

Free vibration analysis of corrugated-face sheet composite sandwich plates

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Abstract Free vibrational characteristics of corrugated sandwich plates consisting of two corrugated composite face sheets and a foam filling core were investigated. Natural frequencies of sandwich plates with sinusoidal and trapezoidal corrugation shapes are obtained and compared with that of flat sandwich plates. Three-dimensional (3D) finite element method (FEM) is employed to compare and contrast the natural frequencies. FEM models of the proposed sandwich plates are constructed using ANSYS 12.0 code. The core of the sandwich plates is assumed to be a soft isotropic material which is adhesively bonded to two stiff laminated composite face sheets. The effects of significant parameters such as corrugation shape, boundary conditions, composite layup and thickness ratio of the structure on the natural frequencies and mode shapes of the corrugated sandwich structures are assessed. Comparison of the obtained numerical results with those values available in

the literatures demonstrates the accuracy of the proposed models. The numerical results indicate that employing corrugated composite face sheets increase the natural frequencies of sandwich plates, significantly.

Keywords Vibration analysis · Corrugated-face sheet · Sandwich plates · Composite face sheets · Finite element method

List of symbols

A	Cross section of the plate
a	Length of the plate
b	Width of the plate
C	Wavelength
E	Young's modulus
F	Amplitude
G	Shear modulus
h	Thickness of the plate
h_c	Thickness of the core
h_f	Thickness of the face sheets
I	Second moment of area
m	Number of waves along X -axis
n	Number of waves along Y -axis
u	Displacement at any point along X -direction
v	Displacement at any point along Y -direction
w	Displacement at any point along Z -direction
α	Angle of fibers orientation
θ	Trapezoidal angle
θ_x	Rotation of the normal to the mid-plane about X -axis
θ_y	Rotation of the normal to the mid-plane about Y -axis
θ_z	Rotation of the normal to the mid-plane about Z -axis
ν	Poisson's ratio
ω	Angular frequency
ϖ	Fundamental frequency
ρ	Density

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1 Introduction

In the past decades, increasing needs to lighter structures with higher mechanical properties persuaded engineers to seek for a new generation of materials to be applicable in many industries such as aerospace, naval and automotive. Attractive properties of multilayered structures which includes high strength-to-weight ratio, good ability of sound and energy absorption, easy installation, ease of manufacturing and low production cost make them an obvious choice for design engineers. Sandwich structures are generally consisted of a lightweight thick core and two stiff thin face sheets. In many cases, the core consists of a foam polymer or honeycomb material which is flexible in all directions, where the composite laminates are commonly used as the face sheets. Dynamic behavior and vibration response of such structures are important to engineers since they have found major application in aerospace and naval industries.

For efficient design and usage of these structures, a thorough understanding of their mechanical behavior is needed. To model the mechanical behaviors of these plates accurately, continuity conditions of the displacements and inter-laminar transverse shear stresses should be satisfied [1]. However, variations of material properties between the core and the face sheets cause the slopes of displacements and transverse shear stresses to change in the face sheet–core interfaces which make it more complicated to analyze [2]. Furthermore, using a soft compressible material for the core such as foam polymer, it should be considered more flexible in all directions than the face sheets. As such, this behavior leads to localize stresses and non-identical displacement patterns through the depth of the panel resulting in the displacements of the upper face sheet to differ from those of the lower one [1].

To date, there are many papers published to analyze the mechanical behavior of the sandwich plates. Exact 3D elasticity solutions for static analysis of the composite sandwich plates were presented by Pagano [3] and Noor et al. [4]. Higher order ESL theories were presented by Kant and Swaminathan [5] for studying mechanical behavior of composite sandwich plates. For thick laminated or sandwich plates, the ESL theories can give inaccurate results and are not suitable for predicting the local behavior of sandwich plates such as delamination, matrix cracking or wrinkling. To model thick laminated plates in a better manner and to evaluate their local behaviors, Carrera and Demasi [6] extended the mixed LW theory for sandwich plates. In these models, the number of unknowns depends on the number of the layers in composite and it becomes large as the number of the layers increases.

Layer-wise theories need so much computational time and efforts that on occasion its application becomes

impractical. To overcome this problem, Zigzag theories with linear or high-order local functions were proposed. Di Sciuva [7] proposed a refined zigzag plate theory in which the unknowns for the in-plane displacements at each layer were assumed in terms of those at the reference plane and the transverse displacement was assumed constant along the plate thickness. Most zigzag theories satisfy the transverse shear continuity conditions, but in these theories, the transverse stresses continuity was not enforced in the governing equations. Post-processing method based on equilibrium consideration has to be adopted to calculate the transverse shear stresses. Shariyat [8] introduced a high-order global–local theory that guarantees the continuity conditions of all displacements and transverse stress components and considered the transverse flexibility of sandwich plates. Kheirikhah et al. [2, 9] presented a new improved higher order theory using third-order plate theory of face sheets and quadratic and cubic functions for transverse and in-plane displacements of the core.

Recently, employing corrugation in the core or face sheets has been found as a solution to reinforce sandwich plates against various loading conditions. Sandwich plates which are reinforced with corrugations can achieve a higher stiffness and strength than flat plates, and can thus improve the mechanical behavior and strength/weight ratio of structures that are made of them.

There are two types of corrugated sandwich plates which are used in different industries. The first one is corrugated-core sandwich plates which consist of two flat face sheets and a corrugated core. This type of sandwich plates has been widely studied and used in last three decades. The second type is corrugated-face sheet sandwich plate which consist of two corrugated-face sheets in which the core is simply a foam filling the gap between the two face sheets. This type of corrugated sandwich plate is relatively new and has not been studied sufficiently at this time. Therefore, the purpose of this paper is to investigate the free vibration analysis of corrugated-face sheet sandwich plates.

Corrugation of face sheets leads to increase the strength-to-weight ratio [10]. Thus, they have the ability to resist against higher loads in comparison to flat plates. However, plate theories such as 3D elasticity, ESL or LW were only presented to analyze the mechanical behavior of flat sandwich and composite structures without the corrugation. One of the simple and valid ways of analyzing the corrugated metallic or laminated plates is to consider them as orthotropic plates of uniform thickness with equivalent rigidities. Based on this theory, some researchers investigated the mechanical behavior of corrugated plates. For instance, Seydel's formulation [11] was used for many years to estimate the equivalent rigidities of such structures. These formulas were then improved by Lau [12] and Briassoulis [13]. An analytical model was presented by

