



Multi-attribute decision-making methods based on Aczel–Alsina power aggregation operators for managing complex intuitionistic fuzzy sets

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Abstract

Aczel–Alsina t-norm and t-conorm were derived by Aczel and Alsina in 1982, which were the modified or extended forms of algebraic t-norm and t-conorm. Furthermore, the power aggregation operator is also a very significant and precious method that is used for evaluating the finest preference from the collection of finite preferences. Inspired by the above theories, we concentrate to derive the theory of Aczel–Alsina power aggregation operators for C-IF information, such as C-IFAAPA, C-IFAAWPA, C-IFAAPG, and C-IFAAWPG operators. Moreover, we derive the theory of idempotency and prove that the property of monotonicity and boundedness failed with the help of some counterexamples. Additionally, we evaluate a MADM approach to derive operators in the presence of C-IF information. Finally, we illustrate practical examples for comparing the derived work with various existing or prevailing operators to show the supremacy and proficiency of the derived information.

Keywords Aczel–Alsina power aggregation operators · Complex intuitionistic fuzzy sets · Multi-attribute decision-making methods

Mathematics Subject Classification 94D05 · 03B52 · 03E72 · 28E10 · 68T27

Abbreviations

C-IF Complex intuitionistic fuzzy
MG Membership grade

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NMG	Non-membership grade
C-IFAAPA	Complex intuitionistic fuzzy Aczel–Alsina power averaging
C-IFAAWPA	Complex intuitionistic fuzzy Aczel–Alsina weighted power averaging
C-IFAAPG	Complex intuitionistic fuzzy Aczel–Alsina power geometric
C-IFAAWPG	Complex intuitionistic fuzzy Aczel–Alsina weighted power geometric
MADM	Multi-attribute decision-making
FS	Fuzzy set
IFS	Intuitionistic fuzzy set
C-FS	Complex fuzzy set
C-IFS	Complex intuitionistic fuzzy set

1 Introduction

Modeling ambiguities in MADM procedures are the main and important part of the decision-making technique to address certain genuine life problems in the environment of modern research areas. Many MADM methods have been derived for evaluating a problem in road signals, artificial intelligence, and the robustness of human opinion in such a way that a collection of preferences is derived under the consideration of many attributes. Simultaneously, the ranking of preferences is derived and eventually, a beneficial preference is selected among the efficient ones. MADM is a dominantly employed cognitive mathematical technique, the major impact of which is to derive among a finite number of preferences using the optimal data given by experts. However, MADM tools manage to be awkward and unreliable, it requires the ambiguity of human reasoning capabilities, evaluating it tricky for experts in the review procedure which give feasible resolve. Because of ambiguity and complications in genuine life problems, as well as unreliable and insufficient information or awkward data, it is complicated for the experts to take a beneficial optimal. The dilemma of complex data has become a main problem for the last few years. To evaluate the above dilemmas, a fuzzy set (FS) (Zadeh 1965) was deliberated in 1965 by modifying a function contained in the crisp set. The function of MG $\overline{\Psi}_R(\tilde{\mu}_\varrho) \in [0, 1]$, wherein the term classical information the value of MG $\overline{\Psi}_R(\tilde{\mu}_\varrho) \in \{0, 1\}$. Furthermore, the fuzzy superior mandelbrot set was derived by Mahmood and Ali (2022), the fuzzy N-soft set was pioneered by Akram et al. (2018), and the multi-fuzzy N-soft set was deliberated by Fatimah and Alcantud (2021). To involve the NMG in the environment of FS is a very challenging task for all experts because, in various genuine life cases, experts faced a certain problem that contained the MG and NMG continuously. For this, intuitionistic FS (IFS) (Atanassov 1986) was derived in 1986. Additionally, IFS managed two grades at a time, called MG “ $\overline{\Psi}_R(\tilde{\mu}_\varrho)$ ” and NMG “ $\overline{\Xi}_R(\tilde{\mu}_\varrho)$ ” with a characteristic: $\overline{\Psi}_R(\tilde{\mu}_\varrho) + \overline{\Xi}_R(\tilde{\mu}_\varrho) \in [0, 1]$, where the idea of FS and crisp set is the simple case of IFSs. Moreover, distance measures for IFS were derived by Garg and Rani (2022), Choquet-integral aggregation operators for IFS were pioneered by Jia and Wang (2022), and certain scholars have utilized different types of information for IFS, such as the extended MAIRA technique (Ecer 2022), image segmentation (Jebadass and Balasubramaniam 2022), twin support vector machine (Liang and Zhang 2022), investigation of the evolutionary procedure (Yu et al. 2022), and bipolar soft sets (Mahmood 2020).

To utilize phase terms in the mathematical shape of MG “ $\overline{\Psi}_R(\tilde{\mu}_\varrho)$ ” is a very challenging task for all scholars. Because in certain places, experts personally felt that phase term is very valuable for some real-life problems, for instance, if we want to purchase a new branded car

based on two pieces of valuable information, called the model of the car and production date of the car, wherewith this kind of information, FS has failed to evaluate it. For this, Ramot et al. (2002) derived the theory of complex FS (C-FS) in 2002 by modifying a function contained in the FS. The C-FS contained a truth grade in the form of a complex number, whose amplitude term and phase term are contained in the unit interval, where the model of the car represented the amplitude term, and the production date of the car represented the phase term of the complex number. The function of MG $\overline{\Psi}_R(\widetilde{\mu}_\varrho) \in [0, 1]$ in FS was extended to the MG $\overline{\Psi}_R(\widetilde{\mu}_\varrho), \overline{\Psi}_I(\widetilde{\mu}_\varrho) \in [0, 1]$ in C-FS, where the complete shape of MG in C-FS is of the shape: $\overline{\Psi}_{\Theta_\Lambda}(\widetilde{\mu}_\varrho) = \overline{\Psi}_R(\widetilde{\mu}_\varrho)e^{i2\pi(\overline{\Psi}_I(\widetilde{\mu}_\varrho))}$. Furthermore, distance measures for C-FS were derived by Liu et al. (2020), the theory of complex fuzzy N-soft set was derived by Mahmood and Ali (2021), complex multi-fuzzy information was invented by Al-Qudah and Hassan (2018), and finally, the theory of complex fuzzy logic was derived by Tamir et al. (2015). To involve the complex-valued NMG in the environment of C-FS is a very challenging task for all experts because, in various genuine life cases, experts faced a certain problem that contained the complex-valued MG and complex-valued NMG continuously. For this, complex IFS (C-IFS) was derived by Alkouri and Salleh (2012). Additionally, C-IFS managed two grades at a time, called complex-valued MG “ $\overline{\Psi}_{\Theta_\Lambda}(\widetilde{\mu}_\varrho) = \overline{\Psi}_R(\widetilde{\mu}_\varrho)e^{i2\pi(\overline{\Psi}_I(\widetilde{\mu}_\varrho))}$,” and complex-valued NMG “ $\overline{\Xi}_{\Theta_\Lambda}(\widetilde{\mu}_\varrho) = \overline{\Xi}_R(\widetilde{\mu}_\varrho)e^{i2\pi(\overline{\Xi}_I(\widetilde{\mu}_\varrho))}$,” with a characteristic: $\overline{\Psi}_R(\widetilde{\mu}_\varrho) + \overline{\Xi}_R(\widetilde{\mu}_\varrho) \in [0, 1], \overline{\Psi}_I(\widetilde{\mu}_\varrho) + \overline{\Xi}_I(\widetilde{\mu}_\varrho) \in [0, 1]$, where the idea of FS, C-FS, and IFS is the simple case of C-IFSs. Moreover, another form of C-IF soft set was derived by Ali et al. (2021), aggregation operators based on generalized C-IFS were pioneered by Garg and Rani (2019), the new ranking technique for C-IFS with the help of new aggregation operators was invented by Garg and Rani (2020a), generalized geometric aggregation operators for C-IFS was deliberated by Garg and Rani (2020b), and robust aggregation information for C-IFS was proposed by Garg and Rani (2020c).

Coping with ambiguity and uncertainty is a very awkward and challenging task for scholars and for handling such sorts of problems, many scholars have derived different types of theories. Similarly, Aczel and Alsina (1982) derived some novel t-norm and t-conorm, called Aczel–Alsina t-norm and t-conorm in 1982. These derived norms received much attention from different scholars and certain people have utilized them in the environment of different fields, for instance, Pamucar et al. (2022) invented the sustainable transportation problems based on Aczel–Alsina information. Furthermore, Senapati et al. (2022a) derived the theory of hesitant fuzzy Aczel–Alsina aggregation operators. The theory of Aczel–Alsina aggregation operators for IFS was invented by Senapati et al. (2022b). Moreover, Aczel–Alsina aggregation operators for interval-valued IFSs were derived by Senapati et al. (2022c), geometric aggregation operators for IFSs using Aczel–Alsina norms were derived by Senapati et al. (2023), Aczel–Alsina aggregation operators for C-IFS was derived by Mahmood et al. (2022), Aczel–Alsina aggregation operators for bipolar C-FS was derived by Mahmood et al. (2023), and finally, the Aczel–Alsina prioritized aggregation operators for IFS was proposed by Sarfraz et al. (2022). Additionally, power aggregation operators (PAOs) (Yager 2001) are a significant and precious method which are used for combining the collection of preferences into one preference, and because of this reason experts can take easily proceed with their procedure. PAOs for IFS were derived by Xu (2011) and the PAOs for C-FS were invented by Hu et al. (2019). Finally, nowadays, Rani and Garg (2018) presented the PAOs for C-IFSs. After a lot of analysis, we noticed that the experts have the following major issues, such as

1. How do we derive new aggregation operators?
2. How do we aggregate the collection of the finite number of preferences into a singleton set?
3. How do we derive the finest or most valuable preferences from the collection of preferences?

Managing the above queries is a very challenging task for individuals because the above three complications are the backbone of every article, and managing the above queries is complicated. Furthermore, it is also a very awkward and challenging task for all scholars to combine the theory of PAOs based on Aczel–Alsina t-norm and t-conorm, whereas the theory of PAOs based on Aczel–Alsina t-norm and t-conorm for FSs, IFSs, CFSs, and C-IFSs were does not derive yet, it means that the derived work in this analysis is very valuable and novel. The major advantages of the proposed work are listed below:

1. Putting the value of $\overline{\Xi}_{\Theta_\Lambda}(\widetilde{\mu}_\varrho) = 0$ in the derived theory, then we will be getting the power Aczel–Alsina aggregation operators for C-FS.
2. Putting the value of $\overline{\Xi}_{\overline{I}}(\widetilde{\mu}_\varrho) = \overline{\Psi}_{\overline{I}}(\widetilde{\mu}_\varrho) = 0$ in the derived theory, then we will be getting the power Aczel–Alsina aggregation operators for IFS.
3. Putting the value of $\overline{\Xi}_{\overline{I}}(\widetilde{\mu}_\varrho) = \overline{\Psi}_{\overline{I}}(\widetilde{\mu}_\varrho) = \overline{\Xi}_{\overline{R}}(\widetilde{\mu}_\varrho) = 0$ in the derived theory, then we will be getting the power Aczel–Alsina aggregation operators for FS.
4. Putting the value of $\overline{\Xi}_{\Theta_\Lambda}(\widetilde{\mu}_\varrho) = 0$ and removing the power aggregation operator in the derived theory, then we will be getting the Aczel–Alsina aggregation operators for C-FS.
5. Putting the value of $\overline{\Xi}_{\overline{I}}(\widetilde{\mu}_\varrho) = \overline{\Psi}_{\overline{I}}(\widetilde{\mu}_\varrho) = 0$ and removing the power aggregation operator in the derived theory, then we will be getting the Aczel–Alsina aggregation operators for IFS.
6. Putting the value of $\overline{\Xi}_{\overline{I}}(\widetilde{\mu}_\varrho) = \overline{\Psi}_{\overline{I}}(\widetilde{\mu}_\varrho) = \overline{\Xi}_{\overline{R}}(\widetilde{\mu}_\varrho) = 0$ and removing the power aggregation operator in the derived theory, then we will be getting the Aczel–Alsina aggregation operators for FS.
7. Putting the value of $\overline{\Xi}_{\Theta_\Lambda}(\widetilde{\mu}_\varrho) = 0$ and removing the Aczel–Alsina operational laws in the derived theory, then we will be getting the power aggregation operators for C-FS.
8. Putting the value of $\overline{\Xi}_{\overline{I}}(\widetilde{\mu}_\varrho) = \overline{\Psi}_{\overline{I}}(\widetilde{\mu}_\varrho) = 0$ and removing the Aczel–Alsina operational laws in the derived theory, then we will be getting the power aggregation operators for IFS.
9. Putting the value of $\overline{\Xi}_{\overline{I}}(\widetilde{\mu}_\varrho) = \overline{\Psi}_{\overline{I}}(\widetilde{\mu}_\varrho) = \overline{\Xi}_{\overline{R}}(\widetilde{\mu}_\varrho) = 0$ and removing the Aczel–Alsina operational laws in the derived theory, then we will be getting the power aggregation operators for FS.

Noticed that the proposed work has a lot of advantages because from the derived operators we can easily obtain a bundle of ideas. Therefore, inspired by the above benefits, we derive the following ideas:

1. To derive the theory of C-IFAAPA and C-IFAAWPA operators.
2. To pioneer the theory of C-IFAAPG and C-IFAAWPG operators.
3. To demonstrate the property of Monotonicity and boundedness failed for the above derived all operators and justify it with the help of an example.
4. To illustrate MADM application based on pioneered operators is to show the feasibility of the presented approaches.
5. To compare the derived work with some prevailing approaches.

Our derived information is computed or divided into the following parts: In Sect. 2, in Sect. 3, we derived the theory of C-IFAAPA, C-IFAAWPA, C-IFAAPG, and C-IFAAWPG

operators and evaluated the significant idempotency property. We also proved that the property of monotonicity and boundedness have failed and justified it with the help of some counterexamples. In Sect. 4, in the presence of the above desirable information, a MADM approach is examined by using the theory of C-IF information. An illustrative practical example related to the consideration of the beneficial preference is selected to evaluate the supremacy and proficiency of the derived information. In Sect. 5, we verified the comparative analysis between the proposed results with various prevailing results. In Sect. 6, we mentioned the final remarks.

2 Preliminaries

Here, we mentioned some prevailing derived ideas, called PAOs, C-IFSs, and their Aczel–Alsina operational laws with some more operational information. Where the universal set is stated by: \widetilde{X}_U .

Definition 1 (Senapati et al. 2022c) Let $\overline{\overline{\Theta}}_{\Lambda_j}, j = 1, 2, \dots, n$, be the finite family of positive information. The mathematical information:

$$PA\left(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_2}, \dots, \overline{\overline{\Theta}}_{\Lambda_n}\right) = \frac{\sum_{j=1}^n \left(1 + \Gamma\left(\overline{\overline{\Theta}}_{\Lambda_j}\right)\right) \overline{\overline{\Theta}}_{\Lambda_j}}{\sum_{j=1}^n \left(1 + \Gamma\left(\overline{\overline{\Theta}}_{\Lambda_j}\right)\right)}. \tag{1}$$

Called PA operator with $\Gamma\left(\overline{\overline{\Theta}}_{\Lambda_j}\right) = \sum_{\substack{s=1 \\ s \neq j}}^n \text{SUP}\left(\overline{\overline{\Theta}}_{\Lambda_j}, \overline{\overline{\Theta}}_{\Lambda_s}\right)$, and the term

$\text{SUP}\left(\overline{\overline{\Theta}}_{\Lambda_j}, \overline{\overline{\Theta}}_{\Lambda_s}\right)$ represents as support of $\overline{\overline{\Theta}}_{\Lambda_j}$ from $\overline{\overline{\Theta}}_{\Lambda_s}$ with some limitations:

1. $\text{SUP}\left(\overline{\overline{\Theta}}_{\Lambda_j}, \overline{\overline{\Theta}}_{\Lambda_s}\right) \in [0, 1]$.
2. $\text{SUP}\left(\overline{\overline{\Theta}}_{\Lambda_j}, \overline{\overline{\Theta}}_{\Lambda_s}\right) = \text{SUP}\left(\overline{\overline{\Theta}}_{\Lambda_s}, \overline{\overline{\Theta}}_{\Lambda_j}\right)$.
3. $\text{SUP}\left(\overline{\overline{\Theta}}_{\Lambda_j}, \overline{\overline{\Theta}}_{\Lambda_s}\right) \geq \text{SUP}\left(\overline{\overline{\Theta}}_{\Lambda_k}, \overline{\overline{\Theta}}_{\Lambda_l}\right)$ when $\left|\overline{\overline{\Theta}}_{\Lambda_j} - \overline{\overline{\Theta}}_{\Lambda_s}\right| \leq \left|\overline{\overline{\Theta}}_{\Lambda_k} - \overline{\overline{\Theta}}_{\Lambda_l}\right|$.

