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The FMEA model based on LOPCOW-ARAS methods with interval-valued Fermatean fuzzy information for risk assessment of R&D projects in industrial robot offline programming systems

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Abstract

Failure mode and effect analysis (FMEA) is a reliability analysis and risk management technique used to analyze the potential causes of system failure modes and their impact on system performance and has been used in many fields. However, the traditional FMEA model often ignores the weight of risk factors and their internal relationship in an uncertain setting. To compensate for the above shortcomings, this article proposes a new FMEA model for risk prioritization by combining the Hamy mean operator, logarithmic percentage change-driven objective weighting (LOPCOW) model, and additive ratio assessment (ARAS) method under interval-valued Fermatean fuzzy (IVFF) environment. To begin with, novel IVFF operations based on Aczel–Alsina norms are defined. Then, some novel Hamy mean operators are proposed based on the defined operations including the IVFF Aczel–Alsina Hamy mean (IVFFAAHAM) operator, IVFF Aczel–Alsina weighted Hamy mean (IVFFAAWHAM) operator and their dual forms. The corresponding properties of these operators are also investigated. Further, the IVFF-LOPCOW method is proposed to find the weight of risk criteria and an improved ARAS method is propounded based on the presented operators for attaining the ranking of failure modes. Afterward, a novel FMEA model based on the integrated IVFF-LOPCOW-ARAS methodology is constructed, which can effectively determine the weight of experts and risk factors, as well as consider the interrelationship in the course of risk analysis. Lastly, a case about the risks in the R&D project of an industrial robot offline programming system is utilized to test the feasibility of the proposed FMEA model. The stability and advantages of the model are also investigated through sensitivity and comparison studies, respectively.

Keywords FMEA · Interval-valued Fermatean fuzzy set · LOPCOW · ARAS · Information fusion

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1 Introduction

Failure mode and effect analysis (FMEA) is a reliability model and quality control method that can prevent potential failures and risks in systems, project development, and quality service processes. FMEA not only reduces risks in product development but also indirectly improves product quality. It can identify the most dangerous liabilities from potential faults, as well as carry out preventive work and design adjustment plans in advance. Owing to its significant superiority for risk and reliability analysis in the field of engineering development, FMEA has been applied by researchers from different fields in many industrial engineering fields, such as decision analysis (Ma et al. 2023; Sarwar et al. 2023), risk assessment (Chen et al. 2023), manufacturing industry (Dhalmahapatra et al. 2022), and product development (Wang et al. 2018).

FMEA expert group evaluates potential failure modes from three aspects: severity (S), detection difficulty (D), and occurrence (O), and then analyzes and identifies potential failure modes by calculating the risk priority number of the failure mode (Huang et al. 2020b; Liu et al. 2013). The traditional risk priority number (RPN) calculation method can directly and conveniently obtain the RPN of the failure mode by multiplying the aforementioned three factors. However, the traditional RPN calculation method does not consider the importance of the Risk factor, which will result in ambiguous results, and different failure modes will also be difficult to determine the risk level of failure modes due to the same RPN value. Furthermore, because of the limitations and uncertainties of human cognition, the complexity of the assessment environment, and the subjective impact of expert assessment, it is difficult for FMEA experts to give an accurate assessment of the risk factors through experience and knowledge. To effectively solve such problems, some uncertain technologies, such as Fuzzy sets (FSs) (Zadeh 1965), intuitionistic FSs (IFSs) (Atanassov 1986), and interval-valued intuitionistic FSs (IVIFSs) (Atanassov and Gargov 1989), have been proposed. The theory of FSs originated by Zadeh is utilized to help decision experts portray uncertainty and ambiguity effectively in the evaluation procedure. Henceforth, the expanded forms including the IFSs, Pythagorean FSs (PFSs) (Yager 2014; Yager and Abbasov 2013), and Fermatean FSs (FFSs) (Senapati and Yager 2020) are propounded to depict the uncertain assessment information with less restrictive conditions. In view of the merits of the mentioned notions in processing uncertainty, they have been applied to draw uncertain risk assessment information and further construct different kinds of FMEA modes with uncertain circumstances (Huang and Xiao 2021; Huang et al. 2020a; Lai et al. 2022; Liang and Li 2023; Liu et al. 2019; Shahri et al. 2021; Yu et al. 2023). However, the aforementioned uncertain conceptions also possess some constraints that fail to describe the risk information more comprehensively.

Recently, an innovative information representation model called IVFF sets have been propounded by (Rani and Mishra 2022) as a combination of interval-valued and FFSs. IVFF sets can express the uncertain risk assessment information more comprehensively than IVIFSs and interval-valued Pythagorean FSs (IVIFSs) (He et al. 2021). Since it is pioneered to process uncertainty and vagueness in practical application, the studies on IVFFSs have been investigated from multiple perspectives; e.g., information aggregation, information measure, decision methodology, and applications in real life (Akram et al. 2022; Luqman and Shahzadi 2023; Palanikumar and Iampan 2022; Qin et al. 2023; Sergi et al. 2022). In addition, Liu et al. (2022) introduced the notion of an interval-valued hesitant Fermatean fuzzy set and then built the corresponding theoretical system including distance measures and aggregation operators. Further, the extended complex proportional assessment (COPRAS) method is also presented for selecting the optimal desalination technology. Rani et al. (2022a) propounded the improved score function for IVFF sets to make up for the deficiencies of the extant functions and propounded a novel COPRAS methodology using the improved score function and Einstein operators. Hezam et al. (2023) built up a novel assessment framework for choosing autonomous smart wheelchairs by utilizing a Dombi operators-based combined compromise solution method under the IVFF environment. Seikh and Mandal (2023) also proposed a hybrid group decision method based on the Dombi operators and preference ranking organization method for enrichment evaluation II (PROMETHEE II) and stepwise weight assessment ratio analysis (SWARA) methods to select an appropriate capable organization for managing the biomedical waste. However, the IVFF sets are not utilized by experts to cope with FMEA problems yet. Meanwhile, the extant aggregation operators for IVFF sets neglect the interrelationship among the input data. Therefore, this study will present a novel FMEA model based on a decision approach with an IVFF environment to unfold risk analysis.

Information aggregation is a vital branch in the field of decision analysis. Researchers proposed a lot of aggregation functions to fuse multiple uncertain and fuzzy data and further achieve the goal of dimensionality reduction. In the procedure of decision analysis, experts often need to aggregate the assessment information offered by multiple experts and then unfold the exploration stage according to collective group assessment. Especially, the interrelationship among the assessment criteria needs to be taken into account for attaining a more reasonable fusion outcome in some realistic applications. The Hamy mean is a famous, stronger, and flexible aggregation function to ponder the correlation among aggregation data (Hara et al. 1998). Research on it has been acquired to fuse diverse different uncertain information and applied them to settle different problems (Liu et al. 2020; Rong et al. 2020; Wu et al. 2018; Xu et al. 2021). The fundamental to propose a new operator is first to define the operational laws for fuzzy numbers. Based on the literature overview, we can find the current IVFF aggregation operator is propounded based on the Archimedean norms. Recently, the Aczel–Alsina norms have been utilized to construct operational rules and define novel aggregation operators because of their flexibility and generalization advantages (Aczél and Alsina 1982; Rong et al. 2022; Senapati et al. 2022a, b, c, 2023). However, the Aczel–Alsina norms are not used to define operations for IVFF numbers (IVFFNs) and the Hamy mean fails to be investigated over IVFF circumstances.

The implementation process of FMEA is to rank the potential failure modes based on some criteria with the help of experts' risk assessment opinions and preferences. Thus, it is viewed as a typical multiple criteria decision-making (MCDM) problem in effect. To effectively and scientifically address the MCDM problems, some powerful models are applied to deal with diverse actual applications and provide much decision support for departments and managers. As a common and efficient decision method, the additive ratio assessment (ARAS) method originated based on the utility degree theory, which can settle decision and assessment issues effectively (Zavadskas et al. 2010). ARAS is a compensatory approach that can determine the order of a set of alternatives over the criteria by a simple relative procedure. Recently, ARAS has been applied to various applications with uncertain information and attained some excellent outcomes for managers. For instance, Rani et al. (2022b) developed an expanded version of the ARAS method based on Heronian mean operators within FFSs to assess the food waste treatment technology, wherein the criteria weight is computed by the improved Fermatean fuzzy method based on the removal effects of criteria (MEREC) algorithm. Mishra et al. (2021) prioritized and assessed the electric vehicle charging station with the aid of the similarity degree-based ARAS method and a combination weight determination technique under the single neutrosophic circumstance. Mishra et al. (2022) propounded a new low-carbon tourism strategy assessment framework by integrating the

ARAS method and similarity measure-based weight computation algorithm with interval-valued intuitionistic fuzzy information. Further, Mishra et al. (2023) presented an integrated assessment method named full consistency method FUCOM-ARAS based on some novel dual probabilistic linguistic Dombi power aggregation operators to select the optimal medical equipment supplier using the dual probabilistic linguistic evaluation information. FUCOM was employed to acquire the subjective weight information of the assessment criteria. More extensions and research of the ARAS method can refer to the following achievements (Estiri et al. 2021; Fan et al. 2023; Karimi and Nikkhah-Farkhani 2022; Mentes and Akyildiz 2023; Muttakin et al. 2022). Up to now, ARAS has not been utilized in FMEA model to deal with risk problems under IVFF setting.

1.1 Motivations of the research

By means of the aforementioned discussion and literature investigation, the motivations of this study are summarized as follows:

FMEA is a powerful reliability analysis and risk management technique and has a widespread application in various fields. IVFF set is a novel extension of previous works like IFS, PFS, IVIFS, and IVPFS that can articulate uncertainty more efficiently. However, the extant works show that there is no study to combine FMEA and IVFF set for unfold risk analysis.
The existing aggregation operators such as the Einstein operator, Hamacher operator, and Dombi operator can be used to aggregate IVFF information effectively. Nevertheless, these operators fail to think about the flexibility and interrelationship concurrently. Besides, the Aczel–Alsina operations have not been extended for IVFFNs. Hence, it is important to define IVFF Aczel–Alsina operations and propose the Hamy mean operators for making up the mentioned defects.
The logarithmic percentage change-driven objective weighting (LOPCOW) is a novel criteria weight determination method that considers the percentage effect among criteria. It has been successfully applied in several real-life problem instances. But so far, it has not been proposed to estimate the criteria importance degree within IVFF sets.
ARAS model possesses higher robustness and stability for determining an optimal solution in the course of decision analysis. However, the previous research shows that there is no investigation for the combination of ARAS and FMEA to the application of uncertain risk analysis. Besides, the prior extensions of ARAS fail to ponder the correlation among the risk criteria.

1.2 Contributions of the research

Given the aforementioned motivations, this research propounds a hybrid group FMEA risk assessment approach within IVFF information for evaluators to prioritize the risk factors by considering the interrelationship among the risk criteria. Accordingly, the contributions and originalities of our study are summarized as follows:

(1) Some novel operational laws for IVFFNs are defined based on the Aczel–Alsina norms.
(2) A series of Hamy mean operators based on the defined operations are propounded to ponder the correlation among the input decision data, including IVFF Aczel–Alsina Hamy mean (IVFFAAHAM) operator, IVFF Aczel–Alsina weighted Hamy mean (IVFFAAWHAM) operator, IVFF Aczel–Alsina dual Hamy mean (IVFFAADHAM)

operator and their dual extensions. The valuable properties and special cases of these novel operators are explored at length.

(3) The LOPCOW method is proposed based on the score function for the determination of risk criteria weight with IVFF information.

(4) An improved IVFF-ARAS method based on the propounded Hamy mean operator is developed to determine the rank of failure modes.

1.3 Organization of the research

In a nutshell, the remaining sections of this study are organized as below. Section 2 succinctly looks back at several fundamental conceptions of the IVFF set, Aczel–Alsina norms, and Hamy mean operator. Section 3 propounds some novel Hamy mean operators based on the IVFF Aczel–Alsina operations. Section 4 proposes a novel FMEA model based on the hybrid LOPCOW-ARAS methodology under the IVFF environment. The case study that analyzes the risks in an R&D project of an industrial robot offline programming system is conducted to verify the applicability of the proposed FMEA mode in Sect. 5. In addition, the analysis of results involving parameter discussion and comparison study is also implemented. We summarize this study and discuss the limitations of the proposed model and future research directions in Sect. 6.

2 Prerequisites

The notion of IVFFSs emerged from the FFSs, which generalizes the membership and non-membership grades from exact numbers to interval numbers. In the following, we present some fundamental ideas about the IVFF sets.

Definition 2.1 (*Senapati and Yager* 2020). Let Y be a non-empty universe of discourse. An FFS $\widetilde{F}$ in Y can be represented as:

$$\widetilde{F} = \left\{\left\langle y, \phi_{\widetilde{F}}(y), \xi_{\widetilde{F}}(y)\right\rangle | y \in Y\right\}. \tag{1}$$

wherein $\phi_{\widetilde{F}}, \xi_{\widetilde{F}} : Y \to [0, 1]$, signify the grade of membership and the grade of non-membership of the object $y \in Y$ to $\widetilde{F}$, respectively, with the condition $0 \le \left(\phi_{\widetilde{F}}(y)\right)^3 + \left(\xi_{\widetilde{F}}(y)\right)^3 \le 1, \forall y \in Y$. The indeterminacy grade of an element $y \in Y$ is indicated as $\pi_{\widetilde{F}}(y) = \sqrt[3]{1 - \left(\phi_{\widetilde{F}}(y)\right)^3 - \left(\xi_{\widetilde{F}}(y)\right)^3}$. For the sake of simplicity, $\left(\phi_{\widetilde{F}}(y), \xi_{\widetilde{F}}(y)\right)$ is defined as a Fermatean fuzzy number (FFN), represented by $\widetilde{\alpha} = (\phi_{\widetilde{\alpha}}, \xi_{\widetilde{\alpha}})$, where $\phi_{\widetilde{\alpha}}, \xi_{\widetilde{\alpha}} \in [0, 1]$, $\pi_{\widetilde{\alpha}} = \sqrt[3]{1 - (\phi_{\widetilde{\alpha}})^3 - (\xi_{\widetilde{\alpha}})^3}$, and $0 \le (\phi_{\widetilde{\alpha}})^3 + (\xi_{\widetilde{\alpha}})^3 \le 1$.

Definition 2.2 (*Jeevaraj* 2021;*Rani and Mishra* 2022). It is supposed that $Int[0, 1]$ be the set of all closed subintervals of the interval $[0, 1]$. The mathematical definition of an IVFFS H on the universe Y can be defined as:

$$H = \left\{\left\langle y, \left[\phi_H^{lf}(y), \phi_H^{uf}(y)\right], \left[\xi_H^{lf}(y), \xi_H^{uf}(y)\right]\right\rangle : y \in Y\right\}, \tag{2}$$

wherein $0 \le \phi_H^{lf}(y) \le \phi_H^{uf}(y) \le 1$, $0 \le \xi_H^{lf}(y) \le \xi_H^{uf}(y) \le 1$, and $0 \le \left(\phi_H^{uf}(y)\right)^3 + \left(\xi_H^{uf}(y)\right)^3 \le 1$. Here, $\phi_H(y) = \left[\phi_H^{lf}(y), \phi_H^{uf}(y)\right]$ and $\xi_H(y) = \left[\xi_H^{lf}(y), \xi_H^{uf}(y)\right]$ stand for the interval-valued membership and non-membership degrees of $y \in Y$, respectively.