Shimansky and Lele [14] to analyze the initial transverse stiffness of sinusoidal corrugated plates. Using reduction formulas, Semenyuk and Neskhodovskaya [15] analyzed the stability of longitudinally corrugated cylindrical shells by replacing the corrugated shell with an equivalent orthotropic shell. Liew et al. [16, 17] used a mesh-free Galerkin method based on the FSDT for the free vibration and nonlinear analysis of stiffened and unstiffened sinusoidal and trapezoidal corrugated plates by simulating corrugated plates with an equivalent orthotropic plate model. They [17] showed that in constant thickness ratio, trapezoidal corrugated plate has greater natural frequencies than sinusoidal one. Free vibration characteristics of trapezoidal corrugated sheets were investigated by Samanta and Mukhopadhyay [18] using three-dimensional FEM analyses and proposing an equivalent orthotropic model. Renhuai and Dong [19] studied the non-linear bending and vibration of corrugated circular plates. The relation of the non-linear period and amplitude for free vibrational corrugated circular plates was obtained using Galerkin method. Wang et al. [20–22] investigated the non-linear and large amplitude vibration of heated corrugated circular plates with shallow sinusoidal corrugations under uniformly static temperature changes. From the investigation, it was found that the non-linear fundamental frequency decreases by increasing the temperature parameter. Lok and Cheng [23] presented a closed-form solution for free and forced vibration of a corrugated truss-type core orthotropic sandwich panel. Rubino et al. [24] measured the dynamic response of fully clamped rectangular sandwich plates with Y-frame and corrugated cores. Liang et al. [25] investigated the dynamic response of a corrugated panel subjected to uniformly distributed pressure pulse. Recently, Kheirikhah et al. [26] studied the natural vibration analysis of trapezoidal corrugated-face sheet sandwich plates using three-dimensional finite element methods. They compared the vibration response of corrugated sandwich plates for different trapezoidal angles and composite layups. But, they only studied trapezoidal corrugated-face sheet sandwich plates.

Many different corrugation shapes have been used for corrugated-core sandwich plates such as: I-beam, Y-beam, triangular, trapezoidal and sinusoidal. But, it seems to date only two corrugation shapes have been used for corrugated-face sandwich plates: trapezoidal and sinusoidal [11, 16–22, 26]. Therefore, in the present paper these two common types of corrugated sandwich plates are studied. However, there are some important points that should be noted about analyzing the corrugated plates using equivalent rigidities and uniform thickness. The first point is that the accuracy of this approach greatly depends on the correct estimation of equivalent rigidities. As a result, the results obtained by this theory have to be compared with numerical methods

such as FEM for the validation of results [16, 17, 21, 22, 26]. The second point is that this approach cannot inspect and predict the local behavior of materials around the corrugation shapes. These limitations lead scientists to look for numerical methods that can create the real physical model of plate with higher accuracy to predict the local behavior of corrugation shapes. Nowadays, FEM is one of numerical methods in engineering applications which is universally used. The FEM is not only able to predict the mechanical behavior of corrugated sandwich structures accurately, but also is able to consider and analyze the local effects and makes it possible to analyze complex structures with different dimensions and various boundary conditions easily.

It seems that there are no published results on free vibration which compare the natural frequencies of trapezoidal and sinusoidal rectangular corrugated-face sheet sandwich plates. Therefore, the goal of this paper is to predict the natural frequencies of the corrugated-face sheet sandwich plates using a 3D FE approach using the ANSYS program. So, sinusoidal and trapezoidal corrugated sandwich plates which consist of two corrugated-face sheets and a foam filling core are introduced in the present research. The fundamental natural frequencies of proposed corrugated sandwich plates are obtained and then compared with flat sandwich plates. The effects of many parameters such as boundary conditions, thickness ratios of the structures and fiber orientation of the composite face sheets are assessed.

2 Finite element modeling

Sandwich plates are composed of three layers: upper and lower face sheets and the core layer. Each face sheet has the thickness of h_f while the thickness of the core is considered to be h_c . Also, length and width of the sandwich plates are considered to be a and b , respectively. Figures 1, 2, 3 and 4 show part of the geometry of a trapezoidal and sinusoidal corrugated sandwich plates and their dimensions with the coordinate system and considered parameters, respectively.

Both sinusoidal and trapezoidal waves employed as the corrugation shape in the present sandwich plates, have the amplitude and wavelength of F and $2C$, as shown in Figs. 2 and 4, respectively. Also, the trapezoidal angle is considered to be θ as shown in Fig. 2.

Two different types of materials are used for the modeling of the present sandwich plates: a soft isotropic foam type material to construct the core section of the sandwich plate and a stiff orthotropic epoxy-based composite material with glass fibers as face sheets.

FE model of the sandwich plate is constructed using the ANSYS 12.0 standard code. First, temporary bottom area of the corrugated core was constructed in the x - y plane and then extruded along the z -axis using 8-node SOLID185

