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# Multiple attribute group decision-making method based on extended bipolar fuzzy MABAC approach

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**Abstract**
In this paper, we apply multiple attribute border approximation area comparison approach to multiple attribute group decision-making with bipolar fuzzy numbers (BFNs). We reconsider the notion of BFNs and propose its corresponding operational rules, score and accuracy functions. Further, we introduce two aggregation operators and develop an MADM approach based on conventional BABAC model with overall BFNs. The proposed technique is valid and accurate for considering the conflicting attributes. We analyse the proposed method by considering a numerical example for the selection of renewable energy power generation project to show the effectiveness of the developed approach. At last, we compare the developed approach with some existing operators to show its efficiency.

**Keywords** Bipolar fuzzy numbers · Original MABAC model · Bipolar MABAC approach · MAGDM

**Mathematics Subject Classification** 03E72 · 97R30 · 97R40

## 1 Introduction

Multiple attribute decision-making (MADM) approach in diverse areas of science and technology draws the attention of the researchers. In the previous familiar works, the researchers have studied a variety of MADM approaches such as TOPOSIS approach (Gebrehiwet and Luo 2018), EDAS approach (Roy et al. 2019), VIKOR (Pramanik and Mallick 2018) approach, ELECTRE (Akram and Arshad 2020), PROMETHEE (Ziemba 2018), and so on. Pamučar and Ćirović (Pamučar and Ćirović 2015) first projected a novel MADM approach called multi-attributive border approximation area comparison (MABAC) which can include conflicting attributes into consideration during decision-making. In a model of MABAC

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structures considering the advantage of only border approximate areas (BAA) to consider the intangibility of decision-makers (DMs) and the vagueness of decision-theoretic environment to get more concrete and feasible aggregate information. Later on, MABAC has been applied in different application areas such as Muravev and Mijic (2020) used hybrid BWM-MABAC model for selecting a integrated provider. Biswas (2020) have studied MABAC method for health performance evaluation of India under a comparative approach in MCDM problems. To see more applications of MABAC approach, readers are referred to the works (Pamučar et al. 2018; Pamučar and StevićŽ 2018a; Sun et al. 2017; Mishra et al. 2020; Wang et al. 2020).

The BFS was first stated by Zhang (1994, 1998), which was another generalization of fuzzy sets (FS) formulated by (Zadeh 1965). BFS tool is not only applied in reasoning as well as BF set theory but also utilized in different application areas such as (Gul 2015) proposed bipolar fuzzy aggregation operators and then used them to model an MADM problem. Later, in the same environment, Wei et al. (2018) used Hamacher operators to develop MADM approach. Jana et al. (2019a) set up a novel MCDM method based on bipolar fuzzy soft operators. Further, Jana et al. (2019b) utilized Dombi operator to aggregate BFNs and they have developed MADM method using bipolar fuzzy Dombi operators. In the same way, Jana et al. (2020a) used Dombi prioritized operators to construct MADM approach in BF environment. Alghamdi et al. (2018) used to study bipolar fuzzy TOPSIS and ELECTRE-I approach for solving bipolar fuzzy MCDM method. A bipolar fuzzy TOPSIS method has been developed by Sarwar et al. Sarwar et al. (2018) to study a decision-making method in the environment of bipolar fuzzy competition graph approach. In the same environment, Shumaiza et al. (2019) utilized VIKOR approach for solving a group decision-making method in trapezoidal bipolar fuzzy environment. Xu et al. (2020) proposed bipolar fuzzy petri nets approach for the studying of knowledge representation and acquisition in non-cooperative behaviors. A PROMETHEE-based MCGDM problems has been studied by Akram et al. (2020) in bipolar fuzzy environment for the selection green suppliers. Akram and Arshad (2020) proposed bipolar fuzzy TOPSIS and bipolar fuzzy ELECTRE-I two MCDM methods and applied them for diagnosis the disease. Jana and Pal (2021a) proposed MCGDM problems using bipolar fuzzy EDAS method for selecting the road construction company. At the same time, Zhao et al. (2021) provided the selection of network security service provider based on CPT-TODIM method for solving MAGDM approach using bipolar fuzzy numbers. Lan et al. (2021) utilized extended traditional aggregation operator into interval-valued bipolar uncertain (IVBUL) aggregation operators and they applied CODAS method to solve MAGDM problems based on the proposed operators for the evaluation of Chinese enterprises' overseas mergers and acquisitions risks. In the same way, Gao et al. (2019) have used IVBUL aggregation operators to solve MADM problems and then applied these operators for evaluating Computer Network Security selection. In the current literature of BFNs, there are no studies of MABAC approach with BFNs. Therefore, here an attempt has been made to fill the research gap by developing an MADM problem for MABAC method with BFNs. In addition, to date, The MABAC approach which was first originated by Pamučar and Ćirović (2015) that computes distance measure between the border approximation area (BAA) and the alternatives and has a various characteristics such as (1) the computing results based on MABAC method are stable; (2) for this purpose using equations are simple; (3) it takes into account inactive values of losses and gains; (4) it can be comfortably combine with other approaches. Therefore, MABAC model is an effective tool to make a good decision-making results. In this study, first developed MABAC model based on conventional MABAC model with BF information and set up a MAGDM process. In this model, we first extend MABAC approach to BFNs. Next, utilize the original MABAC approach for

