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Einstein Heronian mean aggregation operator and its application in decision making problems

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Abstract
In this paper, an aggregation operator for multi-criteria decision-making (MCDM) problems of interval valued intuitionistic fuzzy sets (IVIFSs) is proposed. In the present approach, the Heronian mean (HM) operator and Einstein operational laws are combined to develop interval-valued intuitionistic fuzzy Einstein Heronian mean (IVIFEHM) operator. Properties of proposed aggregation operator are investigated. Further the technique for order preference by similarity to ideal solution (TOPSIS) MCDM model is established using IVIFEHM operator and Jaccard distance measure. The proposed model is demonstrated by solving a numerical example and its efficiency is authenticated by comparing with existing methods.

Keywords IVIFSs · Einstein operations · Heronian mean operator · Jaccard distance · TOPSIS

Mathematics Subject Classification 91B06 · 03E72

1 Introduction

Assuming membership and non-membership functions as intervals, Atanassov and Gargov (1989) proposed the interval-valued intuitionistic fuzzy sets (IVIFSs) for solving decision-making problems. The aggregation operators are the important decision-making tools which gives the overall evaluation of each alternative, and then based on these overall values, the ranking of alternatives is made. Therefore, aggregation operators are one of the important decision-making tools in modern decision theory. In general, the intersection and union in aggregation operators are modeled using algebraic operational rules. Xu (2007), Xu and Chen

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(2007) proposed arithmetic and geometric averaging operators using algebraic operations of IVIFNs for aggregating the information and applied to decision making. In the real-world decision problems, sometimes there exists complex relationships among the criteria. Thus, it is of utmost importance for an aggregation operator to capture those complex interrelationships and generate more accurate aggregation results (Liu et al. 2015). Therefore, researchers initiated using union and intersection with t-operators as they provide unique optimal solution (Wang 2018). Developing aggregation operators using t-operators grabbed more attention in the past decade; different operators were developed using various t-norms and t-conorms [Dombi (Wu et al. 2018, 2020); Hamacher (Liu 2013); Frank (Zhang 2017) and Einstein (Liu and Wang 2020)].

Considering the fact that Einstein norms are conventional examples in the class of Archimedean norms (Klement et al. 2000), several researchers proposed Einstein operations on various generalizations of fuzzy sets. Wang and Liu (2011, 2012) developed Einstein norms on intuitionistic fuzzy sets proposed arithmetic and geometric aggregation operators and studied various properties of these operators. Sireesha and Himabindu (2019) defined Einstein operational laws on interval-valued intuitionistic trapezoidal fuzzy sets and presented an Einstein weighted averaging operator. Liu et al. (2015) defined Einstein laws on IVIFSs and developed several averaging and geometric operators to solve decision making problems.

Due to extreme criterion values and biased assessments, there may lie a negative effect on the results of aggregation; to prevent this, researchers developed aggregation operators using mean operators (Subramanian et al 2020). Different mean operators: Heronian (Yu 2013; Liu and Chen 2017, Hamy (Wu et al. 2018), Muirhead (Liu and Li 2017), Bonferroni (Zhou et al. 2019) and Maclaurin (Ali and Tahir 2020) were extensively used during aggregation. Yu and Wu (2012) noted that the Heronian mean (HM) operator considers the interrelationship of the aggregated arguments and presented HM operator to aggregate IVIFNs. Nevertheless, they also elucidated the advantages of HM over Bonferroni mean, which also considers the interrelationship between the aggregated arguments. Therefore, to combine information HM operator can be used to produce robust aggregation operators.

There are several methods to solve decision making problems. Yoon and Hwang (1981) developed the decision-making model TOPSIS, which is a major area of research interest. The TOPSIS method has made enormous success in dealing with IVIF decision making problems (Bai 2013; Abdullah et al. 2020). Dugenci (2016) proposed a generalized distance measure for IVIFSs and an extension of TOPSIS method is presented, in which the proposed distance measures are used as separation measures and illustrated its application to DM. Wang (2021) defined novel distance measures between IVIFSs and presented an IVIF-TOPSIS method by extending conventional TOPSIS method. Moreover, Jaccard similarity measure provides a very simple and intuitive measure of similarity between data samples and hence been frequently implemented in decision support systems with various domains (Verma and Rajesh 2020). The Jaccard distance is the complement of the Jaccard similarity co-efficient and measures dissimilarity of two sets.

The comprehensive analysis on IVIF decision making problems motivated us to develop an averaging operator using the Heronian mean with Einstein norms. Further, an integrated IVIF-TOPSIS model is presented in which the developed operator is used and the Jaccard distance is used to calculate the separation measure. The paper is ordered as follows: In Sect. 2 definitions and operations on IVIFNs, distance measure and mean operator are reviewed. In the later section, Heronian Mean operator is developed based on Einstein operations for aggregating IVIFNs. In Sect. 4, a TOPSIS model using the proposed aggregating operator and a distance measure is presented. A numerical example is illustrated in Sect. 5 for the demonstration of model. In Sect. 6, to show the authenticity of the model a comparative analysis is given. Finally, the study conclusion is given in Sect. 7.

2 Preliminaries

This section reviews definition and literature about IVIFSs, Heronian mean and Jaccard distance measure.

Definition 1 Interval-valued intuitionistic fuzzy sets (IVIFSs) (Atanassov and Gargov 1989).

Let X be a universe set and $E = \{x_1, x_2, \ldots, x_n\}$ be a subset of its elements, then the interval-valued intuitionistic fuzzy set $\tilde{P}$ has the form

$$\tilde{P} = \left\{< x_i, \left(\left[\mu_{\tilde{P}L}(x_i), \mu_{\tilde{P}U}(x_i)\right], \left[\vartheta_{\tilde{P}L}(x_i), \vartheta_{\tilde{P}U}(x_i)\right]\right) > x_i \in V\right\},$$

where $\mu_{\tilde{P}} : X \rightarrow S[0,1]$ and $\vartheta_{\tilde{P}} : X \rightarrow S[0,1]$ satisfying the condition $0 \leq \mu_{\tilde{P}}^{U}(x_i) + \vartheta_{\tilde{P}}^{U}(x_i) \leq 1$ for $x \in X$ and $S[0,1]$ are closed subintervals of $[0,1]$.

Note: 1. If $\mu_{\tilde{P}}^{L}(x_i) = \mu_{\tilde{P}}^{U}(x_i)$ and $\vartheta_{\tilde{P}}^{L}(x_i) = \vartheta_{\tilde{P}}^{U}(x_i)$ then the IVIFS $\tilde{P}$ reduces to IFS (Atanassov 1994).

2. Let $\tilde{P} = \{< x_i, (\left[\mu_{\tilde{P}}^{L}(x_i), \mu_{\tilde{P}}^{U}(x_i)\right], \left[\vartheta_{\tilde{P}}^{L}(x_i), \vartheta_{\tilde{P}}^{U}(x_i)\right]) > x_i \in V\}$ be an IVIFS, then the pair $(\left[\mu_{\tilde{P}}^{L}(x_i), \mu_{\tilde{P}}^{U}(x_i)\right], \left[\vartheta_{\tilde{P}}^{L}(x_i), \vartheta_{\tilde{P}}^{U}(x_i)\right])$ is called as an IVIFN (Atanassov 1994).

3. For convenience, we can denote an IVIFN by $\widetilde{\delta} = ([a, b], [c, d])$.

Definition 2 Interval hesitancy degree (Atanassov 1994).

For each element x_i, the unknown degree (hesitancy degree) of an intuitionistic fuzzy interval of $x_i \in X$ in $\tilde{P}$ can be computed.

