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ORIGINAL STUDY

Estimating 'noise-floor PSD' by using two or three collocated seismometers: an alternative approach

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Abstract

The revolutionary approach to determining self-noise of the test seismometer by the use of two additional seismometers was presented by Sleeman et al (Bull Seismol Soc Am 96(1):258–271, 2006). Yet nowadays there are common situations where only two seismometers are installed side by side. This article thus outlines the various procedures that can be used in such situations. As it will be shown by examples, these procedures do not provide independent information unlike those given by three-seismometer procedures, however they still provide relevant data that can be used to assess the condition of the tested seismometers. Three equations are presented, that can be used in two-seismometer approach. The Eqs. 1 and 2 are already explained in the article (Tasič and Runovc in J Seismol 16:183–194. https://doi.org/10.1007/s10950-011-9257-4, 2012) The Eq. 3, which represents the "average self-noise", differs from the equation "average self-noise" from the previously described article. Experimentally we found that the latter equation is not consistent with the results obtained for the "average self-noise" with the Sleeman procedure, while the equation for the "average self-noise" presented in this article is consistent with it. It is consistent in the frequency interval, where PSD of the seismic signal is at least 5 dB above the seismometers self-noises and it is very suitable for situations where two seismometers of the same type are compared between each other. This paper also presents an alternative three-seismometers approach. It is derived from the aforementioned Eq. 3. For this reason, at the frequency interval, where PSD of the seismic signal is at least 5 dB above the seismometers self-noises the output from this algorithm should be in accordance with the output from algorithm of Sleeman et al (Bull Seismol Soc Am 96(1):258–271, 2006). If there are deviations in certain frequency interval, this indicates some irregularities in the test itself. Under optimal measuring conditions, the Sleeman procedure is sufficient. However, if the test conditions are suboptimal, a comparison between the two procedures, where one contains division and the other does not, may be used to estimate the frequency range where the results are not reliable. Alternative equations, presented in this paper, are useful to discover unknown errors of the test system. The applicability of the equations is given by examples.

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Keywords Uncorrelated signal · Three-seismometer method · Two-seismometer method · Seismometer self-noise · Average self-noise

Abbreviations

NLNM	New low noise model
ARSO	Slovenian Environment Agency
PSD	Power spectral density estimation
CPSD	Cross power spectral density estimation

1 Introduction

"Self-noise" of the seismometer is its characteristic (e.g. Hutt et al. 2009, Evans et al. 2010, Ringler and Hutt 2010, Sleeman and Melichar 2012). It is usually presented as PSD spectra by manufacturers or it is defined by independent institutions. It is determined using one or two reference seismometers. Holcomb presented the pioneering work in the field of measurement with a reference seismometer (Holcomb 1989, 1990), and a revolutionary approach were demonstrated by Sleeman et al. (2006).

The "noise-floor" represents the lower limit of seismograph detection at particular location and is the sum of seismometer self-noise, acquisition self-noise and "unwanted signal". "Unwanted signal" represents all the undesired disturbances that cause, that "noise-floor" at some frequency is greater than expected values. "Unwanted signal" is usually a function of frequency, but can also be time dependent. With information about noise-floor, the sources of disturbances can be detected and their influence on the seismological measurements can be evaluated. "Noise-floor" can be estimated by using two or three seismometers methods (e.g. Sleeman et al. 2006).

Procedures, when only data from two seismometers are available, are presented in the section "Two collocated seismometers". These procedures do not provide independent information as those given by three-seismometer procedures, but in some cases, however, these procedures have their advantages. Above all, only data from two seismometers are involved in the calculation. Two three-seismometer-methods are presented in section "Three collocated seismometers".

The orientation misalignment between seismometers can increase the estimated noise-floor but this is seemingly increased noise-floor and is due to the orientation misalignment. By transforming of the space of the three component measurements of the particular seismometer into the space of the selected seismometer, this effect is avoided (Tasič and Runovc 2012, 2013). In this way, we also avoid the influence that inaccurate generator constants have on the measurement (Tasic and Runovc 2014).

2 Two collocated seismometers

Situations, when only two collocated seismometers can be tested at the same time are very common. The reasons are different, sometimes only one 6 channel acquisition unit is available or space limitations do not allow proper installation of more than two seismological systems. In some situations, users are only interested in the performance of a particular seismometer relative to a reference one, for example, when seismometer is taken from its location because of unusual behavior and users are not sure from where the error originated. Similarly, users could also have interest to check the new seismometer or seismometer that returned from the repair. To compare or test the behavior of both seismometers, they should be placed on the same pier at a short distance between each other in a temperature stable environment, on well designed vault, without any interruption or movements. It is recommended that the measurement is done in a quiet environment in a seismic sense. If one seismometer is a reference unit, this should be a high quality broad-band seismometer as its self-noise can be expected to be well known and is stable inside of some expected value or accuracy. Let define, that seismometer with the index ‘r’ is reference seismometer. Its expected self-noise PSD is marked as N^0_{rr}. Superscript ‘^{0}‘ mean, that self-noise PSD of seismometer ‘r’ is estimated in the past with different sets of seismometers. However, this is only valid if the noise of the reference unit has been estimated in a stable environment, and in the same environment this seismometer will be continuously used as a reference unit.