Definition 2 (Alkouri and Salleh 2012) The mathematical information:

$$\overline{\overline{\Theta}}_{\Lambda} = \left\{ \left(\overline{\overline{\Psi}}_{\overline{\overline{\Theta}}_{\Lambda}}(\widetilde{\mu}_{\varrho}), \overline{\overline{\Xi}}_{\overline{\overline{\Theta}}_{\Lambda}}(\widetilde{\mu}_{\varrho}) \right) : \widetilde{\mu}_{\varrho} \in \widetilde{X}_U \right\}. \tag{2}$$

Stated as a C-IF set. Further, we considered $\overline{\overline{\Psi}}_{\overline{\overline{\Theta}}_{\Lambda}}(\widetilde{\mu}_{\varrho}) = \overline{\overline{\Psi}}_{\overline{\overline{R}}_{\Lambda}}(\widetilde{\mu}_{\varrho}) e^{i2\pi\left(\overline{\overline{\Psi}}_{\overline{\overline{T}}_{\Lambda}}(\widetilde{\mu}_{\varrho})\right)}$ and $\overline{\overline{\Xi}}_{\overline{\overline{\Theta}}_{\Lambda}}(\widetilde{\mu}_{\varrho}) = \overline{\overline{\Xi}}_{\overline{\overline{R}}_{\Lambda}}(\widetilde{\mu}_{\varrho}) e^{i2\pi\left(\overline{\overline{\Xi}}_{\overline{\overline{T}}_{\Lambda}}(\widetilde{\mu}_{\varrho})\right)}$ as an MG and NMG. Moreover, the main characteristic of the C-IF set is stated by: $0 \leq \overline{\overline{\Psi}}_{\overline{\overline{R}}_{\Lambda}}(\widetilde{\mu}_{\varrho}) + \overline{\overline{\Xi}}_{\overline{\overline{R}}_{\Lambda}}(\widetilde{\mu}_{\varrho}) \leq 1$ and $0 \leq \overline{\overline{\Psi}}_{\overline{\overline{T}}_{\Lambda}}(\widetilde{\mu}_{\varrho}) + \overline{\overline{\Xi}}_{\overline{\overline{T}}_{\Lambda}}(\widetilde{\mu}_{\varrho}) \leq 1$. Assume that the mathematical structure of the refusal grade is stated by: $\overline{\overline{\mathfrak{R}}}_{\overline{\overline{\Theta}}_{\Lambda}}(\widetilde{\mu}_{\varrho}) = \overline{\overline{\mathfrak{R}}}_{\overline{\overline{R}}_{\Lambda}}(\widetilde{\mu}_{\varrho}) e^{i2\pi\left(\overline{\overline{\mathfrak{R}}}_{\overline{\overline{T}}_{\Lambda}}(\widetilde{\mu}_{\varrho})\right)} = \left(1 - \left(\overline{\overline{\Psi}}_{\overline{\overline{R}}_{\Lambda}}(\widetilde{\mu}_{\varrho}) + \overline{\overline{\Xi}}_{\overline{\overline{R}}_{\Lambda}}(\widetilde{\mu}_{\varrho})\right)\right) e^{i2\pi\left(1 - \left(\overline{\overline{\Psi}}_{\overline{\overline{T}}_{\Lambda}}(\widetilde{\mu}_{\varrho}) + \overline{\overline{\Xi}}_{\overline{\overline{T}}_{\Lambda}}(\widetilde{\mu}_{\varrho})\right)\right)}$, with a new shape, which is represented as a C-IF number (C-IFN): $\overline{\overline{\Theta}}_{\Lambda_j} = \left(\overline{\overline{\Psi}}_{\overline{\overline{R}}_j} e^{i2\pi\left(\overline{\overline{\Psi}}_{\overline{\overline{T}}_j}\right)}, \overline{\overline{\Xi}}_{\overline{\overline{R}}_j} e^{i2\pi\left(\overline{\overline{\Xi}}_{\overline{\overline{T}}_j}\right)} \right), j = 1, 2, \dots, n$. Additionally, in the consideration of any

two C-IFNs $\overline{\Theta}_{\Lambda_j} = \left(\overline{\Psi}_{R_j} e^{i2\pi \left(\overline{\Psi}_{I_j} \right)}, \overline{\Xi}_{R_j} e^{i2\pi \left(\overline{\Xi}_{I_j} \right)} \right), j = 1, 2$, we derive the following AA operational laws, such as:

$$\overline{\Theta}_{\Lambda_1} \oplus \overline{\Theta}_{\Lambda_2} = \left(\left(1 - e^{-\left((-\log(1-\overline{\Psi}_{R_1}))^\xi + (-\log(1-\overline{\Psi}_{R_2}))^\xi \right)^{\frac{1}{\xi}}} \right)^{i2\pi \left(1 - e^{-\left((-\log(1-\overline{\Psi}_{I_1}))^\xi + (-\log(1-\overline{\Psi}_{I_2}))^\xi \right)^{\frac{1}{\xi}}} \right)}, \left(e^{-\left((-\log(\overline{\Xi}_{R_1}))^\xi + (-\log(\overline{\Xi}_{R_2}))^\xi \right)^{\frac{1}{\xi}}} \right)^{i2\pi \left(e^{-\left((-\log(\overline{\Xi}_{I_1}))^\xi + (-\log(\overline{\Xi}_{I_2}))^\xi \right)^{\frac{1}{\xi}}} \right)} \right), \tag{3}$$

$$\overline{\Theta}_{\Lambda_1} \otimes \overline{\Theta}_{\Lambda_2} = \left(\left(e^{-\left((-\log(\overline{\Psi}_{R_1}))^\xi + (-\log(\overline{\Psi}_{R_2}))^\xi \right)^{\frac{1}{\xi}}} \right)^{i2\pi \left(e^{-\left((-\log(\overline{\Psi}_{I_1}))^\xi + (-\log(\overline{\Psi}_{I_2}))^\xi \right)^{\frac{1}{\xi}}} \right)}, \left(1 - e^{-\left((-\log(1-\overline{\Xi}_{R_1}))^\xi + (-\log(1-\overline{\Xi}_{R_2}))^\xi \right)^{\frac{1}{\xi}}} \right)^{i2\pi \left(1 - e^{-\left((-\log(1-\overline{\Xi}_{I_1}))^\xi + (-\log(1-\overline{\Xi}_{I_2}))^\xi \right)^{\frac{1}{\xi}}} \right)} \right), \tag{4}$$

$$\overline{\theta}_S \overline{\Theta}_{\Lambda_1} = \left(\left(1 - e^{-\left(\overline{\theta}_S (-\log(1-\overline{\Psi}_{R_1}))^\xi \right)^{\frac{1}{\overline{\theta}_S}}} \right)^{i2\pi \left(1 - e^{-\left(\overline{\theta}_S (-\log(1-\overline{\Psi}_{I_1}))^\xi \right)^{\frac{1}{\overline{\theta}_S}}} \right)}, \left(e^{-\left(\overline{\theta}_S (-\log(\overline{\Xi}_{R_1}))^\xi \right)^{\frac{1}{\overline{\theta}_S}}} \right)^{i2\pi \left(e^{-\left(\overline{\theta}_S (-\log(\overline{\Xi}_{I_1}))^\xi \right)^{\frac{1}{\overline{\theta}_S}}} \right)} \right), \tag{5}$$

$$\overline{\Theta}_{\Lambda_1}^{\overline{\theta}_S} = \left(\left(e^{-\left(\overline{\theta}_S (-\log(\overline{\Psi}_{R_1}))^\xi \right)^{\frac{1}{\overline{\theta}_S}}} \right)^{i2\pi \left(e^{-\left(\overline{\theta}_S (-\log(\overline{\Psi}_{I_1}))^\xi \right)^{\frac{1}{\overline{\theta}_S}}} \right)}, \left(1 - e^{-\left(\overline{\theta}_S (-\log(1-\overline{\Xi}_{R_1}))^\xi \right)^{\frac{1}{\overline{\theta}_S}}} \right)^{i2\pi \left(1 - e^{-\left(\overline{\theta}_S (-\log(1-\overline{\Xi}_{I_1}))^\xi \right)^{\frac{1}{\overline{\theta}_S}}} \right)} \right), \tag{6}$$

Definition 3 In the consideration of any two C-IFNs $\overline{\Theta}_{\Lambda_j} = \left(\overline{\Psi}_{R_j} e^{i2\pi \left(\overline{\Psi}_{I_j} \right)}, \overline{\Xi}_{R_j} e^{i2\pi \left(\overline{\Xi}_{I_j} \right)} \right), j = 1, 2$, we derive the following information, such as:

$$\overline{\omega}_{SV}(\overline{\Theta}_{\Lambda_1}) = \frac{1}{2} (\overline{\Psi}_{R_1} + \overline{\Psi}_{I_1} - \overline{\Xi}_{R_1} - \overline{\Xi}_{I_1}), \overline{\omega}_{SV}(\overline{\Theta}_{\Lambda_1}) \in [-1, 1], \tag{7}$$

$$\overline{\omega}_{AV}(\overline{\Theta}_{\Lambda_1}) = \frac{1}{2} (\overline{\Psi}_{R_1} + \overline{\Psi}_{I_1} + \overline{\Xi}_{R_1} + \overline{\Xi}_{I_1}), \overline{\omega}_{AV}(\overline{\Theta}_{\Lambda_1}) \in [0, 1]. \tag{8}$$

Stated as a score and accuracy values, with some limitations, such as:

1. When $\overline{\overline{\omega_{SV}}}(\overline{\overline{\Theta_{\Lambda_1}}}) > \overline{\overline{\omega_{SV}}}(\overline{\overline{\Theta_{\Lambda_2}}}) \Rightarrow \overline{\overline{\Theta_{\Lambda_1}}} > \overline{\overline{\Theta_{\Lambda_2}}}$;
2. When $\overline{\overline{\omega_{SV}}}(\overline{\overline{\Theta_{\Lambda_1}}}) < \overline{\overline{\omega_{SV}}}(\overline{\overline{\Theta_{\Lambda_2}}}) \Rightarrow \overline{\overline{\Theta_{\Lambda_1}}} < \overline{\overline{\Theta_{\Lambda_2}}}$;
3. When $\overline{\overline{\omega_{SV}}}(\overline{\overline{\Theta_{\Lambda_1}}}) = \overline{\overline{\omega_{SV}}}(\overline{\overline{\Theta_{\Lambda_2}}}) \Rightarrow$;
- (i) When $\overline{\overline{\omega_{AV}}}(\overline{\overline{\Theta_{\Lambda_1}}}) > \overline{\overline{\omega_{AV}}}(\overline{\overline{\Theta_{\Lambda_2}}}) \Rightarrow \overline{\overline{\Theta_{\Lambda_1}}} > \overline{\overline{\Theta_{\Lambda_2}}}$;
- (ii) When $\overline{\overline{\omega_{AV}}}(\overline{\overline{\Theta_{\Lambda_1}}}) < \overline{\overline{\omega_{AV}}}(\overline{\overline{\Theta_{\Lambda_2}}}) \Rightarrow \overline{\overline{\Theta_{\Lambda_1}}} < \overline{\overline{\Theta_{\Lambda_2}}}$;
- (iii) When $\overline{\overline{\omega_{AV}}}(\overline{\overline{\Theta_{\Lambda_1}}}) = \overline{\overline{\omega_{AV}}}(\overline{\overline{\Theta_{\Lambda_2}}}) \Rightarrow \overline{\overline{\Theta_{\Lambda_1}}} = \overline{\overline{\Theta_{\Lambda_2}}}$.

Theorem 1 In the consideration of any family of C-IFNs $\overline{\overline{\Theta_{\Lambda_j}}} = \left(\overline{\overline{\Psi_{R_j}}} e^{i2\pi \left(\overline{\overline{\Psi_{T_j}}} \right)}, \overline{\overline{\Xi_{R_j}}} e^{i2\pi \left(\overline{\overline{\Xi_{T_j}}} \right)} \right), j = 1, 2, \dots, n$, we have derived the following results, such as:

1. $\overline{\overline{\Theta_{\Lambda_1}}} \oplus \overline{\overline{\Theta_{\Lambda_2}}} = \overline{\overline{\Theta_{\Lambda_2}}} \oplus \overline{\overline{\Theta_{\Lambda_1}}}$;
2. $\overline{\overline{\Theta_{\Lambda_1}}} \otimes \overline{\overline{\Theta_{\Lambda_2}}} = \overline{\overline{\Theta_{\Lambda_2}}} \otimes \overline{\overline{\Theta_{\Lambda_1}}}$;
3. $\overline{\overline{\theta_S}}(\overline{\overline{\Theta_{\Lambda_1}}} \oplus \overline{\overline{\Theta_{\Lambda_2}}}) = \overline{\overline{\theta_S}} \overline{\overline{\Theta_{\Lambda_1}}} \oplus \overline{\overline{\theta_S}} \overline{\overline{\Theta_{\Lambda_2}}}$;
4. $(\overline{\overline{\theta_{S_1}}} + \overline{\overline{\theta_{S_1}}}) \overline{\overline{\Theta_{\Lambda_1}}} = \overline{\overline{\theta_{S_1}}} \overline{\overline{\Theta_{\Lambda_1}}} \oplus \overline{\overline{\theta_{S_2}}} \overline{\overline{\Theta_{\Lambda_1}}}$;
5. $(\overline{\overline{\Theta_{\Lambda_1}}} \otimes \overline{\overline{\Theta_{\Lambda_2}}})^{\overline{\overline{\theta_S}}} = \overline{\overline{\Theta_{\Lambda_1}}}^{\overline{\overline{\theta_S}}} \otimes \overline{\overline{\Theta_{\Lambda_2}}}^{\overline{\overline{\theta_S}}}$;
6. $\overline{\overline{\Theta_{\Lambda_1}}}^{\overline{\overline{\theta_{S_1}}}} \otimes \overline{\overline{\Theta_{\Lambda_1}}}^{\overline{\overline{\theta_{S_1}}}} = \overline{\overline{\Theta_{\Lambda_1}}}^{\overline{\overline{\theta_{S_1}} + \overline{\overline{\theta_{S_2}}}}}$.

3 PAOs based on Aczel–Alsina t-norm and t-conorm for C-IFs

Here, we derived the theory of C-IFAAPA, C-IFAAWPA, C-IFAAPG, and C-IFAAWPG operators and evaluated the significant idempotency property. We also proved that the property of monotonicity and boundedness have failed and justified it with the help of some counterexamples.

Definition 4 In the consideration of any family of C-IFNs $\overline{\overline{\Theta_{\Lambda_j}}} = \left(\overline{\overline{\Psi_{R_j}}} e^{i2\pi \left(\overline{\overline{\Psi_{T_j}}} \right)}, \overline{\overline{\Xi_{R_j}}} e^{i2\pi \left(\overline{\overline{\Xi_{T_j}}} \right)} \right), j = 1, 2, \dots, n$, we derive the idea of the C-IFAAPA operator, such as:

$$\text{C-IFAAPA}(\overline{\overline{\Theta_{\Lambda_1}}}, \overline{\overline{\Theta_{\Lambda_2}}}, \dots, \overline{\overline{\Theta_{\Lambda_n}}}) = \overline{\overline{\mathbb{Z}_1}} \overline{\overline{\Theta_{\Lambda_1}}} \oplus \overline{\overline{\mathbb{Z}_2}} \overline{\overline{\Theta_{\Lambda_2}}} \oplus \dots \oplus \overline{\overline{\mathbb{Z}_n}} \overline{\overline{\Theta_{\Lambda_n}}} = \bigoplus_{j=1}^n (\overline{\overline{\mathbb{Z}_j}} \overline{\overline{\Theta_{\Lambda_j}}}), \tag{9}$$

where,

$$\overline{\overline{\mathbb{Z}_j}} = \text{PA}(\overline{\overline{\Theta_{\Lambda_1}}}, \overline{\overline{\Theta_{\Lambda_2}}}, \dots, \overline{\overline{\Theta_{\Lambda_n}}}) = \frac{(1 + \Gamma(\overline{\overline{\Theta_{\Lambda_j}}}))}{\sum_{j=1}^n (1 + \Gamma(\overline{\overline{\Theta_{\Lambda_j}}})}. \tag{10}$$

Called PA operator with $\Gamma(\overline{\Theta}_{\Lambda_j}) = \sum_{s=1, s \neq j}^n \text{SUP}(\overline{\Theta}_{\Lambda_j}, \overline{\Theta}_{\Lambda_s})$, and the term $\text{SUP}(\overline{\Theta}_{\Lambda_j}, \overline{\Theta}_{\Lambda_s}) = 1 - d(\overline{\Theta}_{\Lambda_j}, \overline{\Theta}_{\Lambda_s})$ represents as support of $\overline{\Theta}_{\Lambda_j}$ from $\overline{\Theta}_{\Lambda_s}$ and $d(\overline{\Theta}_{\Lambda_j}, \overline{\Theta}_{\Lambda_s}) = \frac{1}{4} \left(\left| \overline{\Psi}_{R_j} - \overline{\Psi}_{R_s} \right| + \left| \overline{\Psi}_{I_j} - \overline{\Psi}_{I_s} \right| + \left| \overline{\Xi}_{R_j} - \overline{\Xi}_{R_s} \right| + \left| \overline{\Xi}_{I_j} - \overline{\Xi}_{I_s} \right| \right)$ used as a distance measure.