The interval hesitancy degree of $\tilde{P}$ is defined as follows:

$$\pi_{\tilde{P}}(x_i) = [1 - \mu_{\tilde{P}}^{U}(x_i) - \vartheta_{\tilde{P}}^{U}(x_i), 1 - \mu_{\tilde{P}}^{L}(x_i) - \vartheta_{\tilde{P}}^{L}(x_i)]$$

Definition 3 Operations on IVIFNs (Atanassov 1994).

Let $\widetilde{\gamma_1} = ([s_1, t_1], [u_1, v_1])$ and $\widetilde{\gamma_2} = ([s_2, t_2], [u_2, v_2])$ be IVIFNs, the operations on $\widetilde{\gamma_1}$ and $\widetilde{\gamma_2}$ are given as:

$$\widetilde{\gamma_1} \cap \widetilde{\gamma_2} = ([\min(s_1, s_2), \min(t_1, t_2)], [\max(u_1, u_2), \max(v_1, v_2)])$$

$$\widetilde{\gamma_1} \cup \widetilde{\gamma_2} = ([\max(s_1, s_2), \max(t_1, t_2)], [\min(u_1, u_2), \min(v_1, v_2)])$$

$$\widetilde{\gamma_1} \oplus \widetilde{\gamma_2} = ([s_1 + s_2 - s_1 s_2, t_1 + t_2 - t_1 t_2], [u_1 u_2, v_1 v_2])$$

$$\widetilde{\gamma_1} \otimes \widetilde{\gamma_2} = ([s_1 s_2, t_1 t_2], [u_1 + u_2 - u_1 u_2, v_1 + v_2 - v_1 v_2])$$

$$r\, \widetilde{\gamma_1} = \left(\left[1 - (1 - s_1)^r, 1 - (1 - t_1)^r\right], \left[{u_1}^r, {v_1}^r\right]\right), r > 0$$

$$\widetilde{\gamma_1}^{\,r} = \left(\left[{s_1}^r, {t_1}^r\right], \left[1 - (1 - u_1)^r, 1 - (1 - v_1)^r\right]\right), r > 0$$

Definition 4 Triangular operators (Navara 2007).

t-norm: a triangular norm (t-norm) operator T, is a binary operation on [0,1] satisfying the following conditions:

- $T(a, b) = T(b, a)$
- $T(a, T(b, c)) = T(T(a, b), c)$
- $b \leq c \Rightarrow T(a, b) \leq T(a, c)$
- $T(a, 1) = a$

t-conorm: a triangular conorm (t-conorm) operator S, is a binary operation on [0, 1]; is the dual notion to a t-norm satisfying the following conditions:

- $S(a, b) = S(a, b)$
- $S(a, S(b, c)) = S(S(a, b), c)$
- $b \leq c \Rightarrow S(a, b) \leq S(a, c)$
- $S(a, 0) = a$

The representation of t-conorms is dual to those of t-norms i.e., $S(a, b) = 1 - T(1 - a, 1 - b)$.

Example The Einstein t-norm (T) and t-conorm (S) are defined as follows (Beliakov et al. 2007).

$$T(s, t) = \frac{st}{1 + (1 - s)(1 - t)}$$
$$S(s, t) = \frac{s + t}{1 + st}$$

Definition 5 Aggregation operator.

The aggregation operators are mathematical objects that have the function of reducing a set of numbers into a unique number.

Example Generalized Heronian mean operartor (Sykora 2009).

Let $p, q \geq 0$ and $I = [0, 1]$, then the function $\mathrm{GHM}^{p,q} : I^n \to I$ given by

$$\mathrm{GHM}^{p,q}(b_1, b_2, \ldots b_m) = \left(\frac{2}{m(m+1)} \sum_{s=1}^{m} \sum_{t=s}^{m} b_s{}^p b_t{}^q \right)^{\frac{1}{p+q}}$$

is called Generalized Heronian operator.

Definition 6 Einstein operational rules of IVIFNs (Liu et al. 2015).

The Einstein norm operations on IVIFNs $\widetilde{\gamma}_1 = ([s_1, t_1], [u_1, v_1])$ and $\widetilde{\gamma}_2 = ([s_2, t_2], [u_2, v_2])$ are defined as follows:

$$\widetilde{\gamma}_1 \oplus_E \widetilde{\gamma}_2 = \left(\left[\frac{s_1 + s_2}{1 + s_1 s_2}, \frac{t_1 + t_2}{1 + t_1 t_2} \right], \left[\frac{u_1 u_2}{1 + (1 - u_1)(1 - u_2)}, \frac{v_1 v_2}{1 + (1 - v_1)(1 - v_2)} \right] \right)$$
$$\widetilde{\gamma}_1 \otimes_E \widetilde{\gamma}_2 = \left(\left[\frac{s_1 s_2}{1 + (1 - s_1)(1 - s_2)}, \frac{t_1 t_2}{1 + (1 - t_1)(1 - t_2)} \right], \left[\frac{u_1 + u_2}{1 + u_1 u_2}, \frac{v_1 + v_2}{1 + v_1 v_2} \right] \right)$$

for $n > 0$

$$n\widetilde{\gamma}_1 = \left(\left[\frac{(1 + s_1)^n - (1 + s_2)^n}{(1 + s_1)^n + (1 + s_2)^n}, \frac{(1 + t_1)^n - (1 + t_2)^n}{(1 + t_1)^n + (1 + t_2)^n} \right], \left[\frac{2u_1^n}{(2 - u_1)^n + u_1^n}, \frac{2v_1^n}{(2 - v_1)^n + v_1^n} \right] \right)$$
$$\widetilde{\gamma}_1{}^n = \left(\left[\frac{2s_1^n}{(2 - s_1)^n + s_1^n}, \frac{2t_1^n}{(2 - t_1)^n + t_1^n} \right], \left[\frac{(1 + u_1)^n - (1 + u_2)^n}{(1 + u_1)^n + (1 + u_2)^n}, \frac{(1 + v_1)^n - (1 + v_2)^n}{(1 + v_1)^n + (1 + v_2)^n} \right] \right)$$

Definition 7 Jaccard distance (Michael and David 1971).

Jaccard distance is a measure of dissimilarity between two sets, given by

$$d_J\left(\tilde{P}_1, \tilde{P}_2\right) = 1 - S_J\left(\tilde{P}_1, \tilde{P}_2\right),$$

where $S_J\left(\tilde{P}_1, \tilde{P}_2\right) = \frac{\left|\tilde{P}_1 \cap \tilde{P}_2\right|}{\left|\tilde{P}_1 \cup \tilde{P}_2\right|}$ is Jaccard similarity co-efficient.

Definition 8 Jaccard distance measure on IVIFSs (Anusha and Sireesha 2021).

For any IVIFSs $\tilde{P}_1, \tilde{P}_2$ on U.