The self-noise of tested seismometer ‘q’ is then (Tasič and Runovc 2012)

$$N_{qq} = P_{qq} - \frac{P_{rq}P^*_{rq}}{(P_{rr} - N^0_{rr})}. \tag{1}$$

Equation (1) is valid if self noise of acquisition unit is well below the self noise of seismometers and does not affect the measurement. If this is not achievable, then the reference system’s self-noise is a combination of self-noises of both, the seismometer (usually at low frequencies) and the acquisition unit (usually at higher frequencies). If $N^0_{rr} \ll N_{qq}$, then Eq. 1 can be simplified (Tasič and Runovc 2012)

$$N_{qq} = P_{qq} - \frac{P_{rq}P^*_{rq}}{P_{rr}}, \quad \text{for } N_{qq} >> N^0_{rr} \tag{2}$$

This equation can also be applied to determine the accelerometer self-noise using a quieter seismometer (Tasič 2018).

Although mathematically sustainable, the result is questionable when the self-noise of the tested seismometer is much lower than the self-noise of the reference one ($N^0_{rr} \ll N_{qq}$). The reason lies in the process of numerical calculation of the PSD. Because of the type of input data (finite signal length is used, signal is digitized…) the PSD estimations contain errors which affect the calculation of the self-noise for more quiet seismometer. This can be explained by Eqs. 1 and 2. Suppose that within a given frequency interval the following condition applies: $N_{qq} \gg N_{rr}$. In this frequency interval the Eq. 2 can be used to calculate N_{qq}. So if we can use this equation to calculate N_{qq}, in the reverse process in this frequency interval N_{rr} cannot be correctly calculated with Eq. 1, even if N_{qq} is well estimated, since the N_{rr} is so small that the impact of the calculation errors is greater.

When no information about any self-noises of both seismometers are known, approximation of average self-noise can be used. It is a good approximation, if self-noise of both seismometer are expected to be similar or very equal

$$\frac{1}{2}(N_{qq} + N_{rr}) = \sqrt{P_{qq}P_{rr}} - \sqrt{P_{rq}P^*_{rq}}, \quad \text{for } P_{qq}, P_{rr} > N_{rr}, N_{qq}. \tag{3}$$

Equation represents the average “self-noise” of two systems and is valid when PSDs of instrumental noises are below the seismic PSD for at least 5 dB (Fig. 2). Equation 3 is different from the equation in Tasič and Runovc’s article (2012), which states that

$\frac{1}{2}(N_{qq}+N_{rr})_{T\&R2012}=\frac{1}{2}(P_{qq}+P_{rr})-\sqrt{P_{rq}P^{*}_{rq}}$. The latter equation is very good for calculating the transformation matrix between two seismometers (Tasič and Runovc 2014), but experimentally, we found that this equation is not consistent with the results obtained for the "average self-noise" with the Sleeman procedure (see Eq. 7). It is turned out that 0.5-$(N_{qq}+N_{rr})_{T\&R2012} > 0.5(N_{qq}+N_{rr})_{Sleem}$ and that the difference is at least 1 dB. With further tests, we found that if the arithmetic mean '0.5 $(P_{qq}+P_{rr})$' is replaced with the geometric mean '$(P_{qq}P_{rr})^{0.5}$', results are consistent with the data obtained from the Sleeman procedure.

This means that in the 'PSD space', when the seismic signal is significantly higher than the instrumental noise, the geometric mean is valid and not the arithmetic.

Equation 3 is very useful when seismometers are of the same type as it allows us to compare estimated values with the theoretical self-noise. But small deviations cannot be estimated with this equation. It can be wrongly concluded, that they can be estimated if the left side of Eq. 3 is inserted into Eq. 1 instead of N^{0}_{rr}. If doing so and if seismic signal is well above the self noise, than expression $(N_{rr}-N_{qq})$ is negligible and for two systems of equal type a new expression for "average self-noise" is obtained

$$(0.5N_{rr}+0.5N_{qq})_{q}=P_{qq}-\frac{P_{rq}P^{*}_{rq}}{\left(P_{rr}-\left(\sqrt{P_{qq}P_{rr}}-\sqrt{P_{rq}P^{*}_{rq}}\right)\right)},\quad \text{for } P_{qq},P_{rr}>|(N_{rr}-N_{qq})|. \tag{4}$$

This too complicated estimation can be used to determine the validity range of Eq. 3 ("Appendix"). When only two seismometers of equal type are on test bed, with the help of Eq. 4 the frequency interval can be defined, where both equations perform equal outputs and the results are trustworthy (see Fig. 4 as example).