Fig. 1 Geometry of the *trapezoidal* corrugated sandwich plate

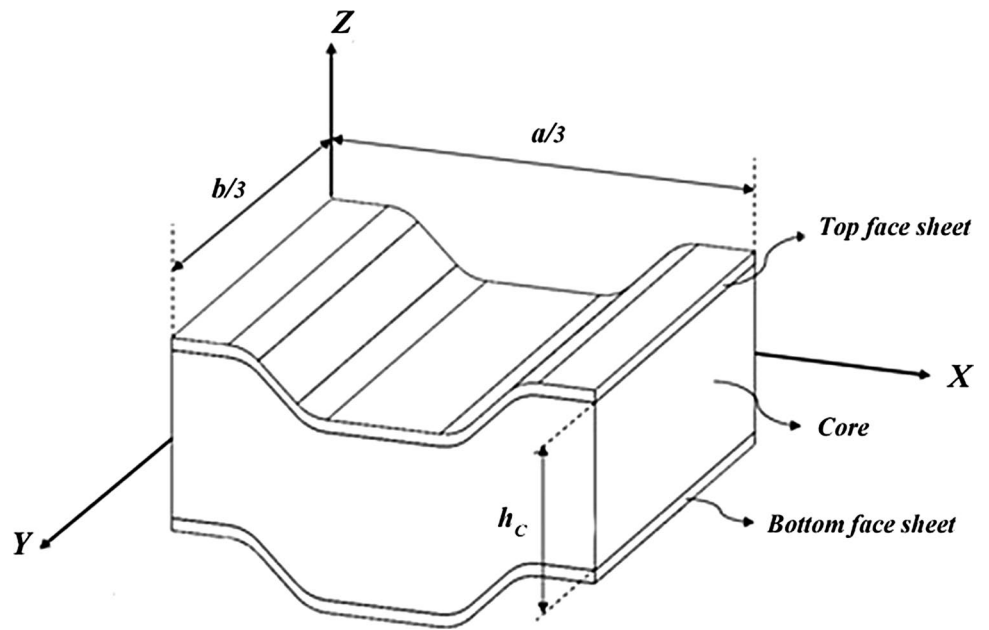


Fig. 2 Cross-sectional view of the *trapezoidal* wave and its parameters

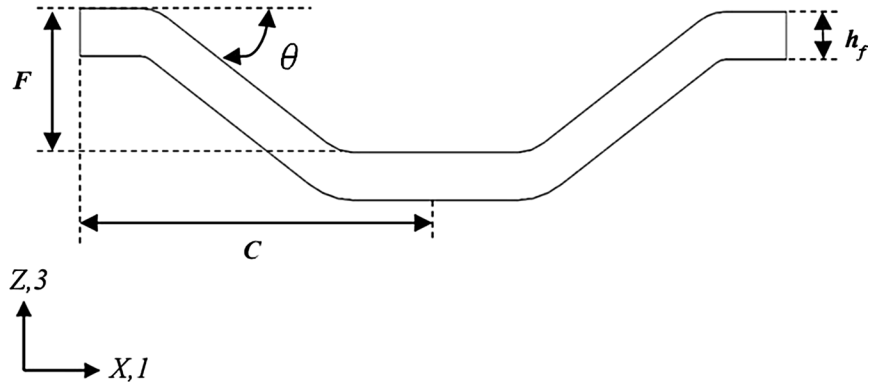


Fig. 3 Geometry of the *sinusoidal* corrugated sandwich plate

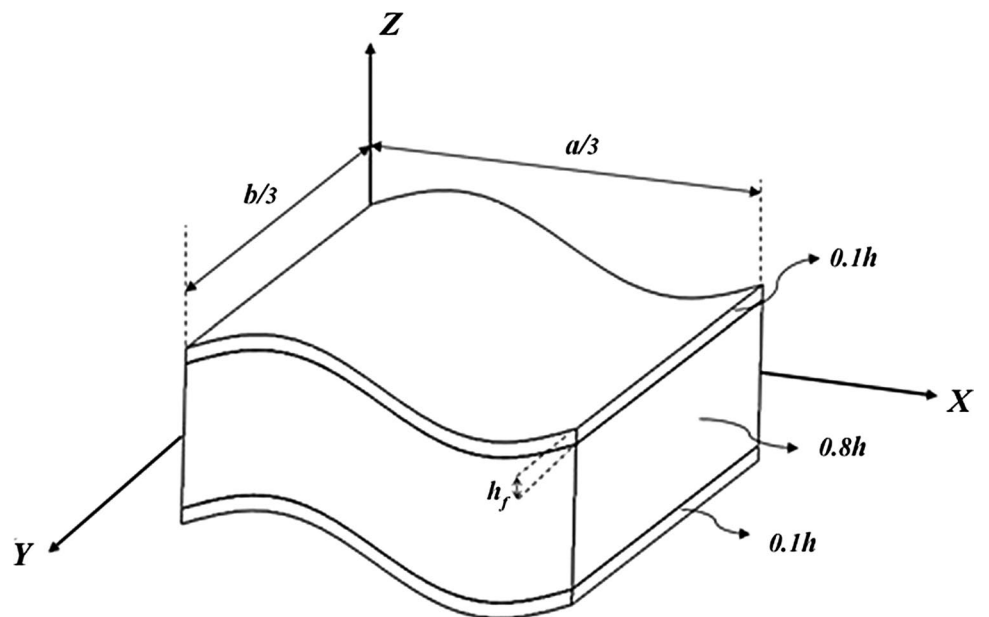


Fig. 4 Cross-sectional view of the *sinusoidal* wave and its parameters

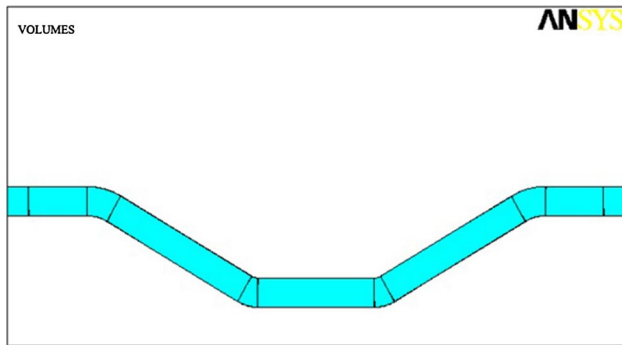
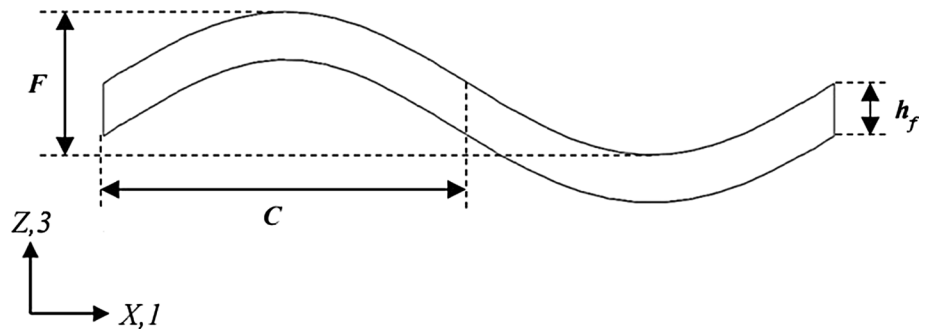


Fig. 5 Front view of the constructed *trapezoidal* corrugated core

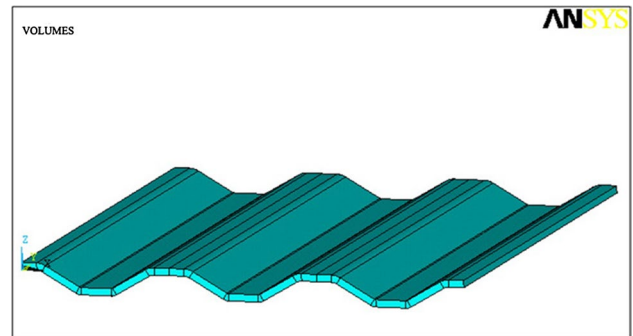


Fig. 6 3D view of the present corrugated core

brick element. This extrusion has the height of h_c . The SOLID185 element is used for 3D modeling of solid structures. It is defined by eight nodes having three degrees of freedom at each node: translations in the nodal x , y , and z directions. The element has stress stiffening, large deflection and large strain capabilities [27]. Figures 5 and 6 show the cross section and the 3D view of the generated core with trapezoidal corrugation shape.

In the next step, the upper and lower surfaces of the core which were generated in the previous step were utilized to construct the top and bottom corrugated-face sheets of the sandwich plate. To do so, the upper surface of the core was extruded and meshed along the normal direction (z -axis) using 8-node SOLID46 layered brick element with 3° of freedom at each node. The SOLID46 element is the layered version of the SOLID185 element with same node numbers and degree of freedoms. This element has ability to model layered composite structures. The layered composite specifications including layer thickness, material properties and layer orientation can be specified in this element [27]. The geometry and node locations for this element are shown in Fig. 7.

This extrusion has the height of h_f . The same procedure was performed on the lower surface of the core along the normal (Z -axis) for generating the bottom face sheet of the sandwich plate. SOLID46 element was employed to generate the composite layers in each face sheet. The same

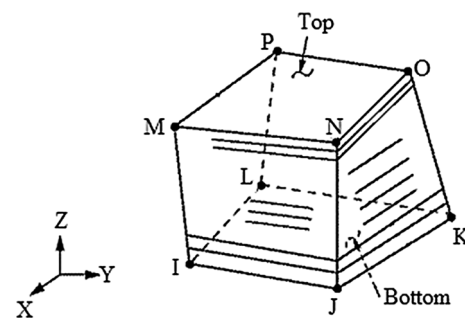


Fig. 7 The geometry and node locations for SOLID46 element [27]

procedure as explained above was repeated to construct the FE model of the sandwich plate with sinusoidal corrugation shape. Figures 8 and 9 show the complete 3D FE model of sinusoidal and trapezoidal corrugated sandwich plates.

The effects of different boundary conditions on the natural frequencies of sandwich plates are also considered in this paper. In particular, two common types of boundary conditions including all edges simply supported (SSSS) and all edges clamped (CCCC) are assumed in the finite element modeling.

