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ORIGINAL STUDY

The application of a combination of weighted least-squares and autoregressive methods in predictions of polar motion parameters

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Abstract This study employs a combination of weighted least-squares extrapolation and an autoregressive model to produce medium-term predictions of polar motion (PM) parameters. The precisions of PM parameters extracted from earth orientation parameter (EOP) products are applied to determine the weight matrix. This study employs the EOP products released by the analysis center of the 'International Global Navigation Satellite System Service and International Earth Rotation and Reference Systems Service' needs to be modified to 'International Global Navigation Satellite System Service (IGS) and International Earth Rotation and Reference Systems Service (IERS)' as primary data. The polar motion parameters and their precisions are extracted from the EOP products to predict the changes in polar motion over spans of 1–360 days. Compared with the combination of least-squares and autoregressive model, this method shows considerable improvement in the prediction of PM parameters.

Keywords Earth orientation products · Weighted least-squares · Autoregressive model · Polar motion prediction

1 Introduction

Earth orientation parameters (EOPs) include the nutation and precession parameters, the polar motion parameters and the length of day (LOD). EOPs contain a wealth of geodynamics information and play a significant role in applications including the determination of high-precision satellite orbits, spacecraft tracking, laser measurements, and deep space exploration. These parameters are needed to achieve mutual conversion between celestial and terrestrial reference frames. Thanks to developments in modern measurement

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techniques, such as very long baseline interferometry (VLBI), satellite laser ranging (SLR), global navigation satellite system (GNSS), and doppler orbitography by radio-positioning integrated on satellite (DORIS) (Dow et al. 2005), increasingly accurate observations of EOPs promote advances in celestial dynamics.

Due to the complexity of the data processing involved, EOP products cannot be calculated in real time using the data obtained by modern earth observation technologies. Thus, the parameters are usually provided after a delay of hours to days. However, for some practical applications, the EOPs must be acquired in advance. For example, in satellite navigation systems, only long-term predictions of EOPs can be used to achieve mutual conversion between the celestial and terrestrial reference frames when satellites enter the autonomous orbit mode. Therefore, a reliable and high precision prediction model is needed. However, EOPs are affected by many factors (Völgyesi 2006) that can be divided into two groups, specifically the effects of (1) different levels and periodic excitation sources and (2) high-frequency variations. These factors cause enormous challenges in the precise prediction of EOPs.

To check the accuracy of different methods of predicting EOPs, the Institute of Geodesy and Geophysics of the University of Vienna conducted two prediction competitions, the Earth Orientation Parameters Prediction Comparison Campaign (EOP PCC) in October 2005 (http://users.cbk.waw.pl/~kalma/EOP_PCC/) and the Earth Orientation Parameters Combination of Prediction Pilot Project (EOPCPPP) in October 2010 (http://eopcppp.cbk.waw.pl). The purposes of these activities were to encourage scholars worldwide to utilize different methods to predict EOPs and to assess the forecasting accuracy and applicability of various prediction methods. The prediction methods can be divided into two categories, specifically single models (including both linear and nonlinear models) and combinations of multi models (Freedman et al. 1994; Schuh et al. 2002; Kosek et al. 2004, 2008; Xu 2012; Guo et al. 2013; Xu and Zhou 2015). One major conclusion that was reached as a result of these competitions is that no single prediction technique is suitable for all EOPs over their entire ranges of variation. The following two conclusions were also reached. (1) Least-squares extrapolation of harmonic models and autoregressive prediction (LS + AR), spectral analysis and LS extrapolation, and neural networks display relatively good performance in predicting PM. (2) Kalman filters, wavelet decomposition and auto-covariance prediction, and adaptive transformation of the atmospheric angular momentum (AAM) yield relatively high-quality results in predicting UT1-UTC and LOD (Kalarus et al. 2007, 2010).