3 Three collocated seismometers

Three collocated seismometers are preferred to be the best practice to estimate self noise of particular seismometer, whenever three systems are available to be installed on the seismic pier. To estimate the self-noise PSD of seismometer 'q' equation developed by Sleeman et al. (2006) is used

$$N_{qq}=P_{qq}-P_{rq}\frac{P_{qk}}{P_{rk}}. \tag{5}$$

with the term P_{qk}/P_{rk} the authors replace the transfer function H_r with the transfer function H_q in the element P_{qr}. Due to division in a expression P_{qk}/P_{rk}, the errors resulting from algorithm calculation of cross-power spectral densities can be magnified. These errors can be reduced by additional numerical methods, for example, with averaging.

Because Eq. 3 is consistent with the Sleeman procedure, when the seismic signal is significantly higher than the instrument noise, a non-divisible expression of N_{qq} can be derived from Eq. 3 ("Appendix"),

$$N_{qq}=\sqrt{P_{qq}P_{rr}}+\sqrt{P_{qq}P_{kk}}-\sqrt{P_{kk}P_{rr}}-\left(\sqrt{P_{qr}P^{*}_{qr}}+\sqrt{P_{qk}P^{*}_{qk}}-\sqrt{P_{kr}P^{*}_{kr}}\right),\quad \text{for } P_{ii}>N_{ii};\ i=q,r,k. \tag{6}$$

The alternative equation is equivalent to the Eq. 5 insofar that the seismic signal is 5 dB above the self-noise. The output from this algorithm should be in accordance with the output from algorithm of Sleeman et al. (2006) and under optimal measuring conditions, the Sleeman procedure is sufficient. But if there are any deviations in some frequency interval between both procedures, Eq. 5 contains division and Eq. 6 does not, this indicates some irregularities in the test itself. For this reason, Eq. 6 may be used to estimate the frequency range where the results, estimated by Eq. 5, are not reliable.

4 Applicability of equations

The Eqs. 1, 2, 3 and 6 are alternatives to the well known Eq. 5, which was developed by Sleeman et al. (2006). A study of the applicability of alternative equations with regard to the 'Sleeman procedure', used as reference, will be performed on data from three collocated seismometers which were installed side by side at the Conrad Observatory (www.zamg.ac.at/about/conrad-observatory) of the Central Institute for Meteorology and Geodynamics (ZAMG). Two seismometers were CMG-3ESPC (one is be marked with index 'A', second is marked with index 'B') and one was STS-2 seismometer. Seismometers CMG-3ESPC are of lower quality than STS-2, they have higher self-noise at a whole frequency band comparing with STS-2 (Tasič and Runovc 2012). Seismometers were connected to the EarthData PR6 acquisition units. The sample rate at acquisition units were set to 200 samples per second and gains were set to be high sensitive, so that self-noise of acquisition units was lower than the presumed self-noise of seismometers. Record in the total length of 24 h from 2010-01-07 was used for this study. It was divided into four equal parts, so a 6-h seismic segment was used in each calculation. To reduce the impact of orientation misalignments and impact of gain differences to the "self-noise" calculation, data from seismometers CMG-3ESPC-'B' and STS-2 are transformed to the space of seismometer CMG-3ESPC-'A' (Tasič and Runovc 2012, 2013). For the "self-noise" evaluation, Welch's method (Welch 1967) for the PSD estimation was applied using a Matlab$^{(C)}$ built-in function with a Hanning window of length 2^{21} and with 50% overlapping time-series segments. For better presentation, the estimated PSDs are first smoothed around equally distributed points in a log space (200 points per decade) and then ensemble average of each individual estimated group is performed.

The applicability of Eq. 6, where data from three seismometers is used, is presented at Fig. 1. The self-noises of two seismometers CMG-3ESPC (A and B) are estimated with both three seismometers approaches (Eqs. 5 and 6). The third seismometer was STS-2. Outputs from both methods are almost equal, with two deviations at 1 Hz and 2 Hz, as long as seismic signal is enough above the self-noise. At 1 Hz and 2 Hz outputs are slightly differently calculated by both methods. These deviations are result of seismometer 'B', which detects strong disturbance at 1 Hz and 2 Hz and transfer this disorder into the calculation. Why this happens will be explained later (Fig. 5). What exactly is the source of disorder at seismometer 'B' is not clear, but most likely it indicates a fault on the cable or connector. As the impact of this disorder is different with respect to the type of the calculation it can be used as indicator indicating a deviation in the calculations.