**Table 1** Characteristic comparisons with some of the existing methods

| Methods | Whether describe fuzzy More information easier | Weather the method Use MABAC method |
|---|---|---|
| Gul (2015) | √ | × |
| Jana et al. (2019a) | √ | × |
| Jana et al. (2019b) | √ | × |
| Jana et al. (2020a) | √ | × |
| Alghamdi et al. (2018) | √ | × |
| Sarwar et al. (2018) | √ | × |
| Xu et al. (2020) | √ | × |
| Akram et al. (2020) | √ | × |
| Akram and Arshad (2020) | √ | × |
| Zhao et al. (2021) | √ | × |
| Wei et al. (2018) | √ | × |
| Proposed method | √ | √ |

MAGDM with BFNs. Finally, introduce a numerical example to discuss the novel approach with BFNs and then organized a comparison study with some fantastic existing operators such as BF-weighted average (BFWA) operator and BF-weighted geometric (BFWG) operator, BF-Dombi weighted average (BFDWA) operator and BF-Dombi weighted geometric (BFDWG) operator, BF-Hamacher weighted average (BFHWA) operator and BF-Hamacher weighted geometric (BFHWG) operator to show its effectiveness and feasibility of the novel approach. The objectives of this paper are to

- extended MABAC approach is developed in connection with BFNs
- utilizing this method a BFMAGDM approach is constructed
- proposed method is described by a numerical example
- superiority of the proposed method is justified to compare with some existing methods.

A more detailed classification of some related research is presented in Table 1.

The outliving part of the paper is presented as follows: Section 2, reexamines some prior literature. In Sect. 3, focus definition, score and accuracy functions, and operational formula for BFNs are given. Section 4 defines some bipolar fuzzy arithmetic aggregation operators. Section 5 discusses original MABAC approach with development procedure. Section 6 proposes MABAC model for MAGDM with BFNs. Section 7 furnishes an example of estimate and selects renewable energy power generation project to discuss the proposed approach. Section 8 shows some analysis of this approach with some existing models. Section 9 includes the future working directions.

## 2 Literature review

Intuitionistic fuzzy set (IFS), stated by Atanassov (1999), portrayed with membership and non-membership function was a powerful extension of fuzzy set (FS) Zadeh (1965) which contained only membership part. In an extensive idea, IFS is a broader concept of FS to express objects of the world more comprehensively from the points of view of comfort, conflict and indeterminacy, respectively, which have been broadly analyzed and applied to aggregation operators by the researchers (Liu 2017; Xu and Wei 2017; Jana and Pal 2021c; Jana et al. 2021). For more further information of operators and terminology, researchers are referred to (Jana et al. 2019c, d, 2020b; Jana and Pal 2021b).

