a linear contribution and nonlinear ones (see, e.g. [86, 87, 101]). The linear contribution is the convolution between the first Volterra kernel and the input. The nonlinear contributions are multidimensional convolutions between the higher-order Volterra kernels and the input. A lot of scientific publications deals with this updating method. Chatterjee and Vyas [21] used this method to update the nonlinear parameter of a Duffing oscillator. The case of multi-input Volterra series was studied by Chatterjee and Vyas [22]. In this article, the third order Volterra kernel was estimated in the frequency domain thanks to a relationship with the previously computed first order one. Chatterjee [20] extended this updating method to the case of multi-tone excitation. Lang et al. [60] used nonlinear systems which can be described by a polynomial form differential equation model to compute an expression for the frequency response function. This method was used by Lang et al. [61] to compute the optimal value of nonlinear viscous dampers for vibration control of multi-degree-of-freedom systems. It was illustrated by a multi-storey shear building model submitted to harmonic or earthquake loadings. Peng et al. [81] compared this method with the HB method using the Duffing oscillator. According to their conclusions, this method is better for strongly nonlinear systems, but contrary to the HB method, it cannot capture the jump phenomenon. Dong et al. [30] computed analytical expressions for the calculation of output power spectral density (PSD) and input–output cross-PSD of nonlinear systems subjected to a Gaussian white noise excitation. These expressions allowed the authors to express the output PSD as well as the input–output cross-PSD as a polynomial function of the input intensity or a polynomial function of the nonlinear characteristic parameters. This formulation was validated in a single degree-of-freedom nonlinear system. Feijoo et al. [33] took advantage of the Volterra model to split the nonlinear equation system into associated linear equations for Volterra operators and to define the associated frequency response functions. These ones are easier to analyse and interpret than the more complicated higher-order frequency response functions. This method applied to only single degree-of-freedom nonlinear system was extended to multi-degree-of-freedom nonlinear (see Feijoo et al. [34] in which the method was illustrated with a two degree-of-freedom example).

2.4 Volterra series in the time domain

Moreover, the Volterra series can be used directly in the time domain to define metrics for the nonlinear model updating. da Silva [93] proposed metrics based on first two Volterra kernels to identify local non-linearity in large FE model. Shiki et al. [91] presented an improvement of this updating method using the first three Volterra kernels. This updating method was used by Shiki et al. [90, 92] for damage detection in nonlinear structures. Another metrics based on first three Volterra kernels was used by Bussetta et al. [12] to update large nonlinear FE model. Third-order Volterra model was used to simulate nonlinear bridge aerodynamics by Wu and Kareem [107, 108]. Some authors evaluate the value of the Volterra kernels thanks to another nonlinear model. Wray and Green [106] shown that a certain class of artificial neural networks are equivalent to Volterra series and gave the expression of the Volterra kernel versus the internal parameters of the network. Guo et al. [41] brought to light that the Volterra series can be considered as a specialisation of the Adomian decomposition and gave the relation between the value of the Volterra kernels and the Adomian polynomials.

