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Existence and stability results for a partial impulsive stochastic integro-differential equation with infinite delay

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Abstract This article presents the result on existence and stability of mild solutions of stochastic partial differential equations with infinite delay in the phase space $\mathcal{B}$ with non-lipschitz coefficients. We use the theory of resolvent operator devolopped in Grimmer (Trans Am Math Soc 273(1):333–349, 1982) to show the existence of mild solutions. An example is provided to illustrate the results of this work.

Keywords Resolvent operators · C_0-semigroup · Neutral stochastic partial integrodifferential equations · Wiener process · Picard iteration · Mild solutions · Stability in mean square

1 Introduction

In this paper, we study the existence of mild solutions for a class of abstract partial impulsive integrodifferential equations with infinite delays:

$$\begin{cases} dx(t) &= \left[Ax(t) + \int_0^t B(t-s)x(s)ds + F(t, x_t) \right] dt + H(t, x_t)dw(t), \quad t \in J := [0, T], \\ \Delta x(t_k) &= x(t_k^+) - x(t_k^-) = I_k(x(t_k)), \quad k = 1, 2, \ldots, m, \\ x_0 &= \varphi \in \mathcal{B}. \end{cases} \tag{1.1}$$

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Here, the state $x(\cdot)$ takes values in a separable real Hilbert spaces $\mathbb{H}$ with inner product $(\cdot,\cdot)$ and norm $\|\cdot\|$, A is the infinitesimal generator of a strongly continous semigroup of bounded linear operators $S(t), t \geq 0$ on $\mathbb{H}$, with $D(A) \subset \mathbb{H}$, and $B(t), t \in J$ is a closed linear operator on $\mathbb{H}$. The history $x_t :]-\infty, 0] \rightarrow \mathbb{H}$, $x_t(\theta) = x(t+\theta)$, for $t \geq 0$, belongs to some abstract phase space $\mathcal{B}$ which will be described axiomatically in Sect. 2. Let $\mathbb{K}$ be another separable Hilbert spaces with inner product $\langle\cdot,\cdot\rangle_{\mathbb{K}}$ and norm $\|\cdot\|_{\mathbb{K}}$. Suppose $\{w(s) : 0 \leq s \leq t\}$ is a given $\mathbb{K}$-valued Wiener process with covariance operator $Q \geq 0$ defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ equipped with a normal filtration $\{\mathcal{F}_t\}_{t\geq 0}$ which is generated by the Wiener process w. We are also employing the same notation $\|\cdot\|$ for the norm $\mathcal{L}(\mathbb{H};\mathbb{K})$, where $\mathcal{L}(\mathbb{H};\mathbb{K})$ denotes the space of all bounded linear operator from $\mathbb{K}$ into $\mathbb{H}$. Assume that $F : \mathbb{R}_+ \times \mathcal{B} \rightarrow \mathbb{H}$ and $H : \mathbb{R}_+ \times \mathcal{B} \rightarrow \mathcal{L}_2^0$ are two appropriate mappings specified later. Here $\mathcal{L}_2^0 = \mathcal{L}_2(\mathbb{K}_0;\mathbb{H})$ denotes the space of all Q-Hilbert-Schmidt operators from $\mathbb{K}_0$ to $\mathbb{H}$ wich will be defined in Sect. 2. The initial data $\varphi = \{\varphi(t) : -\infty < t \leq 0\}$ is an $\mathcal{F}_0$-adapted, $\mathcal{B}$-valued random variable independent of the Wiener process w with finite second moment, $I_k : \mathcal{B} \rightarrow \mathbb{H}$, $k = 1, 2, \ldots, m$ are appropriate functions. The fixed moments of time t_k satisfies $0 < t_1 < \cdots < t_m < T$, $x(t_k^+)$ and $x(t_k^-)$ represent the right and left limits of $x(t)$ at $t = t_k$, respectively, $\Delta x(t_k)$ represents the jump in the state x at time t_k with I_k determining the size of the jump.

In recent years, stochastic differential equations have received more attention. They have been applied to model various phenomena in mechanical, electrical, economics, pysics and several fields in engineering. There have been increasing interest in investigating Impulsive stochastic differential equations. For instance in [10] nonautonomous and random dynamical systems perturbed by impulses is investigated. Yang et al. [16], studied the stability analysis of ISDEs with delays; Yang et al. [26], studied the exponential p-stability of ISDEs with delays. In [12,23], Sakthivel and Luo the existence and asymptotic stability in p-th moment of mild solutions to ISDEs with and without infinite delays through fixed point theory.

Motivated by the works of [12,19,20,23], we will generalize the existence and uniqueness of the solution to impulsive stochastic partial functional integrodifferential equations under non-Lipschitz condition and Lipschitz condition. Moreover, we study the stability through the continuous dependence on the initial values by means of Corollary of Bihari's inequality. Our main results concerning (1.1), relay essentially on techniques employing a strongly continous family of operators $R(t), t \geq 0$ defined on the Hilbert space $\mathbb{H}$ and called their resolvent (the precise definition will be given below). The paper is organized as follows: in Sect. 2, we recall some preliminaries which are used throughout this paper. In Sect. 3, we state the existence and uniqueness of a mild solution. In Sect. 4, we study the stability through the continuous dependence on the initial values. Finally in Sect. 5, an example is given to illustrate our results.