Although IFS and IVIFS have been utilized to solve real-world problems which contain uncertainty, in some cases, where to express information of an object that corresponds to each property, there exists a counter property. To overcome this issues, Zhang (1994, 1998) initiated bipolar fuzzy sets (BFS) another generalization of FS. The MD and NMD of BFS belong to wider range $[-1, 1]$. BFS then considered as a new tool to control uncertainty in real life problems. BFSs were applied in bipolar logical reasoning as well as bipolar set theory (Han et al. 2015; Zhang and Zhang 2004). Lately, Gul (2015) proposed of aggregation operators in BF environment, and then defined bipolar fuzzy weighted averaging (BFWA) and weighted geometric (BFWG) operators, and developed an MADM model. Wei et al. (2018) have applied Hamacher operator to aggregate bipolar fuzzy information and used them to construct MADM problems. Later, Wang et al. (2018) set up Frank–Choquet–Bonferroni mean operators to aggregate bipolar neutrosophic numbers for developing MCDM problems. Also, Gao et al. (2018) defined prioritized Hamacher operators, and then utilized this to construct dual hesitant bipolar average operator and dual hesitant bipolar geometric operator, and then used these operators to model an MADM approach. Wei et al. (2017) have studied hesitant BF weighted averaging (HBFWA), ordered averaging (HBFOWA), hybrid averaging (HBFHWA), and hesitant BF weighted geometric (HBFWG), ordered geometric (HBFOWG) and hybrid geometric (HBFHWG) operators, and then used these to develop an MADM problem. Xu and Wei (2017) introduced dual hesitant bipolar fuzzy arithmetic aggregation (DHBFWA) and geometric aggregation (DHBFWG) operators, and used them to solve MADM problems. Lu et al. (2017) proposed bipolar 2-tuple linguistic concept to define bipolar 2-tuple linguistic hybrid average (B2TLHA) and hybrid geometric (B2TLHG) operators, and developed MADM problems using these operators. Wei et al. (2018) have developed an MADM model for risk assessment of enterprise human capital investment using IVB2TLN arguments. Han et al. (2015) has been introduced bipolar fuzzy rough numbers and gave an application of decision-making approach. Jana and Pal (2018) developed novel decision-making problems based on bipolar intuitionistic fuzzy soft environment. Later in the same environment, Shanthi Anita and Jaypalan (2019) have studied ELECTRE 1 approach-based MCDM model. But in recent times, we have seen some innovative MADM approach. In this view, Wu and Liao (2019) have studied a novel outranking approach which measured gained and lost dominance score (GLDS). The GLDS method measured both "group utility" and the individual regret score at a time. The MABAC method was originated by Pamučar and Ćirović (2015) for studying the problem of transport and distribution resource logistics center selection. After that, Pamučar et al. (2018) modified original MABAC approach. Again, Pamučar and StevićŹ (2018a) studied hybrid IR-AHP-MABAC model. Sun et al. (2017) proposed extended MABAC approach with HFLNs which are applied to calculate patient prioritized level. Yu et al. (2017) developed intuitionistic trapezoidal linguistic (ITLNs) likelihood-based MABAC approach. Mishra et al. (2020) introduced MCDM model based on MABAC approach with IVIFS. Wang et al. (2020) have studied MABAC approach with Q-rung orthopair fuzzy environment. In the current literature, there exist some MADM problems based on aggregation operators (Jana et al. 2019a, b, 2020a; Wei et al. 2018) in bipolar fuzzy environment, some bipolar fuzzy soft algebraic structures based-decision-making approach (Khan et al. 2019; Ibrar et al. 2019), etc. But, there is no research on the proposed approach in my knowledge. This study try to filling the research gap to address MABAC approach for MAGDM with BFN information.