2.5 Identification of the Volterra kernels

Unfortunately, in the general case, the identification of the Volterra kernels is very difficult to do because of ill-posed and convergence problems. In addition, the number of terms to define the kernels is quite large (M^p coefficients to identify pth-order kernel for a system with a memory of M samples, i.e. the input's value affects the next M next samples of the output). To identify the Volterra kernels, some authors use a specific input signal. Bedrosian and Rice [3] presented a method to compute the Volterra series in the frequency domain with harmonic or Gaussian inputs. This method is extended to deal with the multiple sinusoidal inputs and multi-output for both continuous-time and discrete-time in Worden et al. [105]. A simpler and more general version of this algorithm was proposed by Jones [45]. This algorithm is not limited to specific harmonic inputs. Wu and Kareem [107] utilised an impulse function as input to identify the Volterra kernels in the time domain. The proposed method is tested thanks to a numerical example of a long-span suspension bridge. On the other hand, to reduce the number of estimated coefficients the Volterra kernel can be expanded in a specific function basis. The Volterra kernels expanded in a orthogonal basis are named Wiener kernels. da Rosa et al. [85] studied an optimisation of a two-parameter Kautz basis for the orthonormal series expansion of discrete-time Volterra models. da Silva et al. [95] identified the first and second-order Volterra kernels in an Kautz orthogonal basis. Shiki et al. [91] used the same method to identify the first three Volterra kernels. In Scussel and da Silva [88] the Volterra kernels have been identified using only output data. Prazenica and Kurdila [84] proposed to expand the Volterra kernels in a multiwavelet basis using the technique of intertwining. This technique is used to compute first-, second-, and third-order Volterra kernels. This method was extended to the case of nonlinear multi-input multi-output aeroelastic systems by Khawar et al. [55]. Laguerre expansions of the Volterra kernels was studied by Marmarelis [73] and Campello et al. [14]. Orcioni [79] presented an improved cross-correlation method to identify each Wiener kernel with a different input variance and new formulas for conversion from Wiener to Volterra representation are presented. This improved method was compared with the reference one. Kibangoua et al. [56] proposed a new constructive procedure for selecting a generalized orthonormal basis in the case of second-order Volterra systems. Another approach is presented by Brenner et al. [10], they proposed to identify the Volterra kernel thanks to a regularisation method using a multiscale collocation method and to approximate the full matrix of the Volterra kernel by an sparse matrix. The least-squares method was used to identify the sparse Volterra kernels of nonlinear bridge aerodynamics from a numerical simulation and a wind-tunnel experiment (see [108]).

2.6 Restoring force surface (RFS)

A classical and historical time domain approach is the Restoring Force Surface (RFS) method. This method uses the equation of motion written in terms of the input and the output to identify the nonlinear function of the restoring force. The RFS was studied on the point of view of the integration and the differentiation of measured time data (see [102]) as well as the choice of the excitation signal (see [103]). Kerschen et al. [49] dealt with the experimental and numerical identification of nonlinear beam using the RFS method. This method was used by Kerschen et al. [51] for the identification of the VTT benchmark, which it consists of wire rope isolators mounted between a load mass and a base mass. Platten et al. [83] used an enhancement of the RFS method using an extended modal space model to identify multi-degree of freedom nonlinear systems. The RFS method was used by Keshavarzzadeh and Masri [54] to identify the nonlinear system using the generalized form of a Padé–Legendre approximation. Vismara et al. [100] updated the finite element model of the flap of the intermediate experimental space vehicle using the RFS model. da Silva et al. [94] compared the performance of two metrics based on the HB, one using the CRE, two using the RFS and one computing the POD for nonlinear model updating. According to their conclusion, in the case of strong non-linearity only the RFS method is able to update correctly the model.

2.7 Proper orthogonal decomposition (POD)

Another nonlinear updating method in the time domain is based on the proper orthogonal decomposition (POD), also known as Karhunen–Loève (K–L) decomposition or principal component analysis (PCA). The POD method consists of extracting spatially coherent modes from time-series data. This decomposition can be used as an orthogonal basis for efficient representation of the nonlinear system. Lenaerts et al. [65] use this method to update a numerical model of a beam with a local non-linearity thanks to experimental data. Galvanetto and Violaris [37] proposed a new method based on the POD for damage detection in mechanical structures. This method was used with homogeneous plates and composite beams by Thiene et al. [97]. The randomly vibrating systems was studied by Bellizzi and Sampaio [4]. Another kind of decomposition, that is not orthogonal in the Euclidean sense, named smooth decomposition (SD), was presented by Bellizzi and Sampaio [5].