2 Preliminaries

2.1 Wiener process

Let $(\Omega, \mathcal{F}, P, \mathbf{F})$ with $(\mathbf{F} = \{\mathcal{F}_t\}_{t\geq 0})$ be a complete filtered probability space satisfying that $\mathcal{F}_0$ contains all P-null sets of $\mathcal{F}$. An $\mathbb{H}$-valued random variable is an $\mathcal{F}$-mesurable function $x(t) : \Omega \rightarrow \mathbb{H}$ and the collection of random variables $S = \{x(t,\omega) : \Omega \rightarrow \mathbb{H} | \ t \in J\}$ is called a stochastic process. Generally, we juste write $x(t)$ instead of $x(t,\omega)$ and $x(t) : J \rightarrow \mathbb{H}$ in the space of S. Let $\{e_i\}_{i=1}^{\infty}$ be a complete orthormal basis of $\mathbb{K}$. Suppose that

$\{\omega(t) : t \geq 0\}$ is a cylindrical $\mathbb{K}$-valued Wiener process with a finite trace nuclear covariance operator $Q \geq 0$, denote $Tr(Q) = \sum_{n=1}^{\infty} \lambda_i = \lambda < \infty$, which satisfaies that $Qe_i = \lambda_i e_i$. So, actually, $w(t) = \sum_{i=1}^{\infty} \sqrt{\lambda_i}\omega_i(t)e_i$, where $\{\omega_i\}_{i=1}^{\infty}$ are mutually independent one-dimensional standard Wiener processes. We assume that $\mathcal{F}_t = \sigma\{w(s); 0 \leq s \leq t\}$ is the σ-algebra generated by ω and $\mathcal{F}_T = \mathcal{F}$. Let $L(\mathbb{K}; \mathbb{H})$ denote the space of all bounded linear operators from $\mathbb{K}$ into $\mathbb{H}$. For $h_1, h_2 \in L(\mathbb{K}; \mathbb{H})$, we define $(h_1, h_2) = Tr(h_1 Q h_2^*)$, where h_2^* is the adjoint of the operator h_2 and Q is the nuclear operator associated with the Wiener process, where $Q \in L_n^+(\mathbb{K})$, the space of positve nuclear operator in $\mathbb{K}$. For $\psi \in L(\mathbb{K}, \mathbb{H})$ we define

$$\|\psi\|_Q^2 = Tr(\psi Q \psi^*) = \sum_{n=1}^{\infty} \left\| \sqrt{\lambda_n}\ \psi e_n \right\|^2$$

If $\|\psi\|_Q < \infty$, then ψ is called a Q-Hilbert Schmidt operator. Let $L_Q(\mathbb{K}, \mathbb{H})$ denote the space of all Q-Hilbert Schmidt operator ψ. The completion $L_Q(\mathbb{K}, \mathbb{H})$ of $L(\mathbb{K}, \mathbb{H})$ with respect to the topology induced by the norm $\|\cdot\|_Q$ where $\|\psi\|_Q^2 = (\psi, \psi)$ is a Hilbert space with the above norm topology. For more details, we refer the reader to Da Prato and Zabczyk [26]. The collection of all strongly measurable, square integrable, $\mathbb{H}$-valued random variables, denoted by $L_2(\Omega, \mathcal{F}, P; \mathbb{H}) \equiv L_2(\Omega, \mathbb{H})$, is a Banach space equiped with norm

$$\|x(\cdot)\|_{L_2} = \left(\mathbb{E}\,\|x(\cdot, \omega)\|^2\right)^{\frac{1}{2}},$$

where the expectation, $\mathbb{E}$ is defined by $\mathbb{E}x = \int_\Omega x(w)dP$. Let $C(J, L_2(\Omega, \mathbb{H}))$ be the Banach space of all continuous maps from J into $L_2(\Omega, \mathbb{H})$ satisfying the condition

$$\sup_{t \in J} \mathbb{E}\,\|x(t)\|^2 < \infty.$$

An important subspace is given by $L_2^0(\Omega, \mathbb{H}) = \{f \in L_2(\Omega, \mathbb{H}) : f \text{ is } \mathcal{F}_0 - \text{measurable}\}$.

We say that a function $u : [\nu, \tau] \to \mathbb{H}$ is a normalized piecewise continuous function on $[\nu, \tau]$ if u is piecewise continuous and left continuous on $[\nu, \tau]$. We denote by $\mathcal{PC}([\nu, \tau]; \mathbb{H})$ the space formed by the normalized piecewise continuous, $\mathcal{F}_t$-adapted measurable processes from $[\nu, \tau]$ into $\mathbb{H}$. In particular, we introduce the space $\mathcal{PC}$ formed by all $\mathcal{F}_t$-adapted measurable, $\mathbb{H}$-valued stochastic processes $\{u(t) : t \in [0, T]\}$ such that u is continuous at $t \neq t_k$, $u(t_k^-) = u(t_k)$ and $u(t_k^+)$ exists, for $k = 1, \ldots, m$. In this paper, we always assume that $\mathcal{PC}$ is endowed with the norm $\|u\|_{\mathcal{PC}} = \left(\sup_{s \in J} \mathbb{E}\,\|u(s)\|^2\right)^{\frac{1}{2}}$. It is clear that $(\mathcal{PC}, \|\cdot\|_{\mathcal{PC}})$ is a Banach space. Further, let $\mathcal{B}_\mathcal{T}$ be a Banach space $\mathcal{B}_\mathcal{T}((-\infty, T], L_2)$, the family of all $\mathcal{F}_t$-adapted process $\varphi(t, \omega)$ with almost surely continuous in t for fixed $\omega \in \Omega$ with norm defined for any $\varphi \in \mathcal{B}_\mathcal{T}$

$$\|\varphi\|_{\mathcal{B}_\mathcal{T}} = \left(\sup_{0 \leq t \leq T} \mathbb{E}\,\|\varphi\|_t^2\right)^{\frac{1}{2}}.$$

where $\|\varphi\|_t = \sup_{-\infty < \theta \leq 0} \|\varphi\|_{\mathbb{H}}$.

2.2 Partial integrodifferential equations in Banach spaces

In the present section, we recall some definitions, notations and properties needed in what follows. Let Z_1 and Z_2 be Banach spaces. We denote by $\mathcal{L}(Z_1, Z_2)$ the Banch space of bounded linear operators from Z_1 into Z_2 endowed with operator norm and we abbreviate this notation to $\mathcal{L}(Z_1)$ when $Z_1 = Z_2$.