Theorem 2 In the consideration of any family of C-IFNs $\overline{\Theta}_{\Lambda_j} = \left(\overline{\Psi}_{R_j} e^{i2\pi \left(\frac{\overline{\Psi}_{I_j}}{I_j} \right)}, \overline{\Xi}_{R_j} e^{i2\pi \left(\frac{\overline{\Xi}_{I_j}}{I_j} \right)} \right), j = 1, 2, \dots, n$, we derive that the idea of C-IFAAPA operator is given again a C-IFN, such as:

$$\begin{aligned} & \text{C-IFAAPA}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}, \dots, \overline{\Theta}_{\Lambda_n}) \\ &= \left(\begin{aligned} & \left(1 - e^{-\left(\sum_{j=1}^n \overline{Z}_j \left(-\log \left(1 - \overline{\Psi}_{R_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)^{\frac{1}{\xi}} e^{i2\pi \left(1 - e^{-\left(\sum_{j=1}^n \overline{Z}_j \left(-\log \left(1 - \overline{\Psi}_{I_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)^{\frac{1}{\xi}}} \right), \\ & \left(e^{-\left(\sum_{j=1}^n \overline{Z}_j \left(-\log \left(\overline{\Xi}_{R_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)^{\frac{1}{\xi}} e^{i2\pi \left(e^{-\left(\sum_{j=1}^n \overline{Z}_j \left(-\log \left(\overline{\Xi}_{I_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)^{\frac{1}{\xi}}} \right) \end{aligned} \right). \end{aligned} \tag{11}$$

Proof To derive the theory given in Eq. (11), we consider the procedure of mathematical induction. Therefore, we derive the Eq. (11) for the value of $n = 2$, such as:

$$\overline{Z}_1 \overline{\Theta}_{\Lambda_1} = \left(\begin{aligned} & \left(1 - e^{-\left(\overline{Z}_1 \left(-\log \left(1 - \overline{\Psi}_{R_1} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)^{\frac{1}{\xi}} e^{i2\pi \left(1 - e^{-\left(\overline{Z}_1 \left(-\log \left(1 - \overline{\Psi}_{I_1} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)^{\frac{1}{\xi}}} \right), \\ & \left(e^{-\left(\overline{Z}_1 \left(-\log \left(\overline{\Xi}_{R_1} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)^{\frac{1}{\xi}} e^{i2\pi \left(e^{-\left(\overline{Z}_1 \left(-\log \left(\overline{\Xi}_{I_1} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)^{\frac{1}{\xi}}} \right) \end{aligned} \right),$$

$$\overline{\overline{\mathbb{Z}}_2 \Theta_{\Lambda_2}} = \left(\begin{array}{c} \left(\begin{array}{c} 1 - e^{-\left(\overline{\overline{\mathbb{Z}}_2}(-\log(1-\overline{\overline{\Psi}}_{R_2}))\right)^\xi} \right)^{\frac{1}{\xi}} \\ i2\pi \left(1 - e^{-\left(\overline{\overline{\mathbb{Z}}_2}(-\log(1-\overline{\overline{\Psi}}_{I_2}))\right)^\xi} \right)^{\frac{1}{\xi}} \end{array} \right) e \\ \left(\begin{array}{c} e^{-\left(\overline{\overline{\mathbb{Z}}_2}(-\log(\overline{\overline{\Theta}}_{R_2}))\right)^\xi} \right)^{\frac{1}{\xi}} \\ i2\pi \left(e^{-\left(\overline{\overline{\mathbb{Z}}_2}(-\log(\overline{\overline{\Theta}}_{I_2}))\right)^\xi} \right)^{\frac{1}{\xi}} \end{array} \right) \end{array} \right),$$

then

$$\begin{aligned} \text{C-IFAAPA}(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_2}) &= \overline{\overline{\mathbb{Z}}_1 \Theta_{\Lambda_1}} \oplus \overline{\overline{\mathbb{Z}}_2 \Theta_{\Lambda_2}} \\ &= \left(\begin{array}{c} \left(\begin{array}{c} 1 - e^{-\left(\overline{\overline{\mathbb{Z}}_1}(-\log(1-\overline{\overline{\Psi}}_{R_1}))\right)^\xi} \right)^{\frac{1}{\xi}} \\ i2\pi \left(1 - e^{-\left(\overline{\overline{\mathbb{Z}}_1}(-\log(1-\overline{\overline{\Psi}}_{I_1}))\right)^\xi} \right)^{\frac{1}{\xi}} \end{array} \right) e \\ \left(\begin{array}{c} e^{-\left(\overline{\overline{\mathbb{Z}}_1}(-\log(\overline{\overline{\Theta}}_{R_1}))\right)^\xi} \right)^{\frac{1}{\xi}} \\ i2\pi \left(e^{-\left(\overline{\overline{\mathbb{Z}}_1}(-\log(\overline{\overline{\Theta}}_{I_1}))\right)^\xi} \right)^{\frac{1}{\xi}} \end{array} \right) \end{array} \right) \\ \oplus \left(\begin{array}{c} \left(\begin{array}{c} 1 - e^{-\left(\overline{\overline{\mathbb{Z}}_2}(-\log(1-\overline{\overline{\Psi}}_{R_2}))\right)^\xi} \right)^{\frac{1}{\xi}} \\ i2\pi \left(1 - e^{-\left(\overline{\overline{\mathbb{Z}}_2}(-\log(1-\overline{\overline{\Psi}}_{I_2}))\right)^\xi} \right)^{\frac{1}{\xi}} \end{array} \right) e \\ \left(\begin{array}{c} e^{-\left(\overline{\overline{\mathbb{Z}}_2}(-\log(\overline{\overline{\Theta}}_{R_2}))\right)^\xi} \right)^{\frac{1}{\xi}} \\ i2\pi \left(e^{-\left(\overline{\overline{\mathbb{Z}}_2}(-\log(\overline{\overline{\Theta}}_{I_2}))\right)^\xi} \right)^{\frac{1}{\xi}} \end{array} \right) \end{array} \right) \\ \left(\begin{array}{c} \left(\begin{array}{c} 1 - e^{-\left(\sum_{j=1}^2 \overline{\overline{\mathbb{Z}}_j}(-\log(1-\overline{\overline{\Psi}}_{R_j}))\right)^\xi} \right)^{\frac{1}{\xi}} \\ i2\pi \left(1 - e^{-\left(\sum_{j=1}^2 \overline{\overline{\mathbb{Z}}_j}(-\log(1-\overline{\overline{\Psi}}_{I_j}))\right)^\xi} \right)^{\frac{1}{\xi}} \end{array} \right) e \\ \left(\begin{array}{c} e^{-\left(\sum_{j=1}^2 \overline{\overline{\mathbb{Z}}_j}(-\log(\overline{\overline{\Theta}}_{R_j}))\right)^\xi} \right)^{\frac{1}{\xi}} \\ i2\pi \left(e^{-\left(\sum_{j=1}^2 \overline{\overline{\mathbb{Z}}_j}(-\log(\overline{\overline{\Theta}}_{I_j}))\right)^\xi} \right)^{\frac{1}{\xi}} \end{array} \right) \end{array} \right). \end{aligned}$$

Hence, Eq. (11) is satisfied, further, we try to write it for the value of $n = k$, such as:

$$\begin{aligned}
 & \text{C-IFAAPA}(\overline{\Theta_{\Lambda_1}}, \overline{\Theta_{\Lambda_2}}, \dots, \overline{\Theta_{\Lambda_k}}) \\
 &= \left(\begin{array}{l} \left(\begin{array}{l} 1 - e^{-\left(\sum_{j=1}^k \overline{\mathbb{Z}}_j \left(-\log\left(1 - \overline{\Psi}_{R_j}\right)\right)\right)^\xi} \right)^{\frac{1}{\xi}} \\ e^{-\left(\sum_{j=1}^k \overline{\mathbb{Z}}_j \left(-\log\left(1 - \overline{\Psi}_{R_j}\right)\right)\right)^\xi} \end{array} \right)^{\frac{1}{\xi}} i2\pi \left(\begin{array}{l} 1 - e^{-\left(\sum_{j=1}^k \overline{\mathbb{Z}}_j \left(-\log\left(1 - \overline{\Psi}_{I_j}\right)\right)\right)^\xi} \\ e^{-\left(\sum_{j=1}^k \overline{\mathbb{Z}}_j \left(-\log\left(1 - \overline{\Psi}_{I_j}\right)\right)\right)^\xi} \end{array} \right)^{\frac{1}{\xi}} \end{array} \right), \\
 & \left(\begin{array}{l} e^{-\left(\sum_{j=1}^k \overline{\mathbb{Z}}_j \left(-\log\left(\overline{\Xi}_{R_j}\right)\right)\right)^\xi} \\ e^{-\left(\sum_{j=1}^k \overline{\mathbb{Z}}_j \left(-\log\left(\overline{\Xi}_{R_j}\right)\right)\right)^\xi} \end{array} \right)^{\frac{1}{\xi}} i2\pi \left(\begin{array}{l} e^{-\left(\sum_{j=1}^k \overline{\mathbb{Z}}_j \left(-\log\left(\overline{\Xi}_{I_j}\right)\right)\right)^\xi} \\ e^{-\left(\sum_{j=1}^k \overline{\mathbb{Z}}_j \left(-\log\left(\overline{\Xi}_{I_j}\right)\right)\right)^\xi} \end{array} \right)^{\frac{1}{\xi}} \end{array} \right).
 \end{aligned}$$

Moreover, we derive it for the value of $n = k + 1$, such as:

$$\begin{aligned}
 & \text{C-IFAAPA}(\overline{\Theta_{\Lambda_1}}, \overline{\Theta_{\Lambda_2}}, \dots, \overline{\Theta_{\Lambda_{k+1}}}) = \oplus_{j=1}^{k+1} (\overline{\mathbb{Z}}_j \overline{\Theta_{\Lambda_j}}) = \oplus_{j=1}^k (\overline{\mathbb{Z}}_j \overline{\Theta_{\Lambda_j}}) \oplus \overline{\mathbb{Z}}_{k+1} \overline{\Theta_{\Lambda_{k+1}}} \\
 &= \left(\begin{array}{l} \left(\begin{array}{l} 1 - e^{-\left(\sum_{j=1}^k \overline{\mathbb{Z}}_j \left(-\log\left(1 - \overline{\Psi}_{R_j}\right)\right)\right)^\xi} \right)^{\frac{1}{\xi}} \\ e^{-\left(\sum_{j=1}^k \overline{\mathbb{Z}}_j \left(-\log\left(1 - \overline{\Psi}_{R_j}\right)\right)\right)^\xi} \end{array} \right)^{\frac{1}{\xi}} i2\pi \left(\begin{array}{l} 1 - e^{-\left(\sum_{j=1}^k \overline{\mathbb{Z}}_j \left(-\log\left(1 - \overline{\Psi}_{I_j}\right)\right)\right)^\xi} \\ e^{-\left(\sum_{j=1}^k \overline{\mathbb{Z}}_j \left(-\log\left(1 - \overline{\Psi}_{I_j}\right)\right)\right)^\xi} \end{array} \right)^{\frac{1}{\xi}} \end{array} \right), \\
 & \left(\begin{array}{l} e^{-\left(\sum_{j=1}^k \overline{\mathbb{Z}}_j \left(-\log\left(\overline{\Xi}_{R_j}\right)\right)\right)^\xi} \\ e^{-\left(\sum_{j=1}^k \overline{\mathbb{Z}}_j \left(-\log\left(\overline{\Xi}_{R_j}\right)\right)\right)^\xi} \end{array} \right)^{\frac{1}{\xi}} i2\pi \left(\begin{array}{l} e^{-\left(\sum_{j=1}^k \overline{\mathbb{Z}}_j \left(-\log\left(\overline{\Xi}_{I_j}\right)\right)\right)^\xi} \\ e^{-\left(\sum_{j=1}^k \overline{\mathbb{Z}}_j \left(-\log\left(\overline{\Xi}_{I_j}\right)\right)\right)^\xi} \end{array} \right)^{\frac{1}{\xi}} \end{array} \right) \\
 & \oplus \left(\begin{array}{l} \left(\begin{array}{l} 1 - e^{-\left(\overline{\mathbb{Z}}_{k+1} \left(-\log\left(1 - \overline{\Psi}_{R_{k+1}}\right)\right)\right)^\xi} \right)^{\frac{1}{\xi}} \\ e^{-\left(\overline{\mathbb{Z}}_{k+1} \left(-\log\left(1 - \overline{\Psi}_{R_{k+1}}\right)\right)\right)^\xi} \end{array} \right)^{\frac{1}{\xi}} i2\pi \left(\begin{array}{l} 1 - e^{-\left(\overline{\mathbb{Z}}_{k+1} \left(-\log\left(1 - \overline{\Psi}_{I_{k+1}}\right)\right)\right)^\xi} \\ e^{-\left(\overline{\mathbb{Z}}_{k+1} \left(-\log\left(1 - \overline{\Psi}_{I_{k+1}}\right)\right)\right)^\xi} \end{array} \right)^{\frac{1}{\xi}} \end{array} \right), \\
 & \left(\begin{array}{l} e^{-\left(\overline{\mathbb{Z}}_{k+1} \left(-\log\left(\overline{\Xi}_{R_{k+1}}\right)\right)\right)^\xi} \\ e^{-\left(\overline{\mathbb{Z}}_{k+1} \left(-\log\left(\overline{\Xi}_{R_{k+1}}\right)\right)\right)^\xi} \end{array} \right)^{\frac{1}{\xi}} i2\pi \left(\begin{array}{l} e^{-\left(\overline{\mathbb{Z}}_{k+1} \left(-\log\left(\overline{\Xi}_{I_{k+1}}\right)\right)\right)^\xi} \\ e^{-\left(\overline{\mathbb{Z}}_{k+1} \left(-\log\left(\overline{\Xi}_{I_{k+1}}\right)\right)\right)^\xi} \end{array} \right)^{\frac{1}{\xi}} \end{array} \right)
 \end{aligned}$$

$$= \left(\begin{array}{c} \left(\left(1 - e^{-\left(\sum_{j=1}^{k+1} \overline{\overline{Z}}_j \left(-\log \left(1 - \overline{\overline{\Psi}}_{R_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)^{\frac{1}{\xi}} e^{i2\pi \left(1 - e^{-\left(\sum_{j=1}^{k+1} \overline{\overline{Z}}_j \left(-\log \left(1 - \overline{\overline{\Psi}}_{I_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)^{\frac{1}{\xi}}} \right) \\ \left(e^{-\left(\sum_{j=1}^{k+1} \overline{\overline{Z}}_j \left(-\log \left(\overline{\overline{\Psi}}_{R_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} e^{i2\pi \left(e^{-\left(\sum_{j=1}^{k+1} \overline{\overline{Z}}_j \left(-\log \left(\overline{\overline{\Psi}}_{I_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)^{\frac{1}{\xi}}} \right) \end{array} \right).$$