Boundary conditions applied to the edges are defined as follows: Edges parallel to Y -axis:

$$\text{SSSS: } v = w = \theta_y = 0 \tag{1}$$

Fig. 8 Complete three-dimensional FE model of the *trapezoidal* corrugated sandwich plate

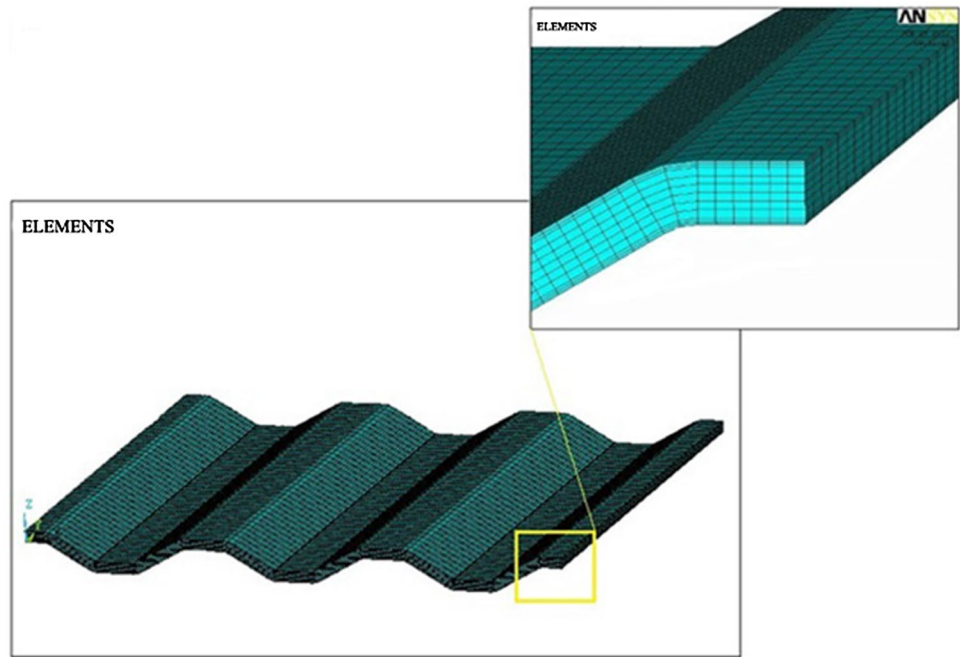
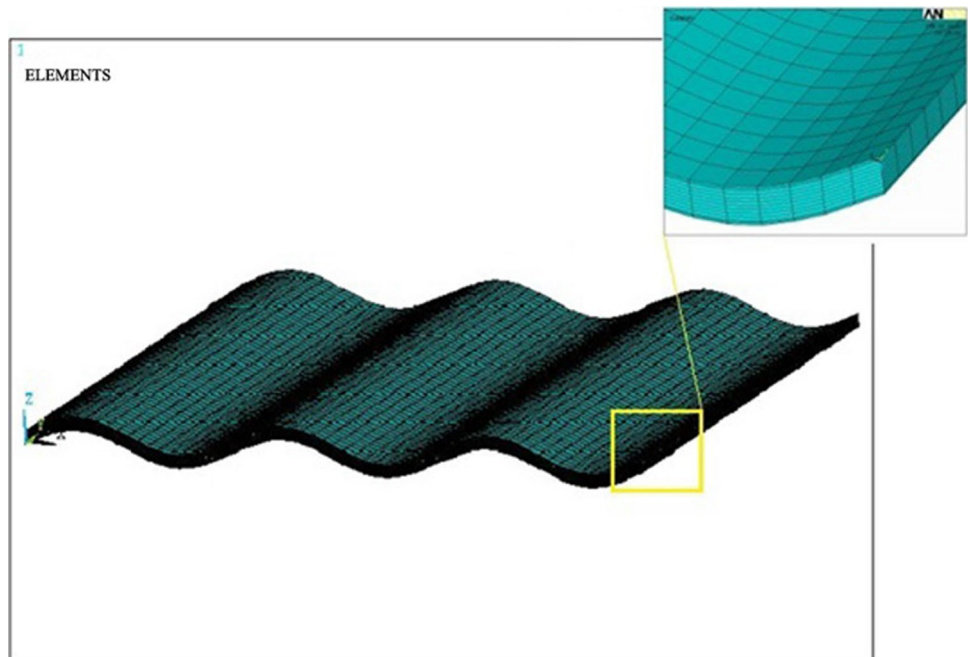


Fig. 9 Complete three-dimensional FE model of the *sinusoidal* corrugated sandwich plate



CCCC: $u = v = w = \theta_x = \theta_y = \theta_z = 0$

Edges parallel to X-axis:

SSSS: $u = w = \theta_x = 0$

CCCC: $u = v = w = \theta_x = \theta_y = \theta_z = 0$

(2) **3 Model verification**

(3) In this section, the accuracy of the modeling and analyzing and validity of the results is studied. Different sets of materials are used in the modeling in this research. Table 1 shows the material properties which are used in the present FE modeling and analysis. The dimensionless fundamental

Table 1 Material properties used in the present finite element modeling

Material set no.	Location	E (GPa)			G (GPa)			V ₁₂	V ₁₃	V ₂₃	ρ (Kg/m ³)
		E ₁	E ₂	E ₃	G ₁₂	G ₁₃	G ₂₃				
1. Wang et al. [21]	Face sheets	131	10.34	10.34	6.895	6.205	6.895	0.22	0.22	0.49	1,627
	Core	6.89e-3	6.89e-3	6.89e-3	3.45e-3	3.45e-3	3.45e-3	0	0	0	97
2. Liew et al. [17]	Plate	30	30	30	11.538	11.538	11.538	0.3	0.3	0.3	7,830
3. Kaw [29]	Face sheets	38.6	8.27	8.27	4.14	4.14	3.28	0.26	0.26	0.26	1,800
	Core	0.3	0.3	0.3	0.108	0.108	0.108	0.39	0.39	0.39	250

Table 2 Convergence of the fundamental frequencies

Number of line division	Number of core and skins elements	Fundamental frequencies
30	5,400	38.62
35	7,350	35.94
40	9,600	32.26
45	12,150	30.53
50	15,000	28.74
60	21,600	25.73
70	29,400	22.12
75	33,750	20.76
80	38,400	18.54
90	48,600	16.43
95	54,150	16.41
100	60,000	16.41

natural frequencies of sandwich plates which is used in this paper can be defined as [17]:

$$\bar{\omega} = \left(\frac{\omega a^2}{h}\right) \sqrt{\left(\frac{\rho}{E_2}\right)}_f \tag{5}$$

where ω is the obtained natural frequency of the sandwich plate, a is the length of sandwich plate, h is the total thickness of sandwich plate and ρ and E₂ are the density and transverse stiffness of the face sheet, respectively.