In what follows, $\mathbb{H}$ is a Banach space, A and $B(t)$ are closed linear operators on $\mathbb{H}$. Y represents the Banach space $D(A)$ equipped with the graph norm defined by

$$|y|_Y := |Ay| + |y| \quad \text{for} \;\; y \in Y.$$

The notations $C([0,+\infty); Y)$, $\mathcal{L}(Y,\mathbb{H})$ stand for the space of all continuous functions from $[0,+\infty)$ into Y, the set of all bounded linear operators from Y into $\mathbb{H}$, respectively. We consider the following Cauchy problem

$$\begin{cases} v^{'}(t) = Av(t) + \displaystyle\int_0^t B(t-s)v(s)ds \;\; \text{for} \;\; t \geq 0 \\ v(0) = v_0 \in \mathbb{H}. \end{cases} \tag{2.1}$$

Definition 2.1 [3] A resolvent operator for Eq. (2.1) is a bounded linear operator valued function $R(t) \in \mathcal{L}(\mathbb{H})$ for $t \geq 0$, having the following properties:

(i) $R(0) = I$ and $|R(t)| \leq Ne^{\beta t}$ for some constants N and β.
(ii) For each $x \in \mathbb{H}$, $R(t)x$ is strongly continuous for $t \geq 0$.
(iii) For $x \in Y$, $R(\cdot)x \in C^1([0,+\infty); \mathbb{H}) \cap C([0,+\infty); Y)$ and

$$\begin{aligned} R'(t)x &= AR(t)x + \int_0^t B(t-s)R(s)xds \\ &= R(t)Ax + \int_0^t R(t-s)B(s)xds \qquad \text{for} \;\; t \geq 0 \end{aligned}$$

For additional details on resolvent operators, we refer the reader to [3,13]. The resolvent operators plays an important role to study the existence of solutions and to give a variation of constants formula for non linear systems. We need to know when the linear system (2.1) has a resolvent operator. Theorem 2.2 gives a satisfactory answer to this problem.

In what follows we suppose the following assumptions:

(H1) A is the generator of a strongly continuous semigroup on $\mathbb{H}$.

(H2) For $t \geq 0$, $B(t)$ is closed linear operator from $D(A)$ to $\mathbb{H}$, and $B(t) \in B(Y,\mathbb{H})$. For any $y \in Y$, the map $t \to B(t)y$ is bounded, differentiable and the derivative $t \to B'(t)y$ is bounded uniformly continuous on $\mathbb{R}^+$

Theorem 2.2 [3] *Assume that the assumptions* **(H1)** *and* **(H2)** *hold. Then there existe a unique resolvent operator of the Cauchy problem Eq.* (2.1).

In the following, we give some results on the existence of solutions for the following integrodifferential equation

$$\begin{cases} v^{'}(t) = Av(t) + \displaystyle\int_0^t B(t-s)v(s)ds + q(t) \;\; \text{for} \;\; t \geq 0 \\ v(0) = v_0 \in \mathbb{H}, \end{cases} \tag{2.2}$$

where $q : [0,+\infty[\to \mathbb{H}$ is a continuous function.

Definition 2.3 [3] A continuous function $v : [0,+\infty) \to \mathbb{H}$ is said to be a strict solution of Eq. (2.2) if

(i) $v \in C^1([0, +\infty); \mathbb{H}) \cap C([0, +\infty); Y)$,
(ii) v satisfies Eq. (2.2) for $t \geq 0$.

Remark 2.4 From this definition, we deduce that $v(t) \in D(A)$, the function $B(t-s)v(s)$ is integrable, for all $t > 0$ and $s \in [0, +\infty)$.

Theorem 2.5 [3] *Assume that* **(H1)–(H2)** *hold. If v is a strict solution of Eq.* (2.2)*, then the following variation of constants formula holds*

$$v(t) = R(t)v_0 + \int_0^t R(t-s)q(s)ds \quad \textit{for} \quad t \geq 0. \tag{2.3}$$

Accordingly, we make the following definition.

Definition 2.6 [3] For $v_0 \in \mathbb{H}$. A function $v : [0, +\infty) \to \mathbb{H}$ is called a mild solution of (2.2) if v satisfies the variation of constants formula (2.3).

The next theorem provides sufficient conditions for the regularity of solutions of Eq. (2.2).

Theorem 2.7 [3] *Let* $q \in C^1([0, +\infty); \mathbb{H})$ *and v be defined by* (2.3)*. If* $v_0 \in D(A)$*, then v is a strict solution of Eq.* (2.2)*.*

In the whole of this work, we suppose that the phase space is axiomatically defined, we use the approach proposed by Hale and Kato in [8]. To establish the axioms of the phase space $\mathcal{B}$, we follows the terminology used in Hino et al. [7]. The axioms of the phase space $\mathcal{B}$ are established for $\mathcal{F}_0$-measurable functions from $]-\infty, 0]$ into $\mathbb{H}$, endowed with a seminorm wich satisfies the following axiomns:

Axiom 2.8 (A1) If $x : (-\infty, T] \to \mathbb{H}$, $T > 0$ is such that $x_0 \in \mathcal{B}$ and $x_{|[0,T]} \in \mathcal{PC}([\nu, \tau]; \mathbb{H})$ then, for every $t \in [0, T]$, the following conditions hold:

(1) $x_t \in \mathcal{B}$,
(2) $\|x(t)\| \leq L \|x_t\|_{\mathcal{B}}$,
(3) $\|x_t\|_{\mathcal{B}} \leq u(t) \sup_{0 \leq s \leq t} |x(s)| + v(t) \|x_0\|_{\mathcal{B}}$, where $L > 0$ is a constant ; $u(\cdot), v(\cdot) : [0, +\infty) \to [1, +\infty)$, $u(\cdot)$ is continuous, $v(\cdot)$ is locally bounded, and $L, u(\cdot), v(\cdot)$ are independent of $x(\cdot)$

(A2) The space $\mathcal{B}$ is complete.

The next result is a consequence of the phase space axioms.