Finally, we get our mentioned information. Further, we aim to justify the information in Eq. (11) with the help of some suitable examples. For this, we use three C-IFNs, such as: $\overline{\overline{\Theta}}_{\Lambda_1} = (0.2e^{i2\pi(0.2)}, 0.2e^{i2\pi(0.2)})$, $\overline{\overline{\Theta}}_{\Lambda_2} = (0.4e^{i2\pi(0.4)}, 0.4e^{i2\pi(0.4)})$ and $\overline{\overline{\Theta}}_{\Lambda_3} = (0.1e^{i2\pi(0.1)}, 0.89e^{i2\pi(0.89)})$ for $\xi = 2$, then by using the information in Eq. (11), we get

$$\begin{aligned} d(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_2}) &= \frac{1}{4}(|0.2 - 0.4| + |0.2 - 0.4| + |0.2 - 0.4| + |0.2 - 0.4|) = 0.2 \\ d(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_3}) &= 0.395; \quad d(\overline{\overline{\Theta}}_{\Lambda_2}, \overline{\overline{\Theta}}_{\Lambda_3}) = 0.395 \\ \text{SUP}(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_2}) &= 1 - d(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_2}) = 1 - 0.2 = 0.8 \\ \text{SUP}(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_3}) &= 0.605; \quad \text{SUP}(\overline{\overline{\Theta}}_{\Lambda_2}, \overline{\overline{\Theta}}_{\Lambda_3}) = 0.605 \end{aligned}$$

Then,

$$\begin{aligned} \Gamma(\overline{\overline{\Theta}}_{\Lambda_1}) &= \sum_{\substack{s=1, \\ s \neq 1}}^n \text{SUP}(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_s}) \\ &= \text{SUP}(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_2}) + \text{SUP}(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_3}) = 0.8 + 0.605 = 1.405 \\ \Gamma(\overline{\overline{\Theta}}_{\Lambda_2}) &= 1.405; \quad \Gamma(\overline{\overline{\Theta}}_{\Lambda_3}) = 1.21 \\ &\sum_{j=1}^n (1 + \Gamma(\overline{\overline{\Theta}}_{\Lambda_j})) = 7.02 \end{aligned}$$

Then,

$$\begin{aligned} \overline{\overline{Z}}_1 &= \frac{(1 + \Gamma(\overline{\overline{\Theta}}_{\Lambda_1}))}{\sum_{j=1}^3 (1 + \Gamma(\overline{\overline{\Theta}}_{\Lambda_j}))} = \frac{1 + 1.405}{7.02} = 0.34259 \\ \overline{\overline{Z}}_2 &= 0.34259; \quad \overline{\overline{Z}}_3 = 0.31481. \end{aligned}$$

Therefore, we get the final result, such as:

$$\text{C-IFAAPA}(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_2}, \overline{\overline{\Theta}}_{\Lambda_3}) = (0.13412e^{i2\pi(0.13412)}, 0.62398e^{i2\pi(0.62398)}).$$

Idempotency 1 In the consideration of any family of C-IFNs $\overline{\overline{\Theta}}_{\Lambda_j} = \left(\overline{\overline{\Psi}}_{R_j} e^{i2\pi \left(\overline{\overline{\Psi}}_{I_j} \right)}, \overline{\overline{\Xi}}_{R_j} e^{i2\pi \left(\overline{\overline{\Xi}}_{I_j} \right)} \right), j = 1, 2, \dots, n$, when $\overline{\overline{\Theta}}_{\Lambda_j} = \overline{\overline{\Theta}}$, then we derive the below theory, such as:

$$\text{C-IFAAPA} \left(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_2}, \dots, \overline{\overline{\Theta}}_{\Lambda_n} \right) = \overline{\overline{\Theta}}. \tag{12}$$

Proof Using the available above information $\overline{\overline{\Theta}}_{\Lambda_j} = \overline{\overline{\Theta}} = \left(\overline{\overline{\Psi}}_{R_j} e^{i2\pi \left(\overline{\overline{\Psi}}_{I_j} \right)}, \overline{\overline{\Xi}}_{R_j} e^{i2\pi \left(\overline{\overline{\Xi}}_{I_j} \right)} \right)$, then we have

$$\begin{aligned} & \text{C-IFAAPA} \left(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_2}, \dots, \overline{\overline{\Theta}}_{\Lambda_n} \right) \\ &= \left(\begin{array}{l} \left(\left(1 - e^{-\left(\sum_{j=1}^n \overline{\overline{Z}}_j \left(-\log \left(1 - \overline{\overline{\Psi}}_{R_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right) e^{i2\pi \left(1 - e^{-\left(\sum_{j=1}^n \overline{\overline{Z}}_j \left(-\log \left(1 - \overline{\overline{\Psi}}_{I_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)} \right) \\ \left(e^{-\left(\sum_{j=1}^n \overline{\overline{Z}}_j \left(-\log \left(\overline{\overline{\Xi}}_{R_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right) e^{i2\pi \left(e^{-\left(\sum_{j=1}^n \overline{\overline{Z}}_j \left(-\log \left(\overline{\overline{\Xi}}_{I_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)} \right) \end{array} \right), \\ &= \left(\begin{array}{l} \left(1 - e^{-\left(\left(-\log \left(1 - \overline{\overline{\Psi}}_{R_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right) e^{i2\pi \left(1 - e^{-\left(\left(-\log \left(1 - \overline{\overline{\Psi}}_{I_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)} \right) \\ \left(e^{-\left(\left(-\log \left(\overline{\overline{\Xi}}_{R_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right) e^{i2\pi \left(e^{-\left(\left(-\log \left(\overline{\overline{\Xi}}_{I_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)} \right) \end{array} \right) \\ &= \left(\begin{array}{l} \left(1 - e^{\log \left(1 - \overline{\overline{\Psi}}_{R_j} \right)} \right) e^{i2\pi \left(1 - e^{\log \left(1 - \overline{\overline{\Psi}}_{I_j} \right)} \right)} \\ \left(e^{\log \left(\overline{\overline{\Xi}}_{R_j} \right)} \right) e^{i2\pi \left(e^{\log \left(\overline{\overline{\Xi}}_{I_j} \right)} \right)} \end{array} \right) = \left(\overline{\overline{\Psi}}_{R_j} e^{i2\pi \left(\overline{\overline{\Psi}}_{I_j} \right)}, \overline{\overline{\Xi}}_{R_j} e^{i2\pi \left(\overline{\overline{\Xi}}_{I_j} \right)} \right). \end{aligned}$$

Monotonicity 2 In the consideration of any family of C-IFNs $\overline{\overline{\Theta}}_{\Lambda_j} = \left(\overline{\overline{\Psi}}_{R_j} e^{i2\pi \left(\overline{\overline{\Psi}}_{I_j} \right)}, \overline{\overline{\Xi}}_{R_j} e^{i2\pi \left(\overline{\overline{\Xi}}_{I_j} \right)} \right), j = 1, 2, \dots, n$, when $\overline{\overline{\Theta}}_{\Lambda_j} \leq \overline{\overline{\Theta}}_{\Lambda_j}' =$

$\left(\overline{\overline{\Psi}}_{R_j} e^{i2\pi \left(\overline{\overline{\Psi}}_{I_j} \right)}, \overline{\overline{\Xi}}_{R_j} e^{i2\pi \left(\overline{\overline{\Xi}}_{I_j} \right)} \right)$, then we have

$$\text{C-IFAAPA} \left(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_2}, \dots, \overline{\overline{\Theta}}_{\Lambda_n} \right) \not\subseteq \text{C-IFAAPA} \left(\overline{\overline{\Theta}}_{\Lambda_1}', \overline{\overline{\Theta}}_{\Lambda_2}', \dots, \overline{\overline{\Theta}}_{\Lambda_n}' \right). \tag{13}$$

To justify the above proposition (Monotonocity 2), we give some practical examples and show that the information in Eq. (13) has failed. For this, we use three C-IFNs, such as: $\overline{\overline{\Theta}}_{\Lambda_1} = (0.2e^{i2\pi(0.2)}, 0.2e^{i2\pi(0.2)})$, $\overline{\overline{\Theta}}_{\Lambda_2} = (0.4e^{i2\pi(0.4)}, 0.4e^{i2\pi(0.4)})$, $\overline{\overline{\Theta}}_{\Lambda_3} = (0.1e^{i2\pi(0.1)}, 0.89e^{i2\pi(0.89)})$ and $\overline{\overline{\Theta}}_{\Lambda_1}' = (0.2e^{i2\pi(0.2)}, 0.2e^{i2\pi(0.2)})$, $\overline{\overline{\Theta}}_{\Lambda_2}' = (0.4e^{i2\pi(0.4)}, 0.4e^{i2\pi(0.4)})$, $\overline{\overline{\Theta}}_{\Lambda_3}' = (0.11e^{i2\pi(0.11)}, 0.11e^{i2\pi(0.11)})$ for $\xi = 2$, then using the information in Eq. (11), we get

$$\begin{aligned} d \left(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_2} \right) &= \frac{1}{4} (|0.2 - 0.4| + |0.2 - 0.4| + |0.2 - 0.4| + |0.2 - 0.4|) = 0.2 \\ d \left(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_3} \right) &= 0.395; \quad d \left(\overline{\overline{\Theta}}_{\Lambda_2}, \overline{\overline{\Theta}}_{\Lambda_3} \right) = 0.395 \\ \text{SUP} \left(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_2} \right) &= 1 - d \left(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_2} \right) = 1 - 0.2 = 0.8 \\ \text{SUP} \left(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_3} \right) &= 0.605; \quad \text{SUP} \left(\overline{\overline{\Theta}}_{\Lambda_2}, \overline{\overline{\Theta}}_{\Lambda_3} \right) = 0.605 \end{aligned}$$

Then,

$$\begin{aligned} \Gamma \left(\overline{\overline{\Theta}}_{\Lambda_1} \right) &= \sum_{\substack{s=1, \\ s \neq 1}}^n \text{SUP} \left(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_s} \right) \\ &= \text{SUP} \left(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_2} \right) + \text{SUP} \left(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_3} \right) = 0.8 + 0.605 = 1.405 \\ \Gamma \left(\overline{\overline{\Theta}}_{\Lambda_2} \right) &= 1.405; \quad \Gamma \left(\overline{\overline{\Theta}}_{\Lambda_3} \right) = 1.21 \\ &\sum_{j=1}^n \left(1 + \Gamma \left(\overline{\overline{\Theta}}_{\Lambda_j} \right) \right) = 7.02 \end{aligned}$$

Then,

$$\begin{aligned} \overline{\overline{\mathbb{Z}}}_1 &= \frac{\left(1 + \Gamma \left(\overline{\overline{\Theta}}_{\Lambda_1} \right) \right)}{\sum_{j=1}^3 \left(1 + \Gamma \left(\overline{\overline{\Theta}}_{\Lambda_j} \right) \right)} = \frac{1 + 1.405}{7.02} = 0.34259 \\ \overline{\overline{\mathbb{Z}}}_2 &= 0.34259; \quad \overline{\overline{\mathbb{Z}}}_3 = 0.31481. \end{aligned}$$

Therefore, we get the final result, such as:

$$\text{C-IFAAPA} \left(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_2}, \overline{\overline{\Theta}}_{\Lambda_3} \right) = \left(0.13412e^{i2\pi(0.13412)}, 0.62398e^{i2\pi(0.62398)} \right)$$

And similarly, we get

$$\text{C-IFAAPA} \left(\overline{\overline{\Theta}}_{\Lambda_1}', \overline{\overline{\Theta}}_{\Lambda_2}', \overline{\overline{\Theta}}_{\Lambda_3}' \right) = \left(0.13144e^{i2\pi(0.13144)}, 0.48398e^{i2\pi(0.48398)} \right)$$

Therefore, based on the above information, we get

$$C\text{-IFAAPA}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}, \dots, \overline{\Theta}_{\Lambda_n}) \not\subseteq C\text{-IFAAPA}(\overline{\Theta}'_{\Lambda_1}, \overline{\Theta}'_{\Lambda_2}, \dots, \overline{\Theta}'_{\Lambda_n})$$

Hence, the monotonicity property has failed in the environment of the C-IFAAPA operator.

Definition 5 In the consideration of any family of C-IFNs $\overline{\Theta}_{\Lambda_j} = \left(\overline{\Psi}_{R_j} e^{i2\pi \left(\frac{\overline{\Psi}_{R_j}}{I_j} \right)}, \overline{\Xi}_{R_j} e^{i2\pi \left(\frac{\overline{\Xi}_{R_j}}{I_j} \right)} \right), j = 1, 2, \dots, n$, we derive the idea of the C-IFAAWPA operator, such as:

$$C\text{-IFAAWPA}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}, \dots, \overline{\Theta}_{\Lambda_n}) = \overline{Z}_1 \overline{\Theta}_{\Lambda_1} \oplus \overline{Z}_2 \overline{\Theta}_{\Lambda_2} \oplus \dots \oplus \overline{Z}_n \overline{\Theta}_{\Lambda_n} = \oplus_{j=1}^n (\overline{Z}_j \overline{\Theta}_{\Lambda_j}) \tag{14}$$

where,

$$\overline{Z}_j = WPA(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}, \dots, \overline{\Theta}_{\Lambda_n}) = \frac{\Phi_j (1 + \Gamma(\overline{\Theta}_{\Lambda_j}))}{\sum_{j=1}^n \Phi_j (1 + \Gamma(\overline{\Theta}_{\Lambda_j}))} \tag{15}$$

Called WPA operator with $\Gamma(\overline{\Theta}_{\Lambda_j}) = \sum_{s=1}^n \text{SUP}(\overline{\Theta}_{\Lambda_j}, \overline{\Theta}_{\Lambda_s}),$ and the term $s \neq j$

$\text{SUP}(\overline{\Theta}_{\Lambda_j}, \overline{\Theta}_{\Lambda_s}) = 1 - d(\overline{\Theta}_{\Lambda_j}, \overline{\Theta}_{\Lambda_s})$ represents as support of $\overline{\Theta}_{\Lambda_j}$ from $\overline{\Theta}_{\Lambda_s}$ and $d(\overline{\Theta}_{\Lambda_j}, \overline{\Theta}_{\Lambda_s})$ used as a distance measure, where $\Phi_j \in [0, 1]$ with $\sum_{j=1}^n \Phi_j = 1$ represents the weight vector.