The applicability of Eq. 1 is presented in Fig. 2. Applying the 'two-seismometer' method, an STS-2 is used as reference unit with predefined "self-noise" data (Tasič and Runovc 2012). The results are expected to be worse with respect to the three seismometer procedure (e.g. Sleeman et al. 2006), but estimated self-noises are still a very good

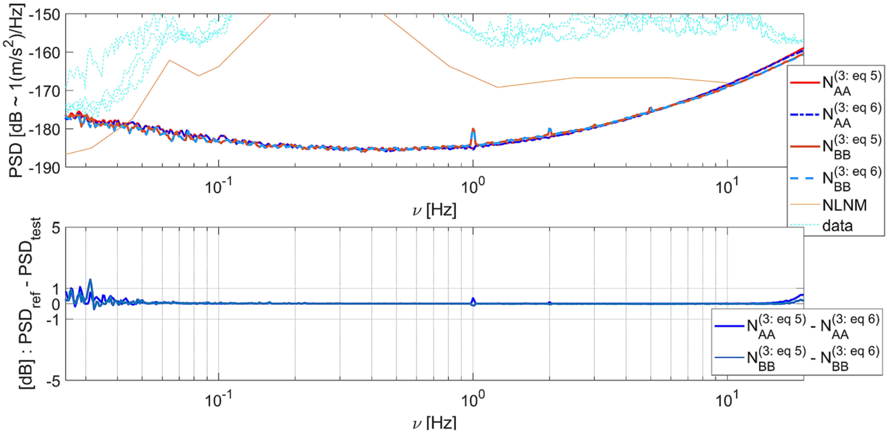

Fig. 1 The self-noises of seismometers CMG-3ESPC 'A' and CMG-3ESPC 'B' are estimated with two "three seismometer methods" (Eqs. 5 and 6). The third seismometer was STS-2. The differences in the results of both approaches are below 1 dB as long as seismic signal is approximately 5 db over the self-noise. Outputs are slightly differently calculated at 1 and 2 Hz by both approaches. These deviations are result of seismometer 'B' detection, which detects strong disturbance at 1 and 2 Hz and transfers this disorder into the calculation

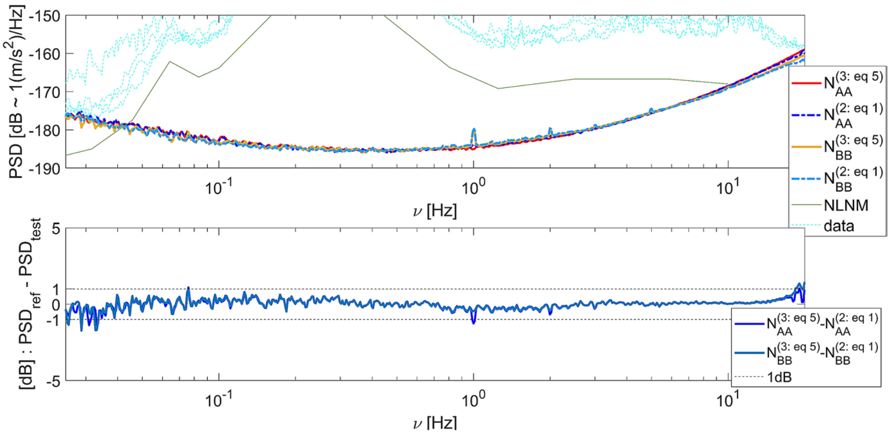

Fig. 2 The two seismometer approach, where self-noises for seismometers CMG-3ESPC 'A' and 'B' are estimated with the help of the only one reference unit (Eq. 1), is compared with the Sleeman method (Sleeman et al. 2006). In both cases, the STS-2 is used as reference unit with predefined "self-noise" (Tasič and Runovc 2012). The results are expected to be worse with respect to the three seismometer procedure, but estimated self-noises are still a very good approximation. The procedures become less reliable when the seismic signal approaches the self-noise to less than 7 dB

approximation: as long as seismic signal is well above their self-noise, they mostly differ by less than 1 dB from the reference procedure (Sleeman et al. 2006). The procedure become less reliable when the seismic signal approaches the self-noise to less than 7 dB. This procedure is appropriate when the self-noise of the tested seismometer is above the self-noise of the reference unit.

Figure 3 shows the estimated "average self-noises" for units CMG-3ESPC (A and B), which were estimated by two different procedures. The first method is based on the Sleeman procedure of three seismometers (Sleeman et al. 2006). The equation is

$$0.5(N_{qq} + N_{rr})_{Sleem} = 0.5(P_{qq} + P_{rr} - (P_{rq}(P_{qk}/P_{rk}) + P_{qr}(P_{rk}/P_{qk})). \tag{7}$$

At the second approach, the Eq. 3 is used to calculate the average, where data from only two seismometers are used as input. As long as seismic signal is above self-noise for 5 dB or more, both methods give the equal result. If only two seismometers are on the same seismic pier and both have very similar self-noise, the most appropriate procedure involves the Eq. 3. The disadvantage of these procedure is, that it does not detect slight differences in self-noises of both units at higher frequencies (Figs. 1, 2).

If only two seismometers of the same type are placed side by side, Eq. 4 can be used to determine the range, where Eq. 3 applies (Fig. 4). As long as the outputs from both procedures are equal, Eq. 3 is a good approximation to Eq. 7. From Fig. 3 and 4 it is evident that at these examples, Eq. 3 can be fully trusted in the frequency range between 0.035 and 15 Hz.