## 3 Preliminaries

In this section, we recall some basic concepts related to bipolar fuzzy sets (BFSs) over the universe of discourse $X$.

### 3.1 Bipolar fuzzy sets

**Definition 1** Wei et al. (2018) Let $B$ be a bipolar fuzzy set (BFS) fixed over $X$ defined as

$$B = \{\langle x, \mu_B^+(x), \nu_B^-(x)\rangle | x \in X\},$$

where positive membership (PM) function be $\mu_B^+$ and negative membership (NM) function be $\nu_B^-(x)$ such that $\mu_B^+ \in [0, 1$ and $\nu_B^- \in [-1, 0]$ of an element $x$ to a BFS $B$, for every $x \in X$. Then, $\delta(x) = 1 - \mu^+(x) + \nu^-(x)$ denotes the indeterminacy degree of an element $x$ of the set $x \in B$. The set $\langle(\mu_B^+, \nu_B^-)\rangle$ denotes bipolar fuzzy numbers (BFNs) or bipolar fuzzy values (BFVs).

Basic operations of BFNs are given below:

**Definition 2** Wei et al. (2018) Let $B = (\langle \mu_B^+(x), \nu_B^-(x)\rangle)$ and $\tilde{C} = (\langle \mu_C^+(x), \nu_C^-(x)\rangle)$ be two BFNs. The operations of two BFNs are given as follows:

(1) $B \subseteq C$, if $\mu_B^+(x) \leq \mu_C^+(x)$, $\nu_B^-(x) \geq \nu_C^-(x)$ for all $x \in X$
(2) $B \cup C = \{\langle x, \max\{\mu_B^+(x), \mu_C^+(x)\}, \min\{\nu_B^-(x), \nu_C^-\}\rangle | x \in X\}$
(3) $B \cap C = \{\langle x, \min\{\mu_B^+(x), \mu_C^+(x)\}, \max\{\nu_B^-(x), \nu_C^-\}\rangle | x \in X\}$
(iv) $\overline{B} = \{\langle x, 1 - \mu_B^+(x), |\nu_B^-(x)| - 1\rangle | x \in X\}$.

**Definition 3** Wei et al. (2018) Let $B = (\mu_B, \nu_B)$ be a BFNs, the score function $\Delta$ of $B$ is defined below:

$$\Delta(B) = \frac{1 + \mu_B^+ + \nu_B^-}{2}, \Delta(B) \in [0, 1] \tag{1}$$

and accuracy function $\nabla$ of $B$ defined as

$$\nabla(B) = \frac{\mu_B^+ - \nu_B^-}{2}, \nabla(B) \in [0, 1]. \tag{2}$$

For the above definition of $\triangle$ and $\nabla$, the order relation between two BFNs $B$ and $C$ is as below:

**Definition 4** (Wei et al. 2018) Let $\tilde{S}$ and $\tilde{T}$ be any two BFEs.

(i) If $\triangle(B) < \triangle(C)$, follows $B \prec C$
(ii) If $\triangle(B) > \triangle(C)$, follows $B \succ C$
(iii) If $\triangle(B) = \nabla(C)$, then

(1) If $\nabla(B) < \nabla(C)$, follows $B \prec C$.
(2) If $\nabla(B) > \nabla(C)$, follows $B \succ C$.
(3) If $\nabla(B) = \nabla(C)$, then $B \sim C$.