Theorem 3 In the consideration of any family of C-IFNs $\overline{\Theta}_{\Lambda_j} = \left(\overline{\Psi}_{R_j} e^{i2\pi \left(\frac{\overline{\Psi}_{R_j}}{I_j} \right)}, \overline{\Xi}_{R_j} e^{i2\pi \left(\frac{\overline{\Xi}_{R_j}}{I_j} \right)} \right), j = 1, 2, \dots, n$, we derive that the idea of C-IFAAWPA operator is given again a C-IFN, such as:

$$C\text{-IFAAWPA}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}, \dots, \overline{\Theta}_{\Lambda_n}) = \left(\left(\left(1 - e^{-\left(\sum_{j=1}^n \overline{Z}_j \left(-\log \left(1 - \frac{\overline{\Psi}_{R_j}}{I_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)^{\frac{1}{\xi}} \right)^{\frac{1}{\xi}} e^{i2\pi \left(1 - e^{-\left(\sum_{j=1}^n \overline{Z}_j \left(-\log \left(1 - \frac{\overline{\Psi}_{R_j}}{I_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)^{\frac{1}{\xi}}} \right), \left(\left(e^{-\left(\sum_{j=1}^n \overline{Z}_j \left(-\log \left(\frac{\overline{\Xi}_{R_j}}{I_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)^{\frac{1}{\xi}} \right)^{\frac{1}{\xi}} e^{i2\pi \left(e^{-\left(\sum_{j=1}^n \overline{Z}_j \left(-\log \left(\frac{\overline{\Xi}_{R_j}}{I_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)^{\frac{1}{\xi}}} \right) \right) \tag{16}$$

Further, we aim to justify the information in Eq. (16) with the help of some suitable examples. For this, we use three C-IFNs, such as: $\overline{\Theta}_{\Lambda_1} = (0.2e^{i2\pi(0.2)}, 0.2e^{i2\pi(0.2)})$, $\overline{\Theta}_{\Lambda_2} = (0.4e^{i2\pi(0.4)}, 0.4e^{i2\pi(0.4)})$ and $\overline{\Theta}_{\Lambda_3} = (0.1e^{i2\pi(0.1)}, 0.89e^{i2\pi(0.89)})$ for $\xi = 2$, then by using the information in Eq. (16), we get

$$\begin{aligned}
 d(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}) &= \frac{1}{4}(|0.2 - 0.4| + |0.2 - 0.4| + |0.2 - 0.4| + |0.2 - 0.4|) = 0.2 \\
 d(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_3}) &= 0.395; \quad d(\overline{\Theta}_{\Lambda_2}, \overline{\Theta}_{\Lambda_3}) = 0.395 \\
 \text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}) &= 1 - d(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}) = 1 - 0.2 = 0.8 \\
 \text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_3}) &= 0.605; \quad \text{SUP}(\overline{\Theta}_{\Lambda_2}, \overline{\Theta}_{\Lambda_3}) = 0.605
 \end{aligned}$$

Then,

$$\begin{aligned}
 \Gamma(\overline{\Theta}_{\Lambda_1}) &= \sum_{\substack{s=1, \\ s \neq 1}}^n \text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_s}) \\
 &= \text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}) + \text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_3}) = 0.8 + 0.605 = 1.405 \\
 \Gamma(\overline{\Theta}_{\Lambda_2}) &= 1.405; \quad \Gamma(\overline{\Theta}_{\Lambda_3}) = 1.21
 \end{aligned}$$

where the weighted vectors are 0.4, 0.3 and 0.3, then we have

$$\sum_{j=1}^n \Phi_j(1 + \Gamma(\overline{\Theta}_{\Lambda_j})) = 2.3465$$

Then,

$$\begin{aligned}
 \overline{\overline{Z}}_1 &= \frac{\Phi_1(1 + \Gamma(\overline{\Theta}_{\Lambda_1}))}{\sum_{j=1}^3 \Phi_j(1 + \Gamma(\overline{\Theta}_{\Lambda_j}))} = 0.40997 \\
 \overline{\overline{Z}}_2 &= 0.30748; \quad \overline{\overline{Z}}_3 = 0.28255.
 \end{aligned}$$

Therefore, we get the final result, such as:

$$\text{C-IFAAWPA}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}, \overline{\Theta}_{\Lambda_3}) = (0.13056e^{i2\pi(0.13056)}, 0.60671e^{i2\pi(0.60671)}).$$

Idempotency 3 In the consideration of any family of C-IFNs $\overline{\Theta}_{\Lambda_j} = (\overline{\Psi}_{R_j} e^{i2\pi(\overline{\Psi}_{I_j})}, \overline{\Xi}_{R_j} e^{i2\pi(\overline{\Xi}_{I_j})})$, $j = 1, 2, \dots, n$, when $\overline{\Theta}_{\Lambda_j} = \overline{\Theta}$, then we derive the below theory, such as:

$$\text{C-IFAAWPA}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}, \dots, \overline{\Theta}_{\Lambda_n}) = \overline{\Theta} \tag{17}$$

Definition 6 In the consideration of any family of C-IFNs $\overline{\overline{\Theta}}_{\Lambda_j} = \left(\overline{\overline{\Psi}}_{R_j} e^{i2\pi \left(\overline{\overline{\Psi}}_{I_j} \right)}, \overline{\overline{\Xi}}_{R_j} e^{i2\pi \left(\overline{\overline{\Xi}}_{I_j} \right)} \right), j = 1, 2, \dots, n$, we derive the idea of the C-IFAAPG operator, such as:

$$C - IFAAPG \left(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_2}, \dots, \overline{\overline{\Theta}}_{\Lambda_n} \right) = \overline{\overline{\Theta}}_{\Lambda_1}^{\overline{\overline{Z}}_1} \otimes \overline{\overline{\Theta}}_{\Lambda_2}^{\overline{\overline{Z}}_2} \otimes \dots \otimes \overline{\overline{\Theta}}_{\Lambda_n}^{\overline{\overline{Z}}_n} = \otimes_{j=1}^n \left(\overline{\overline{\Theta}}_{\Lambda_j}^{\overline{\overline{Z}}_j} \right) \tag{18}$$

where,

$$\overline{\overline{Z}}_j = PA \left(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_2}, \dots, \overline{\overline{\Theta}}_{\Lambda_n} \right) = \frac{\left(1 + \Gamma \left(\overline{\overline{\Theta}}_{\Lambda_j} \right) \right) \overline{\overline{\Theta}}_{\Lambda_j}}{\sum_{j=1}^n \left(1 + \Gamma \left(\overline{\overline{\Theta}}_{\Lambda_j} \right) \right)} \tag{19}$$

Called PA operator with $\Gamma \left(\overline{\overline{\Theta}}_{\Lambda_j} \right) = \sum_{s=1}^n \text{SUP} \left(\overline{\overline{\Theta}}_{\Lambda_j}, \overline{\overline{\Theta}}_{\Lambda_s} \right),$ and the term

$\text{SUP} \left(\overline{\overline{\Theta}}_{\Lambda_j}, \overline{\overline{\Theta}}_{\Lambda_s} \right) = 1 - d \left(\overline{\overline{\Theta}}_{\Lambda_j}, \overline{\overline{\Theta}}_{\Lambda_s} \right)$ represents as support of $\overline{\overline{\Theta}}_{\Lambda_j}$ from $\overline{\overline{\Theta}}_{\Lambda_s}$ and $d \left(\overline{\overline{\Theta}}_{\Lambda_j}, \overline{\overline{\Theta}}_{\Lambda_s} \right)$ used as a distance measure.

Theorem 4 In the consideration of any family of C-IFNs $\overline{\overline{\Theta}}_{\Lambda_j} = \left(\overline{\overline{\Psi}}_{R_j} e^{i2\pi \left(\overline{\overline{\Psi}}_{I_j} \right)}, \overline{\overline{\Xi}}_{R_j} e^{i2\pi \left(\overline{\overline{\Xi}}_{I_j} \right)} \right), j = 1, 2, \dots, n$, we derive that the idea of C-IFAAPG operator is given again a C-IFN, such as:

$$C - IFAAPG \left(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_2}, \dots, \overline{\overline{\Theta}}_{\Lambda_n} \right) = \left(\left(e^{- \left(\sum_{j=1}^n \overline{\overline{Z}}_j \left(-\log \left(\overline{\overline{\Psi}}_{R_j} \right) \right) \right)^{\frac{1}{\xi}}} \right)^{\frac{1}{\xi}} e^{i2\pi \left(e^{- \left(\sum_{j=1}^n \overline{\overline{Z}}_j \left(-\log \left(\overline{\overline{\Psi}}_{I_j} \right) \right) \right)^{\frac{1}{\xi}}} \right)^{\frac{1}{\xi}}}, \left(1 - e^{- \left(\sum_{j=1}^n \overline{\overline{Z}}_j \left(-\log \left(1 - \overline{\overline{\Xi}}_{R_j} \right) \right) \right)^{\frac{1}{\xi}}} \right)^{\frac{1}{\xi}} e^{i2\pi \left(1 - e^{- \left(\sum_{j=1}^n \overline{\overline{Z}}_j \left(-\log \left(1 - \overline{\overline{\Xi}}_{I_j} \right) \right) \right)^{\frac{1}{\xi}}} \right)^{\frac{1}{\xi}}} \right) \tag{20}$$

Further, we aim to justify the information in Eq. (20) with the help of some suitable examples. For this, we use three C-IFNs, such as: $\overline{\overline{\Theta}}_{\Lambda_1} = (0.2e^{i2\pi(0.2)}, 0.2e^{i2\pi(0.2)}), \overline{\overline{\Theta}}_{\Lambda_2} = (0.4e^{i2\pi(0.4)}, 0.4e^{i2\pi(0.4)})$ and $\overline{\overline{\Theta}}_{\Lambda_3} = (0.1e^{i2\pi(0.1)}, 0.89e^{i2\pi(0.89)})$ for $\xi = 2$, then by using the information in Eq. (20), we get

$$d \left(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_2} \right) = \frac{1}{4} (|0.2 - 0.4| + |0.2 - 0.4| + |0.2 - 0.4| + |0.2 - 0.4|) = 0.2$$

$$d \left(\overline{\overline{\Theta}}_{\Lambda_1}, \overline{\overline{\Theta}}_{\Lambda_3} \right) = 0.395; \quad d \left(\overline{\overline{\Theta}}_{\Lambda_2}, \overline{\overline{\Theta}}_{\Lambda_3} \right) = 0.395$$

$$\begin{aligned} \text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}) &= 1 - d(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}) = 1 - 0.2 = 0.8 \\ \text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_3}) &= 0.605; \quad \text{SUP}(\overline{\Theta}_{\Lambda_2}, \overline{\Theta}_{\Lambda_3}) = 0.605 \end{aligned}$$

Then,

$$\begin{aligned} \Gamma(\overline{\Theta}_{\Lambda_1}) &= \sum_{\substack{s=1, \\ s \neq 1}}^n \text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_s}) \\ &= \text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}) + \text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_3}) = 0.8 + 0.605 = 1.405 \\ \Gamma(\overline{\Theta}_{\Lambda_2}) &= 1.405; \quad \Gamma(\overline{\Theta}_{\Lambda_3}) = 1.21 \\ \sum_{j=1}^n (1 + \Gamma(\overline{\Theta}_{\Lambda_j})) &= 7.02 \end{aligned}$$

Then,

$$\begin{aligned} \overline{\overline{Z}}_1 &= \frac{(1 + \Gamma(\overline{\Theta}_{\Lambda_1}))}{\sum_{j=1}^3 (1 + \Gamma(\overline{\Theta}_{\Lambda_j}))} = \frac{1 + 1.405}{7.02} = 0.34259 \\ \overline{\overline{Z}}_2 &= 0.34259; \quad \overline{\overline{Z}}_3 = 0.31481. \end{aligned}$$

Therefore, we get the final result, such as:

$$C - \text{IFAAPG}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}, \overline{\Theta}_{\Lambda_3}) = (0.13412e^{i2\pi(0.13412)}, 0.62398e^{i2\pi(0.62398)}).$$

Idempotency 4 In the consideration of any family of C-IFNs $\overline{\Theta}_{\Lambda_j} = (\overline{\Psi}_{R_j} e^{i2\pi(\overline{\Psi}_{I_j})}, \overline{\Xi}_{R_j} e^{i2\pi(\overline{\Xi}_{I_j})})$, $j = 1, 2, \dots, n$, when $\overline{\Theta}_{\Lambda_j} = \overline{\Theta}$, then we derive the below theory, such as:

$$C - \text{IFAAPG}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}, \dots, \overline{\Theta}_{\Lambda_n}) = \overline{\Theta} \tag{21}$$

Definition 7 In the consideration of any family of C-IFNs $\overline{\Theta}_{\Lambda_j} = (\overline{\Psi}_{R_j} e^{i2\pi(\overline{\Psi}_{I_j})}, \overline{\Xi}_{R_j} e^{i2\pi(\overline{\Xi}_{I_j})})$, $j = 1, 2, \dots, n$, we derive the idea of the C-IFAAWPG operator, such as:

$$C - \text{IFAAWPG}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}, \dots, \overline{\Theta}_{\Lambda_n}) = \overline{\Theta}_{\Lambda_1}^{\overline{\overline{Z}}_1} \otimes \overline{\Theta}_{\Lambda_2}^{\overline{\overline{Z}}_2} \otimes \dots \otimes \overline{\Theta}_{\Lambda_n}^{\overline{\overline{Z}}_n} = \otimes_{j=1}^n (\overline{\Theta}_{\Lambda_j}^{\overline{\overline{Z}}_j}) \tag{22}$$

where,

$$\overline{\overline{Z}}_j = \text{WPA}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}, \dots, \overline{\Theta}_{\Lambda_n}) = \frac{\Phi_j(1 + \Gamma(\overline{\Theta}_{\Lambda_j}))\overline{\Theta}_{\Lambda_j}}{\sum_{j=1}^n \Phi_j(1 + \Gamma(\overline{\Theta}_{\Lambda_j}))} \tag{23}$$

Called WPA operator with $\Gamma(\overline{\Theta}_{\Lambda_j}) = \sum_{\substack{s=1, \\ s \neq j}}^n \text{SUP}(\overline{\Theta}_{\Lambda_j}, \overline{\Theta}_{\Lambda_s})$, and the term $\text{SUP}(\overline{\Theta}_{\Lambda_j}, \overline{\Theta}_{\Lambda_s}) = 1 - d(\overline{\Theta}_{\Lambda_j}, \overline{\Theta}_{\Lambda_s})$ represents as support of $\overline{\Theta}_{\Lambda_j}$ from $\overline{\Theta}_{\Lambda_s}$ and $d(\overline{\Theta}_{\Lambda_j}, \overline{\Theta}_{\Lambda_s})$ used as a distance measure, where $\Phi_j \in [0, 1]$ with $\sum_{j=1}^n \Phi_j = 1$ represents the weight vector.

Theorem 5 In the consideration of any family of C-IFNs $\overline{\Theta}_{\Lambda_j} = \left(\overline{\Psi}_{R_j} e^{i2\pi \left(\overline{\Psi}_{T_j} \right)}, \overline{\Xi}_{R_j} e^{i2\pi \left(\overline{\Xi}_{T_j} \right)} \right), j = 1, 2, \dots, n$, we derive that the idea of C-IFAAWPG operator is given again a C-IFN, such as:

$$\begin{aligned} & \text{C-IFAAWPG}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}, \dots, \overline{\Theta}_{\Lambda_n}) \\ &= \left(\left(e^{-\left(\sum_{j=1}^n \overline{Z}_j \left(-\log \left(\overline{\Psi}_{R_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)^{i2\pi \left(e^{-\left(\sum_{j=1}^n \overline{Z}_j \left(-\log \left(\overline{\Psi}_{T_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)} \right), \\ & \left(\left(1 - e^{-\left(\sum_{j=1}^n \overline{Z}_j \left(-\log \left(1 - \overline{\Xi}_{R_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)^{i2\pi \left(1 - e^{-\left(\sum_{j=1}^n \overline{Z}_j \left(-\log \left(1 - \overline{\Xi}_{T_j} \right) \right)^\xi \right)^{\frac{1}{\xi}}} \right)} \right) \right) \end{aligned} \tag{24}$$

Further, we aim to justify the information in Eq. (16) with the help of some suitable examples. For this, we use three C-IFNs, such as: $\overline{\Theta}_{\Lambda_1} = (0.2e^{i2\pi(0.2)}, 0.2e^{i2\pi(0.2)})$, $\overline{\Theta}_{\Lambda_2} = (0.4e^{i2\pi(0.4)}, 0.4e^{i2\pi(0.4)})$ and $\overline{\Theta}_{\Lambda_3} = (0.1e^{i2\pi(0.1)}, 0.89e^{i2\pi(0.89)})$ for $\xi = 2$, then by using the information in Eq. (16), we get

$$\begin{aligned} d(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}) &= \frac{1}{4} (|0.2 - 0.4| + |0.2 - 0.4| + |0.2 - 0.4| + |0.2 - 0.4|) = 0.2 \\ d(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_3}) &= 0.395; \quad d(\overline{\Theta}_{\Lambda_2}, \overline{\Theta}_{\Lambda_3}) = 0.395 \\ \text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}) &= 1 - d(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}) = 1 - 0.2 = 0.8 \\ \text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_3}) &= 0.605; \quad \text{SUP}(\overline{\Theta}_{\Lambda_2}, \overline{\Theta}_{\Lambda_3}) = 0.605 \end{aligned}$$

Then,

$$\begin{aligned} \Gamma(\overline{\Theta}_{\Lambda_1}) &= \sum_{\substack{s=1, \\ s \neq 1}}^n \text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_s}) \\ &= \text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}) + \text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_3}) = 0.8 + 0.605 = 1.405 \end{aligned}$$

$$\Gamma(\overline{\Theta}_{\Lambda_2}) = 1.405; \quad \Gamma(\overline{\Theta}_{\Lambda_3}) = 1.21$$

where the weighted vectors are 0.4,0.3 and 0.3, then we have

$$\sum_{j=1}^n \Phi_j(1 + \Gamma(\overline{\Theta}_{\Lambda_j})) = 2.3465$$

Then,

$$\begin{aligned} \overline{\mathbb{Z}}_1 &= \frac{\Phi_1(1 + \Gamma(\overline{\Theta}_{\Lambda_1}))}{\sum_{j=1}^3 \Phi_j(1 + \Gamma(\overline{\Theta}_{\Lambda_j}))} = 0.40997 \\ \overline{\mathbb{Z}}_2 &= 0.30748; \quad \overline{\mathbb{Z}}_3 = 0.28255. \end{aligned}$$

Therefore, we get the final result, such as:

$$C - \text{IFAAWPG}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}, \overline{\Theta}_{\Lambda_3}) = (0.48236e^{i2\pi(0.48236)}, 0.41013e^{i2\pi(0.41013)}).$$

Idempotency 5 In the consideration of any family of C-IFNs $\overline{\Theta}_{\Lambda_j} = \left(\overline{\Psi}_{R_j} e^{i2\pi \left(\frac{\overline{\Psi}_{T_j}}{R_j} \right)}, \overline{\Xi}_{R_j} e^{i2\pi \left(\frac{\overline{\Xi}_{T_j}}{R_j} \right)} \right), j = 1, 2, \dots, n$, when $\overline{\Theta}_{\Lambda_j} = \overline{\Theta}$, then we derive the below theory, such as:

$$C - \text{IFAAWPG}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}, \dots, \overline{\Theta}_{\Lambda_n}) = \overline{\Theta} \tag{25}$$

where, the monotonicity property and boundedness has been failed for the idea of C-IFAAPA, C-IFAAWPA, C-IFAAPG, and C-IFAAWPG operators.