Lemma 2.9 *Let* $x : (-\infty, 0] \to \mathbb{H}$ *be a* $\mathcal{F}_t$*-adapted measurable process such that the* $\mathcal{F}_0$*-adapted process* $x_0 = \varphi \in L_2^0(\Omega, \mathcal{B})$ *and* $x_{|[0,T]} \in \mathcal{PC}(J, \mathbb{H})$*, then*

$$\mathbb{E}\, \|x_s\|_{\mathcal{B}} \leq v_T \mathbb{E}\, \|\phi\|_{\mathcal{B}} + u_T \sup_{0 \leq s \leq T} \mathbb{E}\, \|x(s)\| \tag{2.4}$$

where $u_T = \sup_{t \in J}\{u(t)\}$ *and* $v_T = \sup_{t \in J}\{v(t)\}$.

Remark 2.10 In retarded functional differential equations without impulses, the axioms of the abstract phase space $\mathcal{B}$ include the continuity of the function $t \to x_t$; see for instance [7]. Due to the impulsive effect, this property is not satisfied in impulsive delay systems and, for this reason, has been eliminated in our abstract description of $\mathcal{B}$.

Lemma 2.11 *(Bihari inequality)* [18] *Let $T > 0$ and $u_0 \geq 0$, $u(t)$, $v(t)$ be continuous functions on $[0, T]$. Let $K : \mathbb{R}^+ \to \mathbb{R}^+$ be a concave, continuous and nondecreasing function such that $K(r) > 0$ for $r > 0$. If*

$$u(t) \leq u_0 + \int_0^t v(t) K\left(u(s)\right) ds \quad for \quad 0 \leq t \leq T,$$

then

$$u(t) \leq G^{-1}\left(G(u_0) + \int_0^t v(s)ds\right)$$

for all $t \in [0, T]$ such that

$$G(u_0) + \int_0^t v(s)ds \in Dom(G^{-1}),$$

where $G(r) = \int_1^r \frac{ds}{K(s)}(r \geq 0)$ and G^{-1} is the inverse function of the G. In particular,moreover if, $u_0 = 0$ and $\int_{0^+} \frac{ds}{K(s)} = +\infty$, then $u(t) = 0$ for all $t \in [0, T]$.

In order to obtain the stability of solutions, we use the following extended Bihari's inequality

Lemma 2.12 [17] *Let the assumption of Lemma 2.11 hold. If*

$$u(t) \leq u_0 + \int_t^T v(t) K\left(u(s)\right) ds \quad for \quad 0 \leq t \leq T,$$

then

$$u(t) \leq G^{-1}\left(G(u_0) + \int_t^T v(s)ds\right) \quad for\ all \quad t \in [0, T]$$

$$such\ that\ G(u_0) + \int_t^T v(s)ds \in Dom(G^{-1}),$$

where $G(r) = \int_1^r \frac{ds}{K(s)}$, for $r \geq 0$ and G^{-1} is the inverse function of the G.

Corollary 2.13 [17] *Let the assumptions of Lemma 2.11 hold and $v(t) \geq 0$ for $t \in [0, T]$. If for all $\epsilon > 0$, there exists $t_1 \geq 0$ such that for $0 \leq u_0 < \epsilon$, $\int_{t_1}^T v(s)ds \leq \int_{u_0}^{\epsilon} \frac{ds}{K(s)}$ holds, then for every $t \in [t_1, T]$, the estimate $u(t) \leq \epsilon$ holds.*

Lemma 2.14 (Burkholder-Davis-Gundy inequality) [11], *p. 182 For any $p \geq 1$ and for arbitrary $\mathcal{L}_2^0$-valued predictable process $\phi(.)$,*

$$\sup_{0 \leq s \leq t} \mathbb{E}\left\|\int_0^s \phi(l)dw(l)\right\|_{\mathbb{H}}^{2r} \leq C_r \left(\int_0^t (\mathbb{E}\,|\phi(s)\|_{\mathcal{L}_2^0}^{2r})ds\right)^r, \tag{2.5}$$

where $C_r = (r(2r-1))^r$.

Before starting and proving the main results, we present the definition of the mild solution to (1.1).

Definition 2.15 A Stochastic process $\{x(t) \in \mathcal{B}_{\mathcal{T}}, t \in (-\infty, T]\}$, $(0 < T < \infty)$ is called a mild solution of the Eq. (1.1).

(i) $x(t) \in \mathbb{H}$ is $\mathcal{F}_t$-adapted,

(ii) $x(t)$ satisfies the integral equation

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$$x(t)=\begin{cases}\varphi(t) \quad \text{for} \quad t\in]-\infty,0],\\ R(t)\varphi(0)+\int_0^t R(t-s)F(s,x_s)ds+\int_0^t R(t-s)H(s,x_s)dw(s)\\ \quad +\sum_{0<t_k<t} R(t-t_k)I_k(x(t_k)) \quad a.s\ t\in[0,T].\end{cases} \tag{2.6}$$

3 Existence and uniqueness of solution for Eq. (1.1)

In this section we discuss the existence and uniqueness of mild solution of the system 1.1. Now we assume the following assumptions: **(H3)**: the functions $F:\mathbb{R}_+\times\mathcal{B}\to\mathbb{H}$ and $H:\mathbb{R}_+\times\mathcal{B}\to\mathcal{L}_2^0$ satisfy for all $t\in J,\Phi_1,\Phi_2\in\mathcal{B}$

$$\|F(t,\Phi_1)-F(t,\Phi_2)\|^2\vee\|H(t,\Phi_1)-H(t,\Phi_2)\|^2\le K(\|\Phi_1-\Phi_2\|_{\mathcal{B}}{}^2),$$

where $K(.)$ is concave non-decreasing function from $\mathbb{R}^+$ to $\mathbb{R}^+$, $K(0)=0,\ K(u)>0$ for $u>0$ and $\int_{0+}\frac{du}{K(u)}=\infty$. **(H4)** The function $I_k\in C(\mathbb{H},\mathbb{H})$ and there exists some constant h_k such that $\|I_k(\Phi_1(t_k))-I_k(\Phi_2(t_k))\|^2\le h_k\|\Phi_1-\Phi_2\|_{\mathcal{B}}^2$, for each $\Phi_1,\Phi_2\in\mathcal{B},\ k=1,2,\dots,m$. **(H5)**: For all $t\in J$,$F(t,0),\ H(t,0),\ I_k(0)\in L^2$, for $k=1,2,\dots,m$ and there exists a positive constant k_0, such that