In FEM-based analysis, a convergence study must be performed before obtaining the results to obtain the proper elements size. In the present paper, the convergence study is performed for a flat sandwich plate with thickness ratio of a/h = 100. The clamped boundary conditions were applied to all the edges of the plate. Set no. 1 of material properties in Table 1 is used for modeling. Table 2 shows the variation of fundamental frequencies of the plate versus plate length divisions and the total number of elements. It can be seen that by increasing the number of elements, the results converged to a constant value and further increasing does not affect the results. A similar convergence study has been performed for all the analysis and obtained results in this paper.

The validity of the flat and corrugated sandwich plates is inspected here. In order to demonstrate the accuracy of the present FE modeling and analysis, the natural frequencies are compared with existing results obtained by those of plate theories such as layer-wise [28], higher order and mesh-free Galerkin method [21] in different literatures. Different sets of material constants given in Table 1 were used to verify the proposed FE solution.

The validation process consists of two examples. In the first example, the fundamental natural frequencies of the flat square sandwich plate with thickness ratio a/h = 100 obtained by Cetkovic and Vuksanovic [28] and Wang et al. [21] are compared with the present results. The first set of the material constants given in Table 1 were employed. Table 3 compares the numerical results from the current analysis with those of Refs. [21] and [28].

Since there are no published results on the natural frequencies of trapezoidal corrugated sandwich plates, natural frequencies of a trapezoidal corrugated plate have been computed in the second example and compared with those of previously published results by Liew et al. [16]. The verification procedure is performed for both SSSS and CCCC corrugated plates using the second set of material constants given in Table 1. The computed numerical frequencies for the first five natural frequencies are compared in Table 4.

It can be seen that, the natural frequencies obtained using the proposed FE solution in this paper are indeed in good agreement with those of plate theories. The maximum relative errors between the obtained results for examples 1 and 2 using present FEM and the obtained results by Liew et al. [17] are 0.7 % and 2.9 %, respectively.

4 Results and discussion

In this paper, the natural frequencies of flat, trapezoidal and sinusoidal corrugated sandwich plates are obtained by FE solution. Several examples are presented to examine the vibration behavior of the sandwich plates. In all examples, square sandwich plates with the dimension of 0.6 × 0.6 m² and thickness ratios a/h = 50 and 100 are considered. The face sheet thickness ratio is assumed to be h_f = 0.1 h. The

Table 3 Fundamental frequencies of asymmetric [0/90/core/90/0] flat square sandwich plate with $a/h = 100$

Mode	Present	Cetkovic [28]	Wang [21]	Maximum error (%)
1	11.9511	12.1223	11.8674	0.7
2	22.8430	22.9963	22.7200	0.54
3	31.0054	31.1742	30.8859	0.38
4	35.0123	35.2000	34.9339	0.22
5	40.8823	41.0511	40.7379	0.35

amplitude and the wavelength of the sinusoidal and trapezoidal waves employed in this study are considered to be $F = 0.03$ m and $C = 0.1$ m, respectively.

Also, in all examples, material set No. 3 which defines a foam material and an epoxy-glass composite are used to construct the core and face sheets, respectively. The effect of the fiber orientation on the free vibration response of the flat, trapezoidal and sinusoidal corrugated sandwich plates is also regarded in the proposed examples. The layers of the plates are assumed to have a symmetric layup [0/ α /C/ α /0] where α denotes the fibers orientation angle respective to the x -axis. Two common types of boundary conditions including all edges SSSS and all edges CCCC are applied to the proposed examples. Also, to quantify and present a right comparison between natural frequencies of studied sandwich plates, their cross-sectional area and second moment of cross-sectional area are presented in Table 5. It can be seen that the trapezoidal corrugated plate has higher second moment of cross-sectional area than the sinusoidal

one. But, the cross-sectional area of the plates is very close together.

The contour of the first four mode shapes of the sandwich plates with thickness ratio $a/h = 50$ are shown in Fig. 10. The first column shows the contour shapes. The notation (m, n) defines the mode shapes where m and n denote the number of waves along the x - and y -direction, respectively. The second, third and fourth columns illustrate the mode shapes of a trapezoidal, sinusoidal and flat sandwich plate, respectively.

To examine the variation of the fundamental frequencies and to inspect the effects of employed corrugation shapes and fibers embedding direction in a better manner, the variation of obtained fundamental frequencies ($\bar{\omega}$) are plotted against the angle of fibers orientation (α) for the all of the proposed sandwich plates with different boundary conditions, thickness ratios and corrugation shapes. These variations can be seen in Figs. 11, 12, 13 and 14. Each figure is consisted of four diagrams which illustrate the variations of the fundamental frequencies against fiber embedding angle for the first four mode shapes through diagrams (a) to (d), respectively. In each figure, diagram (a) denotes $(m, n) = (1, 1)$, diagram (b) denotes $(m, n) = (2, 1)$, diagram (c) denotes $(m, n) = (3, 1)$ and diagram (d) denotes $(m, n) = (1, 2)$ mode shape. Sandwich plates with thickness ratio $a/h = 100$ and SSSS and CCCC boundary condition are studied in Figs. 11 and 12, respectively. Figures 13 and 14 describe fundamental frequencies fluctuations for sandwich plates with thickness ratio $a/h = 50$ under SSSS and CCCC boundary conditions, respectively. Also, the results

Table 4 Natural frequencies (Hz) of the trapezoidal corrugated plate

Boundary condition	Mode	Present FEM	Analytical Liew et al. [17]	FEM Liew et al. [17]	Maximum error (%)
SSSS	1	13.39	13.74	13.14	1.9
	2	24.22	24.54	23.90	1.33
	3	44.57	45.35	44.05	1.18
	4	45.38	47.19	44.38	2.25
	5	54.48	54.84	53.44	1.94
CCCC	1	27.97	28.58	27.18	2.9
	2	38.96	40.46	39.45	1.24
	3	61.41	63.27	62.12	1.14
	4	69.06	72.59	67.96	1.61
	5	78.72	80.88	78.51	0.26

Table 5 Cross-sectional area (A) and second moment of cross-sectional area of flat, sinusoidal and trapezoidal sandwich plates

a/h	Cross-sectional area (mm^2)			Second moments of area, I_{XX} (mm^4)		
	Flat	Sinusoidal corrugated	Trapezoidal corrugated	Flat	Sinusoidal corrugated	Trapezoidal corrugated
100	3,600.00	3,780.18	3,785.66	10,800.00	554,332.00	624,577.67
50	7,200.00	7,574.14	7,584.31	86,400.00	918,815.93	1209,587.00