Many scholars have carried out studies on methods of predicting polar motion (Zhang 2012; Sun 2013; Lei 2016). Of these methods, LS + AR models represent a relatively stable type of model that is used to predict PM parameters; moreover, many scholars have made different improvements to this method (Sun and Xu 2012; Xu et al. 2012a, b; Yao et al. 2013). Note that the high calculation precision of the PM parameters is almost negligible compare with the prediction accuracy; thus, the effects of model error on the predictions should be emphasized. Therefore, how to address the residuals that result from fitting LS models to data has been examined by different scholars. Differential method processing LS + AR models are used mainly in short-term forecasts (Yan and Yao 2012), whereas WLS + AR models are employed in medium-and-long-term predictions of PM parameters. The main principle used by previous studies to construct the weight matrix is that relatively high weights are assigned when the predicted values lie close to the data; although this practice effectively improves the prediction accuracy, it depends considerably on the experience of the operator (Sun and Xu 2012). Therefore, in this paper, we propose employing the calculation precision of the PM parameters extracted from EOP products as weighting factors to determine the weight matrices of the vectors of observation used in the least-squares extrapolation model. In this study, the fitting parameters of the weighted least-squares model are first calculated; the AR model parameters are then determined from the residuals; and finally, the extrapolated values obtained from the WLS and AR models are combined to obtain the final predictions. Having extracted PM parameters and their precisions from the data products published by the analysis centres of the IGS and the IERS, which are taken to represent basic data, this study predicts medium-term changes in PM and compares them with the results predicted by the LS + AR model to verify the feasibility of the method presented here.

2 Methods

2.1 Weighted least-squares model

The method is based on LS + AR. An introduction to the construction of and the calculations performed by the model is first presented. Existing studies have shown that the major trends of the PM contain a linear term and periodic terms, including the Chandler wobble and the annual and semi-annual terms (Sun 2013). The fitting equation of the LS model can be expressed as:

$$\begin{aligned} \mathrm{X}(t) = {} & a_0 + a_1 \cdot t + a_2 \cdot cos\left(\frac{2\pi t}{T_1}\right) + a_3 \cdot sin\left(\frac{2\pi t}{T_1}\right) + a_4 \cdot cos\left(\frac{2\pi t}{T_2}\right) + a_5 \cdot sin\left(\frac{2\pi t}{T_2}\right) \\ & + a_6 \cdot cos\left(\frac{2\pi t}{T_3}\right) + a_7 \cdot sin\left(\frac{2\pi t}{T_3}\right) \end{aligned} \tag{1}$$

where X(t) is the PM at time t; t is the UTC time (in years); $a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7$ need to be estimated; T_1, T_2, T_3 are the periods of the semi-annual, annual and Chandler wobble terms, respectively. In this paper, $T_1 = 0.5a, T_2 = 1a$ and $T_3 = 1.183a$. Estimates of the model parameters can be calculated using formula (2):

$$\alpha = \left(A^T PA\right)^{-1} A^T PL \tag{2}$$

$$\alpha = \left[a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad a_6 \quad a_7\right]^T \tag{3}$$

$$\mathrm{L} = \left[X(t_1) \quad X(t_2) \quad \cdots \quad X(t_n)\right]^T \tag{4}$$

$$\mathrm{A} = \begin{bmatrix} 1 & t_1 & cos\left(\frac{2\pi t_1}{T_1}\right) & sin\left(\frac{2\pi t_1}{T_1}\right) & cos\left(\frac{2\pi t_1}{T_2}\right) & sin\left(\frac{2\pi t_1}{T_2}\right) & cos\left(\frac{2\pi t_1}{T_3}\right) & sin\left(\frac{2\pi t_1}{T_3}\right) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & t_n & cos\left(\frac{2\pi t_n}{T_1}\right) & sin\left(\frac{2\pi t_1}{T_1}\right) & cos\left(\frac{2\pi t_n}{T_2}\right) & sin\left(\frac{2\pi t_n}{T_2}\right) & cos\left(\frac{2\pi t_n}{T_3}\right) & sin\left(\frac{2\pi t_n}{T_3}\right) \end{bmatrix} \tag{5}$$

where α is the vector of estimated parameter; A is a coefficient matrix; L is a vector of observation; and P is the weight matrix, which is a diagonal matrix. The residuals are then processed using an AR model.