Even at three seismometer methods, when the self-noise of the tested seismometer is much lower than the self-noise of the reference systems, due to the loss of information, relevant self-noise data cannot be obtained for quieter seismometer. The reason lies mainly in error as result of numerical calculation of the PSD and can significantly affect the calculation of an unknown self-noise of quieter seismometer. This is valid for both three-seismometer methods (Eqs. 5 and 6). The Fig. 5 presented the estimated self noise of a quieter seismometer, which is in our case STS-2, obtained from both three-seismometer procedures. If the condition for Eq. 6 is fulfilled, but outputs from both equations differ between each other, the outputs are not reliable in this area. Estimated data are consistent between each other in the frequency interval between 1.02 and 11 Hz. Below 1.02 Hz the input data are no longer reliable, self noise of both CMG-3ESPC seismometers are to high and the calculated self-noise no longer follows the expected values for STS-2 seismometer. The

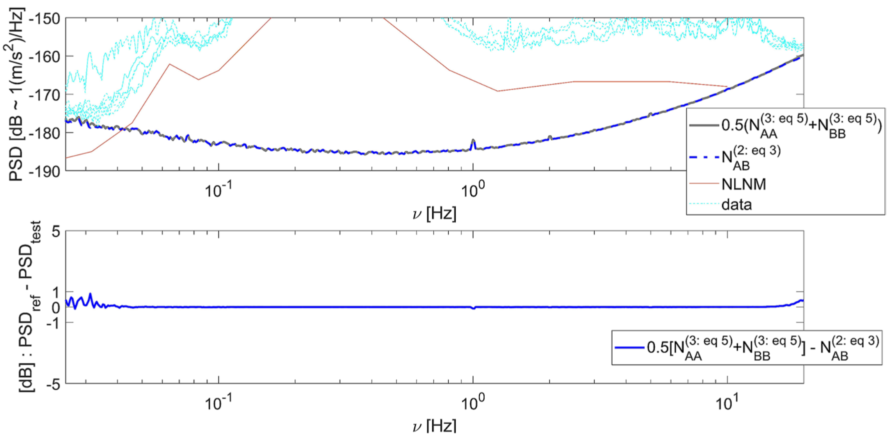

Fig. 3 The self-noise average for two CMG-3ESPC seismometers, calculated with two different approaches. The first approach is based on the Sleeman procedure of three seismometers (Sleeman et al. 2006). The second approach use Eq. 3 and data from only two seismometers. As long as seismic signal is above the self-noises for 5 dB or more, both approaches give almost equal result

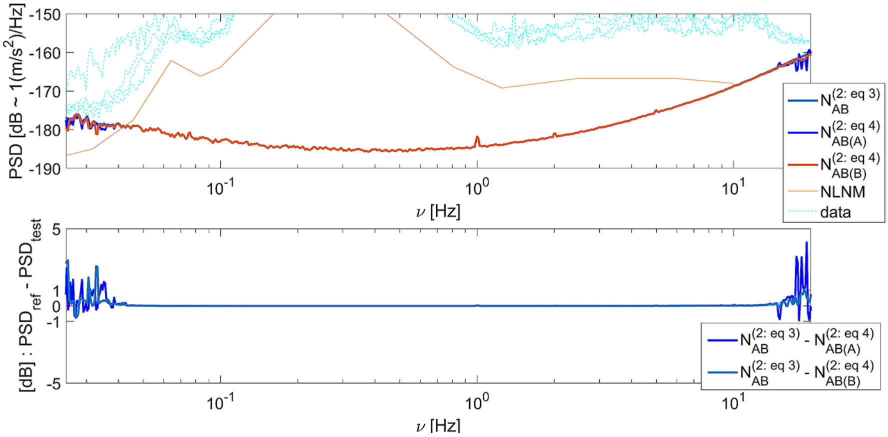

Fig. 4 The self-noise average for two CMG-3ESPC seismometers, calculated with two two-seismometers approaches (Eqs. 3 and 4). The second approach can be calculated in two different ways (Eq. 4), first we denote q = A and second we denote q = B. From Figs. 3 and 4 it is evident that the two seismometers procedure, which use equation Eq. 3, can be fully trusted in the frequency range between 0.035 Hz and 15 Hz