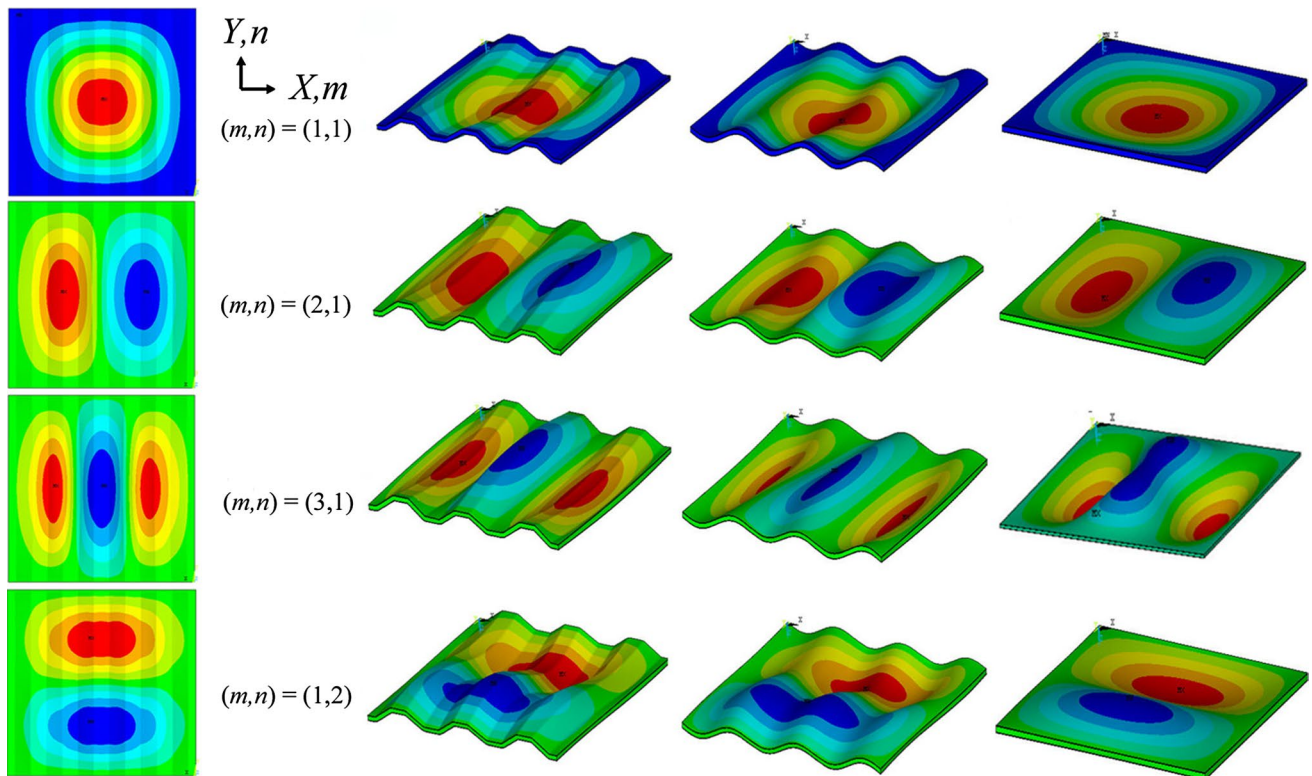


Fig. 10 The contour and mode shapes for the sandwich plates with thickness ratio $a/h = 50$

corresponding to the Figs. 11, 12, 13 and 14 were presented in Table 6.

In Fig. 11, the variation of obtained fundamental frequencies ($\bar{\omega}$) are plotted against the angle of fiber orientation (α) for the sandwich plates with thickness ratio $a/h = 100$ under SSSS boundary. According to this figure, sandwich plates reinforced with sinusoidal and trapezoidal corrugation shape could achieve to quite higher frequencies in comparison to the flat sandwich plates. It can be seen that the trapezoidal and sinusoidal corrugated sandwich plates have quite similar behavior as the angle of fiber orientation increases. However, the sandwich plates with trapezoidal corrugation shape seem to be able to achieve slightly higher fundamental frequencies in comparison with the sinusoidal ones. Also, it can be seen that by increasing α in flat sandwich plates, fundamental frequencies increase. This increment seems to have a higher slope when α jumps from 30° to 60° in mode $(m, n) = (3, 1)$.

Sandwich plates with the length to thickness ratio of $a/h = 100$ under CCCC boundary condition are considered in Fig. 12. This figure shows that the variations of frequencies in trapezoidal corrugated sandwich plates can be assumed to be linear in the first mode shape. But, for other modes, the frequencies tend to reduce after an increase

when α passes almost 60° . Similar to sandwich plates with SSSS boundary conditions, sinusoidal corrugated sandwich plates have the same behavior as trapezoidal corrugated ones. Once again, trapezoidal corrugation allowed the sandwich plates to reach slightly higher fundamental frequency in comparison to the sinusoidal corrugation. But, it seems that an overlap occurs when α jumps from 60° to 90° in modes $(m, n) = (1, 1)$ and $(m, n) = (2, 1)$. In flat sandwich plates, fundamental frequencies tend to linearly increase as α increases.

Completely different behavior was observed when thickness ratio of the structure changed to $a/h = 50$ under SSSS boundary condition as shown in Fig. 13. The first notable point is that the distances between the line indicating the variations of $\bar{\omega}$ for trapezoidal corrugated sandwich plates and the line indicating the variations of $\bar{\omega}$ for sinusoidal corrugated sandwich plates significantly increased in comparison to the sandwich plates with thickness ratio $a/h = 100$ in the Fig. 10 under the same boundary condition. So, it seems that in the lower thickness ratios (a/h) the ability of trapezoidal corrugation shape boosts in improving the dynamic behavior of the sandwich plates and thus is more preferable than sinusoidal corrugation shapes. However, in modes $(m, n) = (2, 1)$ and $(m, n) = (3, 1)$, fundamental frequencies of sandwich plates with trapezoidal

Fig. 11 Variations of fundamental frequencies for simply supported (SSSS) square sandwich plates with thickness ratio $a/h = 100$