4 MADM procedures under-investigated operators

Here, we aim to derive a decision-making procedure based on pioneered operators which are computed for C-IFSS. For this, we described the main procedure of decision-making and try to show that the derived work is massively useful.

For this, we consider the valuable and dominant family of alternatives: $\overline{\Theta}_{\Lambda} = \{ \overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}, \dots, \overline{\Theta}_{\Lambda_m} \}$. Similarly, we use the family of attributes $\overline{\Theta}'_{\Lambda} = \{ \overline{\Theta}'_{\Lambda_1}, \overline{\Theta}'_{\Lambda_2}, \dots, \overline{\Theta}'_{\Lambda_n} \}$ for each alternative based on the weight vector $\overline{\mathbb{Z}} = (\overline{\mathbb{Z}}_1, \overline{\mathbb{Z}}_2, \dots, \overline{\mathbb{Z}}_n)^T \sum_{j=1}^n \overline{\mathbb{Z}}_j = 1$. Based on this information, we assigned the information of C-IFN to each attribute in every alternative, where the value of C-IFN is stated by: $\overline{\Theta}_{\Lambda_{jk}} = \left(\overline{\Psi}_{R_{jk}} e^{i2\pi \left(\frac{\overline{\Psi}_{T_{jk}}}{R_{jk}} \right)}, \overline{\Xi}_{R_{jk}} e^{i2\pi \left(\frac{\overline{\Xi}_{T_{jk}}}{R_{jk}} \right)} \right), j, k = 1, 2, \dots, n, m$. Furthermore, we considered $\overline{\Psi}_{\overline{\Theta}_{\Lambda}}(\tilde{\mu}_{\varrho}) = \overline{\Psi}_{\overline{R}}(\tilde{\mu}_{\varrho}) e^{i2\pi \left(\frac{\overline{\Psi}_{\overline{T}}(\tilde{\mu}_{\varrho})}{\overline{R}} \right)}$ and $\overline{\Xi}_{\overline{\Theta}_{\Lambda}}(\tilde{\mu}_{\varrho}) = \overline{\Xi}_{\overline{R}}(\tilde{\mu}_{\varrho}) e^{i2\pi \left(\frac{\overline{\Xi}_{\overline{T}}(\tilde{\mu}_{\varrho})}{\overline{R}} \right)}$ as an MG and NMG. Moreover, the main characteristic of the C-IF set is stated by:

Table 1 The major steps of the derived algorithm

Proposed algorithm

Key Point 1: Frist, we aim to compute a matrix that contained the finite family of alternatives and attributes in the shape of C-IFNs

Key Point 2: Usually, every decision matrix contained two types of data, like benefit and cost types, if the data is cost type, then normalize it with the help of the below idea:

$$D = \left\{ \begin{array}{l} \left(\overline{\overline{\Psi}}_{R_{jk}} e^{i2\pi \left(\overline{\overline{\Psi}}_{I_{jk}} \right)}, \overline{\overline{\Xi}}_{R_{jk}} e^{i2\pi \left(\overline{\overline{\Xi}}_{I_{jk}} \right)} \right) \text{ Benefit} \\ \left(\overline{\overline{\Xi}}_{R_{jk}} e^{i2\pi \left(\overline{\overline{\Xi}}_{I_{jk}} \right)}, \overline{\overline{\Psi}}_{R_{jk}} e^{i2\pi \left(\overline{\overline{\Psi}}_{I_{jk}} \right)} \right) \text{ Cost} \end{array} \right.$$

But, if the data is benefit types, then do not normalize the data and proceed to the next steps

Key Point 3: Tanking the idea of C-IFAAPA, C-IFAAWPA, C-IFAAPG, and C-IFAAWPG operators and trying to aggregate our considered normalized information

Where the above operators are the modified version of the bundle of ideas because the derived theory is massively valuable and dominant due to computed based on C-IF information

Key Point 4: Tanking the idea in Eqs. (7) and (8) and evaluating the aggregated information

Key Point 5: Examine the order information in the presence of score information to find the best decision Where the above procedure is the form of decision-making and with help of the above algorithm, we can easily derive the best decision from the collection of preferences

$0 \leq \overline{\overline{\Psi}}_R(\tilde{\mu}_\varrho) + \overline{\overline{\Xi}}_R(\tilde{\mu}_\varrho) \leq 1$ and $0 \leq \overline{\overline{\Psi}}_I(\tilde{\mu}_\varrho) + \overline{\overline{\Xi}}_I(\tilde{\mu}_\varrho) \leq 1$. Assume that the mathematical structure of the refusal grade is stated by: $\overline{\overline{\mathfrak{R}}}_{\Theta_\Lambda}(\tilde{\mu}_\varrho) = \overline{\overline{\mathfrak{R}}}_R(\tilde{\mu}_\varrho) e^{i2\pi \left(\overline{\overline{\mathfrak{R}}}_I(\tilde{\mu}_\varrho) \right)} = \left(1 - \left(\overline{\overline{\Psi}}_R(\tilde{\mu}_\varrho) + \overline{\overline{\Xi}}_R(\tilde{\mu}_\varrho) \right) \right) e^{i2\pi \left(1 - \left(\overline{\overline{\Psi}}_I(\tilde{\mu}_\varrho) + \overline{\overline{\Xi}}_I(\tilde{\mu}_\varrho) \right) \right)}$. To proceed with the above procedure, firstly, we aim to compute a decision-making procedure and then try to justify it with the help of some practical examples. The derived algorithm is also stated in the form of Table 1 and their geometrical representation in given in the form of Fig. 1.

Further, here, we stated the flowchart of the above algorithm is to state the proficiency and reliability of the evaluated theory.

Under the evaluation of the procedure, we aim to justify it with the help of some practical examples, which are stated below.

4.1 Practical-example

To demonstrate the derived theory, we use or describe a case study related to entrepreneur to busy a novel branded machine out of five different models represented by $\overline{\overline{\Theta}}_{\Lambda_j}$, $j = 1, 2, 3, 4, 5$, where the considered machines are stated in the form such as $\overline{\overline{\Theta}}_{\Lambda_1}$: ATM machine, $\overline{\overline{\Theta}}_{\Lambda_2}$: security checking machine, $\overline{\overline{\Theta}}_{\Lambda_3}$: biometric machine, $\overline{\overline{\Theta}}_{\Lambda_4}$: milling machine, $\overline{\overline{\Theta}}_{\Lambda_5}$: CNC Lathe machine. The accessibility of the above machines is measured under the presence of the four different criteria, such as $\overline{\overline{\Theta}}'_{\Lambda_1}$: flexibility, $\overline{\overline{\Theta}}'_{\Lambda_2}$: reliability, $\overline{\overline{\Theta}}'_{\Lambda_3}$: prices, and $\overline{\overline{\Theta}}'_{\Lambda_4}$: safety. To resolve the above problems, we select their weight vector 0.1, 0.2, 0.3, 0.4 for each alternative. To evaluate the above theory in the presence of the data in Table 2, where,

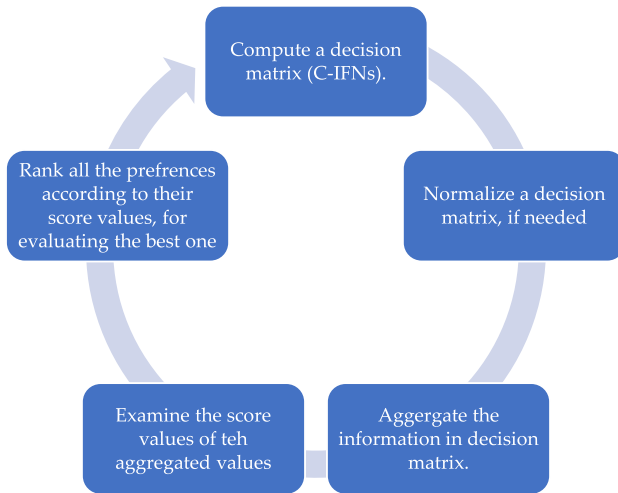


Fig. 1 Geometrical interpretation of the derived algorithm

Table 2 Information matrix (C-IF data)

Alternatives/attributes	$\overline{\overline{\Theta}}'_{\Lambda_1}$	$\overline{\overline{\Theta}}'_{\Lambda_2}$
$\overline{\overline{\Theta}}_{\Lambda_1}$	((0.3, 0.2), (0.1, 0.2))	((0.5, 0.7), (0.3, 0.1))
$\overline{\overline{\Theta}}_{\Lambda_2}$	((0.4, 0.7), (0.3, 0.1))	((0.4, 0.4), (0.2, 0.3))
$\overline{\overline{\Theta}}_{\Lambda_3}$	((0.5, 0.5), (0.2, 0.4))	((0.1, 0.7), (0.5, 0.1))
$\overline{\overline{\Theta}}_{\Lambda_4}$	((0.6, 0.6), (0.2, 0.2))	((0.2, 0.3), (0.4, 0.3))
$\overline{\overline{\Theta}}_{\Lambda_5}$	((0.2, 0.5), (0.1, 0.3))	((0.7, 0.4), (0.1, 0.2))
Alternatives/attributes	$\overline{\overline{\Theta}}'_{\Lambda_3}$	$\overline{\overline{\Theta}}'_{\Lambda_4}$
$\overline{\overline{\Theta}}_{\Lambda_1}$	((0.8, 0.4), (0.1, 0.3))	((0.2, 0.3), (0.4, 0.3))
$\overline{\overline{\Theta}}_{\Lambda_2}$	((0.3, 0.5), (0.5, 0.4))	((0.7, 0.4), (0.1, 0.2))
$\overline{\overline{\Theta}}_{\Lambda_3}$	((0.7, 0.4), (0.3, 0.3))	((0.6, 0.6), (0.2, 0.2))
$\overline{\overline{\Theta}}_{\Lambda_4}$	((0.4, 0.6), (0.3, 0.5))	((0.3, 0.5), (0.5, 0.4))
$\overline{\overline{\Theta}}_{\Lambda_5}$	((0.6, 0.2), (0.1, 0.4))	((0.7, 0.4), (0.3, 0.3))

if we talked about the first entry in Table 2, the machine $\overline{\overline{\Theta}}_{\Lambda_1}$ states that the expert during the decision-making evaluation agrees that it is flexible by 30% under the criteria $\overline{\overline{\Theta}}'_{\Lambda_1}$ and not flexible 10%. Similarly, concerning production date, he feels that 20% is compatible and 20% is not under the presence of the criteria $\overline{\overline{\Theta}}'_{\Lambda_1}$. Thus, the theory ((0.3, 0.2), (0.1, 0.2))

in Table 2 is called C-IFN. Similarly, we evaluated the information in Table 2 under the consideration of the above algorithm. Therefore, to proceed with the above procedure, firstly, we aim to compute a decision-making procedure and then try to justify it with the help of some practical examples. Therefore, the aim key-points are listed below:

Key Point 1: Frist, we aim to compute a matrix that contained the finite family of alternatives and attributes in the shape of C-IFNs, see information in Table 2. Where, the information in Table 2 is arranged in the shape of C-IFNs and it is clear that the theory of C-IFNs has a lot of advantages because they contained the truth and falsity grades in the shape of complex numbers, whose real and imaginary parts are covered in the unit interval. Furthermore, in many situations, we faced two-dimension information, for instance, when we purchase a new car, for this, we needed to visit the car company the owner of the car can provide two types of data regarding each car such as name and production date of the car, where the name of the car stated the real part, and the production date of the car stated the phase term. Noticed that the theory of FS has failed to manage such type of situation.

Key-Point 2: Usually, every decision matrix contained two types of data, like benefit and cost types, if the data is cost type, then normalize it with the help of the below idea:

$$D = \begin{cases} \left(\frac{\overline{\Psi}_{R_{jk}}}{\overline{\Psi}_{R_{jk}}} e^{i2\pi \left(\frac{\overline{\Psi}_{I_{jk}}}{\overline{\Psi}_{I_{jk}}} \right)}, \frac{\overline{\Xi}_{R_{jk}}}{\overline{\Xi}_{R_{jk}}} e^{i2\pi \left(\frac{\overline{\Xi}_{I_{jk}}}{\overline{\Xi}_{I_{jk}}} \right)} \right) \text{ Benefit} \\ \left(\frac{\overline{\Xi}_{R_{jk}}}{\overline{\Xi}_{R_{jk}}} e^{i2\pi \left(\frac{\overline{\Xi}_{I_{jk}}}{\overline{\Xi}_{I_{jk}}} \right)}, \frac{\overline{\Psi}_{R_{jk}}}{\overline{\Psi}_{R_{jk}}} e^{i2\pi \left(\frac{\overline{\Psi}_{I_{jk}}}{\overline{\Psi}_{I_{jk}}} \right)} \right) \text{ Cost} \end{cases}$$

But, if the data is benefit types, then do not normalize the data and proceed to the next steps. Information is given in Table 2, in the form of benefit types, then is no need to normalize, therefore, we consider $\xi = 2$, then by using the information in Eq. (11), we get information in Table 3, which contained the distance measures.

Where, the values of support information, see Table 4, which are obtained from the information in Table 3.

Furthermore, we derive the value of each $\Gamma(\overline{\Theta}_{\Lambda_j}) = \sum_{s=1}^n \text{SUP}(\overline{\Theta}_{\Lambda_j}, \overline{\Theta}_{\Lambda_s})$ for $s \neq j$

every alternative based on their attributes, see Table 5.

Moreover, we use the theory given in Eq. (10) $\overline{\mathbb{Z}}_j = \text{PA}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}, \dots, \overline{\Theta}_{\Lambda_n}) = \frac{(1+\Gamma(\overline{\Theta}_{\Lambda_j}))}{\sum_{j=1}^n (1+\Gamma(\overline{\Theta}_{\Lambda_j}))}$ and try to derive their values, which are given in Table 6.

Moreover, we use the theory given in Eq. (10) $\overline{\mathbb{Z}}_j = \text{PA}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}, \dots, \overline{\Theta}_{\Lambda_n}) = \frac{(1+\Gamma(\overline{\Theta}_{\Lambda_j}))}{\sum_{j=1}^n (1+\Gamma(\overline{\Theta}_{\Lambda_j}))}$ and try to derive their values using the weight vector 0.1, 0.2, 0.3, and 0.4, which are given in Table 7.

Key-Point 3: Tanking the idea of C-IFAAPA, C-IFAAWPA, C-IFAAPG, and C-IFAAWPG operators and trying to aggregate our considered normalized information, see Table 8.

Key Point 4: Tanking the idea in Eqs. (7) and (8) and evaluating the aggregated information, see Table 9.

Key Point 5: Examine the order information in the presence of score information to find the best decision, see Table 10.