$$\|f(t,0)\|^2\vee\|H(t,0)\|^2\vee\|I_k(0)\|^2=k_0.$$

Let us now introduce the successive approximations to Eq. (2.6) as follows

$$x^0(t)=\begin{cases}\varphi(t) \quad \text{for} \quad t\in]-\infty,0],\\ R(t)\varphi(0) \quad \text{for} \quad t\in[0,T],\end{cases} \tag{3.1}$$

and for $n=1,2,\dots,$

$$x^n(t)=\begin{cases}\varphi(t) \quad \text{for} \quad t\in]-\infty,0],\\ R(t)\varphi(0)+\int_0^t R(t-s)F(s,x_s^{n-1})ds+\int_0^t R(t-s)H(s,x_s^{n-1})dw(s)\\ \quad +\sum_{0<t_k<t} R(t-t_k)I_k(x^{n-1}(t_k)) \quad a.s\ t\in[0,T],\end{cases} \tag{3.2}$$

with an arbitrary non-negative initial approximation $x^0\in\mathcal{B}_{\mathcal{T}}$.

Theorem 3.1 *Assume that* $(H1)$–$(H5)$ *hold. Let* $M=\sup_{t\in[0,T]}\|R(t)\|$, *then the system* (1.1) *has unique mild solution* $x(t)$ *in* $\mathcal{B}_T$, *provided*

$$M^2m\sum_{k=1}^m h_k<\frac{1}{4}.$$

Proof Let $x^0 \in \mathcal{B}_{\mathcal{T}}$ be a fixed initial approximation to (3.2). Let $M = \sup_{t\in[0,T]} \|R(t)\|$. Then for any $n \geq 1$, we have,

$$\left\|x^n(t)\right\|^2 \leq 4M^2 \|\varphi(0)\|^2 + 8TM^2 \int_0^t \left[\left\|F(s, x_s^{n-1}) - F(s,0)\right\|^2 + \|F(s,0)\|^2\right] ds$$
$$+ 8M^2 \int_0^t \left[\left\|H(s, x_s^{n-1}) - H(s,0)\right\|^2 + \|H(s,0)\|^2\right] ds$$
$$+ 8M^2 m \sum_{k=1}^{m} \left[\left\|I_k(x^{n-1}(t_k)) - I_k(0)\right\|^2 + \|I_k(0)\|^2\right].$$

□

Thus,

$$\mathbb{E}\left\|x^n(t)\right\|_t^2 \leq \underbrace{4M^2\left[\mathbb{E}\|\varphi(0)\|^2 + 2\left(T(T+1) + M^2 m \sum_{k=1}^{n} h_k\right)\right] k_0}_{Q_1}$$
$$+ 8M^2(T+1)\mathbb{E}\int_0^t K\left(\left\|x^{n-1}\right\|_s^2\right) ds$$
$$+ 8M^2 m \sum_{k=1}^{m} h_k \left\{\mathbb{E}\left\|x^{n-1}\right\|_t^2\right\}.$$

Given that $K(.)$ is concave and $K(0) = 0$, we can find positive constants a and b such that

$$K(u) \leq a + bu, \text{ for all } u \geq 0.$$

Then,

$$\mathbb{E}\left\|x^n(t)\right\|_t^2 \leq \underbrace{Q_1 + 8M^2(T+1)Ta}_{Q_2} + 8M^2(T+1)b\int_0^t \mathbb{E}\left\|x^{n-1}\right\|_s^2 ds$$
$$+ 8M^2 m \sum_{k=1}^{m} h_k \left\{\mathbb{E}\left\|x^{n-1}\right\|_t^2\right\} \quad n = 1, 2, \ldots,$$

Since,

$$\mathbb{E}\left\|x^0(t)\right\|_t^2 \leq M^2 \|\varphi(0)\|^2 = Q_3 < \infty. \tag{3.3}$$

Thus,

$$\mathbb{E}\left\|x^n(t)\right\|^2 \leq Q_4 < \infty \text{ for all } n = 0, 1, 2, \ldots \text{ and } t \in [0, T]. \tag{3.4}$$

This proves the boundedness of $\{x^n(t),\ n \in \mathbb{N}\}$.

Let us next show that $\{x^n(t)\}$ is Cauchy $\in \mathcal{B}_{\mathcal{T}}$. For this, for $n,\ m \geq 1$, we have

$$\left\|x^{n+1}(t) - x^{m+1}(t)\right\|^2 \leq \underbrace{3M^2(T+1)}_{Q_5} \int_0^t K\left(\left\|x^n(s) - x^m(s)\right\|^2\right) ds$$
$$+ \underbrace{3M^2 m \sum_{k=1}^{m} h_k}_{Q_6} \left(\left\|x^n(t) - x^m(t)\right\|^2\right).$$

Thus,

$$\sup_{0\le s\le t} \mathbb{E}\left\|x^{n+1}-x^{m+1}\right\|_s^2 \le Q_5\int_0^t K\left(\sup_{0\le r\le s}\mathbb{E}\left\|x^n-x^m\right\|_r^2\right)ds + Q_6 \sup_{0\le s\le t}\mathbb{E}\left\|x^n-x^m\right\|_s^2 \tag{3.5}$$

Integrating both sides of Eq. (3.5) and applying Jensen's inequality gives that

$$\int_0^t \sup_{0\le l\le s}\mathbb{E}\left\|x^{n+1}-x^{m+1}\right\|_l^2 ds \le Q_5\int_0^t\int_0^s K\left(\sup_{0\le r\le l}\mathbb{E}\left\|x^n-x^m\right\|_r^2\right)dlds \tag{3.6}$$

$$+ Q_6\int_0^t \sup_{0\le l\le s}\mathbb{E}\left\|x^n-x^m\right\|_l^2 ds, \tag{3.7}$$

$$\le Q_5\int_0^t s\int_0^s K\left(\sup_{0\le r\le l}\mathbb{E}\left\|x^n-x^m\right\|_r^2\right)\frac{1}{s}dlds \tag{3.8}$$

$$+ Q_6\int_0^t \sup_{0\le l\le s}\mathbb{E}\left\|x^n-x^m\right\|_l^2 ds, \tag{3.9}$$

$$\le Q_5 t\int_0^t\int_0^s K\left(\sup_{0\le r\le l}\mathbb{E}\left\|x^n-x^m\right\|_r^2\frac{1}{s}dl\right)ds \tag{3.10}$$

$$+ Q_6\int_0^t \sup_{0\le l\le s}\mathbb{E}\left\|x^n-x^m\right\|_l^2 ds. \tag{3.11}$$