2.8 Artificial neural network (ANN)

An artificial neural network (ANN) can define the input–output relationship for complex system. First, data (input and output values) are used to build the ANN. Then, the output of the ANN is validated with another set of input–output data. Hasançebi and Dumlupinar [43] presented an updating technique using ANNs for finite element model of reinforced concrete T-beam bridges. Olivencia Polo et al. [78] proposed a failure mode prediction method of photovoltaic plants based on ANNs. Eski [32] presented a model of a drilling machine using ANNs. Lauret et al. [63] proposed to model the electric load forecasting with ANN, which was computed using the Bayesian approach. This method was compared with a nonlinear regression techniques namely Gaussian process and ANNs model with classical learning method (see [64]). The investigations of Li and Shi [66] were about the comparison of the performance of three different numerical models based on ANNs thanks to three metrics in 1-h-ahead wind speed forecasting. According to the authors, no single neural network model outperforms others universally in terms of all evaluation metrics and the selection of the type of neural networks for best performance is also dependent upon the data sources. To overcome this difficulties, Li et al. [67] proposed to use Bayesian combination method.

2.9 Example of error estimation: symmetric Duffing oscillator

A few simple simulation were performed to illustrate the error estimation presented in this section of the paper. These simulation were done by simulating a updating procedure of a simple symmetric Duffing oscillator [9]. This kind of equation can exhibit very complex behaviour as harmonic distortion, jumps, bifurcation and chaos. Because of this rich behaviour, the Duffing equation is a benchmark in the nonlinear dynamics literature.

In this kind of vibrating system, the nonlinearity comes from polynomial terms in the stiffness terms. For the case of the symmetric Duffing oscillator, the motion equation can be described as:

$$m\ddot{y}(t) + c\dot{y}(t) + k_1 y(t) + k_3 y(t)^3 = u(t) \tag{1}$$

where m is the mass coefficient; c is the damping constant, k_1 is the linear stiffness, and k_3 is called the nonlinear cubic stiffness. Despite the apparent simplicity of this nonlinear differential equation, it can be used to describe the vibrations of systems with geometrical nonlinearities, nonlinear vibration isolators, pendulums, among other systems [59].

For this illustration, the reference parameters of the system were considered to be: $m = 0.078$ kg, $c = 0.50$ N (m/s)$^{-1}$, $k_1 = 1230$ N/m and $k_3 = 8.13 \times 10^7$ N/m^3. These so called reference parameters are the values to be found in an model updating process. In this paper, these values are used to generate pseudo-experimental input and output data. Also, since the focus in on the nonlinear parameters, it is considered that m, c and k_1 are already known since they can be calculated using classical linear model updating techniques.

Three different approaches suitable for single degree of freedom nonlinear systems are illustrated in this section: the harmonic balance (HB), Volterra series in the frequency domain and the restoring force surface (RFS). A more in depth description of other error metrics can be found in the paper [94].

2.9.1 Error estimation in the Duffing oscillator using the Harmonic Balance

The harmonic balance can be used to estimate the frequency response curve (FRC) to steady-state sinusoidal excitation. In an experimental test, this kind of curve is approximated by using a stepped-sine excitation in order to observe the jump phenomenon. In this simulation, the Duffing oscillator is excited by a 0.5 N amplitude stepped sine with a frequency resolution of 0.075 Hz and block duration of 2.5 s from 22 to 25 Hz. By capturing the amplitude of the response for each frequency, the FRC can be estimated and compared to an analytical value in order to fit a value of the nonlinear stiffness k_3.

Analysing the Duffing equation with the harmonic balance, the amplitude relation of the FRC ($H_{\mathrm{FRC}}(\omega, k_3)$) is:

$$H_{\mathrm{FRC}}(\omega, k_3) = \frac{Y}{U}(\omega) = \frac{1}{\sqrt{\left(-m\omega^2 + k_1 + \frac{3}{4}k_3 Y^2\right)^2 + c^2\omega^2}} \tag{2}$$

where Y is the steady-state amplitude of the output and U is the input level. Note that $H_{\mathrm{FRC}}(\omega, k_3)$ is written as a function of k_3 which is considered to be unknown. This analytical curve can be used as a function of the nonlinear parameter k_3 so that one can use this expression in a residue between the experimental FRC ($H_{\exp}(\omega)$) and the Duffing equation FRC ($H_{\mathrm{FRC}}(\omega, k_3)$):

$$J_{\mathrm{HB}}(k_3) = ||H_{\mathrm{FRC}}(\omega, k_3) - H_{\exp}(\omega)|| \tag{3}$$

In the Fig. 1, the error $J_{\mathrm{HB}}(k_3)$ is illustrated as a function of the nonlinear stiffness k_3. It is possible to observe that the global minimum of the error agrees with the reference nonlinear stiffness ($k_3 = 8.13 \times 10^7$).