The function $\pi_H(y) = \left[\pi_H^{lf}(y), \pi_H^{uf}(y)\right]$ denotes the interval-valued Fermatean fuzzy hesitant grade of object $y \in Y$ to H, where $\pi_H^{lf}(x_i) = \sqrt[3]{1 - \left(\phi_H^{uf}(y)\right)^3 - \left(\xi_H^{uf}(y)\right)^3}$ and $\pi_H^{uf}(x_i) = \sqrt[3]{1 - \left(\phi_H^{lf}(y)\right)^3 - \left(\xi_H^{lf}(y)\right)^3}$. For convenience, an IVFFN is denoted by $\psi = \left(\left[\phi_\psi^{lf}, \phi_\psi^{uf}\right], \left[\xi_\psi^{lf}, \xi_\psi^{uf}\right]\right)$, which fulfills $\left(\phi_\psi^{uf}\right)^3 + \left(\xi_\psi^{uf}\right)^3 \leq 1$. Some special cases of IVFF sets are defined as:

(i) If $\phi_H^{lf}(y) = \phi_H^{uf}(y)$ and $\xi_H^{lf}(y) = \xi_H^{uf}(y)$, $\forall y \in Y$, then an IVFF set reduces to an FFS.
(ii) If $\phi_H^{uf}(y) + \xi_H^{uf}(y) \leq 1$, then an IVFF set reduces to an IVIF set.
(iii) If $\left(\phi_H^{uf}(y)\right)^2 + \left(\xi_H^{uf}(y)\right)^2 \leq 1$, then an IVFF set yields to an IVPF set.

Definition 2.3 (*Rani and Mishra* 2022). Let $\psi_1 = \left(\left[\phi_1^{lf}, \phi_1^{uf}\right], \left[\xi_1^{lf}, \xi_1^{uf}\right]\right)$ and $\psi_2 = \left(\left[\phi_2^{lf}, \phi_2^{uf}\right], \left[\xi_2^{lf}, \xi_2^{uf}\right]\right)$ be two IVFFNs. Then, the basic relations between the two IVFFNs ψ_1 and ψ_2 are given below:

(i) $\psi_1 = \psi_2$ iff $\phi_1^{lf} = \phi_2^{lf}$, $\phi_1^{uf} = \phi_2^{uf}$, $\xi_1^{lf} = \xi_2^{lf}$, and $\xi_1^{uf} = \xi_2^{uf}$;
(ii) $\psi_1 \prec \psi_2$ iff $\phi_1^{lf} \leq \phi_2^{lf}$, $\phi_1^{uf} \leq \phi_2^{uf}$, $\xi_1^{lf} \geq \xi_2^{lf}$, and $\xi_1^{uf} \geq \xi_2^{uf}$.

Definition 2.4 (*Rani and Mishra* 2022). Let $\psi = \left(\left[\phi^{lf}, \phi^{uf}\right], \left[\xi^{lf}, \xi^{uf}\right]\right)$, $\psi_1 = \left(\left[\phi_1^{lf}, \phi_1^{uf}\right], \left[\xi_1^{lf}, \xi_1^{uf}\right]\right)$, and $\psi_2 = \left(\left[\phi_2^{lf}, \phi_2^{uf}\right], \left[\xi_2^{lf}, \xi_2^{uf}\right]\right)$ be three IVFFNs and $\lambda > 0$. The following operations are defined as:

(i) $\psi_1 \cup \psi_2 = \left(\left[\max\left\{\phi_1^{lf}, \phi_2^{lf}\right\}, \max\left\{\phi_1^{uf}, \phi_2^{uf}\right\}\right], \left[\min\left\{\xi_1^{lf}, \xi_2^{lf}\right\}, \min\left\{\xi_1^{uf}, \xi_2^{uf}\right\}\right]\right)$;

(ii) $\psi_1 \cap \psi_2 = \left(\left[\min\left\{\phi_1^{lf}, \phi_2^{lf}\right\}, \min\left\{\phi_1^{uf}, \phi_2^{uf}\right\}\right], \left[\max\left\{\xi_1^{lf}, \xi_2^{lf}\right\}, \max\left\{\xi_1^{uf}, \xi_2^{uf}\right\}\right]\right)$;

(iii)
$$\psi_1 \oplus \psi_2 = \left(\left[\sqrt[3]{\left(\phi_1^{lf}\right)^3 + \left(\phi_2^{lf}\right)^3 - \left(\phi_1^{lf}\right)^3\left(\phi_2^{lf}\right)^3}, \sqrt[3]{\left(\phi_1^{uf}\right)^3 + \left(\phi_2^{uf}\right)^3 - \left(\phi_1^{uf}\right)^3\left(\phi_2^{uf}\right)^3}\right], \left[\xi_1^{lf}\xi_2^{lf}, \xi_1^{uf}\xi_2^{uf}\right]\right);$$

(iv)
$$\psi_1 \otimes \psi_2 = \left(\left[\phi_1^{lf}\phi_2^{lf}, \phi_1^{uf}\phi_2^{uf}\right], \left[\sqrt[3]{\left(\xi_1^{lf}\right)^3 + \left(\xi_2^{lf}\right)^3 - \left(\xi_1^{lf}\right)^3\left(\xi_2^{lf}\right)^3}, \sqrt[3]{\left(\xi_1^{uf}\right)^3 + \left(\xi_2^{uf}\right)^3 - \left(\xi_1^{uf}\right)^3\left(\xi_2^{uf}\right)^3}\right]\right);$$

(v) $\lambda\psi = \left(\left[\sqrt[3]{1 - \left(1 - (\phi^{lf})^3\right)^\lambda}, \sqrt[3]{1 - \left(1 - (\phi^{uf})^3\right)^\lambda}\right], \left[\left(\xi^{lf}\right)^\lambda, \left(\xi^{uf}\right)^\lambda\right]\right)$;

(vi) $\psi^\lambda = \left(\left[\left(\phi^{lf}\right)^\lambda, \left(\phi^{uf}\right)^\lambda\right], \left[\sqrt[3]{1 - \left(1 - (\xi^{lf})^3\right)^\lambda}, \sqrt[3]{1 - \left(1 - (\xi^{uf})^3\right)^\lambda}\right]\right)$.

Definition 2.5 (*Rani and Mishra* 2022). Let $\psi = \left(\left[\phi^{lf}, \phi^{uf}\right], \left[\xi^{lf}, \xi^{uf}\right]\right)$ be an IVFFN. Then the score function $\wp(\psi)$ and accuracy function $\Im(\psi)$ are defined as:

$$\wp(\psi) = \frac{1}{2}\left(\left(\phi^{lf}\right)^3 + \left(\phi^{uf}\right)^3 - \left(\xi^{lf}\right)^3 - \left(\xi^{uf}\right)^3\right), \quad \Im(\psi) \in [-1, 1], \tag{3}$$

$$\Im(\psi) = \frac{1}{2}\left(\left(\phi^{lf}\right)^3 + \left(\phi^{uf}\right)^3 + \left(\xi^{lf}\right)^3 + \left(\xi^{uf}\right)^3\right), \quad \Im(\psi) \in [0, 1]. \tag{4}$$

Definition 2.6 (*Rani and Mishra* 2022). Let ψ_1 and ψ_2 be two IVFFNs. Then the comparison rules are given as:

If $\wp(\psi_1) > \wp(\psi_2)$, then $\psi_1 \succ \psi_2$;
If $\wp(\psi_1) < \wp(\psi_2)$, then $\psi_1 \prec \psi_2$;
If $\wp(\psi_1) = \wp(\psi_2)$, then
If $\Im(\psi_1) > \Im(\psi_2)$, then $\psi_1 \succ \psi_2$;
If $\Im(\psi_1) < \Im(\psi_2)$, then $\psi_1 \prec \psi_2$;
If $\Im(\psi_1) = \Im(\psi_2)$, then $\psi_1 \sim \psi_2$.

Definition 2.7 (*Rani and Mishra* 2022). Let $\psi_j = \left(\left[\phi_j^{lf}, \phi_j^{uf}\right], \left[\xi_j^{lf}, \xi_j^{uf}\right]\right)(j = 1(1)p)$ be a family of IVFFNs. Then, the interval-valued Fermatean fuzzy weighted averaging (IVFFWA) and interval-valued Fermatean fuzzy weighted geometric (IVFFWG) operators are given as:

$$IVFFWA(\psi_1, \psi_2, \ldots, \psi_p) = \mathop{\oplus}_{j=1}^{p} \omega_j \psi_j$$
$$= \left(\left[\sqrt[3]{1 - \prod_{j=1}^{p}\left(1 - \left(\phi_j^{lf}\right)^3\right)^{\omega_j}}, \sqrt[3]{1 - \prod_{j=1}^{p}\left(1 - \left(\phi_j^{uf}\right)^3\right)^{\omega_j}}\right], \left[\prod_{j=1}^{p}\left(\xi_j^{lf}\right)^{\omega_j}, \prod_{j=1}^{p}\left(\xi_j^{uf}\right)^{\omega_j}\right]\right), \tag{5}$$

$$IVFFWG(\psi_1, \psi_2, \ldots, \psi_p) = \mathop{\otimes}_{j=1}^{p} (\psi_j)^{\omega_j}$$
$$= \left(\left[\prod_{j=1}^{p}\left(\phi_j^{lf}\right)^{\omega_j}, \prod_{j=1}^{p}\left(\phi_j^{uf}\right)^{\omega_j}\right], \left[\sqrt[3]{1 - \prod_{j=1}^{p}\left(1 - \left(\xi_j^{lf}\right)^3\right)^{\omega_j}}, \sqrt[3]{1 - \prod_{j=1}^{p}\left(1 - \left(\xi_j^{uf}\right)^3\right)^{\omega_j}}\right]\right), \tag{6}$$

wherein $\omega = \left(\omega_1, \omega_2, \ldots, \omega_p\right)^T$ be the weights of $\psi_j (j = 1(1)p)$, satisfying $\omega_j \in [0, 1]$ and $\sum_{j=1}^{p} \omega_j = 1$.

Definition 2.8 (*Aczél and Alsina* 1982). Suppose that a and b are two arbitrary, non-negative real numbers $(x, y > 0)$, then the t-norm and s-norm of Aczel–Alsina can be described as follows:

$$T_{AA}^{\Re}(a, b) = \exp\left\{-\left((-\ln a)^{\Re} + (-\ln b)^{\Re}\right)^{\frac{1}{\Re}}\right\}, \quad \Re > 0, \tag{7}$$

$$S_{AA}^{\Re}(a, b) = 1 - \exp\left\{-\left((-\ln(1-a))^{\Re} + (-\ln(1-b))^{\Re}\right)^{\frac{1}{\Re}}\right\}, \quad \Re > 0. \tag{8}$$

Definition 2.9 (*Hara et al.* 1998). Suppose $c_\varepsilon (\varepsilon = 1, 2, \ldots, p)$ is a set of non-negative real numbers, $\kappa = 1, 2, \ldots, p$. If:

$$HM^{(\kappa)}\left(c_1, c_2, \ldots, c_p\right) = \frac{\sum_{1 \le \varepsilon_1 \le \cdots \le \varepsilon_\kappa \le p}\left(\prod_{j=1}^{\kappa} c_{\varepsilon_j}\right)^{\frac{1}{\kappa}}}{C_p^{\kappa}}, \tag{9}$$

then $HM^{(\kappa)}$ is named as the Hamy mean, where $(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_k)$ traverses all the κ-tuple combinations of $(1, 2, \ldots, p)$, and C_p^κ is the binomial coefficient.

Furthermore, a dual form of HM (i.e., dual Hamy mean (DHM), was developed by Wu et al. (2018) in the following.

Definition 2.10 (*Wu et al.* 2018). Suppose $c_\varepsilon(\varepsilon = 1, 2, \ldots, p)$ is a set of non-negative real numbers, and $\kappa = 1, 2, \ldots, p$. If:

$$DHM^{(\kappa)}(c_1, c_2, \ldots, c_n) = \left(\prod_{1 \le \varepsilon_1 \le \cdots \le \varepsilon_\kappa \le p} \left(\frac{\sum_{j=1}^{\kappa} c_{\varepsilon_j}}{\kappa} \right) \right)^{\frac{1}{C_p^\kappa}}, \tag{10}$$

then $DHM^{(\kappa)}$ is named as the dual Hamy mean.

3 Interval-valued Fermatean fuzzy Aczel-Alsina Hamy mean operators

The current section first defines the novel operations for IVFFNs. Then, a series of Hamy mean operators based on the defined operations are propounded. We also discuss some valuable properties of the presented operators.

3.1 Aczel–Alsina operation for IVFFNs

Definition 3.1 Let $\psi_1 = \left(\left[\phi_1^{lf}, \phi_1^{uf}\right], \left[\xi_1^{lf}, \xi_1^{uf}\right]\right)$ and $\psi_2 = \left(\left[\phi_2^{lf}, \phi_2^{uf}\right], \left[\xi_2^{lf}, \xi_2^{uf}\right]\right)$ be two IVFFNs, λ, $\Re \ge 0$ and then the interval-valued Fermatean fuzzy Aczel–Alsina operational laws are defined as :

(i)

$$\psi_1 \oplus \psi_2 = \left(\begin{array}{c} \left[\begin{array}{c} \sqrt[3]{1-\exp\left\{-\left(\left(-\ln\left(1-\left(\phi_1^{lf}\right)^3\right)\right)^{\Re}+\left(-\ln\left(1-\left(\phi_2^{lf}\right)^3\right)\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}}, \\ \sqrt[3]{1-\exp\left\{-\left(\left(-\ln\left(1-\left(\phi_1^{uf}\right)^3\right)\right)^{\Re}+\left(-\ln\left(1-\left(\phi_1^{uf}\right)^3\right)\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}} \end{array} \right], \\ \left[\begin{array}{c} \sqrt[3]{\exp\left\{-\left(\left(-\ln\left(\left(\xi_1^{lf}\right)^3\right)\right)^{\Re}+\left(-\ln\left(\left(\xi_2^{lf}\right)^3\right)\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}}, \\ \sqrt[3]{\exp\left\{-\left(\left(-\ln\left(\left(\xi_1^{uf}\right)^3\right)\right)^{\Re}+\left(-\ln\left(\left(\xi_2^{uf}\right)^3\right)\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}} \end{array} \right] \end{array} \right), \tag{11}$$

(ii)

$$\psi_1 \otimes \psi_2 = \left(\begin{array}{c} \left[\begin{array}{c} \sqrt[3]{\exp\left\{-\left(\left(-\ln\left(\left(\phi_1^{lf}\right)^3\right)\right)^{\Re}+\left(-\ln\left(\left(\phi_2^{lf}\right)^3\right)\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}}, \\ \sqrt[3]{\exp\left\{-\left(\left(-\ln\left(\left(\phi_1^{uf}\right)^3\right)\right)^{\Re}+\left(-\ln\left(\left(\phi_2^{uf}\right)^3\right)\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}} \end{array} \right], \\ \left[\begin{array}{c} \sqrt[3]{1-\exp\left\{-\left(\left(-\ln\left(1-\left(\xi_1^{lf}\right)^3\right)\right)^{\Re}+\left(-\ln\left(1-\left(\xi_2^{lf}\right)^3\right)\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}}, \\ \sqrt[3]{1-\exp\left\{-\left(\left(-\ln\left(1-\left(\xi_1^{uf}\right)^3\right)\right)^{\Re}+\left(-\ln\left(1-\left(\xi_1^{uf}\right)^3\right)\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}} \end{array} \right] \end{array} \right), \tag{12}$$