Then,

$$\Psi_{n+1,m+1}(t) \le Q_5\int_0^t K\left(\Psi_{n,m}(s)\right)ds + Q_6\Psi_{n,m}(t) \tag{3.12}$$

where

$$\Psi_{n,m}(t) = \frac{\int_0^t \sup_{0\le l\le s}\mathbb{E}\left\|x^n-x^m\right\|_l^2 ds}{t},$$

From (3.5), it is easy to see that

$$\sup_{n,m}(t)\Psi_{n,m}(t) < \infty.$$

So letting $\Psi(t) = \limsup_{n,m\to\infty}\Psi_{n,m}(t)$ and taking into account the Fatou's lemma, we yield that

$$\Psi(t) = \widehat{Q}\int_0^t K\left(\Psi(s)\right)ds, \quad where \;\; \widehat{Q} = \frac{Q_5}{1-Q_6}$$

Now, applying the Lemma 2.11, immediately reveals $\Psi(t) = 0$ for any $t \in [0, T]$. This further means $\{x^n(t), n \in \mathbb{N}\}$ is a Cauchy sequence in $\mathcal{B}_T$. So there is an $x \in \mathcal{B}_T$ such that

$$\lim_{n\to\infty}\int_0^T \sup_{0\le s\le t}\mathbb{E}\left\|x^n-x\right\|_s^2 dt = 0.$$

In addition, by (3.4) is easy to follow that $\mathbb{E}\|x\|_t^2 \le Q_4$. Thus we claim that $x(t)$ is a mild solution to (1.1). On the other hand, by (H3) and letting $x \to \infty$, we can also claim that for $t \in [0, T]$

$$\mathbb{E}\left\|\int_0^t R(t-s)\left[F(s,x_s^{n-1})-F(s,x)\right]ds\right\|^2 \to 0,$$
$$\mathbb{E}\left\|\int_0^t R(t-s)\left[H(s,x_s^{n-1})-H(s,x)\right]dw(s)\right\|^2 \to 0,$$
$$\mathbb{E}\left\|\sum_{0<t_k<t} R(t-t_k)[I_k(x^{n-1}(t_k))-I_k(x(t_k))]\right\|^2 \to 0.$$

Hence, taking limits on both sides of (3.2), we obtain that

$$x(t) = R(t)\varphi(0) + \int_0^t R(t-s)F(s,x_s)ds + \int_0^t R(t-s)H(s,x_s)dw(s) + \sum_{0<t_k<t} R(t-t_k)I_k(x(t_k)).$$

This certainly demonstrates by the Definition 2.13 that $x(t)$ is a mild solution to (1.1) on the interval $[0, T]$ Now, we prove the uniqueness of the solution of (1.1). Let $x_1, x_2 \in \mathcal{B}_T$ be two solution of (1.1) on some interval $(-\infty, T]$. Then, for $t \in (-\infty, T]$, we have

$$\mathbb{E}\,\|x_1-x_2\|_t^2 \leq Q_6\mathbb{E}\,\|x_1-x_2\|_t^2 + Q_5\int_0^t K\left(\mathbb{E}\,\|x_1-x_2\|_s^2\right)ds.$$

Thus,

$$\mathbb{E}\,\|x_1-x_2\|_t^2 \leq \frac{Q_5}{1-Q_6}\int_0^t K\left(\mathbb{E}\,\|x_1-x_2\|_s^2\right)ds.$$

Thus, Bihari's inequality yield that

$$\sup_{t\in[0,T]}\mathbb{E}\,\|x_1-x_2\|_t^2 = 0, \quad 0 \leq t \leq T.$$

Thus, $x_1(t) = x_2(t)$ for all $0 \leq t \leq T$. This acheive the proof.

4 Stability

In this section, we study the stability through the continuous dependence on initial values.

Definition 4.1 A mild solution $x(t)$ of the system (1.1) with initial value ϕ is said to be stable in the mean square if for all $\epsilon > 0$, there exist $\delta > 0$ such that

$$E\,\|x-\widehat{x}\|_t^2 \leq \epsilon, \text{ whenever } E\,\left\|\phi-\widehat{\phi}\right\|_t^2 \leq \delta, \text{ for all } t \in [0,T]. \tag{4.1}$$

where $\widehat{x}(t)$ is another mild solution of the system (1.1) with initial $\widehat{\phi}$.

Theorem 4.2 *Let $x(t)$ and $y(t)$ be mild solution of the system (1.1) with initial values φ_1 and φ_2 respectively. If the assumption of the Theorem 3.1 are satisfied, then the mild solution of the system (1.1) is stable in the mean square.*

Proof Let, $x(t)$ and $y(t)$ be two mild solutions of Eq. (1.1) with initial values φ_1 and φ_2 respectively. Then for $0 \leq t \leq T$

$$\begin{aligned} x(t) - y(t) = R(t)\,[\varphi_1(0) - \varphi_2(0)] + \int_0^t R(t-s)\,[F(s, x_s) - F(s, y_s)]\,ds \\ + \int_0^t R(t-s)\,[H(s, x_s) - H(s, y_s)]\,dw(s) \\ + \sum_{0<t_k<t} R(t-t_k)\,[I_k(x(t_k)) - I_k(x(t_k))] \end{aligned}$$

So, estimating as before, we get

$$\begin{aligned} \mathbb{E}\,\|x-y\|_t^2 \leq 4M^2 E\,\|\varphi_1 - \varphi_2\|^2 + 4M^2(T+1)\int_0^t K\left(E\,\|x-y\|_s^2\right) ds \\ + 4M^2 m \sum_{k=1}^m h_k E\,\|x-y\|_t^2 . \end{aligned}$$