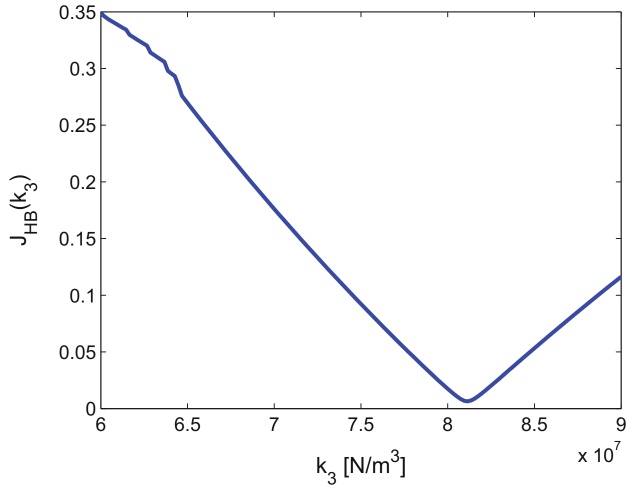

Fig. 1 Calculation of the nonlinear parameter for the Duffing oscillator using the Harmonic Balance

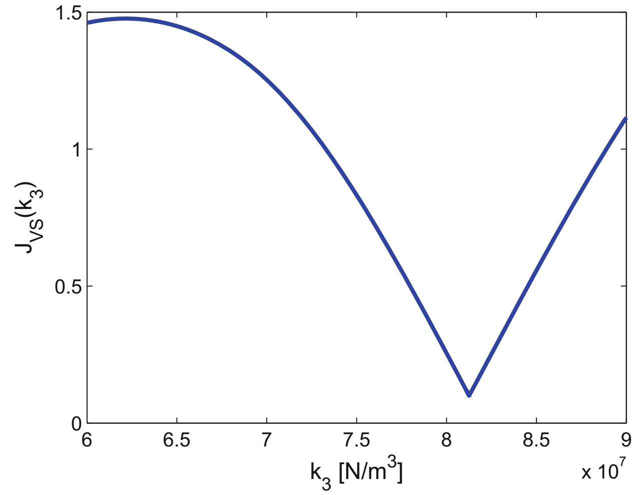

Fig. 2 Calculation of the nonlinear parameter for the Duffing oscillator using the Volterra series in the time domain

2.9.2 *Error estimation in the Duffing oscillator using the Volterra series in the time domain*

To identify the Volterra model of a nonlinear system, the input and output signals need to be measured to have a least-squares approximation of the Volterra kernels. The input signal $u(t)$ need to broadband excite the nonlinear polynomial terms of the system to have a complete characterization. In this case, a sweep sine input was applied with 0.5 N amplitude from 0.1 to 200 Hz during 8.191 s. The signals were discretised using a sampling frequency of 1 kHz, the response $y(t)$ was calculated by using the Newmark algorithm. With the input force applied and the displacement of the Duffing oscillator as the output, the Volterra kernels representing the linear part of the model ($\mathcal{H}_{1,\text{exp}}$) and the nonlinear part of the response ($\mathcal{H}_{3,\text{exp}}$) were calculated. These terms are considered to translate the behaviour of the system in a nonparametric representation.

To calculate an error metric based on the Volterra series, a residue between experimental nonlinear kernel $\mathcal{H}_{3,\text{exp}}(\omega)$ and the kernel calculated with the Duffing oscillator $\mathcal{H}_3(\omega, k_3)$ was applied.

$$J_{\text{VS}}(k_3) = ||\mathcal{H}_3(\omega, k_3) - \mathcal{H}_{3,\text{exp}}(\omega)|| \tag{4}$$

where $J_{\text{VS}}(k_3)$ is the error between the nonlinear kernels. By minimizing this function it is possible to have an approximation of the nonlinear stiffness k_3 based on an experimentally extracted model of the nonlinearity. In the Fig. 2 the error $J_{\text{VS}}(k_3)$ is illustrated as a function of the nonlinear stiffness k_3.