Table 3 Represented the information on distance measures

$\overline{\Theta_{\Lambda_1}}$	$\overline{\Theta_{\Lambda_2}}$	$\overline{\Theta_{\Lambda_3}}$	$\overline{\Theta_{\Lambda_4}}$	$\overline{\Theta_{\Lambda_5}}$
$d(\overline{\Theta_{\Lambda_1}}, \overline{\Theta_{\Lambda_2}}) = 0.25$	$d(\overline{\Theta_{\Lambda_1}}, \overline{\Theta_{\Lambda_2}}) = 0.15$	$d(\overline{\Theta_{\Lambda_1}}, \overline{\Theta_{\Lambda_2}}) = 0.3$	$d(\overline{\Theta_{\Lambda_1}}, \overline{\Theta_{\Lambda_2}}) = 0.25$	$d(\overline{\Theta_{\Lambda_1}}, \overline{\Theta_{\Lambda_2}}) = 0.175$
$d(\overline{\Theta_{\Lambda_1}}, \overline{\Theta_{\Lambda_3}}) = 0.2$	$d(\overline{\Theta_{\Lambda_1}}, \overline{\Theta_{\Lambda_3}}) = 0.2$	$d(\overline{\Theta_{\Lambda_1}}, \overline{\Theta_{\Lambda_3}}) = 0.125$	$d(\overline{\Theta_{\Lambda_1}}, \overline{\Theta_{\Lambda_3}}) = 0.15$	$d(\overline{\Theta_{\Lambda_1}}, \overline{\Theta_{\Lambda_3}}) = 0.2$
$d(\overline{\Theta_{\Lambda_1}}, \overline{\Theta_{\Lambda_4}}) = 0.15$	$d(\overline{\Theta_{\Lambda_1}}, \overline{\Theta_{\Lambda_4}}) = 0.225$	$d(\overline{\Theta_{\Lambda_1}}, \overline{\Theta_{\Lambda_4}}) = 0.1$	$d(\overline{\Theta_{\Lambda_1}}, \overline{\Theta_{\Lambda_4}}) = 0.225$	$d(\overline{\Theta_{\Lambda_1}}, \overline{\Theta_{\Lambda_4}}) = 0.2$
$d(\overline{\Theta_{\Lambda_2}}, \overline{\Theta_{\Lambda_3}}) = 0.25$	$d(\overline{\Theta_{\Lambda_2}}, \overline{\Theta_{\Lambda_3}}) = 0.15$	$d(\overline{\Theta_{\Lambda_2}}, \overline{\Theta_{\Lambda_3}}) = 0.325$	$d(\overline{\Theta_{\Lambda_2}}, \overline{\Theta_{\Lambda_3}}) = 0.2$	$d(\overline{\Theta_{\Lambda_2}}, \overline{\Theta_{\Lambda_3}}) = 0.125$
$d(\overline{\Theta_{\Lambda_2}}, \overline{\Theta_{\Lambda_4}}) = 0.25$	$d(\overline{\Theta_{\Lambda_2}}, \overline{\Theta_{\Lambda_4}}) = 0.125$	$d(\overline{\Theta_{\Lambda_2}}, \overline{\Theta_{\Lambda_4}}) = 0.25$	$d(\overline{\Theta_{\Lambda_2}}, \overline{\Theta_{\Lambda_4}}) = 0.125$	$d(\overline{\Theta_{\Lambda_2}}, \overline{\Theta_{\Lambda_4}}) = 0.075$
$d(\overline{\Theta_{\Lambda_3}}, \overline{\Theta_{\Lambda_4}}) = 0.25$	$d(\overline{\Theta_{\Lambda_3}}, \overline{\Theta_{\Lambda_4}}) = 0.275$	$d(\overline{\Theta_{\Lambda_3}}, \overline{\Theta_{\Lambda_4}}) = 0.125$	$d(\overline{\Theta_{\Lambda_3}}, \overline{\Theta_{\Lambda_4}}) = 0.125$	$d(\overline{\Theta_{\Lambda_3}}, \overline{\Theta_{\Lambda_4}}) = 0.15$

Table 4 Expressed the support information

$\overline{\Theta}_{\Lambda_1}$	$\overline{\Theta}_{\Lambda_2}$	$\overline{\Theta}_{\Lambda_3}$	$\overline{\Theta}_{\Lambda_4}$	$\overline{\Theta}_{\Lambda_5}$
$\text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}) = 0.75$	$\text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}) = 0.85$	$\text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}) = 0.7$	$\text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}) = 0.75$	$\text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_2}) = 0.825$
$\text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_3}) = 0.8$	$\text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_3}) = 0.8$	$\text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_3}) = 0.875$	$\text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_3}) = 0.85$	$\text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_3}) = 0.8$
$\text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_4}) = 0.85$	$\text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_4}) = 0.775$	$\text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_4}) = 0.9$	$\text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_4}) = 0.775$	$\text{SUP}(\overline{\Theta}_{\Lambda_1}, \overline{\Theta}_{\Lambda_4}) = 0.8$
$\text{SUP}(\overline{\Theta}_{\Lambda_2}, \overline{\Theta}_{\Lambda_3}) = 0.75$	$\text{SUP}(\overline{\Theta}_{\Lambda_2}, \overline{\Theta}_{\Lambda_3}) = 0.85$	$\text{SUP}(\overline{\Theta}_{\Lambda_2}, \overline{\Theta}_{\Lambda_3}) = 0.675$	$\text{SUP}(\overline{\Theta}_{\Lambda_2}, \overline{\Theta}_{\Lambda_3}) = 0.8$	$\text{SUP}(\overline{\Theta}_{\Lambda_2}, \overline{\Theta}_{\Lambda_3}) = 0.875$
$\text{SUP}(\overline{\Theta}_{\Lambda_2}, \overline{\Theta}_{\Lambda_4}) = 0.75$	$\text{SUP}(\overline{\Theta}_{\Lambda_2}, \overline{\Theta}_{\Lambda_4}) = 0.875$	$\text{SUP}(\overline{\Theta}_{\Lambda_2}, \overline{\Theta}_{\Lambda_4}) = 0.75$	$\text{SUP}(\overline{\Theta}_{\Lambda_2}, \overline{\Theta}_{\Lambda_4}) = 0.875$	$\text{SUP}(\overline{\Theta}_{\Lambda_2}, \overline{\Theta}_{\Lambda_4}) = 0.925$
$\text{SUP}(\overline{\Theta}_{\Lambda_3}, \overline{\Theta}_{\Lambda_4}) = 0.75$	$\text{SUP}(\overline{\Theta}_{\Lambda_3}, \overline{\Theta}_{\Lambda_4}) = 0.725$	$\text{SUP}(\overline{\Theta}_{\Lambda_3}, \overline{\Theta}_{\Lambda_4}) = 0.875$	$\text{SUP}(\overline{\Theta}_{\Lambda_3}, \overline{\Theta}_{\Lambda_4}) = 0.875$	$\text{SUP}(\overline{\Theta}_{\Lambda_3}, \overline{\Theta}_{\Lambda_4}) = 0.85$

Table 5 Expressed the values of $\Gamma(\overline{\Theta}_{\Lambda_j})$

$\overline{\Theta}_{\Lambda_1}$	$\overline{\Theta}_{\Lambda_2}$	$\overline{\Theta}_{\Lambda_3}$	$\overline{\Theta}_{\Lambda_4}$	$\overline{\Theta}_{\Lambda_5}$
$\Gamma(\overline{\Theta}_{\Lambda_1}') = 2.4$	$\Gamma(\overline{\Theta}_{\Lambda_1}') = 2.425$	$\Gamma(\overline{\Theta}_{\Lambda_1}') = 2.475$	$\Gamma(\overline{\Theta}_{\Lambda_1}') = 2.375$	$\Gamma(\overline{\Theta}_{\Lambda_1}') = 2.425$
$\Gamma(\overline{\Theta}_{\Lambda_2}') = 2.25$	$\Gamma(\overline{\Theta}_{\Lambda_2}') = 2.575$	$\Gamma(\overline{\Theta}_{\Lambda_2}') = 2.125$	$\Gamma(\overline{\Theta}_{\Lambda_2}') = 2.425$	$\Gamma(\overline{\Theta}_{\Lambda_2}') = 2.625$
$\Gamma(\overline{\Theta}_{\Lambda_3}') = 2.3$	$\Gamma(\overline{\Theta}_{\Lambda_3}') = 2.375$	$\Gamma(\overline{\Theta}_{\Lambda_3}') = 2.425$	$\Gamma(\overline{\Theta}_{\Lambda_3}') = 2.525$	$\Gamma(\overline{\Theta}_{\Lambda_3}') = 2.525$
$\Gamma(\overline{\Theta}_{\Lambda_4}') = 2.35$	$\Gamma(\overline{\Theta}_{\Lambda_4}') = 2.375$	$\Gamma(\overline{\Theta}_{\Lambda_4}') = 2.525$	$\Gamma(\overline{\Theta}_{\Lambda_4}') = 2.525$	$\Gamma(\overline{\Theta}_{\Lambda_4}') = 2.575$

Table 6 Represented the values of $\overline{\mathbb{Z}}_j$ without weight vectors

$\overline{\Theta}_{\Lambda_1}$	$\overline{\Theta}_{\Lambda_2}$	$\overline{\Theta}_{\Lambda_3}$	$\overline{\Theta}_{\Lambda_4}$	$\overline{\Theta}_{\Lambda_5}$
$\overline{\mathbb{Z}}_1 = 0.2556$	$\overline{\mathbb{Z}}_1 = 0.2491$	$\overline{\mathbb{Z}}_1 = 0.2565$	$\overline{\mathbb{Z}}_1 = 0.2437$	$\overline{\mathbb{Z}}_1 = 0.242$
$\overline{\mathbb{Z}}_2 = 0.2444$	$\overline{\mathbb{Z}}_2 = 0.26$	$\overline{\mathbb{Z}}_2 = 0.2306$	$\overline{\mathbb{Z}}_2 = 0.2473$	$\overline{\mathbb{Z}}_2 = 0.2562$
$\overline{\mathbb{Z}}_3 = 0.2481$	$\overline{\mathbb{Z}}_3 = 0.2455$	$\overline{\mathbb{Z}}_3 = 0.2528$	$\overline{\mathbb{Z}}_3 = 0.2545$	$\overline{\mathbb{Z}}_3 = 0.2491$
$\overline{\mathbb{Z}}_4 = 0.2519$	$\overline{\mathbb{Z}}_4 = 0.2455$	$\overline{\mathbb{Z}}_4 = 0.2601$	$\overline{\mathbb{Z}}_4 = 0.2545$	$\overline{\mathbb{Z}}_4 = 0.2527$

Table 7 Represented the values of $\overline{\mathbb{Z}}_j$ with weight vectors

$\overline{\Theta}_{\Lambda_1}$	$\overline{\Theta}_{\Lambda_2}$	$\overline{\Theta}_{\Lambda_3}$	$\overline{\Theta}_{\Lambda_4}$	$\overline{\Theta}_{\Lambda_5}$
$\overline{\mathbb{Z}}_1 = 0.1024$	$\overline{\mathbb{Z}}_1 = 0.1001$	$\overline{\mathbb{Z}}_1 = 0.1019$	$\overline{\mathbb{Z}}_1 = 0.0967$	$\overline{\mathbb{Z}}_1 = 0.0963$
$\overline{\mathbb{Z}}_2 = 0.1958$	$\overline{\mathbb{Z}}_2 = 0.2091$	$\overline{\mathbb{Z}}_2 = 0.1833$	$\overline{\mathbb{Z}}_2 = 0.1963$	$\overline{\mathbb{Z}}_2 = 0.2039$
$\overline{\mathbb{Z}}_3 = 0.2982$	$\overline{\mathbb{Z}}_3 = 0.2961$	$\overline{\mathbb{Z}}_3 = 0.3013$	$\overline{\mathbb{Z}}_3 = 0.303$	$\overline{\mathbb{Z}}_3 = 0.2975$
$\overline{\mathbb{Z}}_4 = 0.4036$	$\overline{\mathbb{Z}}_4 = 0.3947$	$\overline{\mathbb{Z}}_4 = 0.4135$	$\overline{\mathbb{Z}}_4 = 0.404$	$\overline{\mathbb{Z}}_4 = 0.4023$

We derive four different types of results under the consideration of four different types of operators. Further, we explained the supremacy of the pioneered information by ignoring the part of the imaginary for the information in Table 2, then the final score values are stated in Table 11.

Finally, we examined the order information in the presence of score information to find the best decision, see Table 12.

Table 8 For $\xi = 2$, we aggregated the information by using different types of operators

Alternatives	C-IFAAPA operator	C-IFAAPG operator
$\overline{\overline{\Theta}}_{\Lambda_1}$	((0.3227, 0.256), (0.4578, 0.491))	((0.6273, 0.6134), (0.1298, 0.1149))
$\overline{\overline{\Theta}}_{\Lambda_2}$	((0.2688, 0.2878), (0.5065, 0.5005))	((0.6757, 0.7196), (0.1619, 0.1353))
$\overline{\overline{\Theta}}_{\Lambda_3}$	((0.3066, 0.3121), (0.557, 0.5047))	((0.5932, 0.7515), (0.1634, 0.1367))
$\overline{\overline{\Theta}}_{\Lambda_4}$	((0.2165, 0.28), (0.606, 0.6065))	((0.611, 0.7122), (0.1885, 0.1889))
$\overline{\overline{\Theta}}_{\Lambda_5}$	((0.3464, 0.1991), (0.4051, 0.5783))	((0.6854, 0.6228), (0.0836, 0.1503))

Alternatives	C-IFAAWPA operator	C-IFAAWPG operator
$\overline{\overline{\Theta}}_{\Lambda_1}$	((0.3376, 0.2448), (0.4869, 0.516))	((0.6091, 0.6308), (0.1473, 0.1262))
$\overline{\overline{\Theta}}_{\Lambda_2}$	((0.3029, 0.2515), (0.476, 0.5325))	((0.6884, 0.7023), (0.1638, 0.1421))
$\overline{\overline{\Theta}}_{\Lambda_3}$	((0.3276, 0.3118), (0.5531, 0.4988))	((0.6255, 0.7514), (0.1558, 0.1206))
$\overline{\overline{\Theta}}_{\Lambda_4}$	((0.1848, 0.2763), (0.641, 0.645))	((0.6011, 0.7178), (0.2081, 0.2056))
$\overline{\overline{\Theta}}_{\Lambda_5}$	((0.3702, 0.184), (0.4312, 0.5874))	((0.7587, 0.6065), (0.0992, 0.1554))

Table 9 Stated different score values

Alternatives	C-IFAAPA operator	C-IFAAPG operator	C-IFAAWPA operator	C-IFAAWPG operator
$\overline{\overline{\Theta}}_{\Lambda_1}$	- 0.185	0.498	- 0.21	0.4832
$\overline{\overline{\Theta}}_{\Lambda_2}$	- 0.225	0.5491	- 0.227	0.5424
$\overline{\overline{\Theta}}_{\Lambda_3}$	- 0.222	0.5223	- 0.206	0.5503
$\overline{\overline{\Theta}}_{\Lambda_4}$	- 0.358	0.4729	- 0.412	0.4526
$\overline{\overline{\Theta}}_{\Lambda_5}$	- 0.219	0.5371	- 0.232	0.5553

Table 10 Stated ranking information

C-IFAAPA operator	$\overline{\overline{\Theta}}_{\Lambda_1} > \overline{\overline{\Theta}}_{\Lambda_5} > \overline{\overline{\Theta}}_{\Lambda_3} > \overline{\overline{\Theta}}_{\Lambda_2} > \overline{\overline{\Theta}}_{\Lambda_4}$
C-IFAAPG operator	$\overline{\overline{\Theta}}_{\Lambda_2} > \overline{\overline{\Theta}}_{\Lambda_5} > \overline{\overline{\Theta}}_{\Lambda_3} > \overline{\overline{\Theta}}_{\Lambda_1} > \overline{\overline{\Theta}}_{\Lambda_4}$
C-IFAAWPA operator	$\overline{\overline{\Theta}}_{\Lambda_3} > \overline{\overline{\Theta}}_{\Lambda_1} > \overline{\overline{\Theta}}_{\Lambda_2} > \overline{\overline{\Theta}}_{\Lambda_5} > \overline{\overline{\Theta}}_{\Lambda_4}$
C-IFAAWPG operator	$\overline{\overline{\Theta}}_{\Lambda_5} > \overline{\overline{\Theta}}_{\Lambda_3} > \overline{\overline{\Theta}}_{\Lambda_2} > \overline{\overline{\Theta}}_{\Lambda_1} > \overline{\overline{\Theta}}_{\Lambda_4}$

Table 11 Stated different score values

Alternatives	C-IFAAPA operator	C-IFAAPG operator	C-IFAAWPA operator	C-IFAAWPG operator
$\overline{\overline{\Theta}}_{\Lambda_1}$	- 0.068	0.2488	- 0.075	0.2309
$\overline{\overline{\Theta}}_{\Lambda_2}$	- 0.119	0.2569	- 0.087	0.2623
$\overline{\overline{\Theta}}_{\Lambda_3}$	- 0.125	0.2149	- 0.113	0.2349
$\overline{\overline{\Theta}}_{\Lambda_4}$	- 0.195	0.2113	- 0.228	0.1965
$\overline{\overline{\Theta}}_{\Lambda_5}$	- 0.029	0.3009	- 0.03	0.3298

Table 12 Stated ranking information

C-IFAAPA operator	$\overline{\overline{\Theta}}_{\Lambda_5} > \overline{\overline{\Theta}}_{\Lambda_1} > \overline{\overline{\Theta}}_{\Lambda_2} > \overline{\overline{\Theta}}_{\Lambda_3} > \overline{\overline{\Theta}}_{\Lambda_4}$
C-IFAAPG operator	$\overline{\overline{\Theta}}_{\Lambda_5} > \overline{\overline{\Theta}}_{\Lambda_2} > \overline{\overline{\Theta}}_{\Lambda_1} > \overline{\overline{\Theta}}_{\Lambda_3} > \overline{\overline{\Theta}}_{\Lambda_4}$
C-IFAAWPA operator	$\overline{\overline{\Theta}}_{\Lambda_5} > \overline{\overline{\Theta}}_{\Lambda_1} > \overline{\overline{\Theta}}_{\Lambda_2} > \overline{\overline{\Theta}}_{\Lambda_3} > \overline{\overline{\Theta}}_{\Lambda_4}$
C-IFAAWPG operator	$\overline{\overline{\Theta}}_{\Lambda_5} > \overline{\overline{\Theta}}_{\Lambda_2} > \overline{\overline{\Theta}}_{\Lambda_3} > \overline{\overline{\Theta}}_{\Lambda_1} > \overline{\overline{\Theta}}_{\Lambda_4}$

We derive the best decision $\overline{\overline{\Theta}}_{\Lambda_5}$ under the consideration of four different types of operators. Further, by using the values of the parameter, we try to derive the influence or stability of the derived operators.