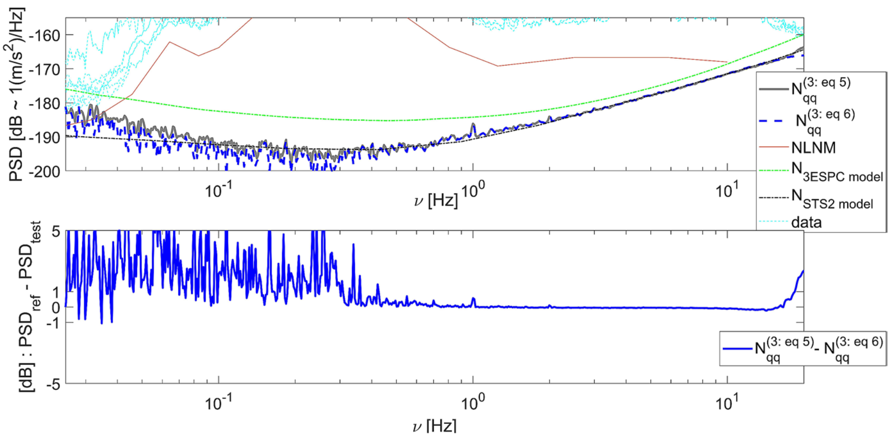

Fig. 5 Two “three seismometer method” outputs are presented, where the self-noise of the tested seismometer (STS-2) is lower than the self-noise of both reference systems (CMG-3ESPC). Estimated data are consistent between each other in the frequency interval between 1.02 Hz and 11 Hz. Below 1.02 Hz the input data are no longer reliable, self-noise of both CMG-3ESPC seismometers are to high. Above 11 Hz, the seismic signal is too close to the self-noises of reference systems

rule of thumb is, that the self-noise of a quieter seismometer can be calculated by three-seismometer-method, if the self-noises of both reference seismometers is less than 5 dB above the self-noise of the tested one. Above 11 Hz, the seismic signal is too close to the self noise of reference systems. Figure 6 shows estimated self-noise of more quiet system with only one reference systems. The results are expected to be worse than in Fig. 5. With only one reference system it is difficult to determine the noise of the test system if its self-nose is lower than the reference system. The rule of thumb is that at “two seismometer

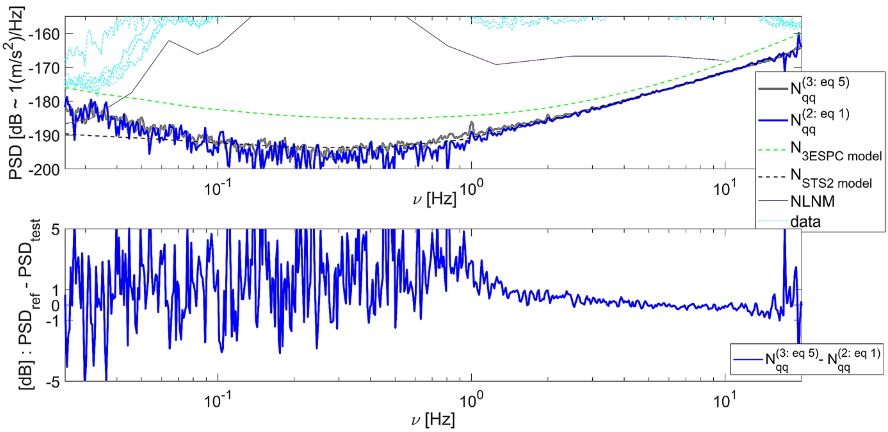

Fig. 6 The example of "two seismometer method" - Eq. 1 is presented, where the self-noise of the tested seismometer (STS-2) is lower than the self-noise of reference systems (CMG-3ESPC "A"). With respect to the data from Fig. 1, the self-noise-model is designed for this reference seismometer. Estimated data is compared with "three seismometer method" (Sleeman et al. 2006) and are conditionally consistent between each other in the frequency interval between 2 Hz and 11 Hz

method"—Eq. 1, the self-noise of reference seismometer should be less than 3 dB above the self-noise of the tested one.

5 Discussion

To check the quality of seismic equipment before any installation on the field, the "huddle-test" is recommended. The properly performed "huddle-test" says a lot about the quality of seismological equipment. It should be performed with at least three seismometers but this is sometimes not possible. Under good test conditions, when systems are in stable environment including stable power supply, and if the location allows the use of three seismometers of equal type to be installed side by side, a procedure with three seismometers, as presented by Sleeman, is the most appropriate. If there are deviations from the expected values, the results can be checked with additional "three-seismometer test" where Eq. 6 is used, as equation contains no division. If the results from both procedures differ from each other and the conditions for using Eq. 6 are fulfilled, this indicates an unknown disturbance on some of the tested systems. In such cases, a two-seismometers algorithms can be only used to identify some differences as in some special cases can points out to some discrepancies.

Let's take a look at an example where different approaches were used. Huddle test were performed with three seismometers, all installed on the same concrete pier. Two of them (marked 'q' and 'r') were of the same type. All seismometers were thermal insulated and connected to the Q330 acquisition units. Amplification at the acquisition units were set to standard, so noise floor of particular system should consisted of the self-noise of acquisition unit at high frequencies and the self-noise of seismometer at low frequencies. Self-noise of both types of seismometers at 0.01 Hz were close to Peterson NLNM (Peterson 1993). It was surprising, however, that at low periods the PSDs of two systems of the same type ('r' and 'q') were significantly higher than the PSDs of the third system. The example, where a signal of 2 h duration is used, is shown at Fig. 7 where the outputs and calculations refer to the seismometer ‘q’. At low frequencies both seismometers of the same type detected unusual signal which was not detected by the third seismometer. Taking that into acount it was evident, that the source was not seismic. When the system was analyzed by three-seismometer procedures, no relevant data were obtained at low frequencies (Fig. 7). The outputs from both procedures started to differ below 0.4 Hz and were not realistic any more. For this reason, two-seismometers procedures were used for additional analysis. The “Two Seismometer Method”—Eq. 3 is used for the seismometers of equal type. From this procedure it is clear that seismometers ‘r’ and ‘q’ detect almost equal relatively strong non-seismic signal. From the “Two Seismometer Method”—Eq. 1, where the reference was the third seismometer (‘k’), have reached the following conclusion. First, seismometers ‘r’ and ‘q’ also detect disturbances at higher frequencies, which are below seismic signal. The effect of the disturbance is visible at least up to 1 Hz. Second, because at low frequencies the output of the algorithm, where Eqs. 1 or 2 is used, does not match the PDS-s of seismometers ‘r’ and ‘q’, it means that also the third seismometer ‘k’ detected the same non-seismic disturbances, but it is much less sensitive to this non-seismic source. The cause of the disturbances was later identified to originate from short pulses of the magnetic field.