2.9.3 *Calculation of the nonlinear parameter in the Duffing oscillator using the restoring force surface*

For this illustration, the same dataset used in the previous section can be applied (with the sweep sine force $u(t)$). In this specific case, it is possible to consider an additional term f_{NL} that represents the nonlinear contribution in the motion equation:

$$m\ddot{y}(t) + c\dot{y}(t) + k_1 y(t) + f_{\text{NL}} = u(t) \tag{5}$$

In a practical parameter identification, considering that the linear parameters m, c and k_1 were previously calculated, the nonlinear force time series can be calculated by doing:

$$f_{\text{NL}} = u(t) - (m\ddot{y}(t) + c\dot{y}(t) + k_1 y(t)) \tag{6}$$

One can observe that the acceleration $\ddot{y}$, velocity $\dot{y}$ and the displacement y need to be known also. Since the acceleration is an usual measurement in modal testing, this signal can be integrated twice to obtain $\dot{y}$ and y. With this it is possible to fit a polynomial curve to represent f_{NL} as a function of the displacement and/or the velocity of oscillation. In this simple case, the f_{NL} was expressed as:

$$f_{\text{NL}} = k_3 y^3 \tag{7}$$

The Fig. 3 shows the comparison between the f_{NL} as a function of the displacement y, as well as the polynomial fit that can be used to calculate k_3.

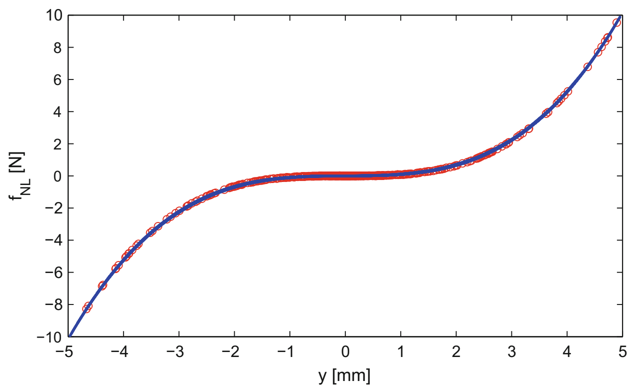

Fig. 3 Calculation of the nonlinear parameter for the Duffing oscillator using the restoring force surface. The red circle represents the experimental data and the blue continuous line is the cubic polynomial that fits the data

3 Algorithm of model identification

The algorithm used to identify the value of the parameters is a key part of the updating process. This algorithm has a important influence over the solution—convergence towards local optimum— as well as over the numerical cost—numbers of computations of the numerical model. The search algorithm can be split into the iterative optimisation algorithms [18] and the global identification algorithms (e.g., genetic, evolutionary, stochastic) [19]. The main advantage of the first kind of algorithm is that the convergence is obtained with a limited number of computations of the numerical model. The main disadvantages of this kind of method stem from the difficulty to compute the objective function gradients, the sequential nature of the process and that the algorithm converge toward local minima, which can be sometimes satisfactory, but not always. With the second kind of algorithm, the global optimum is identified but the number of resolution of the numerical model can be very large. To reduce the computational cost, the search algorithm can use a metamodel. Bonte et al. [7] compared different kinds of search algorithms—sequential approximate optimization, iterative algorithm and metamodel assisted evolutionary strategy—by application to two forging processes. However, because of the lack of data and modelling errors, a local minimum could be a better model of the system (e.g. [109]). This search strategy was enhanced by Caicedo and Zárate [13] by a reducing epistemic uncertainty technique.