(iii)

$$(\psi_1)^{\lambda} = \left(\left[\begin{array}{c} \sqrt[3]{\exp\left\{-\left(\lambda\left(-\ln\left(\phi_1^{lf}\right)^3\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}}, \\ \sqrt[3]{\exp\left\{-\left(\lambda\left(-\ln\left(\phi_1^{uf}\right)^3\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}} \end{array} \right], \left[\begin{array}{c} \sqrt[3]{1-\exp\left\{-\left(\lambda\left(-\ln\left(1-\left(\xi_1^{lf}\right)^3\right)\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}}, \\ \sqrt[3]{1-\exp\left\{-\left(\lambda\left(-\ln\left(1-\left(\xi_1^{uf}\right)^3\right)\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}} \end{array} \right] \right). \tag{13}$$

(iv)

$$(\psi_1)^{\lambda} = \left(\left[\begin{array}{c} \sqrt[3]{\exp\left\{-\left(\lambda\left(-\ln\left(\phi_1^{lf}\right)^3\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}}, \\ \sqrt[3]{\exp\left\{-\left(\lambda\left(-\ln\left(\phi_1^{uf}\right)^3\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}} \end{array} \right], \left[\begin{array}{c} \sqrt[3]{1-\exp\left\{-\left(\lambda\left(-\ln\left(1-\left(\xi_1^{lf}\right)^3\right)\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}}, \\ \sqrt[3]{1-\exp\left\{-\left(\lambda\left(-\ln\left(1-\left(\xi_1^{uf}\right)^3\right)\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}} \end{array} \right] \right). \tag{14}$$

Theorem 3.1 *Let* $\psi_1 = \left(\left[\phi_1^{lf}, \phi_1^{uf}\right], \left[\xi_1^{lf}, \xi_1^{uf}\right]\right)$ *and* $\psi_2 = \left(\left[\phi_2^{lf}, \phi_2^{uf}\right], \left[\xi_2^{lf}, \xi_2^{uf}\right]\right)$ *be two IVFFNs and* $\lambda, \lambda_1, \lambda_2 \geq 0$. *Then, the following properties can be obtained:*

(1) $\psi_1 \oplus \psi_2 = \psi_2 \oplus \psi_1$.
(2) $\psi_1 \otimes \psi_2 = \psi_2 \otimes \psi_1$.
(3) $\lambda(\psi_1 \oplus \psi_2) = \lambda\psi_1 \oplus \lambda\psi_2$.
(4) $\lambda_1\psi_1 \oplus \lambda_2\psi_1 = (\lambda_1 + \lambda_2)\psi_1$.
(5) $(\psi_1 \otimes \psi_2)^\lambda = (\psi_1)^\lambda \otimes (\psi_2)^\lambda$.
(6) $(\psi_1)^{\lambda_1} \otimes (\psi_1)^{\lambda_2} = (\psi_1)^{\lambda_1+\lambda_2}$.

Proof It is trivial via Definition 3.1.

3.2 Interval-valued Fermatean fuzzy Aczel–Alsina Hamy mean operators

This part proposes the definition of the IVFFHAM operator and the IVFFWHAM operator, as well as discusses the related theorems.

Definition 3.2 Let $\psi_j = \left(\left[\phi_j^{lf}, \phi_j^{uf}\right], \left[\xi_j^{lf}, \xi_j^{uf}\right]\right)(j = 1(1)p)$ be a family of IVFFNs. Then the mathematical definition of the IVFFHAM operator can be portrayed as:

$$IVFFAAHAM^{(\kappa)}\left(\psi_1, \psi_2, \ldots, \psi_p\right) = \frac{\bigoplus_{1\leq\varepsilon_1\leq\cdots\leq\varepsilon_\kappa\leq p}\left(\bigotimes_{j=1}^{\kappa} \psi_{\varepsilon_j}\right)^{\frac{1}{\kappa}}}{C_p^\kappa}, \tag{15}$$

wherein $(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_k)$ traverses all the κ-tuple combinations of $(1, 2, \ldots, p)$, and $C_p^\kappa = \frac{p!}{\kappa!(p-\kappa)!}$ is the binomial coefficient with the constraint of $1 \leq \varepsilon_1 \leq \varepsilon_2 \leq \cdots \leq \varepsilon_\kappa \leq p$.

Theorem 3.2 *Suppose that* $\psi_j = \left(\left[\phi_j^{lf}, \phi_j^{uf}\right], \left[\xi_j^{lf}, \xi_j^{uf}\right]\right)(j = 1(1)p)$ *is a family of IVFFNs, the fusion outcome attained by the IVFFHAM operator is still an IVFFN and represented as*:

$$\begin{aligned} &IVFFHAM^{(\kappa)}(\psi_1, \psi_2, \ldots, \psi_p) \\ &= \left(\left[\begin{array}{l} \sqrt[3]{1 - \exp\left\{ -\left[\frac{1}{C_p^\kappa}\left(\sum_{1\leq\varepsilon_1\leq\cdots\leq\varepsilon_\kappa\leq p} \left(-\ln\left(1 - \exp\left\{ -\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\phi_{\varepsilon_j}^{lf}\right)^3 \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\} \right) \right)^{\Re} \right) \right]^{\frac{1}{\Re}} \right\}}, \\ \sqrt[3]{1 - \exp\left\{ -\left[\frac{1}{C_p^\kappa}\left(\sum_{1\leq\varepsilon_1\leq\cdots\leq\varepsilon_\kappa\leq p} \left(-\ln\left(1 - \exp\left\{ -\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\phi_{\varepsilon_j}^{uf}\right)^3 \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\} \right) \right)^{\Re} \right) \right]^{\frac{1}{\Re}} \right\}} \end{array} \right], \right. \\ &\left. \left[\begin{array}{l} \sqrt[3]{\exp\left\{ -\left[\frac{1}{C_p^\kappa}\left(\sum_{1\leq\varepsilon_1\leq\cdots\leq\varepsilon_\kappa\leq p} \left(-\ln\left(1 - \exp\left\{ -\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1 - \left(\xi_{\varepsilon_j}^{lf}\right)^3 \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\} \right) \right)^{\Re} \right) \right]^{\frac{1}{\Re}} \right\}}, \\ \sqrt[3]{\exp\left\{ -\left[\frac{1}{C_p^\kappa}\left(\sum_{1\leq\varepsilon_1\leq\cdots\leq\varepsilon_\kappa\leq p} \left(-\ln\left(1 - \exp\left\{ -\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1 - \left(\xi_{\varepsilon_j}^{uf}\right)^3 \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\} \right) \right)^{\Re} \right) \right]^{\frac{1}{\Re}} \right\}} \end{array} \right] \right). \end{aligned} \tag{16}$$

Proof. Based on the Aczel–Alsina operation laws of IVFFNs, one has:

$$\overset{\kappa}{\underset{j=1}{\otimes}} \psi_{\varepsilon_j} = \left(\left[\begin{array}{l} \sqrt[3]{\exp\left\{ -\left[\sum_{j=1}^{\kappa} \left(-\ln\left(\left(\phi_{\varepsilon_j}^{lf} \right)^3 \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\}}, \\ \sqrt[3]{\exp\left\{ -\left[\sum_{j=1}^{\kappa} \left(-\ln\left(\left(\phi_{\varepsilon_j}^{uf} \right)^3 \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\}} \end{array} \right], \left[\begin{array}{l} \sqrt[3]{1-\exp\left\{ -\left[\sum_{j=1}^{\kappa} \left(-\ln\left(1-\left(\xi_{\varepsilon_j}^{lf} \right)^3 \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\}}, \\ \sqrt[3]{1-\exp\left\{ -\left[\sum_{j=1}^{\kappa} \left(-\ln\left(1-\left(\xi_{\varepsilon_j}^{uf} \right)^3 \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\}} \end{array} \right] \right)$$

and

$$\left(\overset{\kappa}{\underset{j=1}{\otimes}} \psi_{\varepsilon_j} \right)^{\frac{1}{\kappa}} = \left(\left[\begin{array}{l} \sqrt[3]{\exp\left\{ -\left[\frac{1}{\kappa} \sum_{j=1}^{\kappa} \left(-\ln\left(\left(\phi_{\varepsilon_j}^{lf} \right)^3 \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\}}, \\ \sqrt[3]{\exp\left\{ -\left[\frac{1}{\kappa} \sum_{j=1}^{\kappa} \left(-\ln\left(\left(\phi_{\varepsilon_j}^{lf} \right)^3 \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\}} \end{array} \right], \right.$$
$$\left. \times \left[\begin{array}{l} \sqrt[3]{1-\exp\left\{ -\left[\frac{1}{\kappa} \sum_{j=1}^{\kappa} \left(-\ln\left(1-\left(\xi_{\varepsilon_j}^{lf} \right)^3 \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\}}, \\ \sqrt[3]{1-\exp\left\{ -\left[\frac{1}{\kappa} \sum_{j=1}^{\kappa} \left(-\ln\left(1-\left(\xi_{\varepsilon_j}^{lf} \right)^3 \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\}} \end{array} \right] \right).$$

Furthermore:

$$\underset{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}{\oplus} \left(\overset{\kappa}{\underset{j=1}{\otimes}} \psi_{\varepsilon_j} \right)^{\frac{1}{\kappa}}$$
$$= \left(\begin{array}{l} \left[\begin{array}{l} \sqrt[3]{1-\exp\left\{ -\left[\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p} \left(-\ln\left(1-\exp\left\{ -\left[\frac{1}{\kappa} \sum_{j=1}^{\kappa} \left(-\ln\left(\left(\phi_{\varepsilon_j}^{lf} \right)^3 \right) \right)^{\varphi} \right]^{\frac{1}{\Re}} \right\} \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\}} \\ \sqrt[3]{1-\exp\left\{ -\left[\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p} \left(-\ln\left(1-\exp\left\{ -\left[\frac{1}{\kappa} \sum_{j=1}^{\kappa} \left(-\ln\left(\left(\phi_{\varepsilon_j}^{uf} \right)^3 \right) \right)^{\varphi} \right]^{\frac{1}{\Re}} \right\} \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\}} \end{array} \right], \\ \left[\begin{array}{l} \sqrt[3]{\exp\left\{ -\left[\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p} \left(-\ln\left(1-\exp\left\{ -\left[\frac{1}{\kappa} \sum_{j=1}^{\gamma} \left(-\ln\left(1-\left(\xi_{\varepsilon_j}^{lf} \right)^3 \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\} \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\}}, \\ \sqrt[3]{\exp\left\{ -\left[\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p} \left(-\ln\left(1-\exp\left\{ -\left[\frac{1}{\kappa} \sum_{j=1}^{\gamma} \left(-\ln\left(1-\left(\xi_{\varepsilon_j}^{uf} \right)^3 \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\} \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\}} \end{array} \right] \end{array} \right).$$

Then:

$$\frac{\underset{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}{\oplus}\left(\overset{\kappa}{\underset{j=1}{\otimes}}\psi_{\varepsilon_j}\right)^{\frac{1}{\kappa}}}{C_p^\kappa}$$

$$=\left(\begin{array}{l}\left[\sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\phi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}, \sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\phi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}\right],\\ \left[\sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\xi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}, \sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\xi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}\right]\end{array}\right).$$

Accordingly:

$$IVFFHAM^{(\kappa)}(\psi_1,\psi_2,\ldots,\psi_p)$$

$$=\left(\begin{array}{l}\left[\sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\phi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}, \sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\phi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}\right],\\ \left[\sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\xi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}, \sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\xi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}\right]\end{array}\right).$$

Hence, we can prove that Eq. (16) holds. Furthermore, we need to prove that the fusion outcomes attained by the IVFFHAM operator are IVFFNs. Based on the definition of IVFFS, we have $0 \le \phi_{\varepsilon_j}^{lf}, \phi_{\varepsilon_j}^{uf}, \xi_{\varepsilon_j}^{lf}, \xi_{\varepsilon_j}^{uf} \le 1$ and $0 \le \left(\phi_{\varepsilon_j}^{uf}\right)^3 + \left(\xi_{\varepsilon_j}^{uf}\right)^3 \le 1$. Then, we can acquire that the interval-valued membership and non-membership grades of the IVFFHAM operator belong to [0, 1]. Besides, because $\left(\phi_{\varepsilon_j}^{uf}\right)^3 \le 1-\left(\xi_{\varepsilon_j}^{uf}\right)^3$ holds, we can get that the sum of the interval lower bound of membership grade and the interval lower bound of non-membership grade is less than or equal to 1. Consequently, the proof of Theorem 3.2 is finished.

In view of Theorem 3.2, we explore a series of meanwhile properties of the IVFFHAM operator.

Property 3.1 (*Idempotency*) Let $\psi_j = \left(\left[\phi_j^{lf}, \phi_j^{uf}\right], \left[\xi_j^{lf}, \xi_j^{uf}\right]\right)(j = 1(1)p)$ be a family of IVFFNs. If $\psi_j = \psi = \left(\left[\phi^{lf}, \phi^{uf}\right], \left[\xi^{lf}, \xi^{uf}\right]\right)$, for all $j = 1, 2, \cdots, p$, then one has:

$$IVFFAAHAM^{(\kappa)}(\psi_1, \psi_2, \ldots, \psi_p) = \psi.$$

Proof. Since $\psi_j = \psi = \left(\left[\phi^{lf}, \phi^{uf}\right], \left[\xi^{lf}, \xi^{uf}\right]\right)$, then using Theorem 3.2, we have.

$$IVFFHAM^{(\kappa)}(\psi_1, \psi_2, \ldots, \psi_p)$$

$$= \left(\begin{array}{c}\left[\begin{array}{c}\sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\phi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}},\\ \sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\phi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}\end{array}\right],\\ \left[\begin{array}{c}\sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\xi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}},\\ \sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\xi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}\end{array}\right]\end{array}\right)$$

$$= \left(\begin{array}{c}\left[\begin{array}{c}\sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\phi^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}},\\ \sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\phi^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}\end{array}\right],\\ \left[\begin{array}{c}\sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\xi^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}},\\ \sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\xi^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}\end{array}\right]\end{array}\right)$$

$$
= \left(\left[\sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^{\kappa}}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\left(-\ln\left(\left(\phi^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}, \sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^{\kappa}}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\left(-\ln\left(\left(\phi^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}} \right], \left[\sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^{\kappa}}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\left(-\ln\left(1-\left(\xi^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}, \sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^{\kappa}}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\left(-\ln\left(1-\left(\xi^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}} \right] \right)
$$

$$
= \left(\left[\sqrt[3]{1-\exp\left\{-\left[\left(-\ln\left(1-\exp\left\{-\left[\left(-\ln\left(\left(\phi^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}}, \sqrt[3]{1-\exp\left\{-\left[\left(-\ln\left(1-\exp\left\{-\left[\left(-\ln\left(\left(\phi^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}} \right], \left[\sqrt[3]{\exp\left\{-\left[\left(-\ln\left(1-\exp\left\{-\left[\left(-\ln\left(1-\left(\xi^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}}, \sqrt[3]{\exp\left\{-\left[\left(-\ln\left(1-\exp\left\{-\left[\left(-\ln\left(1-\left(\xi^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}} \right] \right)
$$

$$
= \left(\left[\phi^{lf}, \phi^{uf}\right], \left[\xi^{lf}, \xi^{uf}\right]\right) = \psi.
$$