The Jaccard distance between $\tilde{P}_1$ and $\tilde{P}_2$ is

$$d_J\left(\tilde{P}_1, \tilde{P}_2\right) = 1 - \frac{\left| \begin{bmatrix} \min\left(\mu_{\tilde{P}_1}{}^L(x_i), \mu_{\tilde{P}_2}{}^L(x_i)\right), \\ \min\left(\mu_{\tilde{P}_1}{}^U(x_i), \mu_{\tilde{P}_2}{}^U(x_i)\right) \end{bmatrix}, \begin{bmatrix} \max\left(\vartheta_{\tilde{P}_1}{}^L(x_i), \vartheta_{\tilde{P}_2}{}^L(x_i)\right), \\ \max(\vartheta_{\tilde{P}_1}{}^U(x_i), \vartheta_{\tilde{P}_2}{}^U(x_i)) \end{bmatrix}, \begin{bmatrix} \min\left(\pi_{\tilde{P}_1}^L(x_i), \pi_{\tilde{P}_2}^L(x_i)\right), \\ \max(\pi_{\tilde{P}_1}^U(x_i), \pi_{\tilde{P}_2}^U(x_i)) \end{bmatrix} \right|}{\left| \begin{bmatrix} \max\left(\mu_{\tilde{P}_1}{}^L(x_i), \mu_{\tilde{P}_2}{}^L(x_i)\right), \\ \max\left(\mu_{\tilde{P}_1}{}^U(x_i), \mu_{\tilde{P}_2}{}^U(x_i)\right) \end{bmatrix}, \begin{bmatrix} \min\left(\vartheta_{\tilde{P}_1}{}^L(x_i), \vartheta_{\tilde{P}_2}{}^L(x_i)\right), \\ \min(\vartheta_{\tilde{P}_1}{}^U(x_i), \vartheta_{\tilde{P}_2}{}^U(x_i)) \end{bmatrix}, \begin{bmatrix} \min(\pi_{\tilde{P}_1}^L(x_i), \pi_{\tilde{P}_2}^L(x_i)), \\ \max\left(\pi_{\tilde{P}_1}^U(x_i), \pi_{\tilde{P}_2}^U(x_i)\right) \end{bmatrix} \right|}$$

3 Proposed Einstein Heronian mean operator for IVIFSs

In fuzzy set theory, the t-operators are important applications in fuzzy decision making. It has been observed that a suitable choice of triangular operators on different applications can considerably enhance or deteriorate the system's performance (Roy and Wang 1998). As, the Heronian mean considers the interrelationship of the aggregated arguments and relieves the calculation redundancy, an aggregation operator using Heronian mean is developed.

3.1 Interval-valued intuitionistic fuzzy Einstein Heronian mean (IVIFEHM) operator

The IVIFEHM operator is defined using the generalized Heronian mean operator combined with Einstein operational rules, given as:

Definition 9 Let $\tilde{\gamma}_i = ([s_i, t_i], [u_i, v_i])$ be a set of i IVIFNs. The IVIFEHM operator on $\tilde{\gamma}_1, \tilde{\gamma}_2, \ldots, \tilde{\gamma}_n$:

$$\text{IVIFEHM}^{p,q}(\tilde{\gamma}_1, \tilde{\gamma}_2, \ldots, \tilde{\gamma}_n) = \left(\frac{2}{n(n+1)} \oplus_{i=1}^{n} \oplus_{j=i}^{n} \left(\tilde{\gamma}_i{}^p \otimes \tilde{\gamma}_i{}^q\right)\right)^{\frac{1}{p+q}}.$$

Theorem 1 *Let $\tilde{\gamma}_i = ([s_i, t_i], [u_i, v_i])$ bei IVIFNs and $p, q \geq 0$. Then the aggregated value of $\tilde{\gamma}_i$'s by IVIFEHM operator is also an IVIFN, and is*:

$$\text{IVIFEHM}^{p,q}(\tilde{\gamma}_1, \tilde{\gamma}_2, \ldots, \tilde{\gamma}_n) = \left(\frac{2}{n(n+1)} \oplus_{i=1}^{n} \oplus_{j=i}^{n} \left(\tilde{\gamma}_i{}^p \otimes \tilde{\gamma}_i{}^q\right)\right)^{\frac{1}{p+q}}$$

$$= \left(\begin{array}{l} \left[\frac{2\left(\prod_{i=1}^{n}\prod_{j=i}^{n} e_{ij}^{\frac{2}{n(n+1)}} - \prod_{i=1}^{n}\prod_{j=i}^{n} f_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}}{\left(\prod_{i=1}^{n}\prod_{j=i}^{n} e_{ij}^{\frac{2}{n(n+1)}} + 3\prod_{i=1}^{n}\prod_{j=i}^{n} f_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} + \left(\prod_{i=1}^{n}\prod_{j=i}^{n} e_{ij}^{\frac{2}{n(n+1)}} - \prod_{i=1}^{n}\prod_{j=i}^{n} f_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}}, \frac{2\left(\prod_{i=1}^{n}\prod_{j=i}^{n} g_{ij}^{\frac{2}{n(n+1)}} - \prod_{i=1}^{n}\prod_{j=i}^{n} h_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}}{\left(\prod_{i=1}^{n}\prod_{j=i}^{n} g_{ij}^{\frac{2}{n(n+1)}} + 3\prod_{i=1}^{n}\prod_{j=i}^{n} h_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} + \left(\prod_{i=1}^{n}\prod_{j=i}^{n} g_{ij}^{\frac{2}{n(n+1)}} - \prod_{i=1}^{n}\prod_{j=i}^{n} h_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}} \right], \\ \left[\frac{\left(\prod_{i=1}^{n}\prod_{j=i}^{n} k_{ij}^{\frac{2}{n(n+1)}} + 3\prod_{i=1}^{n}\prod_{j=i}^{n} l_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} - \left(\prod_{i=1}^{n}\prod_{j=i}^{n} k_{ij}^{\frac{2}{n(n+1)}} - \prod_{i=1}^{n}\prod_{j=i}^{n} l_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}}{\left(\prod_{i=1}^{n}\prod_{j=i}^{n} k_{ij}^{\frac{2}{n(n+1)}} + 3\prod_{i=1}^{n}\prod_{j=i}^{n} l_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} + \left(\prod_{i=1}^{n}\prod_{j=i}^{n} k_{ij}^{\frac{2}{n(n+1)}} - \prod_{i=1}^{n}\prod_{j=i}^{n} l_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}}, \frac{\left(\prod_{i=1}^{n}\prod_{j=i}^{n} n_{ij}^{\frac{2}{n(n+1)}} + 3\prod_{i=1}^{n}\prod_{j=i}^{n} m_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} - \left(\prod_{i=1}^{n}\prod_{j=i}^{n} n_{ij}^{\frac{2}{n(n+1)}} - \prod_{i=1}^{n}\prod_{j=i}^{n} m_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}}{\left(\prod_{i=1}^{n}\prod_{j=i}^{n} n_{ij}^{\frac{2}{n(n+1)}} + 3\prod_{i=1}^{n}\prod_{j=i}^{n} m_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} + \left(\prod_{i=1}^{n}\prod_{j=i}^{n} n_{ij}^{\frac{2}{n(n+1)}} - \prod_{i=1}^{n}\prod_{j=i}^{n} m_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}} \right] \end{array} \right),$$

where $e_{ij} = (2 - a_i)^p (2 - a_j)^q + 3{a_i}^p a_j^q$

$$f_{ij} = (2 - a_i)^p (2 - a_j)^q - {a_i}^p a_j^q$$

$$g_{ij} = (2 - b_i)^p (2 - b_j)^q + 3{b_i}^p b_j^q$$

$$h_{ij} = (2 - b_i)^p (2 - b_j)^q - {b_i}^p b_j^q$$

$$k_{ij} = (1 + c_i)^p (1 + c_j)^q + 3(1 - c_i)^p (1 - c_j)^q$$

$$l_{ij} = (1 + c_i)^p (1 + c_j)^q - (1 - c_i)^p (1 - c_j)^q$$

$$n_{ij} = (1 + d_i)^p (1 + d_j)^q + 3(1 - d_i)^p (1 - d_j)^q$$

$$m_{ij} = (1 + d_i)^p (1 + d_j)^q - (1 - d_i)^p (1 - d_j)^q$$

Proof

$$\tilde{\gamma}_i^{E^p} = \left(\left[\frac{2a_i^p}{\left((2 - a_i)^p + a_i^p\right)}, \frac{2b_i^p}{\left((2 - b_i)^p + b_i^p\right)} \right], \left[\frac{(1 + c_i)^p - (1 - c_i)^p}{(1 + c_i)^p + (1 - c_i)^p}, \frac{(1 + d_i)^p - (1 - d_i)^p}{(1 + d_i)^p + (1 - d_i)^p} \right] \right)$$

$$\tilde{\gamma}_j^{E^q} = \left(\left[\frac{2a_j^q}{\left((2 - a_j)^q + a_j^q\right)}, \frac{2b_j^q}{\left((2 - b_j)^q + b_j^q\right)} \right], \left[\frac{(1 + c_j)^q - (1 - c_j)^q}{(1 + c_j)^q + (1 - c_j)^q}, \frac{(1 + d_j)^q - (1 - d_j)^q}{(1 + d_j)^q + (1 - d_j)^q} \right] \right)$$