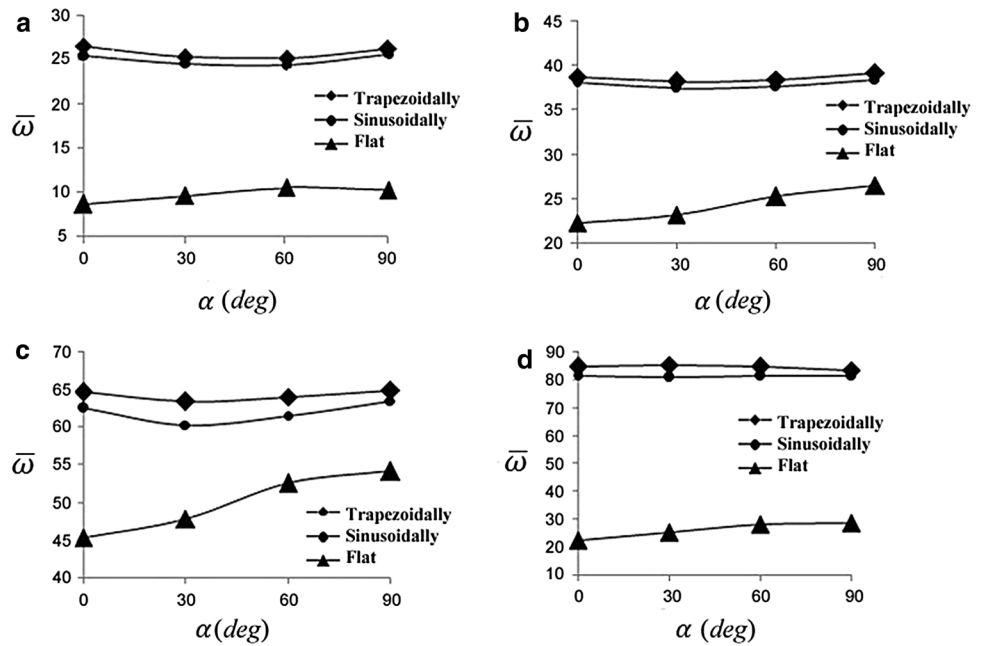
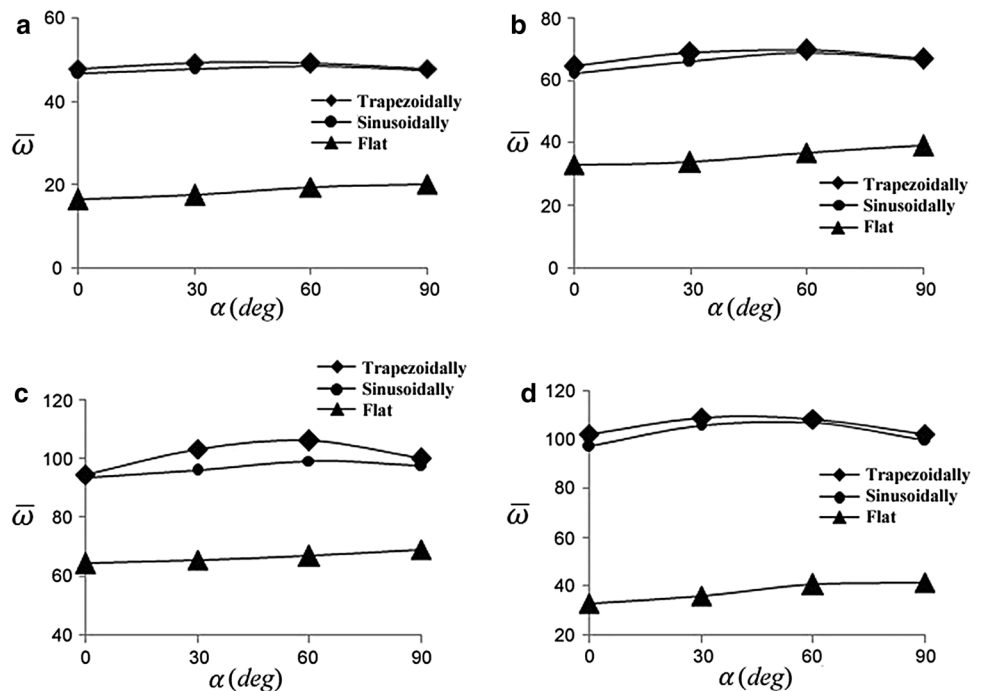


Fig. 12 Variations of fundamental frequencies for clamped (CCCC) square sandwich plates with thickness ratio $a/h = 100$



corrugation shape can be assumed to linearly increase as α increases. In sandwich plates with sinusoidal corrugation shape, the fundamental frequencies tend to slightly increase when α passes 60° . The second important point is that the frequencies of the sandwich plates significantly increase by increasing the thickness of the structure. This is because increasing in the thickness leads to increase the stiffness of the structure.

The other point that should be noted is that in the Fig. 13(c), two lines which indicate the variations of fundamental frequencies of the flat and sinusoidal corrugated sandwich plates intersect together when α passes about 20° . This event shows that flat sandwich plates have a better behavior in the mode $(m, n) = (3, 1)$.

Boundary conditions changed to CCCC as in Fig. 14 while the thickness ratio remains the same as in Fig. 13.

Fig. 13 Variations of fundamental frequencies for simply supported (SSSS) square sandwich plates with thickness ratio $a/h = 50$

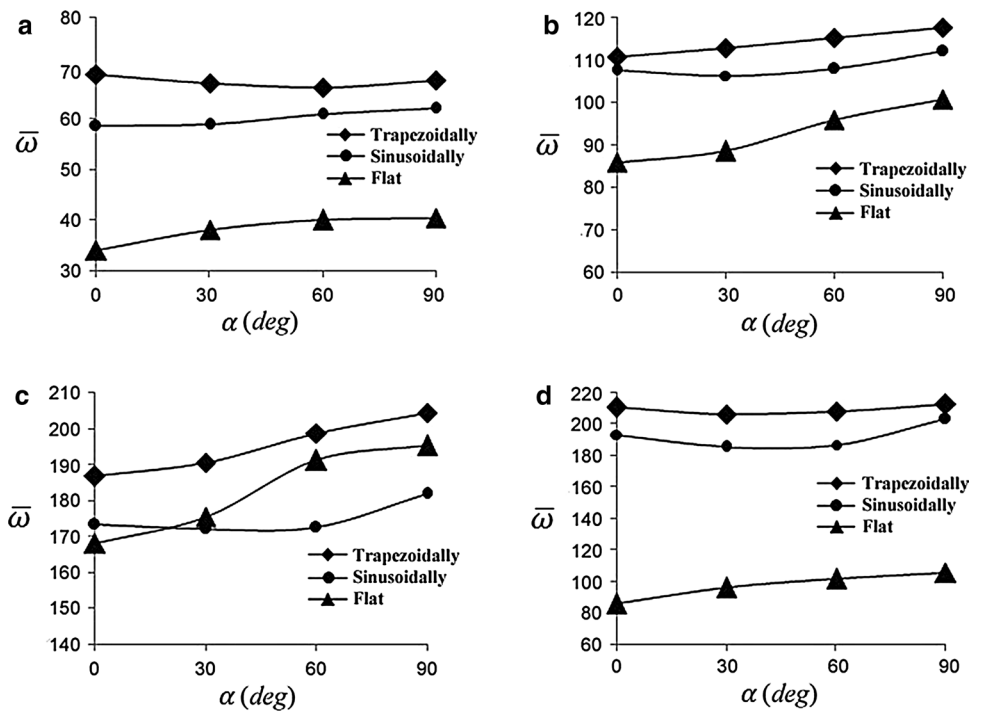
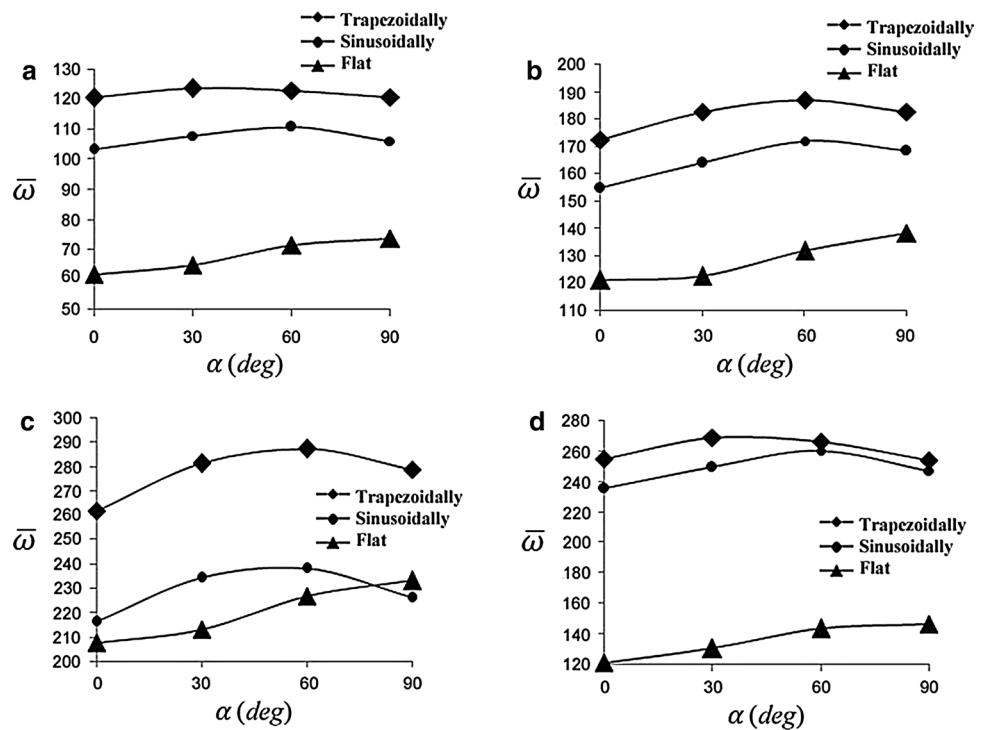