Property 3.2 (*Monotonicity*) Let $\psi_j = \left(\left[\phi_j^{lf}, \phi_j^{uf}\right], \left[\xi_j^{lf}, \xi_j^{uf}\right]\right)$ and $\overline{\psi}_j = \left(\left[\overline{\phi}_j^{lf}, \overline{\phi}_j^{uf}\right], \left[\overline{\xi}_j^{lf}, \overline{\xi}_j^{uf}\right]\right)$ be two collections of IVFFNs. If $\phi_j^{lf} \ge \overline{\phi}_j^{lf}$, $\phi_j^{uf} \ge \overline{\phi}_j^{uf}$, $\xi_j^{lf} \le \overline{\xi}_j^{lf}$, and $\xi_j^{uf} \ge \overline{\xi}_j^{uf}$, then we have:

$$
IVFFAAHAM^{(\kappa)}(\psi_1, \psi_2, \ldots, \psi_p) \ge IVFFAAHAM^{(\kappa)}(\overline{\psi}_1, \overline{\psi}_2, \ldots, \overline{\psi}_p).
$$

Proof. Since $\phi_j^{lf} > \overline{\phi}_j^{lf}$, we have $\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\phi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re} \le \sum_{j=1}^{\kappa}\left(-\ln\left(\left(\overline{\phi}_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}$. Further, we can deduce:

$$
1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\phi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\} \le 1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\overline{\phi}_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\},
$$

and

$$\frac{1}{C_p^\kappa}\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\phi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}$$
$$\ge\frac{1}{C_p^\kappa}\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\overline{\phi}_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}.$$

Furthermore:

$$\sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\phi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}$$
$$\ge\sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\phi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}.$$

The unequal relation for the non-membership grade can be attained in the same manner as:

$$\sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\xi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}$$
$$\le\sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\overline{\xi}_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}.$$

Hence, we have $IVFFAAHAM^{(\kappa)}\left(\psi_1,\ \psi_2,\ \ldots,\ \psi_p\right) > IVFFAAHAM^{(\kappa)}\left(\overline{\psi}_1,\ \overline{\psi}_2,\ \ldots,\ \overline{\psi}_p\right)$. Furthermore, employing the score function of IVFFN, we can obtain that $IVFFAAHAM^{(\kappa)}\left(\psi_1,\ \psi_2,\ \ldots,\ \psi_p\right) = IVFFAAHAM^{(\kappa)}\left(\overline{\psi}_1,\ \overline{\psi}_2,\ \ldots,\ \overline{\psi}_p\right)$ holds when $\phi_j^{lf}=\overline{\phi}_j^{lf}$, $\phi_j^{uf}=\overline{\phi}_j^{uf}$, $\xi_j^{lf}=\overline{\xi}_j^{lf}$, and $\xi_j^{uf}=\overline{\xi}_j^{uf}$. Based on the above illustrations, we can prove that the monotonicity property $IVFFAAHAM^{(\kappa)}\left(\psi_1,\ \psi_2,\ \ldots,\ \psi_p\right) \ge IVFFAAHAM^{(\kappa)}\left(\overline{\psi}_1,\ \overline{\psi}_2,\ \ldots,\ \overline{\psi}_p\right)$ always holds.

Property 3.3 (*Boundness*): Let $\psi_j=\left(\left[\phi_j^{lf},\phi_j^{uf}\right],\left[\xi_j^{lf},\xi_j^{uf}\right]\right)(j=1(1)p)$ be a family of IVFFNs. If $\psi_{\min}=\min_{1\le j\le p}\left\{\psi_j\right\}$ and $\psi_{\max}=\max_{1\le j\le p}\left\{\psi_j\right\}$, then we have:

$$\psi_{\min}\le IVFFAAHAM^{(\kappa)}\left(\psi_1,\ \psi_2,\ \ldots,\ \psi_p\right)\le\psi_{\max}.$$

Proof. Employing the monotonicity of the IVFFHAM operator, we have:

$$\psi_{\min}\le IVFFAAHAM^{(\kappa)}\left(\psi_1,\ \psi_2,\ \ldots,\ \psi_p\right)\ge IVFFAAHAM^{(\kappa)}(\psi_{\min},\ \psi_{\min},\ \ldots,\ \psi_{\min}),$$
$$\psi_{\min}\le IVFFAAHAM^{(\kappa)}\left(\psi_1,\ \psi_2,\ \ldots,\ \psi_p\right)\le IVFFAAHAM^{(\kappa)}(\psi_{\max},\ \psi_{\max},\ \ldots,\ \psi_{\max}).$$

Further, based on the Idempotency of the IVFFHAM operator, we have:

$$IVFFAAHAM^{(\kappa)}(\psi_{\min}, \psi_{\min}, \ldots, \psi_{\min}) = \psi_{\min},$$

$$IVFFAAHAM^{(\kappa)}(\psi_{\max}, \psi_{\max}, \ldots, \psi_{\max}) = \psi_{\max}.$$

Hence, we can attain that the boundness property $\psi_{\min} \le IVFFAAHAM^{(\kappa)}\left(\psi_1, \psi_2, \ldots, \psi_p\right) \le \psi_{\max}$ holds.

Property 3.4 (*Commutativity*) Let $\psi_j = \left(\left[\phi_j^{lf}, \phi_j^{uf}\right], \left[\xi_j^{lf}, \xi_j^{uf}\right]\right)$ and $\overline{\psi}_j = \left(\left[\overline{\phi}_j^{lf}, \overline{\phi}_j^{uf}\right], \left[\overline{\xi}_j^{lf}, \overline{\xi}_j^{uf}\right]\right)$ be two collections of IVFFNs. If $\left(\overline{\psi}_1, \overline{\psi}_2, \cdots, \overline{\psi}_p\right)$ is any permutation of $\left(\psi_1, \psi_2, \cdots, \psi_p\right)$, then:

$$IVFFAAHAM^{(\kappa)}\left(\psi_1, \psi_2, \ldots, \psi_p\right) = IVFFAAHAM^{(\kappa)}\left(\overline{\psi}_1, \overline{\psi}_2, \ldots, \overline{\psi}_p\right).$$

Proof Since $\left(\overline{\psi}_1, \overline{\psi}_2, \cdots, \overline{\psi}_p\right)$ is any permutation of $\left(\psi_1, \psi_2, \cdots, \psi_p\right)$ holds, then we have:

$$\frac{\underset{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}{\oplus}\left(\overset{\kappa}{\underset{j=1}{\otimes}}\psi_{\varepsilon_j}\right)^{\frac{1}{\kappa}}}{C_p^\kappa} = \frac{\underset{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}{\oplus}\left(\overset{\kappa}{\underset{j=1}{\otimes}}\overline{\psi}_{\varepsilon_j}\right)^{\frac{1}{\kappa}}}{C_p^\kappa}.$$

Therefore, the property $IVFFAAHAM^{(\kappa)}\left(\psi_1, \psi_2, \ldots, \psi_p\right) = IVFFAAHAM^{(\kappa)}\left(\overline{\psi}_1, \overline{\psi}_2, \ldots, \overline{\psi}_p\right)$ always keeps.

Next, several special cases of the proposed IVFFAAHAM operator are acquired as follows:

(1) When $\Re = 1$, then the IVFFAAHAM operator can be generated into the IVFFHAM operator.
(2) When $\Re = 1$ and $\kappa = 1$, then the IVFFAAHAM operator can be reduced to the IVFFA operator (Rani and Mishra 2022).
(3) When $\Re = 1$ and $\kappa = p$, then the IVFFAAHAM operator can be yielded to the IVFFG operator (Rani and Mishra 2022).
(4) When $\kappa = 1$, then the IVFFAAHAM operator can be simplified into the IVFFAAA operator.
(5) When $\kappa = p$, then the IVFFAAHAM operator can be generated into the IVFFAAG operator.

We can find that the IVFFAAHAM operator fails to take into account the importance of the fused data, thus the novel weighted Hamy mean operator by utilizing the IVFF Aczel–Alsina operations is presented in the following.

Definition 3.3 Let $\psi_j = \left(\left[\phi_j^{lf}, \phi_j^{uf}\right], \left[\xi_j^{lf}, \xi_j^{uf}\right]\right)$ $(j = 1(1)p)$ be a family of IVFFNs, and $\omega = \left(\omega_1, \omega_2, \ldots, \omega_p\right)^T$ be the weights of $\psi_j (j = 1(1)p)$, satisfying $\omega_j \in [0, 1]$ and $\sum_{j=1}^p \omega_j = 1$. Then, the mathematical definition of the IVFFAAWHAM operator can be portrayed as:

$$IVFFAAWHAM^{(\kappa)}\left(\psi_1, \psi_2, \ldots, \psi_p\right) = \frac{\underset{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}{\oplus}\left(\overset{\kappa}{\underset{j=1}{\otimes}}\omega_{\varepsilon_j}\psi_{\varepsilon_j}\right)^{\frac{1}{\kappa}}}{C_p^\kappa}, \tag{17}$$

wherein $(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_k)$ traverses all the κ-tuple combinations of $(1, 2, \ldots, p)$, and $C_p^\kappa = \frac{p!}{\kappa!(p-\kappa)!}$ is the binomial coefficient with the constraint of $1 \le \varepsilon_1 \le \varepsilon_2 \le \cdots \le \varepsilon_\kappa \le p$.

According to the introduced Aczel–Alsina operation laws of IVFFNs, the aggregation results of the IVFFWHAM operator are expressed as follows.

Theorem 3.3 Suppose that $\psi_j = \left(\left[\phi_j^{lf}, \phi_j^{uf}\right], \left[\xi_j^{lf}, \xi_j^{uf}\right]\right)(j = 1(1)p)$ is a family of IVFFNs, the fusion outcome attained by the IVFFWHAM operator is still an IVFFN and represented as:

$$IVFFWHAM^{(\kappa)}(\psi_1, \psi_2, \ldots, \psi_p)$$

$$= \left(\begin{array}{l} \left[\sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum\limits_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum\limits_{j=1}^{\kappa}\omega_{\varepsilon_j}\left(-\ln\left(\left(\phi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}, \sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum\limits_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum\limits_{j=1}^{\kappa}\omega_{\varepsilon_j}\left(-\ln\left(\left(\phi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}} \right], \\ \left[\sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum\limits_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum\limits_{j=1}^{\kappa}\omega_{\varepsilon_j}\left(-\ln\left(1-\left(\xi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}, \sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum\limits_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum\limits_{j=1}^{\kappa}\omega_{\varepsilon_j}\left(-\ln\left(1-\left(\xi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}} \right] \end{array} \right). \tag{18}$$

Proof Analogous with Theorem 3.2, so we omitted it here.

In addition, the IVFFWHAM operator fulfills the property of monotonicity and boundness but fails to meet the property of idempotency. Thus, the proof is omitted here.

3.3 Interval-valued Fermatean fuzzy Aczel–Alsina dual Hamy mean operators

In the following subsection, the novel dual Hamy mean operators including the IVFFDHAM operator and the IVFFWDHAM operator are propounded based on the IVFF Aczel–Alsina operations.

Definition 3.4 Let $\psi_j = \left(\left[\phi_j^{lf}, \phi_j^{uf}\right], \left[\xi_j^{lf}, \xi_j^{uf}\right]\right)$ $(j = 1(1)p)$ be a family of IVFFNs. Then, the mathematical definition of the IVFFDHAM operator can be portrayed as:

$$\text{IVFFAADHAM}^{(\kappa)}(\psi_1, \psi_2, \ldots, \psi_p) = \left(\bigotimes_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(\frac{\bigoplus_{j=1}^{\kappa}\psi_{\varepsilon_j}}{\kappa}\right)\right)^{\frac{1}{C_p^\kappa}}, \tag{19}$$

wherein $(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_k)$ traverses all the κ-tuple combinations of $(1, 2, \ldots, p)$, and $C_p^\kappa = \frac{p!}{\kappa!(p-\kappa)!}$ is the binomial coefficient with the constraint of $1 \le \varepsilon_1 \le \varepsilon_2 \le \cdots \le \varepsilon_\kappa \le p$.

Theorem 3.4 It is assumed that $\psi_j = \left(\left[\phi_j^{lf}, \phi_j^{uf}\right], \left[\xi_j^{lf}, \xi_j^{uf}\right]\right)$ $(j = 1(1)p)$ is a collection of IVFFNs, the aggregation resultant attained by the IVFFDHAM operator is still an IVFFN and represented as:

$$IVFFDHAM^{(\kappa)}(\psi_1, \psi_2, \ldots, \psi_p) = \left(\left[\begin{array}{l} \sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\phi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}, \\ \sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\phi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}} \end{array}\right], \left[\begin{array}{l} \sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\xi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}, \\ \sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\xi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}} \end{array}\right]\right). \tag{20}$$

Proof On the basis of the Aczel–Alsina operation laws of IVFFNs, one has:

$$\mathop{\oplus}_{j=1}^{\kappa} \psi_{\varepsilon_j} = \left(\left[\begin{array}{l} \sqrt[3]{1-\exp\left\{-\left[\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\phi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}}, \\ \sqrt[3]{1-\exp\left\{-\left[\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\phi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}} \end{array}\right], \times \left[\begin{array}{l} \sqrt[3]{\exp\left\{-\left[\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\xi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}}, \\ \sqrt[3]{\exp\left\{-\left[\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\xi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}} \end{array}\right]\right)$$

and

$$\frac{\overset{\kappa}{\underset{j=1}{\oplus}}\psi_{\varepsilon_j}}{\kappa}\left(\left[\begin{array}{l}\sqrt[3]{1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\phi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}},\\ \sqrt[3]{1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\phi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}}\end{array}\right],\left[\begin{array}{l}\sqrt[3]{\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\xi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}},\\ \sqrt[3]{\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\xi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}}\end{array}\right]\right)$$

.