Then,

$$\tilde{\gamma}_i^{E^p} \otimes_E \tilde{\gamma}_j^{E^q} = \left(\begin{array}{c} \left[\frac{2a_i^p a_j^q}{(2-a_i)^p (2-a_j)^q + a_i^p a_j^q}, \frac{2b_i^p b_j^q}{(2-b_i)^p (2-b_j)^q + b_i^p b_j^q} \right], \\ \left[\frac{(1+c_i)^p (1+c_j)^q - (1-c_i)^p (1-c_j)^q}{(1+c_i)^p (1+c_j)^q + (1-c_i)^p (1-c_j)^q}, \frac{(1+d_i)^p (1+d_j)^q - (1-d_i)^p (1-d_j)^q}{(1+d_i)^p (1+d_j)^q + (1-d_i)^p (1-d_j)^q} \right] \end{array} \right)$$

Thus,

$$\frac{2}{n(n+1)}\otimes_E\left(\tilde{\gamma}_l^{E^p}\otimes_E\tilde{\gamma}_j^{E^q}\right)$$

$$=\left(\begin{array}{c}\left[\begin{array}{c}\dfrac{\left((2-a_i)^p(2-a_j)^q+3a_i^pa_j^q\right)^{\frac{2}{n(n+1)}}-\left((2-a_i)^p(2-a_j)^q-a_i^pa_j^q\right)^{\frac{2}{n(n+1)}}}{\left((2-a_i)^p(2-a_j)^q+3a_i^pa_j^q\right)^{\frac{2}{n(n+1)}}+\left((2-a_i)^p(2-a_j)^q-a_i^pa_j^q\right)^{\frac{2}{n(n+1)}}},\\ \dfrac{\left((2-b_i)^p(2-b_j)^q+3b_i^pb_j^q\right)^{\frac{2}{n(n+1)}}-\left((2-b_i)^p(2-b_j)^q-b_i^pb_j{}^q\right)^{\frac{2}{n(n+1)}}}{\left((2-b_i)^p(2-b_j)^q+3b_i^pb_j^q\right)^{\frac{2}{n(n+1)}}+\left((2-b_i)^p(2-b_j)^q-b_i^pb_j^q\right)^{\frac{2}{n(n+1)}}}\end{array}\right],\\ \left[\begin{array}{c}\dfrac{2\left((1+c_i)^p(1+c_j)^q-(1-c_i)^p(1-c_j)^q\right)^{\frac{2}{n(n+1)}}}{\left((1+c_i)^p(1+c_j)^q+3(1-c_i)^p(1-c_j)^q\right)^{\frac{2}{n(n+1)}}+\left((1+c_i)^p(1+c_j)^q-(1-c_i)^p(1-c_j)^q\right)^{\frac{2}{n(n+1)}}},\\ \dfrac{2\left((1+d_i)^p(1+d_j)^q-(1-d_i)^p(1-d_j)^q\right)^{\frac{2}{n(n+1)}}}{\left((1+d_i)^p(1+d_j)^q+3(1-d_i)^p(1-d_j)^q\right)^{\frac{2}{n(n+1)}}+\left((1+d_i)^p(1+d_j)^q-(1-d_i)^p(1-d_j)^q\right)^{\frac{2}{n(n+1)}}}\end{array}\right]\end{array}\right)$$

Let $e_{ij}=(2-a_i)^p\left(2-a_j\right)^q+3a_i^pa_j{}^q$

$$f_{ij}=(2-a_i)^p\left(2-a_j\right)^q-a_i^pa_j{}^q$$

$$g_{ij}=(2-b_i)^p\left(2-b_j\right)^q+3b_i^pb_j{}^q$$

$$h_{ij}=(2-b_i)^p\left(2-b_j\right)^q-b_i^pb_j{}^q$$

$$k_{ij}=(1+c_i)^p\left(1+c_j\right)^q+3(1-c_i)^p\left(1-c_j\right)^q$$

$$l_{ij}=(1+c_i)^p\left(1+c_j\right)^q-(1-c_i)^p\left(1-c_j\right)^q$$

$$n_{ij}=(1+d_i)^p\left(1+d_j\right)^q+3(1-d_i)^p\left(1-d_j\right)^q$$

$$m_{ij}=(1+d_i)^p\left(1+d_j\right)^q-(1-d_i)^p\left(1-d_j\right)^q$$

Then,

$$\frac{2}{n(n+1)}\otimes_E\left(\tilde{\gamma}_i^{E^p}\otimes_E\tilde{\gamma}_j^{E^q}\right)$$
$$=\left(\left[\frac{e_{ij}{}^{\frac{2}{n(n+1)}}-f_{ij}{}^{\frac{2}{n(n+1)}}}{e_{ij}{}^{\frac{2}{n(n+1)}}+f_{ij}{}^{\frac{2}{n(n+1)}}},\frac{g_{ij}{}^{\frac{2}{n(n+1)}}-h_{ij}{}^{\frac{2}{n(n+1)}}}{g_{ij}{}^{\frac{2}{n(n+1)}}+h_{ij}{}^{\frac{2}{n(n+1)}}}\right],\left[\frac{2l_{ij}{}^{\frac{2}{n(n+1)}}}{k_{ij}{}^{\frac{2}{n(n+1)}}+l_{ij}{}^{\frac{2}{n(n+1)}}},\frac{2m_{ij}{}^{\frac{2}{n(n+1)}}}{n_{ij}{}^{\frac{2}{n(n+1)}}+m_{ij}{}^{\frac{2}{n(n+1)}}}\right]\right)$$

Thereafter,

$$\oplus E_{j=i}^n\left(\frac{2}{n(n+1)}\otimes_E\left(\tilde{\gamma}_i^{E^p}\otimes_E\tilde{\gamma}_j^{E^q}\right)\right)$$

$$=\left(\begin{array}{c}\left[\dfrac{\prod_{j=i}^n e_{ij}{}^{\frac{2}{n(n+1)}}-\prod_{j=i}^n f_{ij}{}^{\frac{2}{n(n+1)}}}{\prod_{j=i}^n e_{ij}{}^{\frac{2}{n(n+1)}}+\prod_{j=i}^n f_{ij}{}^{\frac{2}{n(n+1)}}},\dfrac{\prod_{j=i}^n g_{ij}{}^{\frac{2}{n(n+1)}}-\prod_{j=i}^n h_{ij}{}^{\frac{2}{n(n+1)}}}{\prod_{j=i}^n g_{ij}{}^{\frac{2}{n(n+1)}}+\prod_{j=i}^n h_{ij}{}^{\frac{2}{n(n+1)}}}\right],\\ \left[\dfrac{2\prod_{j=i}^n l_{ij}{}^{\frac{2}{n(n+1)}}}{\prod_{j=i}^n k_{ij}{}^{\frac{2}{n(n+1)}}+\prod_{j=i}^n l_{ij}{}^{\frac{2}{n(n+1)}}},\dfrac{2\prod_{j=i}^n m_{ij}{}^{\frac{2}{n(n+1)}}}{\prod_{j=i}^n n_{ij}{}^{\frac{2}{n(n+1)}}+\prod_{j=i}^n m_{ij}{}^{\frac{2}{n(n+1)}}}\right]\end{array}\right)$$

