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Piero della Francesca, mathematician of the Renaissance

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Abstract From the very beginning, the artistic production of Piero della Francesca is characterised by a rigorous perspective structure, a perfection of geometric volumes and a representation of figures immersed in an almost abstract atmosphere of diffuse light. In addition to painting, Piero also wrote mathematical treatises on algebra, geometry and perspective: respectively, *Trattato d'abaco*, *De prospectiva pingendi* and *De quinque corporibus regularibus*. On the merits of this scientific work, Piero is considered a Renaissance mathematician: he formalized the rules of perspective, which had been until then the exclusive property of artistic technique. After Piero, the French mathematician Girard Desargues resumed these studies and stated the rules of projective geometry.

Keywords Piero della Francesca · Perspective · Geometry · Algebra · Polyhedrons · Art

Piero della Francesca was born in Borgo Sansepolcro, today in the province of Arezzo, sometime between 1406 and 1416 (the exact year is unknown because of a fire in the Sansepolcro archives) and was educated in Florence together with Domenico Veneziano, with whom he collaborated on the lost frescoes in the presbytery of the church of S. Egidio in Florence. His first works, painted around 1450, already show the artist's distinguishing characteristics: rigorous perspective structuring, refinement of geometric volumes, the representation of grandiose figures immersed in an atmosphere of diffused luminosity, subtle and almost abstract [2].

During the 1440s Piero resided in various Italian courts: Urbino, Ferrara, probably Bologna and Rimini, where he worked in the Tempio Malatestiano on the votive fresco with the portrait of Sigismondo Pandolfo Malatesta. In 1452, upon the death of painter Bicci di Lorenzo, who had begun decorating the church of S. Francesco in Arezzo, Piero was called by the Bacci family to complete the frescoes of the presbytery, with the famous cycle representing the Legend of the True Cross.

Between 1458 and 1459 he was active in Rome, called there by Pope Pius II, where he created several frescoes in the Palazzo Apostolico, destroyed in the sixteenth century to make room for the first of the four Raphael Rooms. The 2 years of 1467–1468 found him in Perugia and Bastia Umbra, where he had taken refuge to avoid the plague. It was to this period that the *Flagellation of Christ* is dated (rediscovered in 1893 in the Cathedral of Urbino and today conserved in the Ducal Palace there). Considered to be one of the finest examples of the use of perspective, and a paradigm of the synthesis of mathematics and painting, it is still today the object of study and research.

A guest between 1469 and 1472 of the court of the Duke of Urbino, Federico da Montefeltro, he produced a number of works, including the *Double Portrait of the Dukes of Urbino* depicting Federico da Montefeltro and his wife Battista Sforza in profile. To Federico's son, Guidobaldo da Montefeltro—whose preceptor was the mathematician Luca Pacioli—Piero dedicated his *Libellus de quinque corporibus regularibus* [5], styling it as an encouragement to the young nobleman's inclination towards scientific research (Fig. 1). The booklet deals with topics of Euclidean geometry, reprising ancient themes of the Platonic–Pythagorean tradition. Here for the first time the regular

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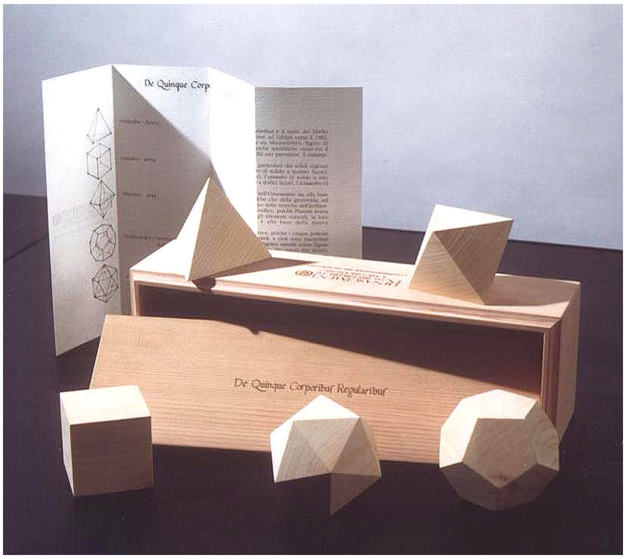

Fig. 1 Art work *De quinque corporibus regolaribus* created by Pierluigi Ghianda. Reproduced by permission

polyhedra are depicted using the technique of perspective, and the relationships among them are studied.

In his final years, according to Giorgio Vasari, he was stricken by a serious illness that affected his eyes, and he was forced to withdraw from his activities, but in that same period he applied himself to the study of perspective. Piero della Francesca died in Sansepolcro on 12 October 1492, on exactly the day celebrated for the discovery of America.

In addition to his artistic work, Piero was also the author of treatises of mathematics and perspective geometry: *Trattato d'abaco* [4], *De prospectiva pingendi* [3] and *Libellus de quinque corporibus regularibus*, mentioned earlier, all of which were probably composed between 1480 and 1490 (although some well-known scholars have dated the *Trattato d'abaco* to as early as 1450). What is found in all three of these works is a synthesis of Euclidean geometry, typical of the school of the erudite, and abacus mathematics, used instead by men of commerce.