4.2 Influence/stability of parameter

Using the different values of parameter ξ , it is very necessary to check the stability and influence of the proposed work with the help of parameter ξ . Already, we proved that the derived operators are massively powerful and dominant to evaluate different kinds of problems. The main aim of this section is to check the stability and influence of the proposed work with the help of parameter ξ . The main analysis for different values of ξ , the final ranking information is described in Table 13.

Information in Table 13, we noticed that the idea of the C-IFAAPA operator is stated two different types of results such as $\overline{\overline{\Theta}}_{\Lambda_5}$ for the value of $\xi = 1$, and $\overline{\overline{\Theta}}_{\Lambda_1}$ for the value of $\xi = 3, 5, 7, 10, 11$. Moreover, the idea of C-IFAAPG operator stated the same results such as $\overline{\overline{\Theta}}_{\Lambda_2}$ for the different value of parameter $\xi = 1, 3, 5, 7, 10, 11$. Similarly, the idea of the C-IFAAWPA operator has stated two different types of results as $\overline{\overline{\Theta}}_{\Lambda_3}$ for the value of $\xi = 1$, and $\overline{\overline{\Theta}}_{\Lambda_1}$ for the value of $\xi = 3, 5, 7, 10, 11$. Finally, the idea of the C-IFAAWPG operator has stated three different types of results as $\overline{\overline{\Theta}}_{\Lambda_3}$ for the value of $\xi = 1$, $\overline{\overline{\Theta}}_{\Lambda_5}$ for the value of $\xi = 3, 7$, and $\overline{\overline{\Theta}}_{\Lambda_2}$ for the value of $\xi = 7, 10, 11$. We derived various kinds of results for four different kinds of operators, but many of them given the most repeated value is $\overline{\overline{\Theta}}_{\Lambda_1}$. Further, by excluding the phase term from the information in Table 2, then we check the influence of the proposed work with the help of parameter ξ . The main analysis for different values of ξ , the final ranking information is described in Table 14.

Table 13 Aggregated information for ξ (with phase term)

Parameter	Operator	Score values	Ranking values
$\xi = 1$	C-IFAAPA	$-0.2498, -0.2684, -0.2583,$ $-0.3881, -0.2441$	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAPG	$0.5438, 0.581, 0.5797, 0.5026,$ 0.5803	$\overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAWPA	$-0.2778, -0.2686, -0.2357,$ $-0.4354, -0.2527$	$\overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAWPG	$0.5277, 0.5781, 0.607, 0.4735,$ 0.5903	$\overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4}$
$\xi = 3$	C-IFAAPA	$-0.1332, -0.1858, -0.1943,$ $-0.3319, -0.2019$	$\overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAPG	$0.4671, 0.5251, 0.4762, 0.447,$ 0.4993	$\overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAWPA	$-0.1558, -0.1918, -0.1848,$ $-0.3898, -0.2167$	$\overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAWPG	$0.4553, 0.5163, 0.5014, 0.4336,$ 0.5188	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4}$
$\xi = 5$	C-IFAAPA	$-0.066, -0.1257, -0.1551,$ $-0.2926, -0.1794$	$\overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAPG	$0.4294, 0.4937, 0.4189, 0.4076,$ 0.4485	$\overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAWPA	$-0.0828, -0.1382, -0.153,$ $-0.349, -0.1943$	$\overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAWPG	$0.423, 0.4842, 0.4385, 0.4023,$ 0.4632	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4}$
$\xi = 7$	C-IFAAPA	$-0.0278, -0.0872, -0.1287,$ $-0.2664, -0.1646$	$\overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAPG	$0.4071, 0.4746, 0.3874, 0.3814,$ 0.4189	$\overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAWPA	$-0.0402, -0.1011, -0.1301,$ $-0.3166, -0.1786$	$\overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAWPG	$0.4041, 0.4658, 0.4029, 0.3795,$ 0.4299	$\overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4}$
$\xi = 10$	C-IFAAPA	$0.0046, -0.0526, -0.1029, -$ $0.2415, -0.1496$	$\overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAPG	$0.3865, 0.457, 0.3608, 0.3567,$ 0.3933	$\overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAWPA	$-0.004, -0.0651, -0.1061,$ $-0.2822, -0.1618$	$\overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_4}$

Table 13 (continued)

Parameter	Operator	Score values	Ranking values
$\xi = 11$	C-IFAAWPG	0.3859, 0.4495, 0.3723, 0.3564, 0.4011	$\overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAPA	0.0118, - 0.0447, - 0.0968, - 0.2357, - 0.1458	$\overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAPG	0.3816, 0.4528, 0.3548, 0.3509, 0.3875	$\overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4} > \overline{\Theta}_{\Lambda_3}$
	C-IFAAWPA	0.004, - 0.0565, - 0.1, - 0.2737, - 0.1574	$\overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAWPG	0.3814, 0.4458, 0.3654, 0.3508, 0.3947	$\overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$

Information in Table 14, we noticed that the idea of the C-IFAAPA operator is stated two different types of results such as $\overline{\Theta}_{\Lambda_5}$ for the value of parameter $\xi = 1, 3, 5$, and $\overline{\Theta}_{\Lambda_1}$ for the value of parameter $\xi = 7, 10, 11$. Moreover, the idea of C-IFAAPG operator stated the same results such as $\overline{\Theta}_{\Lambda_5}$ for the different value of parameter $\xi = 1, 3, 5, 7, 10, 11$. Similarly, the idea of the C-IFAAWPA operator has stated two different types of results as $\overline{\Theta}_{\Lambda_5}$ for the value of parameter $\xi = 1, 3, 5, 7$, and $\overline{\Theta}_{\Lambda_1}$ for the value of parameter $\xi = 10, 11$. Finally, the idea of the C-IFAAWPG operator has stated the same results such as $\overline{\Theta}_{\Lambda_5}$ for the value of parameter $\xi = 1, 3, 5, 7, 10, 11$. Here, check the stability of the derived work by excluding only the phase term from the information in Table 2. Therefore, we obtained $\overline{\Theta}_{\Lambda_5}$ by using four different types, it means that the phase term affects the overall results. Moreover, we compare the derived operators with certain prevailing operators to mention the supremacy and worth of the presented information.

5 Comparative/sensitive analysis

Here, we aim to evaluate the comparison between proposed and prevailing information. For this, we use the above illustrative practical example related to the consideration of the beneficial preference selected to evaluate the supremacy and proficiency of the derived information which is verified by comparing their final ranking results with various prevailing results. To compare the derived information with some prevailing information, we consider the prevailing information of Senapati et al. (2022b) to derive the theory of Aczel–Alsina aggregation operators for IFS. Additionally, PAOs for IFS were derived by Xu (2011) and the PAOs for C-FS were invented by Hu et al. (2019). Finally, nowadays, Rani and Garg (2018) presented the PAOs for C-IFSs. After, a brief discussion, the main comparative investigation is given in Table 15 based on the information in Table 2 (with phase terms).

All proposed operators and the theory of Mahmood et al. (2023) are given different ranking information. The information derived by Senapati et al. (2022b) such as the theory of Aczel–Alsina aggregation operators for IFS, PAOs for IFS were derived by Xu (2011), and the PAOs for C-FS were invented by Hu et al. (2019) lot of limitations because these all operators are computed based on IFSs and C-FS which are the special case of the derived

Table 14 Aggregated information for ξ (without phase term)

Parameter	Operator	Score values	Ranking values
$\xi = 1$	C-IFAAPA	- 0.1069, - 0.1441, - 0.1467, - 0.2124, - 0.0465	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAPG	0.2776, 0.2766, 0.2602, 0.2269, 0.3338	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAWPA	- 0.1197, - 0.1155, - 0.1304, - 0.2412, - 0.0434	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAWPG	0.2628, 0.2896, 0.2811, 0.2068, 0.3555	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4}$
$\xi = 3$	C-IFAAPA	- 0.0379, - 0.0965, - 0.112, - 0.1791, - 0.019	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAPG	0.2298, 0.2422, 0.2787, 0.1985, 0.2731	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAWPA	- 0.0424, - 0.0648, - 0.1023, - 0.2145, - 0.0215	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAWPG	0.2118, 0.2428, 0.1954, 0.1878, 0.3022	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4}$
$\xi = 5$	C-IFAAPA	- 0.0024, - 0.064, - 0.0956, - 0.1554, - 0.0074	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAPG	0.2074, 0.2229, 0.1357, 0.1798, 0.2397	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAWPA	- 0.0049, - 0.038, - 0.0892, - 0.1895, - 0.0099	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAWPG	0.1913, 0.2199, 0.1475, 0.1742, 0.2618	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4}$
$\xi = 7$	C-IFAAPA	0.0163, - 0.0441, - 0.0853, - 0.1398, - 0.001	$\overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAPG	0.1946, 0.2113, 0.1138, 0.1674, 0.2228	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAWPA	0.0146, - 0.0234, - 0.0809, - 0.1699, - 0.0031	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAWPG	0.1808, 0.2075, 0.1227, 0.1642, 0.2395	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4}$
$\xi = 10$	C-IFAAPA	0.0313, - 0.0266, - 0.0755, - 0.1254, 0.0045	$\overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAPG	0.183, 0.2005, 0.0964, 0.1554, 0.2095	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAWPA	0.0301, - 0.0112, - 0.0726, - 0.1496, 0.003	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$

Table 14 (continued)

Parameter	Operator	Score values	Ranking values
$\xi = 11$	C-IFAAWPG	0.172, 0.1968, 0.1028, 0.1538, 0.2216	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAPA	0.0345, - 0.0226, - 0.0731, - 0.122, 0.0057	$\overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAPG	0.1803, 0.1979, 0.09260.1525, 0.2066	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4} > \overline{\Theta}_{\Lambda_3}$
	C-IFAAWPA	0.335, - 0.0085, - 0.0705, - 0.1447, 0.0044	$\overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$
	C-IFAAWPG	0.1701, 0.1943, 0.0985, 0.1512, 0.2177	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4} > \overline{\Theta}_{\Lambda_3}$

Table 15 Comparison information (with phase term information in Table 2)

Methods	Score values	Ranking values
Senapati et al. (2022b)	$\times \times - \times \times - \times \times - \times \times$	$\times \times - \times \times - \times \times - \times \times$
Xu (Xu 2011)	$\times \times - \times \times - \times \times - \times \times$	$\times \times - \times \times - \times \times - \times \times$
Hu et al. (2019)	$\times \times - \times \times - \times \times - \times \times$	$\times \times - \times \times - \times \times - \times \times$
Rani and Garg (2018)	0.2762, 0.2681, 0.2933, 0.1207, 0.2761	$\overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_4}$
C-IFAAPA operator	- 0.185, - 0.225, - 0.222, - 0.358, - 0.219	$\overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_4}$
C-IFAAPG operator	0.498, 0.5491, 0.5223, 0.4729, 0.5371	$\overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4}$
C-IFAAWPA operator	- 0.21, - 0.227, - 0.206, - 0.412, - 0.232	$\overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_4}$
C-IFAAWPG operator	0.4832, 0.5424, 0.5503, 0.4526, 0.5553	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4}$

information in Table 2. But the theory derived by Rani and Garg (2018), such as the PAOs for C-IFSSs has easily evaluated our considered information in Table 2. After, a brief discussion, the main comparative investigation is given in Table 16 based on the information in Table 2 (without phase terms).

All proposed operators, the theory of Rani and Garg (2018), the theory of Senapati et al. (2022b), and the theory of Xu (2011) are given the same ranking results, where the best optimal is $\overline{\Theta}_{\Lambda_5}$. But the theory of PAOs for C-FS was invented by Hu et al. (2019) and has a lot of limitations because these all operators are computed based on C-FS which is the special case of the derived information in Table 2 (without phase term). After, a brief discussion, we concluded that the derived operators played a very important role in the environment of fuzzy decision-making technique and hence our investigation is massively superior to a lot of existing information.

Table 16 Comparison information (without phase term information in Table 2)

Methods	Score values	Ranking values
Senapati et al. (2022b)	− 0.0742, − 0.0858, − 0.1154, − 0.2274, − 0.0307	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$
Xu (2011)	0.1629, 0.1203, 0.1257, 0.0303, 0.2297	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_4}$
Hu et al. (2019)	× × − × × − × × − × ×	× × − × × − × × − × ×
Rani and Garg (2018)	0.1629, 0.1203, 0.1257, 0.0303, 0.2297	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_4}$
C-IFAAPA operator	− 0.068, − 0.119, − 0.125, − 0.195, − 0.029	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$
C-IFAAPG operator	0.2488, 0.2569, 0.2149, 0.2113, 0.3009	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$
C-IFAAWPA operator	− 0.075, − 0.087, − 0.113, − 0.228, − 0.03	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_4}$
C-IFAAWPG operator	0.2309, 0.2623, 0.2349, 0.1965, 0.3298	$\overline{\Theta}_{\Lambda_5} > \overline{\Theta}_{\Lambda_2} > \overline{\Theta}_{\Lambda_3} > \overline{\Theta}_{\Lambda_1} > \overline{\Theta}_{\Lambda_4}$

6 Conclusion

PAOs were derived by Yager which is very famous and valuable to cope with awkward and unreliable information. Further, Aczel–Alsina t-norm and t-conorm were evaluated by Aczel and Alsina, which is used for computing any kind of operator. Under the presence of the above information, we derived the following information, such as:

1. We derived the theory of the C-IFAAPA operator and C-IFAAWPA operator.
2. We exposed the idea of the C-IFAAPG operator and C-IFAAWPG operator.
3. We derived the theory of idempotency and prove that the property of monotonicity and boundedness failed with the help of some counterexamples.
4. We evaluated a MADM approach to derive operators in the presence of C-IF information.
5. We demonstrated some practical examples for comparing the derived work with various existing or prevailing operators to show the supremacy and proficiency of the derived information.

In the future, we concentrate to revise the theory of complex bipolar FSs (Gulistan et al. 2021), C-IF graph (Yaqoob et al. 2019), complex neutrosophic graph (Yaqoob and Akram 2018), m-polar fuzzy aggregation operators (Ali et al. 2023), Dombi aggregation operators (Akram et al. 2020, 2021), interval-valued q-rung orthopair fuzzy soft information (Ali et al. 2022), Fermatean fuzzy bipolar soft information (Ali and Ansari 2022) and try to utilize it in the field of artificial intelligence, machine learning, pattern recognition, and clustering analysis.

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Declarations

Conflict of interest About the publication of this manuscript the authors declare that they have no conflict of interest.

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