And

$$\oplus E_{i=1}^n\oplus E_{j=i}^n\left(\frac{2}{n(n+1)}\otimes_E\left(\tilde{\gamma}_i^{E^p}\otimes_E\tilde{\gamma}_j^{E^q}\right)\right)$$

$$
=\left(\begin{array}{l}\left[\dfrac{\prod_{i=1}^{n}\prod_{j=i}^{n} e_{ij}^{\frac{2}{n(n+1)}}-\prod_{i=1}^{n}\prod_{j=i}^{n} f_{ij}^{\frac{2}{n(n+1)}}}{\prod_{i=1}^{n}\prod_{j=i}^{n} e_{ij}^{\frac{2}{n(n+1)}}+\prod_{i=1}^{n}\prod_{j=i}^{n} f_{ij}^{\frac{2}{n(n+1)}}}, \dfrac{\prod_{i=1}^{n}\prod_{j=i}^{n} g_{ij}^{\frac{2}{n(n+1)}}-\prod_{i=1}^{n}\prod_{j=i}^{n} h_{ij}^{\frac{2}{n(n+1)}}}{\prod_{i=1}^{n}\prod_{j=i}^{n} g_{ij}^{\frac{2}{n(n+1)}}+\prod_{i=1}^{n}\prod_{j=i}^{n} h_{ij}^{\frac{2}{n(n+1)}}}\right], \\ \left[\dfrac{2\prod_{i=1}^{n}\prod_{j=i}^{n} l_{ij}^{\frac{2}{n(n+1)}}}{\prod_{i=1}^{n}\prod_{j=i}^{n} k_{ij}^{\frac{2}{n(n+1)}}+\prod_{i=1}^{n}\prod_{j=i}^{n} l_{ij}^{\frac{2}{n(n+1)}}}, \dfrac{2\prod_{i=1}^{n}\prod_{j=i}^{n} m_{ij}^{\frac{2}{n(n+1)}}}{\prod_{i=1}^{n}\prod_{j=i}^{n} n_{ij}^{\frac{2}{n(n+1)}}+\prod_{i=1}^{n}\prod_{j=i}^{n} m_{ij}^{\frac{2}{n(n+1)}}}\right]\end{array}\right)
$$

Thus,

$$
\left(\oplus_E{}_{i=1}^{n}\oplus_E{}_{j=i}^{n}\left(\frac{2}{n(n+1)}\otimes_E\left(\tilde{\gamma}_i^{E^p}\otimes_E\tilde{\gamma}_j^{E^q}\right)\right)\right)^{\frac{1}{p+q}}
$$

$$
=\left(\begin{array}{l}\left[\begin{array}{l}\dfrac{2\left(\prod_{i=1}^{n}\prod_{j=i}^{n} e_{ij}^{\frac{2}{n(n+1)}}-\prod_{i=1}^{n}\prod_{j=i}^{n} f_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}}{\left(\prod_{i=1}^{n}\prod_{j=i}^{n} e_{ij}^{\frac{2}{n(n+1)}}+3\prod_{i=1}^{n}\prod_{j=i}^{n} f_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}+\left(\prod_{i=1}^{n}\prod_{j=i}^{n} e_{ij}^{\frac{2}{n(n+1)}}-\prod_{i=1}^{n}\prod_{j=i}^{n} f_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}}, \\ \dfrac{2\left(\prod_{i=1}^{n}\prod_{j=i}^{n} g_{ij}^{\frac{2}{n(n+1)}}-\prod_{i=1}^{n}\prod_{j=i}^{n} h_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}}{\left(\prod_{i=1}^{n}\prod_{j=i}^{n} g_{ij}^{\frac{2}{n(n+1)}}+3\prod_{i=1}^{n}\prod_{j=i}^{n} h_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}+\left(\prod_{i=1}^{n}\prod_{j=i}^{n} g_{ij}^{\frac{2}{n(n+1)}}-\prod_{i=1}^{n}\prod_{j=i}^{n} h_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}}\end{array}\right], \\ \left[\begin{array}{l}\dfrac{\left(\prod_{i=1}^{n}\prod_{j=i}^{n} k_{ij}^{\frac{2}{n(n+1)}}+3\prod_{i=1}^{n}\prod_{j=i}^{n} l_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}-\left(\prod_{i=1}^{n}\prod_{j=i}^{n} k_{ij}^{\frac{2}{n(n+1)}}-\prod_{i=1}^{n}\prod_{j=i}^{n} l_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}}{\left(\prod_{i=1}^{n}\prod_{j=i}^{n} k_{ij}^{\frac{2}{n(n+1)}}+3\prod_{i=1}^{n}\prod_{j=i}^{n} l_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}+\left(\prod_{i=1}^{n}\prod_{j=i}^{n} k_{ij}^{\frac{2}{n(n+1)}}-\prod_{i=1}^{n}\prod_{j=i}^{n} l_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}}, \\ \dfrac{\left(\prod_{i=1}^{n}\prod_{j=i}^{n} n_{ij}^{\frac{2}{n(n+1)}}+3\prod_{i=1}^{n}\prod_{j=i}^{n} m_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}-\left(\prod_{i=1}^{n}\prod_{j=i}^{n} n_{ij}^{\frac{2}{n(n+1)}}-\prod_{i=1}^{n}\prod_{j=i}^{n} m_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}}{\left(\prod_{i=1}^{n}\prod_{j=i}^{n} n_{ij}^{\frac{2}{n(n+1)}}+3\prod_{i=1}^{n}\prod_{j=i}^{n} m_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}+\left(\prod_{i=1}^{n}\prod_{j=i}^{n} n_{ij}^{\frac{2}{n(n+1)}}-\prod_{i=1}^{n}\prod_{j=i}^{n} m_{ij}^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}}\end{array}\right]\end{array}\right)
$$

where $e_{ij}=(2-a_i)^p\left(2-a_j\right)^q+3a_i^pa_j^q$

$$f_{ij}=(2-a_i)^p\left(2-a_j\right)^q-a_i^pa_j^q$$

$$g_{ij}=(2-b_i)^p\left(2-b_j\right)^q+3b_i^pb_j^q$$

$$h_{ij}=(2-b_i)^p\left(2-b_j\right)^q-b_i^pb_j^q$$

$$k_{ij}=(1+c_i)^p\left(1+c_j\right)^q+3(1-c_i)^p\left(1-c_j\right)^q$$

$$l_{ij}=(1+c_i)^p\left(1+c_j\right)^q-(1-c_i)^p\left(1-c_j\right)^q$$

$$n_{ij}=(1+d_i)^p\left(1+d_j\right)^q+3(1-d_i)^p\left(1-d_j\right)^q$$

$$m_{ij}=(1+d_i)^p\left(1+d_j\right)^q-(1-d_i)^p\left(1-d_j\right)^q$$

Hence, the given formula for the novel aggregation operator IVIFEHM is obtained.

The following properties of the proposed aggregation operator are verified and proofs are omitted.

Property 1 *Idempotency*

If $\widetilde{\gamma_i}=([s_i,t_i],[u_i,v_i])=\widetilde{\gamma}$ are equal $i=1,2,\ldots,n$ then $\mathrm{IVIFEHM}^{p,q}\left(\widetilde{\gamma_1},\widetilde{\gamma_2},\ldots,\widetilde{\gamma_n}\right)=\widetilde{\gamma}$.