3.1 Iterative optimisation algorithms

The classical iterative optimisation algorithms (conjugate gradient, BFGS, etc.) can be used to identify the parameters value. Castro et al. [17] studied the optimisation of intermediate die shapes of forging process with a method based on a modified sequential unconstrained minimisation technique and a gradient method. The calculation of objective function gradients are computed by the direct differentiation method in [35, 111] or by the adjoint state in [24]. Sekhar and Ganguli [89] compares modified Newton method, rank-1 Broyden update, and rank-2 BFGS update methods for the numerical analysis in the helicopter trim problem. Kim et al. [57] used sequential gradient method to solve nonlinear inverse problem of laminar-forced convective flow.

3.2 Bayesian method

Another algorithm of model identification process is based on the Bayesian method [11]. With the Bayesian inference, the uncertainties (parameter values and noise) are included into the identification process. The process identifies a set of parameters with a degree of belief and error bar. Lauret et al. [62] used this kind of identification method to estimate two convective heat-transfer coefficients of a roof-mounted radiant barrier system. Zárate et al. [110] presented a Bayesian approach for location of acoustic emission sources in the shell of liquid filled tanks. A different approach using a novel Markov chain Monte Carlo algorithm with the Bayesian method for the identification of a nonlinear dynamical system was proposed by Green [39].

3.3 Genetic algorithm (GA)

The genetic algorithm (GA) can be used to update the model. The GAs are based on the law of the survival of the fittest. The nonlinear model of a ball joint system was updated through the genetic algorithm in Davoodi et al. [27]. An evolutionary search model based on the GA was proposed by Castro et al. [16] to update thermomechanical nonlinear finite element model of hot metal-forming processes. Canyurt and Hajela [15] described the parallel implementation of cellular genetic algorithm for multicriteria design optimization problems. The main idea of the cellular genetic algorithm is to treat the values of model's parameters as being distributed over a 2-D grid of cells, which cell defines a set parameter's values. The optimisation of densification process of metal powders with GA was considered by Keshavarz et al. [53]. This process was modelled by thermomechanical nonlinear finite element method.

3.4 Bees algorithm (BA)

The Bees algorithm (BA) is based on the food research strategy of the honeybee swarms [82]. Few scouts explore randomly for source food (acceptable solution). Then, more bees are sent to the source food for neighbourhood searching. Moradi et al. [75] applied this search algorithm to update finite element model of structures. In addition, they compared this algorithm with the GA, the particle swarm optimization (PSO) and the inverse eigensensitivity method (IEM). According to their conclusion, BA showed to have more accurate results than those of GA and IEM, it was similar to PSO. Sun et al. [96] used BA to update a finite element model thanks to surrogate models—response surface methodology, Kriging, radial basis function and support vector regression. The proposed strategy is into the framework of sheet metal forming process. Karaboga and Gorkemli [47] proposed a quick artificial bee colony which is a new version of AB algorithm. This algorithm models the behaviour of onlooker bees more accurately and improves the performance of standard AB in terms of local search ability.

4 Final remarks

The numerical modelling of the nonlinear mechanical system is a useful tool. On the other hand, the identification of the model (the value of the parameters) versus the real system (experimental data) is a complex process. Each part of the identification of the numerical model is important. Obviously, the choice of the numerical model is very important; it depends on the knowledge and the understanding of the user. The updating of the nonlinear model can be divided into the computation of the metric function and the search algorithm. The important number of investigation about the computation of the difference between the results of the numerical model and experimental data shows the complexity of the problem. The differences between the numerical results and the experimental data can be split into the error modelling and the uncertainties in the experimental system. Generally, the metric function is defined to evaluate the error modelling. However, the difference between these two kind of errors is fuzzy and the definition of an appropriate metric function is still open. In view of the optimum updating of the nonlinear model, the search algorithm have to consider the uncertainties in the experimental data, the lack of data, the complexity of the system as well as the doubts over the choice of the metric functions. Moreover, the search algorithm has an important influence over the computational cost of the updating process as well as the reliability of the updated model. The computational cost can be reduce by the evaluation of a metamodel.

To put in a nutshell, the nonlinear updating process is still current subject of investigation and it will remain as such in the next years. The toolbox of the nonlinear updating will be enhanced to increase the reliability of the updating process and to reduce the computational cost.

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