Thus,

$$\begin{aligned} \mathbb{E}\,\|x-y\|_t^2 \leq \frac{4M^2}{1 - 4M^2 m \sum_{k=1}^m h_k}\,\|\varphi_1 - \varphi_2\|^2 \\ + \frac{4M^2(T+1)}{1 - 4M^2 m \sum_{k=1}^m h_k}\int_0^t K\left(\mathbb{E}\,\|x-y\|_s^2\right) ds. \end{aligned}$$

□

Let $K_1(u) = \frac{4M^2(T+1)}{1-4M^2 m \sum_{k=1}^m h_k} K\,(u)$, where K is a concave increasing function from $\mathbb{R}^+$ to $\mathbb{R}^+$ such that $K(0) = 0$, $K(u) > 0$ for $u > 0$ and $\int_{0^+} \frac{du}{K(u)} = +\infty$. Then, $K_1(u)$ is concave from $\mathbb{R}^+$ to $\mathbb{R}^+$ such that $K_1(0) = 0, K_1(u) \geq K(u)$ for $0 \leq u \leq 1$ and $\int_{0^+} \frac{du}{K_1(u)} = +\infty$. Now for any $\epsilon > 0$, $\epsilon_1 = \frac{1}{2}\epsilon$, we have $\lim_{s\to 0} \int_s^{\epsilon_1} \frac{du}{K_1(u)} = \infty$. Then, there is a positive constant $\delta < \epsilon_1$, such that $\int_\delta^{\epsilon_1} \frac{du}{K_1(u)} \geq T$.

Let

$$\begin{aligned} u_0 = \frac{4M^2}{1 - 4M^2 m \sum_{k=1}^m h_k}\,\|\varphi_1 - \varphi_2\|^2 , \\ u(t) = E\,\|x-y\|_t^2 , \quad v(t) = 1, \end{aligned}$$

when $u_0 \leq \delta \leq \epsilon_1$. Then from corollary 2.10, we deduce that

$$\int_{u_0}^{\epsilon_1} \frac{du}{K_1(u)} \geq \int_\delta^{\epsilon_1} \frac{du}{K_1(u)} \geq T = \int_0^T v(t)ds.$$

It follows, for any $t \in [0, T]$, the estimate $u(t) \leq \epsilon_1$ hold. This completes the proof.

Remark 4.3 If $m = 0$ in (1.1), then the system behave as stochastic partial functional integrodifferential equation with infinte delay of the form

$$\begin{cases} dx(t) = \left[Ax(t) + \int_0^t B(t-s)[x(s)ds + F(t, x_t)\right] dt + H(t, x_t)dw(t), \quad 0 \leq t \leq T, \\ x(t) \quad = \varphi \in C_{B_0}^b\,(] - \infty, 0], \mathbb{H})\,. \end{cases} \tag{4.2}$$

By applying Theorem 3.1 under the hypotheses (H1–H5), the system (4.2) guarantees the existence and uniquiness of the mild solution.

Remark 4.4 If the system (4.2) satisfies the Definition, 4.1, then by Theorem 4.2, the mild solution of the system (4.2) is stable in the mean square.

5 Application

We conclude this work with an example of the form

$$\begin{cases} \frac{\partial}{\partial t}v(t,\xi) = \frac{\partial^2}{\partial \xi^2}v(t,\xi) + \int_0^t b(t-s)\frac{\partial^2}{\partial \xi^2}v(s,\xi)ds \\ \quad +f(t, v(t-\tau,\xi))dt + \sigma(t, v(t-\tau,\xi))dw(t) \ \text{ for } t \\ \quad \geq 0 \text{ and } \xi \in [0,\pi],\ \tau > 0, t \in J \ := [0,T], \\ \Delta v(t_k) = v(t_k^+) - v(t_k^-) = I_k(v(t_k)) \ \text{ for } \ t = t_k \ \text{ and } \ k = 1,2,...,m, \\ v(t,0) = v(t,\pi) = 0 \ \text{ for } \ t \in J, \\ v(\theta,\xi) = \varphi(t,\xi) \text{ for } t \in]-\infty,0] \text{ and } \xi \in [0,\pi], \end{cases} \tag{5.1}$$

where $w(t)$ denotes a $\mathbb{R}$-valued Brownian motion, $f, h : \mathbb{R}^+ \times \mathbb{R} \to \mathbb{R}$ are continuous functions, $b : \mathbb{R}^+ \to \mathbb{R}$ is continuous and $v_0 : [-r,0] \times [0,\pi] \to \mathbb{R}$ is a given continuous function such that $v_0(.) \in L^2([0,\pi])$ is $\mathcal{F}_0$-measurable and satisfies $E\,\|v_0\|^2 < \infty$.

Let $\mathbb{H} = L^2(0,\pi)$ with the norm $\|.\|$ and $e_n := \sqrt{\frac{2}{\pi}}\sin(nx)$, $(n = 1,2,3,\ldots)$ denote the completed orthonormal basis in $\mathbb{H}$.

Let $w(t) := \sum_{n=1}^{\infty}\sqrt{\lambda_n}\beta_n(t)e_n(\lambda_n > 0)$, where $\beta_n(t)$ are one dimensional standard Brownian motion mutually independent on a usual complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$.

Define $A : \mathbb{H} \to \mathbb{H}$ by $A = \frac{\partial^2}{\partial z^2}$, with domain $D(A) = H^2(0,\pi) \cap H_0^1(0,\pi)$.

Then $Ah = -\sum_{n=1}^{\infty} n^2 < h, e_n > e_n$, $h \in D(A)$, where e_n, $n = 1,2,3,\ldots,$ is also the orthonormal set of eigenvectors of A. It is well-known that A is the infinitesimal generator of a strongly continuous semigroup on $\mathbb{H}$, thus **(H1)** is true.

Let $F : D(A) \subset \mathbb{H} \to \mathbb{H}$ be the operator defined by $F(t)(z) = b(t)Az$ for $t \geq 0$ and $z \in D(A)$.