2.2 Determination of the weight matrix

The key feature of the WLS method is that it incorporates a weight matrix into the estimation of parameters based on LS models. An appropriate weighting method effectively improves the precision of the model parameters, thus resulting in a more stable base sequence for the AR model. In the actual process of surveying adjustment, using the calculated precision of the parameters as the posterior factor is a commonly weighting method. Therefore, this paper extracts the precision of PM parameters from the EOP products released by different analysis centers for use as a variance factor in building the weight matrix. The weight matrix is expressed as follows:

$$\mathrm{P}_w = \begin{bmatrix} \frac{\sigma_0^2}{\sigma_{1,w}^2} & 0 & 0 & 0 \\ 0 & \frac{\sigma_0^2}{\sigma_{2,w}^2} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \frac{\sigma_0^2}{\sigma_{n,w}^2} \end{bmatrix} \tag{6}$$

where w denotes the polar motion (PMX, PMY); n is the length of the basic data; the unit weight variance σ_0^2 is assigned a value of 1; and $\sigma_{n,w}^2$ represents the variances of the calculated PM parameters on the nth day.

2.3 AR model

An AR model describes the relationship among the variables of a random series $z_t(t = 1, 2, \ldots N)$, which can be expressed as follows:

$$z_t = \sum_{i=1}^{p} \varphi_i z_{t-i} + \varepsilon_t \tag{7}$$

where $\varphi_1, \varphi_2, \ldots \varphi_p$ are autoregressive coefficients obtained using the least squares method; p is the order of the model; and ε represents zero-mean white noise. In this paper, the AR model is constructed by the fitting residuals of the WLS model.

A key step in applying AR model is determining the order p. Three main methods of estimating the orders of AR model exist. Specifically, these rules are the final prediction error (FPE) criterion, the akaike information criterion (AIC), and the criterion autoregressive transfer (CAT). In practice, these three methods are virtually equivalent to each other. The FPE criterion is adopted to determine the order of AR model.

$$\mathrm{FPE}(p) = \frac{(N+p)}{(N-p)} P_p \tag{8}$$

where

$$P_p = \frac{1}{N-p} \sum_{t=p+1}^{N} \left(z_t - \sum_{j=1}^{p} \varphi_j z_{t-j} \right)^2 \tag{9}$$

Here $p = 1, 2, \ldots N$, and P_p is the residual mean squared deviation of the model fitting sequence in AR(p). When FPE (p) reaches its minimum value, p is taken to represent the order of the best model.

2.4 Prediction error estimates

The mean absolute error (MAE) is an indicator of prediction accuracy. It is calculated as:

$$\mathrm{MAE}_i = \frac{1}{n}\sum_{j=1}^{n}\left|S_j^i - O_j^i\right| \tag{10}$$

where i is the prediction interval; S_j^i, O_j^i are the predicted and released values, respectively, on day j; and n is the number of predictions used to calculate the statistics.

3 Examples and analysis

The products released by IGS and IERS contain the values and precision of PM parameters, and the time interval used in the products is 1 day. However, the EOP products released by the IGS Analysis Centre are determined using GNSS technology, whereas those of the IERS Analysis Centre are calculated using multiple technologies (such as GNSS, VLBI, SLR, and DORIS); the precisions of the PM parameters are not the same. In this study, the EOP products released by IGS and IERS are used as basic experimental data to predict the medium-term PM and to verify the effectiveness of the method proposed. As recent studies have shown (Sun et al. 2015), the highest accuracies are obtained in predictions using of the LS + AR models when an input sequence of 10 years is used. Therefore, this thesis takes 10 years as an elementary sequence to predict PM using an interval of 1 day and a span of 1–360 days.