Fig. 14 Variations of fundamental frequencies for clamped (CCCC) square sandwich plates with thickness ratio $a/h = 50$



These diagrams once again confirms that trapezoidal corrugations have the ability to significantly improve the dynamic behavior and vibration response of the sandwich plates in comparison to sinusoidal corrugations and are definitely more preferable in sandwich structures with lower thickness ratios. This time, the intersection between the

lines in mode $(m, n) = (3, 1)$ occurs when α goes almost higher than 80° .

All the above figures demonstrate that increasing the angle of embedded glass fibers from 0° to 90° has a positive effect on the vibration response of flat sandwich plates. In most cases, this effect is more significant when α jumps

Table 6 Fundamental frequencies of symmetric square sandwich plates with lay-up $[0/\alpha/\text{core}/\alpha/0]$

Plate type	Boundary condition	Layer angle (α)	$a/h = 50$				$a/h = 100$			
			(1, 1)	(2, 1)	(3, 1)	(1, 2)	(1, 1)	(2, 1)	(3, 1)	(1, 2)
Flat	SSSS	0	34.12	85.85	168.22	85.96	8.69	22.32	45.27	22.49
		30	38.09	88.53	175.60	96.15	9.74	23.18	47.71	25.32
		60	40.06	96.02	191.15	101.82	10.61	25.23	52.45	28.25
		90	40.50	100.74	195.28	105.79	10.35	26.54	54.08	28.94
	CCCC	0	61.72	121.09	207.72	121.28	16.41	33.11	64.42	33.35
		30	65.03	122.47	213.09	130.64	17.45	33.82	65.62	36.36
		60	71.44	131.78	226.94	143.75	19.40	37.01	66.98	40.77
		90	73.73	138.27	233.22	146.91	20.02	39.06	69.22	41.75
Sinusoidal corrugated	SSSS	0	58.63	107.55	173.46	193.50	25.63	38.12	62.48	81.19
		30	58.94	106.3	172.05	185.42	24.71	37.45	60.24	80.87
		60	60.95	108.01	172.57	186.76	24.57	37.66	61.44	81.32
		90	62.16	112.06	181.86	203.39	25.78	38.43	63.41	81.51
	CCCC	0	103.28	154.62	216.36	235.40	46.66	62.14	93.37	97.48
		30	108.10	163.98	234.47	249.55	47.72	65.99	96.14	106.11
		60	110.83	171.97	238.16	260.33	48.56	68.61	99.37	107.24
		90	106.32	168.40	226.31	247.13	47.28	66.58	97.65	100.05
Trapezoidal corrugated	SSSS	0	68.79	110.83	186.70	210.41	26.55	38.72	64.54	84.46
		30	67.07	112.96	190.59	206.53	25.41	37.87	63.28	85.37
		60	66.29	115.29	198.74	207.96	25.24	38.39	63.94	84.54
		90	67.73	117.72	204.30	212.23	26.18	39.12	64.73	83.347
	CCCC	0	120.70	172.47	261.86	255.07	47.87	64.32	94.78	102.66
		30	123.74	182.50	281.22	268.95	49.30	68.69	103.38	109.19
		60	123.24	186.89	287.16	266.17	49.10	69.57	106.17	108.60
		90	120.99	182.63	278.88	253.84	47.79	66.70	99.971	102.56

from 30° to 60° . In sinusoidal corrugated sandwich plates with thickness ratio of 100, changing the value of α has no significant effect on fundamental frequencies. However, some fluctuations can be seen when α jumps from 0° to 30° in $(m, n) = (3, 1)$ and $(m, n) = (1, 2)$ mode shapes. By changing the thickness to $a/h = 50$, fluctuations were found to have occurred in a higher range. It can be seen that the clamped sandwich plates at $\alpha = 60^\circ$ could achieve to rather higher frequencies. In the sandwich plates with trapezoidal corrugation shape and thickness ratio $a/h = 100$, quite similar behavior as sinusoidal corrugated sandwich plates was detected. By decreasing the a/h ratio of trapezoidal sandwiches to 50 and under SSSS boundary condition, fundamental frequencies tend to increase linearly.

Figures 11, 12, 13, 14 showed that the sandwich plates with trapezoidal corrugation shape seem to be able to achieve slightly higher fundamental frequencies in comparison with the sinusoidal ones. This manner is same with that obtained by Leiw et al. [17] for corrugated laminate plates. Also, it should be noted that under a same material and geometrical parameters, the natural frequency (ω) of a plate can be assumed to be proportional to $\sqrt{I/A}$ where I

and A are the second moment of cross-sectional area and the cross-sectional area of the plate, respectively [30]. It can be seen from Table 5 that the trapezoidal sandwich plates have higher value of I and the approximately same cross-sectional area in comparison to the sinusoidal one which leads to a higher I/A ratio for trapezoidal sandwich plates. Thus, higher natural frequency of the trapezoidal sandwich plates can be attributed to the higher value of I/A ratio.

5 Conclusion

Free vibration response of flat and corrugated sandwich plates has been studied in this paper. Two common types of corrugation shapes which were widely studied in the literatures including trapezoidal and sinusoidal corrugation shapes were employed on the core and top and bottom face sheets to reinforce the proposed sandwich structures. A three-dimensional FEM approach is presented to analyze and predict accurately the natural frequencies of sandwich structures using the ANSYS 12.0 standard code. The

effects of many parameters such as boundary conditions, composite layups and thickness ratio of the sandwich plates are studied on the free vibration behavior of the proposed models. The aim of this article was to obtain and compare the fundamental frequencies of the flat and corrugated sandwich plates with the same plane dimension. According to the obtained results, increasing the value of fiber orientation angle (α) from 0° to 90° had a positive effect on dynamic behavior of flat sandwich plates. It was found that employing trapezoidal and sinusoidal corrugation shapes at the face sheets and the core section of the sandwich plates significantly increase the natural frequency and improve the dynamic behavior of the sandwich plates. The numerical results indicated that trapezoidal corrugated sandwich plates have the ability to reach relatively higher natural frequencies. The effect of trapezoidal shape on improving the dynamic behavior is more significant for sandwich plates with lower thickness ratio whereas the sinusoidal corrugated sandwich plate could reach quite lower natural frequencies.

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