Furthermore:

$$\underset{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}{\otimes}\left(\frac{\overset{\kappa}{\underset{j=1}{\oplus}}\psi_{\varepsilon_j}}{\kappa}\right)$$

$$=\left(\begin{array}{l}\left[\begin{array}{l}\sqrt[3]{\exp\left\{-\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\phi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}},\\ \sqrt[3]{\exp\left\{-\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\phi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}}\end{array}\right],\\ \left[\begin{array}{l}\sqrt[3]{1-\exp\left\{-\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\xi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}},\\ \sqrt[3]{1-\exp\left\{-\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\xi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}}\end{array}\right]\end{array}\right).$$

Then:

$$\left(\underset{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}{\otimes}\left(\frac{\overset{\kappa}{\underset{j=1}{\oplus}}\psi_{\varepsilon_j}}{\kappa}\right)\right)^{\frac{1}{C_p^\kappa}}$$

$$=\left(\begin{array}{c}\left[\begin{array}{c}\sqrt[3]{\exp\left\{-\left(\frac{1}{C_p^\kappa}\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\phi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right)^{1/\varphi}\right\}},\\ \sqrt[3]{\exp\left\{-\left(\frac{1}{C_p^\kappa}\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\phi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right)^{1/\varphi}\right\}}\end{array}\right],\\ \left[\begin{array}{c}\sqrt[3]{1-\exp\left\{-\left(\frac{1}{C_p^\kappa}\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\xi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right)^{1/\varphi}\right\}},\\ \sqrt[3]{1-\exp\left\{-\left(\frac{1}{C_p^\kappa}\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\xi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\varphi}\right]^{1/\varphi}\right\}\right)\right)^{\varphi}\right)^{1/\varphi}\right\}}\end{array}\right]\end{array}\right).$$

Accordingly:

$$IVFFDHAM^{(\kappa)}(\psi_1,\psi_2,\ldots,\psi_p)$$

$$=\left(\begin{array}{c}\left[\begin{array}{c}\sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\phi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}},\\ \sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(1-\left(\phi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}\end{array}\right],\\ \left[\begin{array}{c}\sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\xi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}},\\ \sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^\kappa}\left(\sum_{1\le\varepsilon_1\le\cdots\le\varepsilon_\kappa\le p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum_{j=1}^{\kappa}\left(-\ln\left(\left(\xi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}\end{array}\right]\end{array}\right).$$

Hence, we can prove that Eq. (20) holds. Furthermore, we need to prove that the fusion outcomes attained by the IVFFDHAM operator are IVFFNs. Based on the definition of IVFFS, we have $0 \le \phi_{\varepsilon_j}^{lf}, \phi_{\varepsilon_j}^{uf}, \xi_{\varepsilon_j}^{lf}, \xi_{\varepsilon_j}^{uf} \le 1$ and $0 \le \left(\phi_{\varepsilon_j}^{uf}\right)^3 + \left(\xi_{\varepsilon_j}^{uf}\right)^3 \le 1$. Then, we can acquire the interval-valued membership and non-membership grades of the IVFFHAM operator belong to [0, 1]. Moreover, since $\left(\phi_{\varepsilon_j}^{uf}\right)^3 \le 1-\left(\xi_{\varepsilon_j}^{uf}\right)^3$ holds, then we can get that the sum of the interval lower bound of the membership grade and the interval lower bound of the non-membership grade is less than or equal to 1. Consequently, the proof of Theorem 3.4 is finished.

A series of meanwhile properties of the IVFFDHAM operator are explored using Theorem 3.4.

Property 3.5 (*Idempotency*) Let $\psi_j = \left(\left[\phi_j^{lf}, \phi_j^{uf}\right], \left[\xi_j^{lf}, \xi_j^{uf}\right]\right)$ $(j = 1(1)p)$ be a set of IVFFNs. If $\psi_j = \psi = \left(\left[\phi^{lf}, \phi^{uf}\right], \left[\xi^{lf}, \xi^{uf}\right]\right)$, for all $j = 1, 2, \cdots, p$, then one has:

$$\text{IVFFAADHAM}^{(\kappa)}\left(\psi_1, \psi_2, \ldots, \psi_p\right) = \psi.$$

Property 3.6 (*Monotonicity*) Let $\psi_j = \left(\left[\phi_j^{lf}, \phi_j^{uf}\right], \left[\xi_j^{lf}, \xi_j^{uf}\right]\right)$ and $\overline{\psi}_j = \left(\left[\overline{\phi}_j^{lf}, \overline{\phi}_j^{uf}\right], \left[\overline{\xi}_j^{lf}, \overline{\xi}_j^{uf}\right]\right)$ be two collections of IVFFNs. If $\phi_j^{lf} \geq \overline{\phi}_j^{lf}$, $\phi_j^{uf} \geq \overline{\phi}_j^{uf}$, $\xi_j^{lf} \leq \overline{\xi}_j^{lf}$, and $\xi_j^{uf} \geq \overline{\xi}_j^{uf}$, then we have:

$$\text{IVFFAADHAM}^{(\kappa)}\left(\psi_1, \psi_2, \ldots, \psi_p\right) \geq \text{IVFFAADHAM}^{(\kappa)}\left(\overline{\psi}_1, \overline{\psi}_2, \ldots, \overline{\psi}_p\right).$$

Property 3.7 (*Boundness*): Let $\psi_j = \left(\left[\phi_j^{lf}, \phi_j^{uf}\right], \left[\xi_j^{lf}, \xi_j^{uf}\right]\right)$ $(j = 1(1)p)$ be a family of IVFFNs. If $\psi_{\min} = \min_{1 \leq j \leq p}\left\{\psi_j\right\}$ and $\psi_{\max} = \max_{1 \leq j \leq p}\left\{\psi_j\right\}$, then we have:

$$\psi_{\min} \leq \text{IVFFAADHAM}^{(\kappa)}\left(\psi_1, \psi_2, \ldots, \psi_p\right) \leq \psi_{\max}.$$

Property 3.8 (*Commutativity*) Let $\psi_j = \left(\left[\phi_j^{lf}, \phi_j^{uf}\right], \left[\xi_j^{lf}, \xi_j^{uf}\right]\right)$ and $\overline{\psi}_j = \left(\left[\overline{\phi}_j^{lf}, \overline{\phi}_j^{uf}\right], \left[\overline{\xi}_j^{lf}, \overline{\xi}_j^{uf}\right]\right)$ be two collections of IVFFNs. If $\left(\overline{\psi}_1, \overline{\psi}_2, \cdots, \overline{\psi}_p\right)$ is any permutation of $\left(\psi_1, \psi_2, \cdots, \psi_p\right)$. Then:

$$\text{IVFFAADHAM}^{(\kappa)}\left(\psi_1, \psi_2, \ldots, \psi_p\right) = \text{IVFFAADHAM}^{(\kappa)}\left(\overline{\psi}_1, \overline{\psi}_2, \ldots, \overline{\psi}_p\right).$$

In what follows, several special cases of the proposed IVFFAADHAM operator are acquired as follows:

(1) When $\Re = 1$, then the IVFFAADHAM operator can be generated into the IVFFDHAM operator.
(2) When $\Re = 1$ and $\kappa = 1$, then the IVFFAADHAM operator lessens into the IVFFG operator (Rani and Mishra 2022).
(3) When $\Re = 1$ and $\kappa = p$, then the IVFFAADHAM operator reduces into the IVFFA operator (Rani and Mishra 2022).
(4) When $\kappa = 1$, then the IVFFAADHAM operator can be simplified into the IVFFAAG operator.
(5) When $\kappa = p$, then the IVFFAADHAM operator can be generated into the IVFFAAA operator.

We found that the IVFFAADHAM operator fails to take into account the importance of the fused data. Thus, the novel weighted dual Hamy mean operator by utilizing the IVFFAA operations is presented in the following.

Definition 3.5 Let $\psi_j = \left(\left[\phi_j^{lf}, \phi_j^{uf}\right], \left[\xi_j^{lf}, \xi_j^{uf}\right]\right)$ $(j = 1(1)p)$ be a family of IVFFNs and $\omega = \left(\omega_1, \omega_2, \ldots, \omega_p\right)^T$ be the weights of $\psi_j (j = 1(1)p)$ that satisfy $\omega_j \in [0, 1]$ and $\sum_{j=1}^{p} \omega_j = 1$. Then, the mathematical definition of the IVFFAAWHAM operator can be portrayed as:

$$IVFFAAWDHAM^{(\kappa)}(\psi_1, \psi_2, \ldots, \psi_p) = \left(\bigotimes_{1 \le \varepsilon_1 \le \cdots \le \varepsilon_\kappa \le p} \left(\frac{\bigoplus_{j=1}^{\kappa} \omega_{\varepsilon_j} \psi_{\varepsilon_j}}{\kappa} \right) \right)^{\frac{1}{C_p^\kappa}}, \quad (21)$$

wherein $(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_k)$ traverses all the κ-tuple combinations of $(1, 2, \ldots, p)$, and $C_p^\kappa = \frac{p!}{\kappa!(p-\kappa)!}$ is the binomial coefficient with the constraint of $1 \le \varepsilon_1 \le \varepsilon_2 \le \cdots \le \varepsilon_\kappa \le p$.

Theorem 3.5 Suppose that $\psi_j = \left(\left[\phi_j^{lf}, \phi_j^{uf} \right], \left[\xi_j^{lf}, \xi_j^{uf} \right] \right)$ $(j = 1(1)p)$ is a family of IVFFNs, the fusion outcome attained by the IVFFHAM operator is still an IVFFN and represented as:

$$IVFFWDHAM^{(\kappa)}(\psi_1, \psi_2, \ldots, \psi_p)$$

$$= \left(\begin{array}{c} \left[\sqrt[3]{\exp\left\{ -\left[\frac{1}{C_p^\kappa} \left(\sum_{1 \le \varepsilon_1 \le \cdots \le \varepsilon_\kappa \le p} \left(-\ln\left(1 - \exp\left\{ -\left[\frac{1}{\kappa} \sum_{j=1}^{\kappa} \omega_{\varepsilon_j} \left(-\ln\left(1 - \left(\phi_{\varepsilon_j}^{lf} \right)^3 \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\} \right) \right)^{\Re} \right) \right]^{\frac{1}{\Re}} \right\}}, \sqrt[3]{\exp\left\{ -\left[\frac{1}{C_p^\kappa} \left(\sum_{1 \le \varepsilon_1 \le \cdots \le \varepsilon_\kappa \le p} \left(-\ln\left(1 - \exp\left\{ -\left[\frac{1}{\kappa} \sum_{j=1}^{\kappa} \omega_{\varepsilon_j} \left(-\ln\left(1 - \left(\phi_{\varepsilon_j}^{uf} \right)^3 \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\} \right) \right)^{\Re} \right) \right]^{\frac{1}{\Re}} \right\}} \right], \\ \left[\sqrt[3]{1 - \exp\left\{ -\left[\frac{1}{C_p^\kappa} \left(\sum_{1 \le \varepsilon_1 \le \cdots \le \varepsilon_\kappa \le p} \left(-\ln\left(1 - \exp\left\{ -\left[\frac{1}{\kappa} \sum_{j=1}^{\kappa} \omega_{\varepsilon_j} \left(-\ln\left(\left(\xi_{\varepsilon_j}^{lf} \right)^3 \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\} \right) \right)^{\Re} \right) \right]^{\frac{1}{\Re}} \right\}}, \sqrt[3]{1 - \exp\left\{ -\left[\frac{1}{C_p^\kappa} \left(\sum_{1 \le \varepsilon_1 \le \cdots \le \varepsilon_\kappa \le p} \left(-\ln\left(1 - \exp\left\{ -\left[\frac{1}{\kappa} \sum_{j=1}^{\kappa} \omega_{\varepsilon_j} \left(-\ln\left(\left(\xi_{\varepsilon_j}^{uf} \right)^3 \right) \right)^{\Re} \right]^{\frac{1}{\Re}} \right\} \right) \right)^{\Re} \right) \right]^{\frac{1}{\Re}} \right\}} \right] \end{array} \right). \quad (22)$$

Proof Analogous with Theorem 3.2, so it is omitted here.

In addition, the IVFFWDHAM operator fulfills the property of monotonicity and boundness but fails to the property of idempotency. Thus, the proof is omitted here.

4 An innovative FMEA model based on IVFF-LOPCOW-ARAS

In this section, an innovative multiple criteria group decision-making (MCGDM) methodology named IVFF-LOPCOW-ARAS is presented by combining the proposed IVFFWHAM operator, IVFFWDHAM operator, LOPCOW and ARAS methods under the IVFF environment. In the proposed IVFF-LOPCOW-ARAS method, weights of experts are determined by a similarity degree-based method, while criterion weights are determined by the LOPCOW method. Then, an enhanced ARAS approach is introduced based on the proposed IVFFWHAM and IVFFWDHAM operators. The most advantageous of the proposed IVFF-LOPCOW-ARAS approach is that it takes into consideration the correlation among the IVFF assessment data. Specifically, the presented IVFF-LOPCOW-ARAS method is portrayed in Fig. 1. The concrete procedures of the developed method are expounded below.

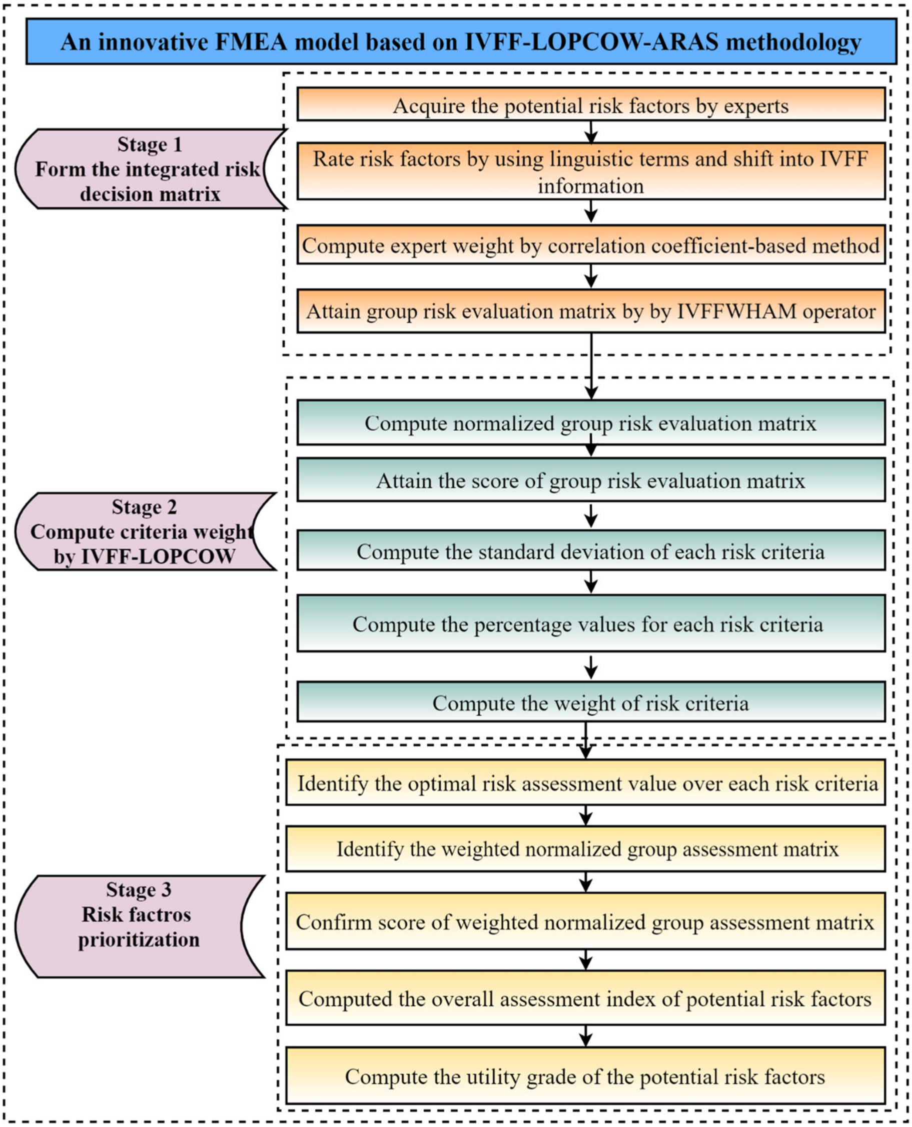

Fig. 1 A schematic diagram of the propounded FMEA model based on IVFF-LOPCOW-ARAS

4.1 Establishment of integrated IVFF decision matrix

It is supposed that the MCGDM problem is made up of q schemes denoted as $D = \{D_1, D_2, \cdots, D_q\}$ and p assessment criteria indicated as $C = \{C_1, C_2, \cdots, C_p\}$. The decision committee consists of γ experts who are denoted as $E = \{E_1, E_2, \cdots, E_\gamma\}$. Let $Q^{(r)} = \left(\chi_{ij}^{(r)}\right)_{q\times p}$ be the IVFF assessment matrices given by expert E_r in the following:

$$Q^{(r)} = \left(\chi_{ij}^{(r)}\right)_{q\times p} = \begin{pmatrix} \chi_{11}^{(r)} & \chi_{12}^{(r)} & \cdots & \chi_{1p}^{(r)} \\ \chi_{21}^{(r)} & \chi_{22}^{(r)} & \cdots & \chi_{2p}^{(r)} \\ \vdots & \vdots & \ddots & \vdots \\ \chi_{q1}^{(r)} & \chi_{q2}^{(r)} & \cdots & \chi_{qp}^{(r)} \end{pmatrix}, \quad \forall r = 1(1)\gamma,$$

wherein $\chi_{ij}^{(r)} = \left(\left[\phi_{ij}^{lf(r)}, \phi_{ij}^{uf(r)}\right], \left[\xi_{ij}^{lf(r)}, \xi_{ij}^{uf(r)}\right]\right)$ denotes the assessment (i.e. judgment) of the scheme $D_i (i = 1, 2, \cdots, q)$ over the evaluation criterion C_j $(j = 1, 2, \cdots, p)$ provided by the expert E_r $(r = 1, 2, \cdots, \gamma)$.