6 Conclusion

By using data from three seismometers, the noise floor of a single system can be accurately analyzed by using two procedures, the ‘Sleeman procedure’ (Sleeman et al. 2006) and the procedure described in this article. The last procedure has limitations as it is suitable in

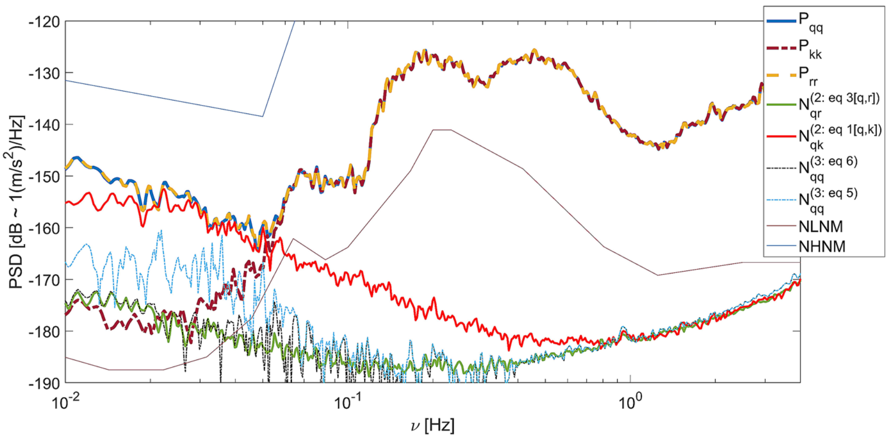

Fig. 7 “Huddle-test” was performed with three thermal insulated broadband seismometers, with theoretical self-noise at 0.01 Hz close to NLNM. Two of them (marked ‘q’ and ‘r’) were of the same type and detect different signal at low frequencies than the third one. All calculations in the figure refer to seismometer ‘q’. The outputs from both “three-seismometers-algorithms” started to differ below 0.4 Hz and were not realistic any more. At long periods “average self-noise” of the same type of seismometers (‘q’ and ‘r’) is strongly below the detected signal which means that these units detect equal relatively strong non-seismic signal. “Two-seismometer-approach”-Eq. 1, where the reference was the third seismometer (‘k’), shows, that seismometers ‘q’ (and ‘r’) detect disturbances also at higher frequencies and also the third seismometer ‘k’ detected at long periods the small portion of non-seismic disturbances

the frequency interval where the seismic signal is above the noise floor by at least 5 db. Despite these shortcomings, using both procedures increases the reliability of the noise-floor estimation for a particular system.

When estimated self-noise PSD for particular instrument does not fit the expected values, this indicates some deviation in the measurement. Estimated deviations may indicate a malfunction of seismometer, they may indicate the problems with peripheral systems (the power supply, problem with cable,…) or the impact of micro-location on the measurement (influence of EM field…). In some cases deviations can be more accurately analyzed by using of "two-seismometer" algorithms.

The two units huddle test is recommended to be performed, when unusual signals are detected at the site of seismic station and there is no technical or spatial capability to place three seismometers, and we are not sure what the actual cause is. Two seismometers "huddle-test" is not as independent as test with more seismometers, but does still resolve some questions, relating to, if they exist, unexpected test results.

The "two-seismometers" algorithms requires more caution, both in the use of equations and in the interpretation of results. For example, the incorrectly defined reference self-noise, when Eq. 1 is used, can also be a source of error, especially when the test system's self-noise is equal to that of the reference system. In such cases it is advisable to calculate "average self- noise". But with this last algorithm some details (small deviations) can be lost.

However, the golden rule is that the reference systems must be of better or of the same quality as the test systems in order to obtain reliable data, regardless of whether a two- or three-seismometer algorithm is used.