3.1 Experiments based on products released by IGS

The values and the precisions derived from the PM parameters products issued by IGS (ftp://cddis.gsfc.nasa.gov/pub/gps/products/), which extend from January 1, 2002 to December 31, 2011, are employed to construct the prediction model and build the weight matrix. For each prediction (1–360 days), 600 and 1000 experiments are performed using the LS + AR and WLS + AR methods. The predictions extend from January 1, 2012 to August 18, 2014 and January 1, 2012 to September 22, 2015, and the sampling interval is 1 day. Tables 1 and 2 list the MAE values for the different prediction intervals, and these values are shown graphically in Figs. 1 and 2.

Table 1 Statistical accuracy results in 600 IGS forecasts

MAE	PMX (mas)			PMY (mas)		
Prediction day	LS + AR	WLS + AR	Accuracy improvement (%)	LS + AR	WLS + AR	Accuracy improvement (%)
1	0.216	0.215	0.074	0.106	0.106	0
5	1.484	1.480	0.23	0.665	0.660	0.739
10	2.831	2.793	1.353	1.244	1.226	1.51
30	8.210	8.024	2.26	3.875	3.613	6.765
60	15.886	15.106	4.907	8.114	7.022	13.467
100	23.285	20.704	11.083	14.159	10.262	27.525
120	27.145	22.273	17.95	16.218	11.193	30.986
150	31.700	24.281	23.405	17.826	12.486	29.957
200	33.499	25.094	25.092	15.553	11.851	23.803
250	30.509	23.914	21.616	13.475	9.726	27.819
300	30.179	25.690	14.874	15.579	10.894	30.069
360	36.631	25.753	29.697	18.685	13.376	28.413

Table 2 Statistical of accuracy results in 1000 IGS forecasts

MAE	PMX (mas)			PMY (mas)		
Prediction day	LS + AR	WLS + AR	Accuracy improvement (%)	LS + AR	WLS + AR	Accuracy improvement (%)
1	0.212	0.211	0.308	0.175	0.174	0.06
5	1.482	1.457	1.695	1.092	1.081	0.978
10	2.831	2.720	3.913	2.056	2.024	1.567
30	8.380	7.818	6.709	6.255	5.875	6.072
60	16.666	14.619	12.279	13.667	11.713	14.295
100	25.904	20.023	22.702	24.593	18.373	25.289
120	29.763	21.616	27.372	28.483	20.576	27.758
150	33.988	23.419	31.097	32.161	22.873	28.88
200	35.434	23.472	33.759	30.309	21.760	28.208
250	34.594	23.404	32.347	28.380	20.871	26.459
300	35.652	27.121	23.928	33.023	24.328	26.33
360	41.881	29.845	28.739	39.246	28.954	26.224