Step 1. *Work out the weight of decision experts*:

The weights of the experts are a key attention for the procedure of fusing the assessment information. In this step, the correlation coefficient-based approach is employed to determine the weight of the experts.

(a) Compute the score matrices of experts as:

$$\widetilde{\wp}^{(r)} = \left(\widetilde{\wp}\left(\chi_{ij}^{(r)}\right)\right)_{q\times p} = \begin{pmatrix} \widetilde{\wp}\left(\chi_{11}^{(r)}\right) & \widetilde{\wp}\left(\chi_{12}^{(r)}\right) & \cdots & \widetilde{\wp}\left(\chi_{p1}^{(r)}\right) \\ \widetilde{\wp}\left(\chi_{21}^{(r)}\right) & \widetilde{\wp}\left(\chi_{22}^{(r)}\right) & \cdots & \widetilde{\wp}\left(\chi_{p2}^{(r)}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{\wp}\left(\chi_{q1}^{(r)}\right) & \widetilde{\wp}\left(\chi_{q2}^{(r)}\right) & \cdots & \widetilde{\wp}\left(\chi_{qp}^{(r)}\right) \end{pmatrix}, \quad \forall r = 1(1)\gamma, \tag{23}$$

where $\widetilde{\wp}\left(\chi_{ij}^{(r)}\right) = \frac{1}{2}\left(\frac{1}{2}\left(\left(\phi_{ij}^{lf(r)}\right)^3 + \left(\phi_{ij}^{uf(r)}\right)^3 - \left(\xi_{ij}^{lf(r)}\right)^3 - \left(\xi_{ij}^{uf(r)}\right)^3\right) + 1\right)$ $(i = 1(1)q,$ $j = 1(1)p,\ r = 1(1)\gamma)$.

(b) Attain the averaging score matrices of experts as:

$$\widetilde{\widetilde{\wp}} = \left(\widetilde{\widetilde{\wp}}\left(\chi_{ij}\right)\right)_{q\times p} = \begin{pmatrix} \widetilde{\widetilde{\wp}}(\chi_{11}) & \widetilde{\widetilde{\wp}}(\chi_{12}) & \cdots & \widetilde{\widetilde{\wp}}\left(\chi_{p1}\right) \\ \widetilde{\widetilde{\wp}}(\chi_{21}) & \widetilde{\widetilde{\wp}}(\chi_{22}) & \cdots & \widetilde{\widetilde{\wp}}\left(\chi_{p2}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{\widetilde{\wp}}\left(\chi_{q1}\right) & \widetilde{\widetilde{\wp}}\left(\chi_{q2}\right) & \cdots & \widetilde{\widetilde{\wp}}\left(\chi_{qp}\right) \end{pmatrix}, \tag{24}$$

where $\widetilde{\widetilde{\wp}}\left(\chi_{ij}\right) = \frac{1}{\gamma}\sum_{r=1}^{\gamma} \widetilde{\wp}\left(\chi_{ij}^{(r)}\right)$ $(i = 1(1)q,\ j = 1(1)p)$.

(c) Work out the correlation coefficient between score matrices and averaging score matrix by:

$$CC_r = \sum_{i=1}^{q} \frac{\sum_{j=1}^{p} \widetilde{\wp}\left(\chi_{ij}^{(r)}\right)\widetilde{\widetilde{\wp}}\left(\chi_{ij}\right)}{\sqrt{\sum_{j=1}^{p}\left(\widetilde{\wp}\left(\chi_{ij}^{(r)}\right)\right)^2}\sqrt{\sum_{j=1}^{p}\left(\widetilde{\widetilde{\wp}}\left(\chi_{ij}\right)\right)^2}}, \quad \forall r = 1(1)\gamma, \tag{25}$$

(d) The weight of the experts can be calculated by:

$$\theta_r = \frac{CC_r}{\sum_{l=1}^{\gamma} CC_l}, \quad \forall r = 1(1)\gamma, \tag{26}$$

where θ_r represents the weight of every expert with $\theta_r \geq 0$ and $\sum_{r=1}^{\gamma} \theta_r = 1$.

Step 2. *Attain the integrated evaluation matrices*:

The assessment values of the experts are fused by utilizing the IVFFWHAM operator or IVFFWDHAM operator to acquire the integrated IVFF evaluation matrices $A = \left(\chi_{ij}\right)_{m \times n}$, wherein:

$$\chi_{ij} = \left(\left[\phi_{ij}^{lf}, \phi_{ij}^{uf}\right], \left[\xi_{ij}^{lf}, \xi_{ij}^{uf}\right]\right) = IVFFWHAM^{(\kappa)}\left(\chi_{ij}^{(r)}, \chi_{ij}^{(r)}, \ldots, \chi_{ij}^{(r)}\right)$$
$$= \left(\begin{array}{l} \left[\sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^{\kappa}}\left(\sum\limits_{1\leq\varepsilon_1\leq\cdots\leq\varepsilon_\kappa\leq\gamma}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum\limits_{j=1}^{\kappa}\theta_{\varepsilon_j}\left(-\ln\left(\left(\phi_{\varepsilon_{ij}}^{lf(r)}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}, \right. \\ \left. \sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^{\kappa}}\left(\sum\limits_{1\leq\varepsilon_1\leq\cdots\leq\varepsilon_\kappa\leq\gamma}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum\limits_{j=1}^{\kappa}\theta_{\varepsilon_j}\left(-\ln\left(\left(\phi_{\varepsilon_{ij}}^{uf(r)}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}\right], \\ \left[\sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^{\kappa}}\left(\sum\limits_{1\leq\varepsilon_1\leq\cdots\leq\varepsilon_\kappa\leq\gamma}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum\limits_{j=1}^{\kappa}\theta_{\varepsilon_j}\left(-\ln\left(1-\left(\xi_{\varepsilon_{ij}}^{lf(r)}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}, \right. \\ \left. \sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^{\kappa}}\left(\sum\limits_{1\leq\varepsilon_1\leq\cdots\leq\varepsilon_\kappa\leq\gamma}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum\limits_{j=1}^{\kappa}\theta_{\varepsilon_j}\left(-\ln\left(1-\left(\xi_{\varepsilon_{ij}}^{uf(r)}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}\right] \end{array}\right), \tag{27a}$$

$$\chi_{ij} = \left(\left[\phi_{ij}^{lf}, \phi_{ij}^{uf}\right], \left[\xi_{ij}^{lf}, \xi_{ij}^{uf}\right]\right) = IVFFWDHAM^{(\kappa)}\left(\chi_{ij}^{(r)}, \chi_{ij}^{(r)}, \ldots, \chi_{ij}^{(r)}\right)$$
$$= \left(\begin{array}{l} \left[\sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^{\kappa}}\left(\sum\limits_{1\leq\varepsilon_1\leq\cdots\leq\varepsilon_\kappa\leq p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum\limits_{j=1}^{\kappa}\omega_{\varepsilon_j}\left(-\ln\left(1-\left(\phi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}, \right. \\ \left. \sqrt[3]{\exp\left\{-\left[\frac{1}{C_p^{\kappa}}\left(\sum\limits_{1\leq\varepsilon_1\leq\cdots\leq\varepsilon_\kappa\leq p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum\limits_{j=1}^{\kappa}\omega_{\varepsilon_j}\left(-\ln\left(1-\left(\phi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}\right], \\ \left[\sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^{\kappa}}\left(\sum\limits_{1\leq\varepsilon_1\leq\cdots\leq\varepsilon_\kappa\leq p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum\limits_{j=1}^{\kappa}\omega_{\varepsilon_j}\left(-\ln\left(\left(\xi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}, \right. \\ \left. \sqrt[3]{1-\exp\left\{-\left[\frac{1}{C_p^{\kappa}}\left(\sum\limits_{1\leq\varepsilon_1\leq\cdots\leq\varepsilon_\kappa\leq p}\left(-\ln\left(1-\exp\left\{-\left[\frac{1}{\kappa}\sum\limits_{j=1}^{\kappa}\omega_{\varepsilon_j}\left(-\ln\left(\left(\xi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\Re}\right]^{\frac{1}{\Re}}\right\}\right)\right)^{\Re}\right)\right]^{\frac{1}{\Re}}\right\}}\right] \end{array}\right). \tag{27b}$$

4.2 Computation of criterion weights

The weight of risk criterion provides a crucial role in the course of risk factor prioritization ranking. As one of the latest and most powerful weight determination approaches, LOPCOW was proposed by Ecer and Pamucar (2022) to compute the risk criteria importance based on percentage values of criteria. Owing to its advantages that offer simple and effective attainment of risk criterion weights, it has been applied to ascertain the importance of criteria in various actual applications. For instance, Ulutas et al. (2023) built a hybrid decision model by fusing MEREC, PSI, LOPCOW and multiple criteria ranking by alternative trace method to recommend the most effective natural fiber insulation material. Ecer et al. (2023) introduced a novel integrated decision framework named VIKOR-LOPCOW for selecting suitable unmanned aerial vehicles to support sustainable agricultural development. Demir et al. (2023) utilized the LOPCOW method to determine the weight of criteria in the evaluation of open data management systems used in the development of e-government. Niu et al. (2022) proposed an extended Fermatean cubic fuzzy LOPCOW method and further constructed a hybrid group decision method. Owing to the applications and merits of the LOPCOW model, we propound the IVFF-LOPCOW method to objectively ascertain risk criterion weights in this study. Let $\omega = \left(\omega_1, \omega_2, \cdots, \omega_p\right)^T$ be the weight vector of the risk criteria, which meets $\omega_j \in [0, 1]$ as well as $\sum_{j=1}^{p} \omega_j = 1$. The detailed steps of the IVFF-LOPCOW approach are illustrated in the following:

Step 1. *Obtain the normalized group evaluation matrix* $\mathbb{N} = \left(\eta_{ij}\right)_{m\times n}$:

$$\eta_{ij} = \begin{cases} \chi_{ij} = \left(\left[\phi_{ij}^{lf}, \phi_{ij}^{uf}\right], \left[\xi_{ij}^{lf}, \xi_{ij}^{uf}\right]\right), \ j \in C_b \\ \left(\chi_{ij}\right)^c \left(\left[\xi_{ij}^{lf}, \xi_{ij}^{uf}\right], \left[\phi_{ij}^{lf}, \phi_{ij}^{uf}\right]\right), \ j \in C_c \end{cases}, \quad \forall i = 1(1)q, \ j = 1(1)p. \tag{28}$$

Step 2. *Compute the score of the normalized group evaluation matrix*:

$$\widetilde{\wp}\left(\eta_{ij}\right) = \frac{1}{2}\left(\frac{1}{2}\left(\left(\phi_{ij}^{lf}\right)^3 + \left(\phi_{ij}^{uf}\right)^3 - \left(\xi_{ij}^{lf}\right)^3 - \left(\xi_{ij}^{uf}\right)^3\right) + 1\right), \quad \forall i = 1(1)q, \ j = 1(1)p. \tag{29}$$

Step 3. *Calculative the standard deviation*:

$$\vartheta_j = \sqrt{\frac{\sum_{i=1}^{q} \left(\widetilde{\wp}\left(\eta_{ij}\right) - \sum_{i=1}^{q} \widetilde{\wp}\left(\eta_{ij}\right) \Big/ q\right)}{q}}, \quad \forall j = 1(1)p. \tag{30}$$

Step 4. *Compute the percentage values for each criterion*:

$$PV_j = \left| \ln\left(\frac{\sqrt{\sum_{i=1}^{m} \left(\widetilde{\wp}\left(\eta_{ij}\right)\right)^2 \Big/ q}}{\vartheta_j}\right) \cdot 100 \right|, \quad \forall j = 1(1)p. \tag{31}$$

Step 5. *Find final weights of the criteria*:

$$\omega_j = \frac{PV_j}{\sum_{l=1}^{p} PV_l}, \quad \forall j = 1(1)p. \tag{32}$$

4.3 Prioritization of the schemes

The IVFF-ARAS method is presented to determine the prioritization of the considered schemes:

Step 1. *Determine the best assessment value of the schemes*:

$$\eta_0 = \begin{cases} \max\limits_{1\le i\le q} \eta_{ij}, \ j \in C_b \\ \min\limits_{1\le i\le q} \eta_{ij}, \ j \in C_c \end{cases}. \tag{33}$$

Step 2. *Discover the weighted normalized group assessment matrix* $\mathbb{R} = \left(\varsigma_{ij}\right)_{m\times n}$:

$$\varsigma_{ij} = \omega_j \cdot \psi_{ij} = \left(\left[\sqrt[3]{1-\exp\left\{-\left(\omega_j\left(-\ln\left(1-\left(\phi_{\varepsilon_j}^{lf}\right)^3\right)\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}}, \sqrt[3]{1-\exp\left\{-\left(\omega_j\left(-\ln\left(1-\left(\phi_{\varepsilon_j}^{uf}\right)^3\right)\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}} \right], \left[\sqrt[3]{\exp\left\{-\left(\omega_j\left(-\ln\left(\xi_{\varepsilon_j}^{lf}\right)^3\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}}, \sqrt[3]{\exp\left\{-\left(\omega_j\left(-\ln\left(\xi_{\varepsilon_j}^{uf}\right)^3\right)^{\Re}\right)^{\frac{1}{\Re}}\right\}} \right] \right),$$

$$\forall i = 1(1)q,\ j = 1(1)p. \tag{34}$$

Step 3. *Figure out the score index of the weighted normalized group assessment value*:

$$\widetilde{\wp}\left(\varsigma_{ij}\right) = \frac{1}{2}\left(\frac{1}{2}\left(\left(\phi_{ij}^{lf}\right)^3 + \left(\phi_{ij}^{uf}\right)^3 - \left(\xi_{ij}^{lf}\right)^3 - \left(\xi_{ij}^{uf}\right)^3\right) + 1\right),\quad \forall i = 1(1)q,\ j = 1(1)p. \tag{35}$$

Step 4. *Determine the comprehensive performance value of each scheme*:

$$T_i = \sum_{j=1}^{p} \widetilde{\wp}\left(\varsigma_{ij}\right),\quad \forall i = 1(1)q. \tag{36}$$

Step 5. *Ascertain the utility grade of the assessment schemes*:

$$\Xi_i = \frac{T_i}{T_0},\quad \forall i = 1(1)q, \tag{37}$$

where Ξ_i $(0 \le \Xi_i \le 1)$ is the utility grade of the scheme D_i, while T_0 is the utility grade of the optimal performance value η_0.