Let $\gamma > 0$, define the phase space $\mathcal{B} = \left\{\varphi \in C((-\infty,0],\mathbb{H}) : \lim_{\theta\to-\infty} e^{\theta\gamma}\varphi(\theta) \text{ exists in } \mathbb{H}\right\}$ and let $\|\varphi\|_{\mathcal{B}} = \sup_{\theta\in(-\infty,0]}\left\{e^{\gamma\theta}\|\varphi\|_{L_2}\right\}$. Then $(\mathcal{B}, \|\cdot\|_{\mathcal{B}})$ is a Banach space and satisfies (A1)–(A2) with $L = 1, u(t) = e^{-\gamma t}$, $v(t) = \max\left\{1, e^{-\gamma t}\right\}$. Therefore, for $(t,\varphi) \in J \times \mathcal{B}$, where $\varphi(\theta)(\xi) = \varphi(\theta,\xi)$, $(\theta,\xi) \in (-\infty,0] \times [0,\pi]$, let $x(t)(\xi) = v(t,\xi)$ and define the functions $f : J \times \mathcal{B} \to \mathbb{H}$ and $h : J \times \mathcal{B} \to \mathcal{L}_2^0(\mathbb{H},\mathbb{H})$ for the infinite delay as follows: $f(t,\psi)(\xi) = \int_{-\infty}^0 k_2(t,\xi,\theta)G_1(\psi(\theta))d\theta$, $h(t,\psi)(\xi) = \int_{-\infty}^0 k_3(t,\xi,\theta)G_2(\psi(\theta))d\theta$, where

- (I) the functions k_2, k_3 are continuous in $J \times [0,\pi] \times (-\infty,0]$ and satisfy $\int_{-\infty}^0 k_2^2(t,\xi,\theta)d\theta = p_2(t,\xi) < \infty$, $\left(\int_0^\pi p_2^2(t,\xi)d\xi\right) < 1$, $\int_{-\infty}^0 k_3^2(t,\xi,\theta)d\theta = p_3(t,\xi) < \infty$, $\left(\int_0^\pi p_3^2(t,\xi)d\xi\right) < 1$,
- (II) the functions G_i, $i = 1,2$ is continuous in $J \times [0,\pi] \times (-\infty,0)$ and satisfies $0 \leq G_1(\psi_1(\theta,\xi)) - G_1(\psi_2(\theta,\xi)) \leq K_a^{\frac{1}{2}}(\|\psi_1(\theta,\cdot) - \psi_2(\theta,\cdot)\|_{L^2}^2)$, $0 \leq G_2(\psi_1(\theta,\xi)) -$

$G_2(\psi_2(\theta,\xi)) \leq K_b^{\frac{1}{2}}(\|\psi_1(\theta,\cdot)-\psi_2(\theta,\cdot)\|_{L^2}^2)$, for $(\theta,\xi)\in(-\infty,0]\times[0,\pi]$, where $K_a(\cdot), K_b(\cdot), K_a^{\frac{1}{2}}(\cdot), K_b^{\frac{1}{2}}(\cdot)\ :\ [0,\infty[\ \to(0,\infty)$ are nondecreasing and concave.

Under the above assumptions, we can rewritten Eq. (5.1) as the abstract form of Eq. (1.1).

$$\begin{cases} dx(t) = Ax(t) + \int_0^t B(t-s)x(s)dsdt \\ \qquad\qquad + f(t,x_t)dt + h(t,x_t)dw(t), \quad t\in J\ :=\ [0,T], \\ \Delta x(t_k) = x(t_k^+) - x(t_k^-) = I_k(x(t_k)), \quad k=1,2,\ldots,m, \\ \quad x_0 = \varphi \in \mathcal{B}. \end{cases} \tag{5.2}$$

Moreover, if b is bounded and C^1 function such that $b^{'}$ is bounded and uniformly continuous, then **(H2)** is satisfied and hence, by Theorem 2.2, Eq. (1.1) has a resolvent operator $(R(t))_{t\geq 0}$ on $\mathbb{H}$. By assumption (I) and (II) we have

$$\begin{aligned} \|f(t,\psi_1)-f(t,\psi_2)\|_{\mathbb{H}}^2 &= \int_0^\pi \left(\int_{-\infty}^0 k_2(t,\xi,\theta)\,(G_1(\psi_1(\theta))-G_1(\psi_2(\theta)))\,d\theta\right)^2 d\xi \\ &\leq \int_0^\pi \left(\int_{-\infty}^0 k_2(t,\xi,\theta)K_a^{\frac{1}{2}}(\|\psi_1(\theta,.)-\psi_2(\theta,\cdot)\|_{L^2}^2)d\theta\right)^2 d\xi \\ &\leq \int_0^\pi\left(\int_{-\infty}^0 k_2(t,\xi,\theta)K_a^{\frac{1}{2}}(e^{2\alpha\theta}\|\psi_1(\theta,\cdot)-\psi_2(\theta,\cdot)\|_{L^2}^2)d\theta\right)^2 d\xi \\ &\leq \left(\int_0^\pi p_2^2(t,\xi)d\xi\right) K_a(\|\psi_1-\psi_2\|_{\mathcal{B}}^2) \\ &\leq K_a(\|\psi_1-\psi_2\|_{\mathcal{B}}^2). \end{aligned}$$

In the same way we obtain the following estimation

$$\|h(t,\phi_1)-h(t,\psi_2)\|_{\mathbb{H}}^2 \leq K_b(\|\psi_1-\psi_2\|_{\mathcal{B}}^2).$$

The next results as consequence of Theorems 3.1 and 4.2, respectively.

Proposition 5.1 *Assume that the hypothesis **(H1)–(H5)** hold. Then there exists a mild solution x of the system* (5.1) *provided*

$$\tilde{Q} = \max\{Q_1, Q_5\} < 1.$$

is satisfied.

Proposition 5.2 *Assume that the conditions of Proposition* 5.1 *hold. Then the mild solution x of the system* (5.1) *is stable in the quadratic mean.*

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