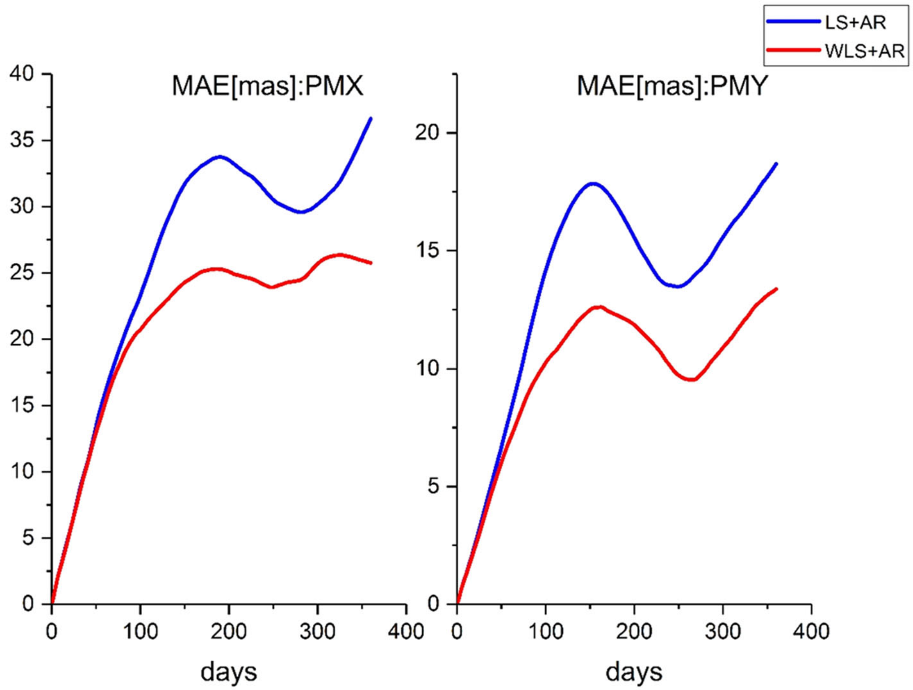

Fig. 1 MAE values calculated for 600 IGS forecasts over different intervals

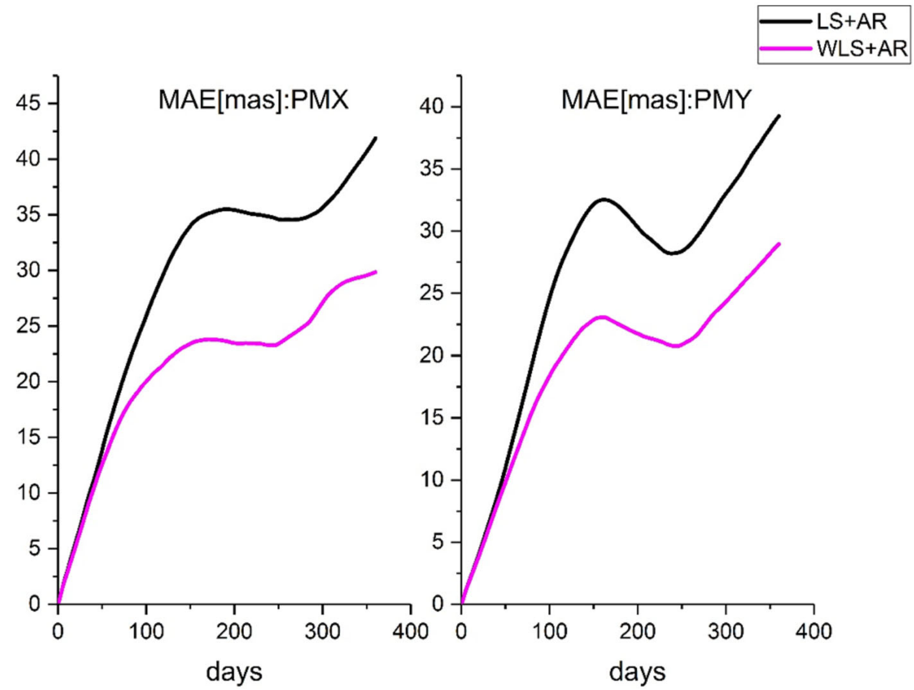

Fig. 2 MAE values calculated for 1000 IGS forecasts over different intervals

3.2 Experiments based on products released by IERS

As another example, this study chooses the values and the precision of PM parameters issued by IERS 08 C04 EOP product (ftp://ftp.iers.org/products/eop/long-term/c04_08/iau2000/), which extends from January 1, 2002 to December 31, 2011. These data are used to construct the prediction model and build the weight matrix. 600 and 1000 experiments are performed using the LS + AR and WLS + AR methods for each prediction (1–360 days), and the sampling interval is uniformly 1 day. The prediction dates are the same as the previous two experiments. The results are presented in Tables 3, 4 and Figs. 3, 4.