Step 6. *Attain the priority of the assessed schemes*:

$$T^* = \left\{ D_i \middle| \max_{1\le i\le q} \Xi_i,\ i = 1, 2, \cdots, q \right\}. \tag{38}$$

where the considered options are ranked in ascending order of the utility grade Ξ_i to determine the most satisfactory option.

5 Case study

The current section employed a case about the risk assessment in the R&D project of an industrial robot offline programming system to validate the feasibility and practicability of the novel FMEA model based on the IVFF-LOPCOW-ARAS methodology. Further, we explore the characteristics and advantages of the proposed FMEA model employing the parameters analysis and comparison study.

5.1 Decision analysis

The rapid development of Industry 4.0 has provided an opportunity for the transformation of the manufacturing industry. Enterprises can optimize the disposal of various links in their production process and improve their competitiveness by utilizing information and intelligent technology. As an important part of the manufacturing process, industrial robots provide scientific and technological support for the rapid development of the industry as well as the digital transformation of enterprises. In the development process of industrial robots, the offline programming system of industrial robots, as a core link in the research and manufacturing of industrial robots, is related to the quality and service life of robots. Therefore, identifying and analyzing potential risks in the development process of industrial robot offline programming systems has certain auxiliary value for improving the efficiency and quality of system development. A new energy vehicle company optimizes the offline programming system for industrial robots in the production chain to improve production efficiency. Considering the high costs involved in the system development process, the R&D department invited professionals to analyze and evaluate potential risks during the research and development process to reduce the cost and time consumption of research and development. First, the department invited three experts who come from the fields of system R&D and machinery manufacturing to consist of the FMEA team denoted as $\{E_1, E_2, E_3\}$. Then, based on the industrial robot offline programming system research and development project and the enterprise's new product research and development report, the FMEA team carries out the risk identification of a new energy vehicles company's industrial robot offline programming system research after consultation and discussion carefully and further determined the potential risk factors denoted as $\{RF_1, RF_2, RF_3, RF_4, RF_5\}$. The detailed explanations of these risk factors are shown in Table 1. This study applies the risk severity (O), risk frequency (S), and risk detectability (D) in the FMEA model as risk criteria to rank potential risks. Considering the uncertainty in the risk assessment process, this paper applies the proposed FMEA model

Table 1 Risks in the R&D project of industrial robot offline programming system

Risk categories	Risk descriptions
Demand risk (RF_1)	It refers to the risks caused by incomplete requirement information, unprofessional requirement analysts, frequent changes in requirements, and incomplete budget requirement assessment during the research and development process
Design risk (RF_2)	It refers to the risks caused by factors such as unreasonable system architecture design, overly complex software operation interfaces, lack of professional technical personnel, and lack of copyright awareness among developers during the research and development process
Development risk (RF_3)	It refers to the risks caused by factors such as non-standard code writing, incomplete source code preservation mechanism, and unstable system operation during the research and development process
Testing risks (RF_4)	It refers to the risks caused by factors such as the inability to conduct testing due to functional limitations during the research and development process, insufficient reserved testing time, lack of complete testing process instructions, and incomplete functional testing
Launch risk (RF_5)	It refers to the risks caused by factors such as complex system functions, insufficient preparation for system launch, and unstable system response speed during the research and development process

Table 2 Linguistic variables and their corresponding IVFF elements (Seikh and Mandal 2023)

Linguistic variables	IVFF numbers
Absolute-high-importance (AHI)	([0.85, 0.95], [0.05, 0.15])
Very-high-importance (VHI)	([0.75, 0.85], [0.15, 0.25])
High-importance (HI)	([0.65, 0.75], [0.25, 0.35])
Equal-importance (EI)	([0.5, 0.5], [0.5, 0.5])
Low-importance (LI)	([0.25, 0.35], [0.65, 0.75])
Very-low-importance (VLI)	([0.15, 0.25], [0.75, 0.85])
Absolutely-low-importance (ALI)	([0.05, 0.15], [0.85, 0.95])

Table 3 Linguistic risk assessment matrix given by the three experts

Risk factors	O			S			D		
	E_1	E_2	E_3	E_1	E_2	E_3	E_1	E_2	E_3
RF_1	HI	EI	LI	LI	LI	EI	EI	LI	LI
RF_2	VHI	AHI	HI	AHI	VHI	VHI	LI	VLI	VLI
RF_3	VHI	LI	HI	VHI	AHI	AHI	EI	LI	LI
RF_4	AHI	VHI	HI	HI	HI	EI	VHI	HI	VHI
RF_5	EI	EI	LI	VHI	HI	VHI	HI	VHI	HI

based on IVFF-LOPCOW-ARAS to evaluate and analyze the potential risks in the development process of industrial robot offline programming systems. The risk implementation procedures are presented in Table 1.

First of all, the risk factors and the risk criteria are determined based on experts' discussions and the FMEA model. The experts in the FMEA team are invited to assess the risk factors by utilizing the linguistic terms displayed in Table 2. In the risk assessment procedure, each risk factor RF_i is evaluated by the risk criteria O, S, and D. Hence, the risk assessment information of the potential five risk factors was assessed based on the three risk criteria and attained by the FMEA team using the linguistic terms displayed in Table 3. Further, the linguistic assessment information given by the FMEA team was shifted into IVFFNs, which were listed by the FMEA team in Tables 4, 5 and 6.

Based on the IVFF risk assessment matrices, we use the correlation coefficient-based approach to ascertain the importance degree of the experts. First, the score matrices of experts' risk assessment matrices and the averaging score matrix are attained by Eqs. (23) and (24), respectively. Then the correlation coefficients between score matrices and averaging score matrix are computed by Eq. (25) and listed as follows: $CC_1 = 4.9674$, $CC_2 = 4.9612$. Further, the importance degrees of expert are acquired by Eq. (26) and the results are displayed as: $\theta_1 = 0.3339$, $\theta_2 = 0.3334$, and $\theta_3 = 0.3327$. Lastly, the integrated IVFF risk evaluation matrix $A = \left(\chi_{ij}\right)_{m\times n}$ can be determined by the IVFFWHAM operator or the IVFFWDHAM operator displayed in Eq. (27). The computational outcome of integrated IVFF risk evaluation matrix is listed in Table 7.

In what follows, we shall identify the importance grade of the risk criteria by the proposed IVFF-LOPCOW method. We first normalize the integrated IVFF risk evaluation matrix by

Table 4 IVFF risk assessment matrix given by the expert E_1

Risk factors	O	S	D
RF_1	([0.65, 0.75], [0.25, 0.35])	([0.25, 0.35], [0.65, 0.75])	([0.50, 0.50], [0. 50, 0.50])
RF_2	([0.75, 0.85], [0.15, 0.25])	([0.85, 0.95], [0.05, 0.15])	([0.25, 0.35], [0.65, 0.75])
RF_3	([0.75, 0.85], [0.15, 0.25])	([0.75, 0.85], [0.15, 0.25])	([0.50, 0.50], [0. 50, 0.50])
RF_4	([0.85, 0.95], [0.05, 0.15])	([0.65, 0.75], [0.25, 0.35])	([0.75, 0.85], [0.15, 0.25])
RF_5	([0.50, 0.50], [0. 50, 0.50])	([0.75, 0.85], [0.15, 0.25])	([0.65, 0.75], [0.25, 0.35])

Table 5 IVFF risk assessment matrix given by the expert E_2

Risk factors	*O*	*S*	*D*
RF_1	([0.50, 0.50], [0. 50, 0.50])	([0.25, 0.35], [0.65, 0.75])	([0.25, 0.35], [0.65, 0.75])
RF_2	([0.85, 0.95], [0.05, 0.15])	([0.75, 0.85], [0.15, 0.25])	([0.15, 0.25], [0.75, 0.85])
RF_3	([0.25, 0.35], [0.65, 0.75])	([0.85, 0.95], [0.05, 0.15])	([0.25, 0.35], [0.65, 0.75])
RF_4	([0.75, 0.85], [0.15, 0.25])	([0.65, 0.75], [0.25, 0.35])	([0.65, 0.75], [0.25, 0.35])
RF_5	([0.50, 0.50], [0. 50, 0.50])	([0.65, 0.75], [0.25, 0.35])	([0.75, 0.85], [0.15, 0.25])

transforming the cost-type risk criteria to benefit-type criteria using Eq. (28). Then we get the score values of the integrated IVFF risk evaluation matrix by Eq. (29). Further, the standard deviation and percentage values for each risk criteria are attained by Eqs. (30) and (31), severally. The final weight of risk criteria could be figured out by Eq. (32). The mentioned computational outcomes of criteria weight are shown in Table 8.

In light of the results of the integrated risk assessment matrix and criterion weights, we present the improved ARAS method based on the IVFFAAHAM operators to determine the prioritization of the potential risks under the IVFF environment. First, the optimal risk assessment values for each risk criterion were identified by Eq. (33) and showed as η_0 =([0.8752, 0.9231], [0.1438 0.2172]), ([0.9138, 0.9627], [0.0685, 0.1489]), ([0.8630, 0.9138], [0.1489, 0.2246]. Then, we found the weighted normalized group risk assessment matrix with the aid of Eq. (34). The results are displayed in Table 9.

Next, the score of the weighted normalized group risk assessment matrix is obtained via Eq. (35) and listed in Table 10. Furthermore, the comprehensive assessment value T_i is determined by Eq. (36) as $T_0 = 2.3985$, $T_1 = 1.4632$, $T_2 = 1.9564$, $T_3 = 1.9654$, $T_4 = 2.2230$, and $T_5 = 2.0347$. The utility degree of each risk factor is computed by Eq. (37) as $\Xi_1 = 0.610$, $\Xi_2 = 0.8157$, $\Xi_3 = 0.8194$, $\Xi_4 = 0.9268$, and $\Xi_5 = 0.8483$.

Table 6 IVFF risk assessment matrix given by the expert E_3

Risk factors	O	S	D
RF_1	([0.25, 0.35], [0.65, 0.75])	([0.50, 0.50], [0. 50, 0.50])	([0.25, 0.35], [0.65, 0.75])
RF_2	([0.65, 0.75], [0.25, 0.35])	([0.75, 0.85], [0.15, 0.25])	([0.15, 0.25], [0.75, 0.85])
RF_3	([0.65, 0.75], [0.25, 0.35])	([0.85, 0.95], [0.05, 0.15])	([0.25, 0.35], [0.65, 0.75])
RF_4	([0.65, 0.75], [0.25, 0.35])	([0.50, 0.50], [0. 50, 0.50])	([0.75, 0.85], [0.15, 0.25])
RF_5	([0.25, 0.35], [0.65, 0.75])	([0.75, 0.85], [0.15, 0.25])	([0.65, 0.75], [0.25, 0.35])

Table 7 Integrated IVFF risk evaluation matrix

Risk factors	O	S	D
RF_1	([0.7123, 0.7352], [0.4217, 0.4533])	([0.5921, 0.6621], [0.4856, 0.5698])	([0.5922, 0.6621], [0.4855, 0.5697])
RF_2	([0.8752, 0.9230], [0.1438, 0.2173])	([0.8948, 0.9409], [0.1081, 0.1805])	([0.4691, 0.5718], [0.5766, 0.6756])
RF_3	([0.7871, 0.8447], [0.2820, 0.3637])	([0.9138, 0.9627], [0.0685, 0.1489])	([0.5922, 0.6621], [0.4855, 0.5697])
RF_4	([0.8752, 0.9231], [0.1438, 0.2472])	([0.7977, 0.8381], [0.2682, 0.3187])	([0.8630, 0.9138], [0.1489, 0.2246])
RF_5	([0.6831, 0.7031], [0.4374, 0.4716])	([0.8630, 0.9138], [0.1489, 0.2246])	([0.8435, 0.8948], [0.1805, 0.2540])

Table 8 Calculation outcomes of risk criteria weights

Risk factors	O	S	D
RF_1	0.6477	0.5496	0.5497
RF_2	0.8609	0.8856	0.4475
RF_3	0.7550	0.9129	0.5497
RF_4	0.8609	0.7611	0.8478
RF_5	0.6195	0.8478	0.8236
Standard deviations (ϑ_j)	0.1021	0.1313	0.1613
Percentage values (PV_j)	200.1753	180.9881	141.4066
Final weights (ω_j)	0.3831	0.3463	0.2706

Table 9 Weighted normalized IVFF risk evaluation matrix

Risk factors	O	S	D
η_0	([0.8210, 0.8768], [0.2446, 0.330])	([0.860, 0.9247], [0.1522, 0.2625])	([0.7863, 0.8462], [0.2917, 0.3806])
RF$_1$	([0.6527, 0.6752], [0.5342, 0.5629])	([0.5322, 0.5981], [0.6021, 0.6737])	([0.5190, 0.5837], [0.6266, 0.6949])
RF$_2$	([0.8209, 0.8768], [0.2446, 0.330])	([0.8374, 0.8944], [0.2096, 0.3005])	([0.4082, 0.5004], [0.7004, 0.7759])
RF$_3$	([0.7273, 0.7876], [0.3988, 0.4797])	([0.860, 0.9247], [0.1522, 0.2625])	([0.5190, 0.5837], [0.6266, 0.6949])
RF$_4$	([0.8210, 0.8768], [0.2446, 0.330])	([0.7318, 0.7743], [0.3979, 0.4480])	([0.7863, 0.8462], [0.7648, 0.8230])
RF$_5$	([0.6243, 0.6437], [0.5485, 0.5794])	([0.8013, 0.860], [0.9984, 0.9955])	([0.2917, 0.3806], [0.3304, 0.4122])

Table 10 The score of the weighted normalized group risk assessment matrix

Risk factors	O	S	D
Optimal performance value (η_0)	0.7942	0.8513	0.7530
RF$_1$	0.5638	0.4602	0.4393
RF$_2$	0.7942	0.8166	0.3456
RF$_3$	0.6749	0.8513	0.4393
RF$_4$	0.7942	0.6758	0.7530
RF$_5$	0.5376	0.7724	0.7246

Finally, the prioritization order of the risk factors can be ascertained by Eq. (38) as $RF_4 \succ RF_5 \succ RF_3 \succ RF_2 \succ RF_1$.

5.2 Sensitivity analysis

The stability and robustness of the FMEA model are very important for the experts to improve efficiency during the process of risk analysis. Hence, this part investigates the stability of the risk prioritization outcomes obtained by the introduced FMEA model based on IVFF-LOPCOW-ARAS. In the proposed model, the two parameters κ and $\Re$, which were used in the IVFFAAWHAM operator and the IVFFAAWDHAM operator, need to be further discussed. Also, their influences on the final risk priority raking outcomes have to be revealed.