Property 2 *Monotonicity*

Let $\widetilde{\gamma_i} = ([s_i, t_i], [u_i, v_i])$ and $\widetilde{\gamma_i}^{'} = \left(\left[s_i^{'}, t_i^{'}\right], \left[u_i^{'}, v_i^{'}\right]\right)$ be tow collections of IVIFNs. If $s_i \leq s_i^{'}$, $t_i \leq t_i^{'}$ and $u_i \geq u_i^{'}$, $v_i \geq v_i^{'}$ then $\mathrm{IVIFEHM}^{p,q}\left(\widetilde{\gamma_1}, \widetilde{\gamma_2}, \ldots, \widetilde{\gamma_n}\right) \leq \mathrm{IVIFEHM}^{p,q}\left(\widetilde{\gamma_i}^{'}, \widetilde{\gamma_i}^{'}, \ldots, \widetilde{\gamma_i}^{'}\right)$.

Property 3 *Boundedness*

Let $\widetilde{\gamma_i} = ([s_i, t_i], [u_i, v_i])$ $(i = 1, 2, \ldots, n)$ be a set of IVIFNs.
If $\widetilde{\gamma_i}^{+} = ([\max(s_i), \max(t_i)], [\min(u_i), \min(v_i)])$
And $\widetilde{\gamma_i}^{-} = ([\min(s_i), \min(t_i)], [\max(u_i), \max(v_i)])$
Then $\widetilde{\gamma_i}^{-} \leq \mathrm{IVIFEHM}^{p,q}\left(\widetilde{\gamma_1}, \widetilde{\gamma_2}, \ldots, \widetilde{\gamma_n}\right) \leq \widetilde{\gamma_i}^{+}$.

4 Proposed TOPSIS model using Einstein Heronian mean aggregation operator

Here, we presented a TOPSIS model for IVIF MCDM problem.

Let $O = O_1, O_2, \ldots, O_n$ be a set of n alternatives and $U = u_1, u_2, \ldots, u_m$ is set of m criteria. The performance of each alternative O_i and criterion u_j is denoted by $\tilde{y}_{ij}$. Let $\tilde{D} = \left[\tilde{y}_{ij}\right]_{n\times m}$ be the fuzzy decision matrix.

i.e., $\tilde{D} = \left[\tilde{y}_{ij}\right]_{n\times m} = \left[\begin{bmatrix} ([s_{11}, t_{11}], [u_{11}, v_{11}]) & \cdots & ([s_{1m}, t_{1m}], [u_{1m}, v_{1m}]) \\ \vdots & \ddots & \vdots \\ ([s_{n1}, t_{n1}], [u_{n1}, v_{n1}]) & \cdots & ([s_{nm}, t_{nm}], [u_{nm}, v_{nm}]) \end{bmatrix}\right]$.

To select the best alternative below given steps are followed:

Step 1: calculate the comprehensive evaluation of each alternative using the IVIFEHM operator using definition 9. And represent the obtained value as

$$\mathrm{IVIFEHM}^{p,q}(\tilde{y}_{i1}, \tilde{y}_{i2}, \ldots, \tilde{y}_{im}) = \tilde{y}_i = ([s_i, t_i], [u_i, v_i]), i = 1, 2 \ldots n$$

Step 2: from the evaluated values of step1, identify the Positive Ideal Solution (PIS) and Negative ideal solution (NIS) as:

$$\tilde{y}^{+} = ([\max(s_i), \max(t_i)], [\min(u_i), \min(v_i)]), i = 1, 2, \ldots, n$$

And $\tilde{y}^{-} = ([\min(s_i), \min(t_i)], [\max(u_i), \max(v_i)]), i = 1, 2, \ldots, n.$

Step 3: calculate the Jaccard distance of each evaluated value of alternative $\tilde{y}_i$ to the PIS $\tilde{y}_i^{+}$ using Definition 8.

$$d_J\left(\tilde{y}_i, \tilde{y}_i^{+}\right) = 1 - \frac{\left\|\begin{bmatrix} \min\left(\mu_{\tilde{y}_i}{}^{L}, \mu_{\tilde{y}^{+}}{}^{L}\right), \\ \min\left(\mu_{\tilde{y}_i}{}^{U}, \mu_{\tilde{y}^{+}}{}^{U}\right) \end{bmatrix}, \begin{bmatrix} \max\left(\vartheta_{\tilde{y}_i}{}^{L}, \vartheta_{\tilde{y}^{+}}{}^{L}\right), \\ max(\vartheta_{\tilde{y}_i}{}^{U}, \vartheta_{\tilde{y}^{+}}{}^{U}) \end{bmatrix}, \begin{bmatrix} \min\left(\pi_{\tilde{y}_i}{}^{L}, \pi_{\tilde{y}^{+}}{}^{L}\right), \\ max(\pi_{\tilde{y}_i}{}^{U}, \pi_{\tilde{y}^{+}}{}^{U}) \end{bmatrix}\right\|}{\left\|\begin{bmatrix} \max\left(\mu_{\tilde{y}_i}{}^{L}, \mu_{\tilde{y}_i^{+}}{}^{L}\right), \\ \max\left(\mu_{\tilde{y}_i}{}^{U}, \mu_{\tilde{y}_i^{+}}{}^{U}\right) \end{bmatrix}, \begin{bmatrix} \min\left(\vartheta_{\tilde{y}_i}{}^{L}, \vartheta_{\tilde{y}_i^{+}}{}^{L}\right), \\ min(\vartheta_{\tilde{y}_i}{}^{U}, \vartheta_{\tilde{y}_i^{+}}{}^{U}) \end{bmatrix}, \begin{bmatrix} \min\left(\pi_{\tilde{y}_i}{}^{L}, \pi_{\tilde{y}_i^{+}}{}^{L}\right), \\ max(\pi_{\tilde{y}_i}{}^{U}, \pi_{\tilde{y}_i^{+}}{}^{U}) \end{bmatrix}\right\|}$$

Similarly, calculate the Jaccard distance of each evaluated value of alternative $\tilde{y}_i$ to the NIS $\tilde{y}_i^-$.

$$d_J\left(\tilde{y}_i, \tilde{y}_i^-\right) = 1 - \frac{\left\| \begin{bmatrix} \min\left(\mu^L_{\tilde{y}_i}, \mu^L_{\tilde{y}^-}\right), \\ \min\left(\mu^U_{\tilde{y}_i}, \mu^U_{\tilde{y}^-}\right) \end{bmatrix}, \begin{bmatrix} \max\left(\vartheta^L_{\tilde{y}_i}, \vartheta^L_{\tilde{y}^-}\right), \\ max(\vartheta^U_{\tilde{y}_i}, \vartheta^U_{\tilde{y}^-}) \end{bmatrix}, \begin{bmatrix} \min\left(\pi^L_{\tilde{y}_i}, \pi^L_{\tilde{y}^-}\right), \\ max(\pi^U_{\tilde{y}_i}, \pi^U_{\tilde{y}^-}) \end{bmatrix} \right\|}{\left\| \begin{bmatrix} \max\left(\mu^L_{\tilde{y}_i}, \mu^L_{\tilde{y}_i^-}\right), \\ \max\left(\mu^U_{\tilde{y}_i}, \mu^U_{\tilde{y}_i^-}\right) \end{bmatrix}, \begin{bmatrix} \min\left(\vartheta^L_{\tilde{y}_i}, \vartheta^L_{\tilde{y}_i^-}\right), \\ min(\vartheta^U_{\tilde{y}_i}, \vartheta^U_{\tilde{y}_i^-}) \end{bmatrix}, \begin{bmatrix} \min\left(\pi^L_{\tilde{y}_i}, \pi^L_{\tilde{y}_i^-}\right), \\ max(\pi^U_{\tilde{y}_i}, \pi^U_{\tilde{y}_i^-}) \end{bmatrix} \right\|}$$