Appendix

Equation

$$0.5(N_{qq} + N_{rr}) = \sqrt{P_{qq}P_{rr}} - \sqrt{P_{rq}P^*_{rq}}, \quad \text{for } P_{qq}, P_{rr} > N_{rr}, N_{qq}. \tag{8}$$

represent the "average self-noise" of systems 'r' and 'q' if seismic signal is above self-noise for at least 5 dB. Let now assume that three seismometers, 'q', 'r' and 'k' are on the same seismic pier. Equation 8 is rewritten as

$$0.5(N_{qq} + N_{kk}) = \sqrt{P_{qq}P_{kk}} - \sqrt{P_{kq}P^*_{kq}}, \tag{9}$$

$$0.5(N_{rr} + N_{kk}) = \sqrt{P_{rr}P_{kk}} - \sqrt{P_{kr}P^*_{kr}}. \tag{10}$$

First sum Eqs. 8 and 9, and from that we subtract the Eq. 10. New estimator for self-noise of seismometer 'q' is expressed as

$$N_{qq} = \sqrt{P_{qq}P_{rr}} - \sqrt{P_{rq}P^*_{rq}} + \sqrt{P_{qq}P_{kk}} - \sqrt{P_{kq}P^*_{kq}} - (\sqrt{P_{rr}P_{kk}} - \sqrt{P_{kr}P^*_{kr}}),\ P_{qq}, P_{rr}, P_{kk} > N_{rr}, N_{qq}, N_{kk}. \tag{11}$$

The equation is valid if the same condition as in Eq. 8 is fulfilled.

The similar is valid for Eq. 12. The equation

$$(0.5\mathrm{N}_{\mathrm{rr}} + 0.5\mathrm{N}_{\mathrm{qq}})_q = \mathrm{P}_{\mathrm{qq}} - \frac{\mathrm{P}_{\mathrm{rq}}\mathrm{P}^*_{\mathrm{rq}}}{\mathrm{P}_{\mathrm{rr}} - (\sqrt{\mathrm{P}_{\mathrm{qq}}\mathrm{P}_{\mathrm{rr}}} - \sqrt{\mathrm{P}_{\mathrm{rq}}\mathrm{P}^*_{\mathrm{rq}}})}, \quad \text{for } \mathrm{P}_{\mathrm{qq}}, \mathrm{P}_{\mathrm{rr}} > |(\mathrm{N}_{\mathrm{rr}} - \mathrm{N}_{\mathrm{qq}})|. \tag{12}$$

produce equal result as the Eq. 8 if seismic signal is above self-noise for at least 5 dB. This can be shown by writing the right side of Eq. 12 as follows

$$\mathrm{X} = \mathrm{P}_{\mathrm{qq}} - \frac{\mathrm{P}_{\mathrm{qr}}\mathrm{P}^*_{\mathrm{qr}}}{\mathrm{P}_{\mathrm{rr}} - (0.5(\mathrm{N}_{\mathrm{rr}} + \mathrm{N}_{\mathrm{qq}}))}. \tag{13}$$

From this equation it follows

$$\mathrm{X}(0.5(\mathrm{N}_{\mathrm{rr}} + \mathrm{N}_{\mathrm{qq}}) - \mathrm{P}_{\mathrm{rr}})) = \mathrm{P}_{\mathrm{qr}}\mathrm{P}^*_{\mathrm{qr}} + \mathrm{P}_{\mathrm{qq}}(0.5(\mathrm{N}_{\mathrm{rr}} + \mathrm{N}_{\mathrm{qq}}) - \mathrm{P}_{\mathrm{rr}}). \tag{14}$$

By using relations $\mathrm{P}_{\mathrm{qq}} = \mathrm{P}_{\mathrm{xx}} + \mathrm{N}_{\mathrm{qq}}$, $\mathrm{P}_{\mathrm{rr}} = \mathrm{P}_{\mathrm{xx}} + \mathrm{N}_{\mathrm{rr}}$ and $\mathrm{P}_{\mathrm{qr}} = \mathrm{P}_{\mathrm{xx}}$, Eq. 14 is rewritten as

$$\mathrm{X}(0.5(\mathrm{N}_{\mathrm{qq}} - \mathrm{N}_{\mathrm{rr}}) - \mathrm{X}\mathrm{P}_{\mathrm{xx}} = \mathrm{N}_{\mathrm{qq}}0.5(\mathrm{N}_{\mathrm{qq}} - \mathrm{N}_{\mathrm{rr}}) - \mathrm{P}_{\mathrm{xx}}0.5(\mathrm{N}_{\mathrm{qq}} + \mathrm{N}_{\mathrm{rr}}). \tag{15}$$

Under the assumption, that expression $(\mathrm{N}_{\mathrm{rr}} - \mathrm{N}_{\mathrm{qq}})$ is negligible with respect to the seismic signal, then $\mathrm{X} = 0.5(\mathrm{N}_{\mathrm{rr}} + \mathrm{N}_{\mathrm{qq}})$.

As long as seismic signal is above the self-noise of at least 5 db, outputs from Eqs. 8 and 12 are comparable. However, when this condition is not fulfilled the outputs from Eqs. 8 and 12 differ. When only two seismometers of equal type are on test bed, Eq. 12 can be used, to define the frequency interval, where Eq. 8 is valid.

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