Table 3 Statistical of accuracy results in 600 IERS forecasts

MAE	PMX (mas)			PMY (mas)		
Prediction day	LS + AR	WLS + AR	Accuracy improvement (%)	LS + AR	WLS + AR	Accuracy improvement (%)
1	0.224	0.223	0.23	0.108	0.108	0
5	1.492	1.479	0.88	0.662	0.654	1.182
10	2.895	2.836	2.018	1.237	1.213	1.974
30	8.785	8.190	6.77	3.873	3.540	8.587
60	17.603	15.323	12.95	8.143	6.996	14.091
100	25.168	20.725	17.653	14.207	11.114	21.774
120	28.619	21.680	24.246	16.265	12.706	21.879
150	33.230	24.343	26.744	17.840	13.974	21.673
200	36.156	24.995	30.869	15.459	12.271	20.623
250	33.790	24.508	27.468	13.287	9.564	28.018
300	32.471	25.898	20.242	15.427	11.918	22.747
360	37.064	26.997	27.161	18.691	15.557	16.765

Table 4 Statistical of accuracy results for 1000 IERS forecasts

MAE	PMX (mas)			PMY (mas)		
Prediction day	LS + AR	WLS + AR	Accuracy improvement (%)	LS + AR	WLS + AR	Accuracy improvement (%)
1	0.224	0.223	0.526	0.183	0.183	0.024
5	1.484	1.459	1.712	1.100	1.089	1.001
10	2.842	2.738	3.659	2.054	2.027	1.299
30	8.716	8.032	7.851	6.254	5.955	4.782
60	17.459	15.250	12.652	13.656	12.518	8.329
100	26.305	20.661	21.455	24.278	21.065	13.237
120	29.746	22.268	25.141	28.008	24.067	14.068
150	33.524	25.122	25.062	31.456	26.488	15.793
200	34.771	25.490	26.691	30.076	25.681	14.611
250	33.630	25.745	23.446	27.763	24.092	13.22
300	36.098	30.768	14.765	31.855	28.775	9.67
360	41.769	33.951	18.717	39.216	35.485	9.514