Step 4: calculate the closeness co-efficient of each alternative to PIS using:

$$R(O_i) = \frac{d_J\left(\tilde{y}_i, \tilde{y}^+\right)}{d_J\left(\tilde{y}_i, \tilde{y}_i^+\right) + d_J\left(\tilde{y}_i, \tilde{y}^-\right)}, i = 1, 2, \ldots, n$$

Step 5: rank the alternatives as
If $R(O_i) < R\left(O_j\right)$ then $O_i < O_j$
If $R(O_i) > R\left(O_j\right)$ then $O_i > O_j$
If $R(O_i) = R\left(O_j\right)$ then $O_i = O_j$

5 Numerical example

The applicability of the proposed method is demonstrated by taking an example from Wu et al. (2020). The problem is about selecting the best forest ecological tourism area out of the given five possible areas (alternatives) $O_i, i = 1, 2, 3, 4, 5$. These areas are evaluated by means of four criteria, $U_i, i = 1, 2, 3, 4$: U_1 is the tourism and leisure value; U_2 is the material production value; U_3 is the scientific research and cultural value; U_4 is the climatic regulation value. The preference values given by expert in the form of IVIFNs are given in Table 1.

Step 1: the comprehensive evaluation of each alternative by using the IVIFEHM operator is calculated and are shown in Table 2.

Step 2: the PIS and NIS are obtained as:
$\tilde{y}^+ = ([0.37, 0.57], [0.15, 0.33])$ and $\tilde{y}^- = ([0.2, 0.37], [0.29, 0.46])$,

Step 3: the Jaccard distance from the aggregated value of each alternative $\tilde{y}_i, i = 1, 2, 3, 4, 5$ obtained in step 1 to the IVIF PIS and IVIF NIS is calculated and presented in Table 3.

Table 1 Decision matrix

	U_1	U_2	U_3	U_4
O_1	([0.4,0.6], [0.2,0.3])	([0.3,0.5], [0.1,0.3])	([0.3,0.5], [0.1,0.2])	([0.1,0.3], [0.3,0.4])
O_2	([0.2,0.5], [0.1,0.4])	([0.3,0.6], [0.2,0.4])	([0.4,0.6], [0.1,0.3])	([0.1,0.4], [0.3,0.5])
O_3	([0.5,0.7], [0.2,0.3])	([0.3,0.6], [0.2,0.3])	([0.2,0.4], [0.3,0.4])	([0.4,0.5], [0.1,0.2])
O_4	([0.4,0.4], [0.2,0.4])	([0.3,0.4], [0.2,0.3])	([0.2,0.4], [0.4,0.3])	([0.2,0.3], [0.1,0.2])
O_5	([0.2,0.6], [0.2,0.4])	([0.2,0.4], [0.4,0.6])	([0.1,0.5], [0.3,0.4])	([0.3,0.6], [0.2,0.3])

Table 2 The aggregated values of the alternatives by IVIFEHM operator ($p = 2, q = 1$)

	IVIFEHM
O_1	$\tilde{y}_1$ =([0.31,0.5], [0.15,0.33])
O_2	$\tilde{y}_2$= ([0.27,0.52], [0.15,0.42])
O_3	$\tilde{y}_3$ = ([0.37,0.57], [0.2,0.34])
O_4	$\tilde{y}_4$ = ([0.31,0.37], [0.18,0.34])
O_5	$\tilde{y}_5$ = ([0.2,0.48], [0.29,0.46])

Table 3 Jaccard distance of $\tilde{y}_i$ from PIS and NIS

Jaccard distance of $\widetilde{\boldsymbol{y}}_{\boldsymbol{i}}$ to $\widetilde{\boldsymbol{y}}_{\boldsymbol{i}}^{+}$		Jaccard distance of $\widetilde{\boldsymbol{y}}_{\boldsymbol{i}}$ to $\widetilde{\boldsymbol{y}}_{\boldsymbol{i}}^{-}$	
$d_J(\tilde{y}_1, \tilde{y}^+)$	0.054	$d_J(\tilde{y}_1, \tilde{y}^-)$	0.169
$d_J(\tilde{y}_2, \tilde{y}^+)$	0.087	$d_J(\tilde{y}_2, \tilde{y}^-)$	0.164
$d_J(\tilde{y}_3, \tilde{y}^+)$	0.037	$d_J(\tilde{y}_3, \tilde{y}^-)$	0.200
$d_J(\tilde{y}_4, \tilde{y}^+)$	0.102	$d_J(\tilde{y}_4, \tilde{y}^-)$	0.139
$d_J(\tilde{y}_5, \tilde{y}^+)$	0.173	$d_J(\tilde{y}_5, \tilde{y}^-)$	0.045

Step 4: the closeness co-efficient of alternatives to PIS is calculated and are presented in Table 4.

Step 5: ranking of alternatives is given in Table 5.

Thus, the best forest ecological tourism area is O_3 which is coinciding with the result obtained in Wu et al. (2020). Further, the obtained results are compared with other MCDM

Table 4 Closeness co-efficient of alternatives

Alternatives	Closeness coefficient
O_1	0.24
O_2	0.34
O_3	0.15
O_4	0.42
O_5	0.79

Table 5 Ranking of alternatives

Alternatives	Rank
O_1	2
O_2	3
O_3	1
O_4	4
O_5	5

Table 6 Ranking results for different p and q values

p and q	Closeness coefficient of alternatives	Rank
$p = 2$ and $q = 1$	$O_1 = 0.24$, $O_2 = 0.34$ $O_3 = 0.15$, $O_4 = 0.42$, $O_5 = 0.79$	$O_3 > O_1 > O_2 > O_4 > O_5$
$p = 2$ and $q = 2$	$O_1 = 0.25$, $O_2 = 0.35$ $O_3 = 0.08$, $O_4 = 0.41$, $O_5 = 0.77$	$O_3 > O_1 > O_2 > O_4 > O_5$
$p = 1$ and $q = 2$	$O_1 = 0.24$, $O_2 = 0.31$ $O_3 = 0.08$, $O_4 = 0.36$, $O_5 = 0.77$	$O_3 > O_1 > O_2 > O_4 > O_5$
$p = 1$ and $q = 5$	$O_1 = 0.30$, $O_2 = 0.37$ $O_3 = 0.05$, $O_4 = 0.43$, $O_5 = 0.72$	$O_3 > O_1 > O_2 > O_4 > O_5$
$p = 5$ and $q = 5$	$O_1 = 0.30$, $O_2 = 0.39$ $O_3 = 0.07$, $O_4 = 0.48$, $O_5 = 0.73$	$O_3 > O_1 > O_2 > O_4 > O_5$

methods (Wu et al. 2018; Bai 2013; Nayagam et al. 2013). From the comparisons it is observed that the proposed method is working on par with the existing method.

5.1 Effect of variables p and q on the result

In this section, we further investigated the effect of p and q in the IVIFEHM operator on the ranking results. For this the values of p, q are randomly chosen and closeness co-efficient of the alternatives is calculated and alternatives are ranked. It is observed that the result is same for any values of p and q unlike the Heronian mean operator with algebraic operations when applied for $p = 1$ and $q = 5$ and $p = 5$ and $q = 5$ (Liu 2017). Some of the tested cases are presented in Table 6.

6 Comparative study

To study the efficiency of the proposed method, we have taken another example from Wu et al. (2018) and applied the procedure. The closeness coefficients and the ranking of alternatives of the proposed method are given in the Table 7.