The *Trattato d'abaco*, a handbook of applied mathematics, in which the parts on geometry and algebra are much more extensive than usual for the day, is composed of 574 propositions of an applicative nature, such as the calculation of the volumes of barrels and other containers, and the geometric rules for topographers and inlayers, which he solves by algebraic methods. Among the topics addressed, in addition to dealing with first- and second-degree equations, binomials and biquadratic equations, Piero also treats—beginning with practical problems and committing some errors—those of degree three and higher, the definitive solutions to which would be found only in the next century thanks to the studies of the algebraists Scipione del Ferro, Niccolò Tartaglia, Girolamo Cardano and Ludovico Ferrari.

Among the problems proposed by Piero in which he recurs to the use of equations of a degree higher than two, are three problems of financial mathematics related to the calculation of the accumulation of interest. One of these requires the calculation of the unit monthly earnings, expressed in the number of *denari* earned by one *libbra* in a month, of a capital of 100 *libbre* given a capital of 150 *libbre* for a period of 3 years. Piero, in contrast to what other mathematicians did, solved the problem in the following way: given x the monthy interest in *denari* to be calculated, in 1 year 1 *libbra* will render $12x$ *denari*; since 1 *soldo* equals 12 *denari*, 1 *libbra* will render x *soldi*, and finally, since 1 *libbra* is equal to 20 *soldi*, in a year the interest rate will thus be equal to $x/20$ *libbre*. At this point, beginning with the annual interest rate of $x/20$ *libbre*, he calculates year by year the final amount. At the end of the first year the interest will be equal to $100 \cdot x/20 = 5x$ *libbre*, and thus the total amount will be $100 + 5x$ *libbre*; at the end of the second year the amount of interest earned will be $(100 + 5x)\cdot x/20 = 5x + x^2/4$ and consequently the total amount will be $100 + 5x + 5x + x^2/4$; analogously, at the end of the third year the interest will be equal to $(100 + 5x + 5x + x^2/4)\cdot x/20 = 5x + x^2/2 + x^3/80$ and the total amount at the end of the 3 years will thus be $100 + 10x + x^2/4 + 5x + x^2/2 + x^3/80 = 100 + 15x + 3x^2/4 + x^3/80$. At this point to find x it is sufficient to solve the following equation: $100 + 15x + 3x^2/4 + x^3/80 = 150$, that is, $1200x + 60x^2 + x^3 = 4000$. To solve it Piero applies the solving formula for the equation of degree that he proposes in the volume (valid for applications of this type but not in general), obtaining the correct solution:

$$x = \sqrt[3]{\left(\frac{1200}{60}\right)^3 + \frac{4000}{1}} - \frac{1200}{60} = \sqrt[3]{12{,}000} - 20.$$

De prospectiva pingendi, written at a time when he had already been stricken by illness, was the first truly systematic treatment of perspective in the Renaissance. Here Piero pursues his line of theoretical study, codifying—he was the first to do so—the rules of the modern science of perspective, theorising about the three parts into which paintings was to be divided: *disegno, commensuratio et colorare* (drawing, commensuration and colouring), in which *commensuratio* consisted in geometric, proportional and perspective measuring. Divided into three books, this treatise developed in a mathematical way the problems of perspective representation, offering working examples of not only geometrical and architectural shapes, but all kinds of natural shapes. The first two books illustrate perspective techniques for plane figures and geometric solids, while the third book illustrates techniques for more complex figures, in order of increasing difficulty.

The *Libellus de quinque corporibus regularibus*—which Luca Pacioli included in its entirety, though without ever crediting Piero, in his *De divina proportione*, in which appear the drawings of the regular, *abscisi* and puntuti polyhedra attributed to Leonardo da Vinci—is composed of 140 propositions, and studies the five regular geometric solids that can be inscribed in a sphere (the Platonic solids): tetrahedron, cube, octahedron, dodecahedron and icosahedron. Although inspired by Plato's *Timaeus*, Piero leaves aside the philosophical implications, restricting himself to the purely mathematical-geometrical characters of the figures and their relation to drawing and the figurative arts. Two aspects are found in the volume: on one hand we find the logical order of the expressions and the rigorous use of the theorems, and on the other we find one that is more technical due to the kinds of figures treated (solids and polyhedra), the absence of classical proofs, and the use of arithmetic and algebraic rules applied to the calculations.

Among the problems addressed we find the calculation of the surface of cross vaults and the computation of the volume of groin vaults. To solve this problem Piero considers the intersection of two equal cylinders perpendicular to each other, and solves by imposing certain proportions, arriving at proving (not without gaps between steps) that the volume of the double vault turns out to be $V = \frac{2}{3}d^3$, where d is the diameter of the cylinders. The correctness of the formula can be easily verified by means of integral calculus.

On the merits of his scientific production, of a specialised nature in the areas of mathematics and painting, Piero della Francesca can be considered to all effects a Renaissance mathematician [1]: to him is owed the formalisation of the rules of perspective (the 'degradation' of real objects on the plane) which up to that time had been the sole patrimony of artistic technique. It is with Piero della Francesca, who Giorgio Vasari called 'the greatest geometer of his times, that technique and formalisation began to travel along separate roads, the former remaining the patrimony of artists, while the latter became the exclusive realm of mathematicians. In fact, after Piero, the first to reprise the study of the representation of geometric figures on the plane was the French Girard Desargues (1591–1661) [2], who in his *Brouillon Projet d'une atteinte aux événements des rencontres du Cône avec un Plan* (1639) stated the rules of a new science: projective geometry.

Translated from the Italian by Kim Williams.

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