The first sensitivity analysis discusses the impact of the parameter $\Re$ on the prioritization of the risk factors. The risk ranking results of the risk factors obtained using diverse values of $\Re$ are attained and displayed in Table 11. From these outcomes, it was found that the utility grade of risk factors RF_i increases with the increase of the parameter $\Re$. In addition, we can observe that the risk ranking outcomes obtained from different values of $\Re$ are all the same. This implies that the proposed FMEA model based on IVFF-LOPCOW-ARAS is stable and robust in terms of the parameter $\Re$.

Table 11 Utility grades and rankings of the risk factors obtained by different values of the parameter $\Re$

Parameter $\Re$	Utility grade of risk factors					Rankings
	RF_1	RF_2	RF_3	RF_4	RF_5	
$\Re = 1$	0.5571	0.8092	0.7804	0.9012	0.8060	$RF_4 \succ RF_5 \succ RF_3 \succ RF_2 \succ RF_1$
$\Re = 3$	0.610	0.8157	0.8194	0.9268	0.8483	$RF_4 \succ RF_5 \succ RF_3 \succ RF_2 \succ RF_1$
$\Re = 5$	0.6184	0.8150	0.8335	0.9332	0.8573	$RF_4 \succ RF_5 \succ RF_3 \succ RF_2 \succ RF_1$
$\Re = 7$	0.6201	0.8126	0.8375	0.9368	0.8611	$RF_4 \succ RF_5 \succ RF_3 \succ RF_2 \succ RF_1$
$\Re = 9$	0.6199	0.8103	0.8383	0.9390	0.8629	$RF_4 \succ RF_5 \succ RF_3 \succ RF_2 \succ RF_1$
$\Re = 11$	0.6192	0.8082	0.8382	0.9404	0.8638	$RF_4 \succ RF_5 \succ RF_3 \succ RF_2 \succ RF_1$
$\Re = 13$	0.6184	0.8066	0.8378	0.9413	0.8643	$RF_4 \succ RF_5 \succ RF_3 \succ RF_2 \succ RF_1$
$\Re = 15$	0.6178	0.8054	0.8373	0.9420	0.8646	$RF_4 \succ RF_5 \succ RF_3 \succ RF_2 \succ RF_1$

Table 12 Utility grades and rankings of the risk factors obtained by different values of the parameter κ

Parameter κ	Utility grade of risk factors					Rankings
	RF_1	RF_2	RF_3	RF_4	RF_5	
$\kappa = 1$	0.570	0.7884	0.8082	0.9004	0.7774	$RF_4 \succ RF_3 \succ RF_2 \succ RF_5 \succ RF_1$
$\kappa = 2$	0.610	0.8157	0.8194	0.9268	0.8483	$RF_4 \succ RF_5 \succ RF_3 \succ RF_2 \succ RF_1$
$\kappa = 3$	0.4565	0.7742	0.6788	0.8764	0.7811	$RF_4 \succ RF_5 \succ RF_2 \succ RF_3 \succ RF_1$

The second sensitivity analysis discusses the impact of the parameter κ on the prioritization of the risk factors. The parameter κ stands for the number of risk criteria with mutual relationships between them. Hence, we explored the impact of considering the interrelationships between different quantity criteria on the final ranking of the risk factors. The corresponding utility grade and priority of the risk factors concerning diverse values of the parameter κ are portrayed in Table 12. From Table 12, it was discovered that although the rankings of the risk factors obtained by different values of κ are not consistent, the most important risk factor was "testing risks" (RF_4). This implies that RF_4 is the most important thing concerned by the managers. They should strengthen risk monitoring and early warning here to further reasonably avoid accidents caused by risks. The main reason why the risk ranking obtained for $\kappa = 2$ is different from the other two situations is because this case considers the pairwise relationship between the risk criteria. It is important to take into account the correlation among the criteria during the procedure of risk analysis because different risk factors may need to consider multiple influences concerning the risk criteria. Besides, the managers can select an appropriate value of the parameter κ according to a requirement of practical applications.

5.3 Comparison analysis

The comparative analysis with some previous IVFF aggregation operators and methods is carried out to reflect the superiority and strength of the proposed FMEA model based on

IVFF-LOPCOW-ARAS for risk assessment and risk factors prioritization. The compared method includes the IVFFWA operator-based method (Rani and Mishra 2022), IVFFHWA operator-based method (Luqman and Shahzadi 2023), IVFFHWG operator-based method (Luqman and Shahzadi 2023), IVFF-WASPAS method (Rani and Mishra 2022), and IVFF-COPRAS method (Rani et al. 2022a). Based on the fused risk assessment information and the importance of the risk criteria, the aforesaid methods are adopted to determine the prioritization of the potential risk factors, the utility degree or score and ranking of the risk factors are attained and displayed in Table 13.

In light of the comparison outcomes, we can derive that the most important risk that needs to be taken seriously is "testing risks" (RF_4). It was attained by the mentioned approaches. This implies that the introduced FMEA model based on IVFF-LOPCOW-ARAS is efficient for risk analysis. In addition, the risk rankings of the potential risk factors are not the same. Further analysis of the extant methods with our approach is discussed and summarized in the following:

- *Comparison with the aggregated operator-based methods.* The methods based on the IVFFWA operator (Rani and Mishra 2022), the IVFFHWA operator (Rani and Mishra 2022), and the IVFFHWA operator (Luqman and Shahzadi 2023) are commonly used methods in decision analysis procedures. Although the aggregated operator-based methods could obtain the comprehensive score of the potential risk factors directly, they neglected the utility value and the correlation of the risk criteria in ascertaining a final utility grade of the risk factors. The inconsistency ranking outcome will be present when the criteria possess the inherent interrelationship. Nevertheless, the proposed IVFF-LOPCOW-ARAS methodology can not only obtain the utility grade of risk factors based on the improved ARAS method but also think over the interconnection among the considered risk criteria. It improves the rationality and accuracy of the final risk ranking result.
- *Comparison with the IVFF-WASPAS method* (Rani and Mishra 2022). The IVFF-WASPAS method was presented based on the IVFFWA operator and the IVFFWG operator. The IVFFWA operator and the IVFFWG operator were defined based on algebraic operations, which not only lacked flexibility but also overlooked the inherent interrelationships among risk criteria. Besides, the final risk ranking determined by the IVFF-WASPAS method took into account the weighted sum and weighted product measures, which failed to consider the influence of the optimal preference on the other risk preferences. In the proposed method, the flexibility and interrelationship among the risk criteria are taken into consideration by the proposed IVFF Hamy mean operators. The IVFF-ARAS method is propounded to rapidly acquire a reasonable ranking of the potential risk factors.
- *The comparison with the IVFF-COPRAS method* (Rani et al. 2022a). This method was proposed based on the IVFFEWA operators and the CRITIC method using a novel score function. Although the IVFF-COPRAS method can attain the ranking of risk factors based on the utility degree of risk factors, it ignores the interrelationship among the risk criteria. Because the IVFF-COPRAS approach was propounded based on the IVFF Einstein operators, it assumed that the fused IVFF information is independent of each other. Hence, the proposed IVFF-LOPCOW-ARAS methodology is more flexible and universal than the IVFF-COPRAS method.

Based upon the aforesaid comparison discussions and analysis, the important features of the compared methods are summarized in Table 14. The proposed FMEA model based on IVFF-LOPCOW-ARAS provides an efficient and feasible risk analysis model for experts or risk managers. It not only thinks about the correlation among the risk criteria but also offers a

Table 13 Decision results obtained by different decision approaches

Parameter $\Re$	Assessment index of risk factors					Rankings
	RF_1	RF_2	RF_3	RF_4	RF_5	
IVFFWA operator (Rani and Mishra 2022)	0.1822	0.640	0.6136	0.6578	0.5595	$RF_4 \succ RF_2 \succ RF_3 \succ RF_5 \succ RF_1$
IVFFHWA operator (Luqman and Shahzadi 2023)	0.2384	0.6203	0.5986	0.6605	0.5570	$RF_4 \succ RF_2 \succ RF_3 \succ RF_5 \succ RF_1$
IVFFHWG operator (Luqman and Shahzadi 2023)	0.0245	0.1939	0.2228	0.3213	0.2310	$RF_4 \succ RF_5 \succ RF_3 \succ RF_2 \succ RF_1$
IVFF-WASPAS (Rani and Mishra 2022)	0.1745	0.5199	0.5331	0.6490	0.5178	$RF_4 \succ RF_3 \succ RF_2 \succ RF_5 \succ RF_1$
IVFF-COPRAS (Rani et al. 2022a)	33.67%	95.22%	91.55%	100%	84.63%	$RF_4 \succ RF_2 \succ RF_3 \succ RF_5 \succ RF_1$
Propounded method	0.610	0.8157	0.8194	0.9268	0.8483	$RF_4 \succ RF_2 \succ RF_3 \succ RF_5 \succ RF_1$

Table 14 Features comparison from different approaches

Methods	MCDM procedure	Weight of experts	Weight of criteria	Flexibility in fusion process	Consider correlation among criteria	Optimal option	Standards
IVFFWA operator (Rani and Mishra 2022)	MCDM	No	Linear model	No	No	RF_4	Method based on IVFFWA operator
IVFFHWA operator (Luqman and Shahzadi 2023)	MCDM	No	Assume	Yes	No	RF_4	Method based on IVFFHWA operator
IVFFHWG operator (Luqman and Shahzadi 2023)	MCDM	No	Assume	Yes	No	RF_4	Method based on IVFFHWG operator
IVFF-WASPAS (Rani and Mishra 2022)	MCDM	No	Linear model	No	No	RF_4	IVFF-WASPAS method
IVFF-COPRAS (Rani et al. 2022a)	MCDM	No	CRITIC	No	No	RF_4	IVFF-COPRAS method based on IVFFEWA operator
Propounded method	Group	Correlation coefficient-based method	LOPCOW method	Yes	Yes	RF_4	IVFF-ARAS method based on IVFF-LOPCOW method

novel IVFF criteria weight determination model under an uncertain environment. Concretely, the merits of the proposed FMEA model novel can be summarized as follows:

(1) The propounded novel FMEA model within IVFF sets can effectively deal with the uncertainty and ambiguity that arise in the process of risk analysis.
(2) The novel FMEA model is presented based on a group decision methodology that takes into account the situation of completely unknown weight information. Hence, the proposed FMEA model based on IVFF-LOPCOW-ARAS is more universal and feasible than the prior FMEA model.
(3) In the proposed FMEA model, the importance of FMEA members is determined by the correlation coefficient-based method with IVFF information, which affords a more reliable weight outcome and makes the integrated risk assessment information more reasonable.
(4) The developed FMEA model can seize the interrelationship among the considered risk criteria in the course of determining the prioritization of risk factors.

6 Conclusions

The FMEA methodology is a synthetic and powerful tool that has been applied to various applications for evaluating and prioritizing potential failure modes. Nevertheless, the existing extensions of FMEA possess several deficiencies during the procedure of risk assessment and ranking. To surmount the mentioned shortcomings and enhance the efficiency of the classical FMEA model, this study introduced the innovative FMEA model based on IVFF-LOPCOW-ARAS by combining the proposed IVFFWHAM operator, IVFFWDHAM operator, LOPCOW, and ARAS methods under the IVFF environment. In the proposed FMEA model, IVFFNs were utilized by the FMEA team to articulate the uncertain preference and evaluations for the potential risk factors over each risk criterion. Then, the weights of the FMEA team members were determined by a correlation coefficient-based method, while the risk criterion weights were obtained by the improved IVFF-LOPCOW method. Further, the enhanced ARAS approach with the proposed IVFFWHAM and IVFFWDHAM operators was presented for the assessment and prioritization of risk factors. It can simultaneously take into consideration the correlation between risk factors as well as the flexibility of the integration process. Next, the actual practicability and feasibility of the presented FMEA framework were confirmed by an application to assess the risks in the R&D project of industrial robot offline programming systems. Lastly, the sensitivity and comparison studies were performed to expound the stability and preponderance of the FMEA model based on IVFF-LOPCOW-ARAS.

The benefits of the presented FMEA model were investigated by comparing it with other similar approaches. However, the current work has some weaknesses that need to be explored. First, the FMEA model prioritizes the risk factors through the three risk criteria, which possess certain boundedness for different applications. Thus, it is an interesting work to consider other risk criteria from different perspectives to unfold the risk analysis more comprehensively. Then, the psychological characteristics of the FMEA team were neglected in the proposed FMEA framework. This might lead to a biased risk-ranking outcome. Therefore, the establishment of a new FEMA model by combining the behavioral decision theory is also a distinguished topic for practical application risk management.

Further research will be devoted to the following directions. First, the mentioned weaknesses of the proposed method should be addressed to fill the gaps in the FMEA model. Then, the presented FMEA model can be applied to other uncertain and ambiguous risk assessment and prioritization problems with IVFF information. Further, some novel extensions of classical FMEA mode should be investigated under diverse uncertain and vague environments, such as linguistic Z-numbers (Chai et al. 2023; Liu et al. 2023), R numbers (Liu et al. 2021; Zhao et al. 2022), probabilistic double hierarchy linguistic term sets (Wang et al. 2022; Xian et al. 2023), and so forth. In addition, it is also an interesting research direction to build a novel FMEA framework by incorporating the consensus-reaching process and social network large-scale group decision framework with complex linguistic information (Gai et al. 2023; Gou et al. 2021; Ji et al. 2023; Sun et al. 2023).

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval All persons who meet authorship criteria are listed as authors, and all authors certify that they have participated sufficiently in the work to take public responsibility for the content, including participation in the concept, design, analysis, writing, or revision of the manuscript. Furthermore, each author certifies that this material or similar material has not been and will not be submitted to or published in any other publication before its appearance in the journal.

Appendix: List of abbreviations

ARAS	Additive ratio assessment
COPRAS	COmplex proportional assessment
FFSs	Fermatean FSs
FMEA	Failure modes and effects analysis
FSs	Fuzzy sets
FUCOM	Full consistency method
IFSs	Intuitionistic FSs
IVFF	Interval-valued Fermatean fuzzy
IVFFAAHAM	Interval-valued Fermatean fuzzy Aczel–Alsina Hamy mean
IVFFAAHAM	Interval-valued Fermatean fuzzy Aczel–Alsina dual Hamy mean
IVFFAAWHAM	Interval-valued Fermatean fuzzy Aczel–Alsina weighted Hamy mean
IVFFAAWHAM	Interval-valued Fermatean fuzzy Aczel–Alsina weighted dual Hamy mean

IVFFWA	Interval-valued Fermatean Fuzzy weighted averaging
IVFFWG	Interval-valued Fermatean Fuzzy weighted geometric
IVIFSs	Interval-valued intuitionistic FSs
IVPFSs	Interval-valued pythagorean FSs
LOPCOW	Logarithmic percentage change-driven objective weighting
MCDM	Multi-criteria decision-making
MCGDM	Multi-criteria group decision-making
MEREC	MEthod based on the removal effects of criteria
PSI	Preference selection index
VIKOR	Vsekriterijumska optimizacija I KOmpromisno Resenje (VIKOR)
CRITIC	CRiteria interaction through inter-criteria correlation
PFSs	Pythagorean FSs
PROMETHEE II	Preference ranking organization method for enrichment evaluations II
RPN	Risk priority number
SWARA	Step wise weight assessment ratio analysis
WASPAS	Weighted aggregated sum product assessment

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