It is observed that the ranking result obtained is identical to the existing approach Wu et al. (2018) and as well as Bai (2013), Nayagam et al. (2013). Moreover, the proposed approach is not getting affected with the variables p, q.

Table 7 Closeness coefficients and ranking of alternatives of example Wu et al. (2018)

	A_1	A_2	A_3	A_4	A_5	Ranking
$p = 1, q = 2$	0.524	0.498	1	0.223	0.226	$O_3 > O_1 > O_2 > O_5 > O_4$

7 Conclusion

Aggregation of information plays an imperative role in decision-making. The operators used in the aggregation should be cautiously chosen so that there is no flaw in final decision. As mean operators are been proven for effective aggregation, in this paper, an aggregation approach is developed using Heronian mean operator. An IVIFEHM operator is proposed and is used in TOPSIS to solve the IVIF MCDM problems. As, the operator has the characteristics of Heronian mean operator and Einstein operations it helps in avoiding loss of original decision information during the process of aggregation. In this process, Jaccard distance which considers the interval hesitancy degree is used as a separation measure to calculate the relative closeness coefficient of each alternative for ranking. To observe the effectiveness of the proposed approach two numerical examples from literature are taken and are studied. The results show that the proposed model is effective in dealing MCDM problems. Further, we have presented a study on the effect of the variables p and q in ranking the alternatives.

References

Abdullah L, Goh C, Zamri N, Othman M (2020) Application of interval valued intuitionistic fuzzy TOPSIS for flood management. J Intell Fuzzy Syst 38(1):873–881

Ali Z, Tahir M (2020) Maclaurin symmetric mean operators and their applications in the environment of complex q-rung Orthopair fuzzy sets. Comput Appl Math 3:161

Anusha V, Sireesha V (2021) Ranking interval valued intuitionistic fuzzy sets by a new distance measure. Adv Math Sci J 10(3):1249–1258

Atanassov KT (1994) Operators over interval valued intuitionistic fuzzy sets. Fuzzy Sets Syst 64:159–174

Atanassov KT, Gargov G (1989) Interval valued intuitionistic fuzzy sets. Fuzzy Sets Syst 31(3):343–349

Bai Z (2013) An interval-valued intuitionistic fuzzy TOPSIS method based on an improved score function. Sci World J. https://doi.org/10.1155/2013/879089

Beliakov G, Pradera A, Calvo T (2007) Aggregation functions: a guide for practitioners. Studies in fuzziness and soft computing series, series, vol 221. Springer, Berlin

Düğenci M (2016) a new distance measure for interval valued intuitionistic fuzzy sets and its application to group decision making problems with incomplete weights information. Appl Soft Comput. https://doi.org/10.1016/j.asoc.2015.12.026

Klement EP, Mesiar R, Endre P (2000) Triangular norms. In: Trends in logic book series (TREN), vol 8

Liu P (2013) Some Hamacher aggregation operators based on the interval-valued intuitionistic fuzzy numbers and their application to group decision making. IEEE Trans Fuzzy Syst 22:83–97

Liu P (2017) Multiple attribute group decision making method based on interval-valued intuitionistic fuzzy power Heronian aggregation operators. Comput Indust Eng 108:199–212

Liu P, Chen SM (2017) Group decision making based on Heronian aggregation operators of intuitionistic fuzzy numbers. IEEE Trans Cybern 47(9):2514–2530

Liu P, Li DF (2017) Some Muirhead mean operators for intuitionistic fuzzy numbers and their applications to group decision making. PLoS ONE 12(1):e0168767. https://doi.org/10.1371/journal.pone.0168767

Liu P, Wang P (2020) Multiple attribute group decision making method based on intuitionistic fuzzy Einstein interactive operations. Int J Fuzzy Syst 22:790–809

Liu P, Li Y, Chen Y (2015) Some generalized Einstein aggregation operators based on the interval-valued intuitionistic fuzzy numbers and their application to group decision making. Sci Iran E 22(6):2684–2701

Michael L, David W (1971) Distance between sets. Nature 234:34–35

Navara M (2007) Triangular norms and conorms. Scholarpedia 2(3):2398. https://doi.org/10.4249/scholarpedia.2398

Nayagam VLG, Geetha S, Ponalagusamy R (2013) Multi-criteria interval valued intuitionistic fuzzy decision making using a new score function. In: Knowledge and information management conference, Meriden, UK

Roy S, Wang BH (1998) On generalized Hamacher families of triangular operators. Int J Approx Reason 19:419–439

Sireesha V, Himabindu K (2019) Decision-making with unknown criteria weight information in the framework of interval valued trapezoidal intuitionistic fuzzy sets. Int J Eng Innov Technol 9(1):4424–4429

Subramanian P, Garg H, Hüseyin K, Akin Osman A (2020) The mean operators and generalized products of fuzzy soft matrices and their applications in MCGDM. Comput Appl Math 39:68

Sykora S (2009) Mathematical means and averages: generalized Heronian means. Stan's Libr. https://doi.org/10.3247/SL3Math09.002

Verma V, Kumar R (2020) A comparative analysis of similarity measures akin to the Jaccard index in collaborative recommendations: empirical and theoretical perspective. Soc Netw Anal Min. https://doi.org/10.1007/s13278-020-00660-9

Wang Q (2018) Interval-valued intuitionistic fuzzy Einstein geometric Choquet integral operator and its application to multi attribute group decision-making. Math Probl Eng. https://doi.org/10.1155/2018/9364987

Wang F (2021) TOPSIS method for teaching effect evaluation of college English with interval-valued intuitionistic fuzzy information. J Math. https://doi.org/10.1155/2021/5517198

Wang W, Liu X (2011) Intuitionistic fuzzy geometric aggregation operators based on Einstein operations. Int J Intell Syst 26(11):1049–1075

Wang W, Liu X (2012) Intuitionistic fuzzy information aggregation using Einstein operations. IEEE Trans Fuzzy Syst 20(5):923–938

Wu L, Wei G, Gao H, Wei Y (2018) Some interval-valued intuitionistic fuzzy Dombi Hamy mean operators and their application for evaluating the elderly tourism service quality in tourism destination. Mathematics 6(12):294. https://doi.org/10.3390/math6120294

Wu L, Wei G, Wu J, Wei C (2020) Some interval-valued intuitionistic fuzzy Dombi Heronian mean operators and their application for evaluating the ecological value of forest ecological tourism demonstration areas. Int J Environ Res Public Health 17(3):829. https://doi.org/10.3390/ijerph17030829

Xu Z (2007) Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making. Control Decis 22(2):215–219

Xu Z, Chen J (2007) On geometric aggregation over interval-valued intuitionistic fuzzy information. In: Proceedings of the fourth international conference on fuzzy systems and knowledge discovery, Washington, DC, USA.

Yoon K, Hwang CL (1981) Multiple attribute decision making: methods and applications, series, vol 186. Springer, Berlin

Yu D (2013) Intuitionistic fuzzy geometric Heronian mean aggregation operators. Appl Soft Comput 13(2):1235–1246

Yu D, Wu Y (2012) Interval-valued intuitionistic fuzzy Heronian mean operators and their application in multi-criteria decision making. Afr J Bus Manag 6(11):4158–4168

Zhang Z (2017) Interval-valued intuitionistic fuzzy Frank aggregation operators and their applications to multiple attribute group decision making. Neural Comput Appl 28(6):1471–1501

Zhou J, Tomas B, Dalia S (2019) Normalized weighted Bonferroni Harmonic mean-based intuitionistic fuzzy operators and their application to the sustainable selection of search and rescue robots. Symmetry 11(2):218. https://doi.org/10.3390/sym11020218

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