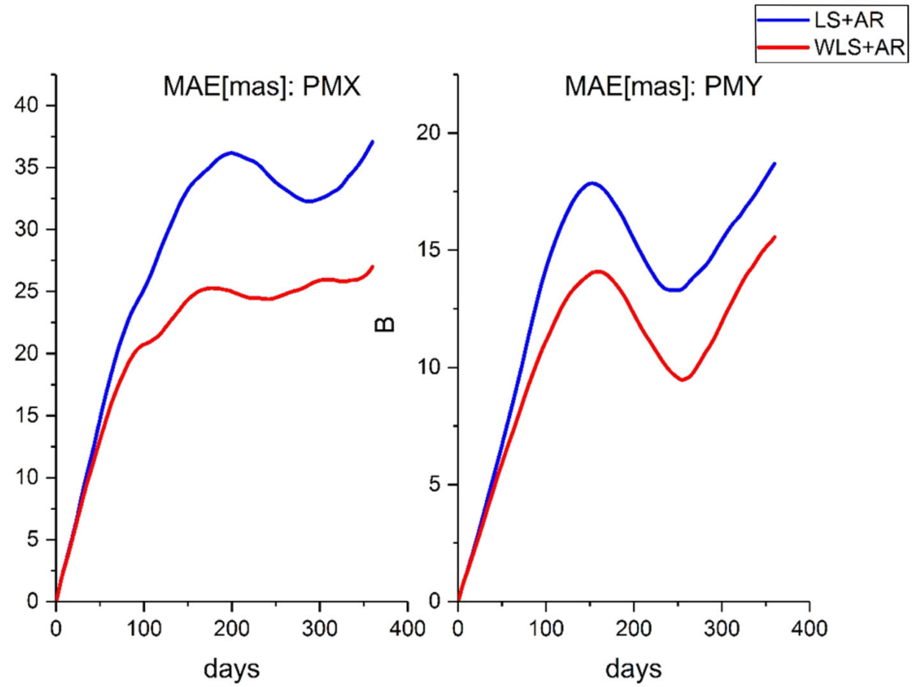

Fig. 3 MAE values calculated in 600 IERS forecasts

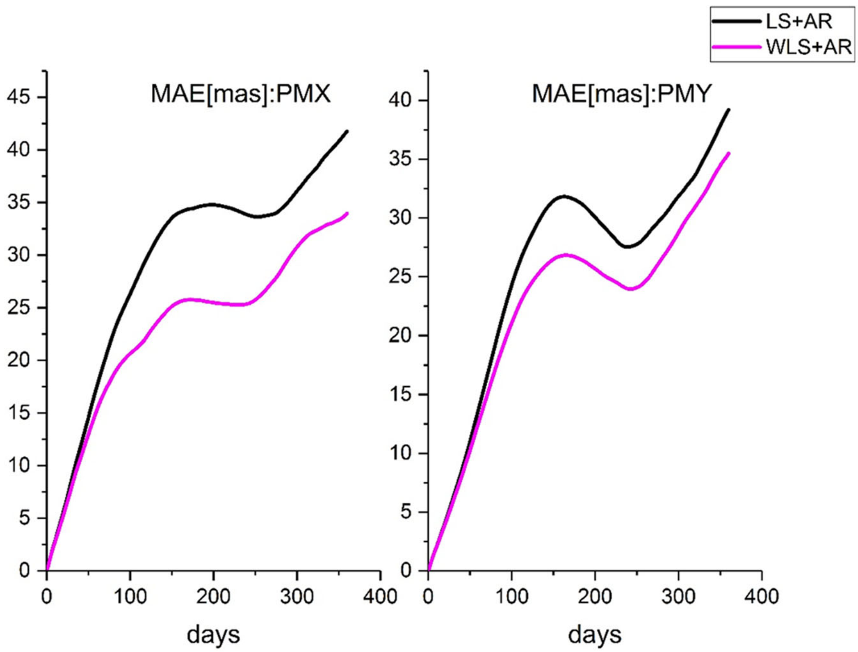

Fig. 4 MAE values calculated for 1000 IERS forecasts

The above results indicate that the prediction accuracy of the WLS + AR model shows improvement at different levels in both the PMX and PMY directions. The improvements in the forecasting accuracy for short-term predictions (1–30 days in the future) are not obvious; the maximum improvement in the polar motion parameters in the PMX direction is less than 8.19%, whereas that in the PMY direction is within 8.587%. However, when the prediction span exceeds 30 days, the accuracy of the prediction displays clear improvements. For the group of 600 experiments, the prediction accuracy in the PMX direction calculated using the IGS and IERS data increases by an average of 16.09 and 21.43%, whereas that in the PMY direction using the IGS and IERS data increases by 23.64 and 19.19%, respectively. In the group of 1000 experiments, the prediction accuracy in the PMX direction calculated using the IGS and IERS data increases by an average of 24.14 and 19.04%, whereas that in the PMY direction using the IGS and IERS data increases by 22.94 and 11.07%, respectively. Compared with the LS + AR model, the WLS + AR model generally better in predicting polar motion parameters and yields more accurate medium-term forecasts.

4 Conclusion

This paper proposes a method of producing medium-term predictions of polar motion parameters using a WLS + AR model. In this method, the calculation precision of the PM parameters is employed to produce weighting factors to build the weight matrix of the vector of observations in the least-squares extrapolation model. More accurate parameters and extrapolated values of the LS model while and more stable residuals to form and calculate the AR model can be obtained using this method. The EOP products released by the analysis center of IGS and IERS are used as the basic data to predict the polar motion parameters in groups of 600 and 1000 experiments with a 1-day prediction interval and a span of 360 days. The results of the experiments show that, compared with the LS + AR model, the WLS + AR model proposed in this paper effectively improves the accuracy with which polar motion parameters can be predicted.

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