Wei et al. (2018) defined operations between two BFNs as follows:

**Definition 5** Wei et al. (2018) Let $B = (\langle \mu_B^+(x), \nu_B^-(x)\rangle)$ and $C = (\langle \mu_B^+(x), \nu_C^-(x)\rangle)$ be two BFNs over $X$, then for all $x \in X$:

(1) $B \wedge C = \{\langle x, \min\{\mu_B^+(x), \mu_C^+(x)\}, \max\{\nu_B^-(x), \nu_C^-\}\rangle\}$
(2) $B \vee C = \{\langle x, \max\{\mu_B^+(x), \mu_C^+(x)\}, \min\{\nu_B^-(x), \nu_C^-\}\rangle\}$
(3) $B \oplus C = \big(\langle \mu_B^+(x) + \mu_C^+(x) - \mu_B^+(x)\mu_C^+(x), -|\nu_B^-(x)||\nu_C^-(x)|\rangle\big)$
(4) $B \otimes C = \big(\langle \mu_B^+(x)\mu_B^+(x), \nu_B^-(x) + \nu_C^-(x) - \nu_B^-(x)\nu_C^-(x)\rangle\big)$
(5) $\lambda B = \big(1 - (1 - \mu_B^+(x))^\lambda, -|\nu_B(x)|^\lambda\big)$
(6) $B^\lambda = \big(\mu_B^\lambda(x), -1 + |1 + \nu_B^-(x))|^\lambda\big)$.

Utilizing Definition 5, Wei et al. (2018) introduced these operations:

**Definition 6** Wei et al. (2018) Let $B = (\langle \mu_B^+, \nu_B^-\rangle)$ and $C = (\langle \mu_B^+, \nu_C^-\rangle)$ be two BFNs over $X$ and $\gamma, \gamma_1, \gamma_2 > 0$, then

(1) $B \oplus C = C \oplus B$
(2) $B \otimes C = C \otimes B$
(3) $\gamma(B \oplus C) = \gamma B \oplus \gamma C$
(4) $(B \otimes C)^\gamma = B^\gamma \otimes C^\gamma$
(5) $\gamma_1 B \oplus \gamma_2 B = (\gamma_1 + \gamma_2)B$
(8) $B^{\gamma_1} \otimes B^{\gamma_2} = B^{(\gamma_1+\gamma_2)}$
(7) $(B^{\gamma_1})^{\gamma_2} = B^{\gamma_1\gamma_2}$.

## 4 Some bipolar fuzzy aggregation operators

**Definition 7** Jana et al. (2019a) Let $b_\tau = (\mu_\tau^+, \nu_\tau^-)$ $(\tau = 1, 2, \ldots \eta)$ be a group of BFNs. A bipolar fuzzy weighted average (BFWA) operator of dimension $\eta$ is a function $\Omega^\eta \to \Omega$ that correlated with weight vector $\psi = (\psi_1, \psi_2, \ldots, \psi_\eta)^T$ for which $\psi > 0$ and $\sum_{b=1}^{\eta} \psi_b = 1$, as $BFWA_\psi(b_1, b_2, \ldots, b_\eta) = \bigoplus_{\tau=1}^{\eta}(\psi_\tau b_\tau)$

$$= \left(1 - \prod_{\tau=1}^{\eta}(1 - \mu_\tau^+)^{\psi_\tau}, - \prod_{\tau=1}^{\eta}\left(|\nu_\tau^-|\right)^{\psi_\tau}\right). \tag{3}$$

**Definition 8** Jana et al. (2019a) Let $b_\tau = (\mu_b^+, \nu_\tau^-)$ $(\tau = 1, 2, \ldots \eta)$ be a group of BFNs. A bipolar fuzzy weighted geometric (BFWG) operator of dimension $\eta$ is a function $\Omega^\eta \to \Omega$ that correlated with weight vector $\psi = (\psi_1, \psi_2, \ldots, \psi_\eta)^T$ for which $\psi > 0$ and $\prod_{\tau=1}^{\eta} \psi_\tau = 1$, as $BFWG_\psi(b_1, b_2, \ldots, b_\eta) = \bigoplus_{\tau=1}^{\eta}(\psi_\tau b_\tau)$

$$= \left(\prod_{\tau=1}^{\eta} \mu_\tau^{+\psi_\tau}, -1 + \prod_{\tau=1}^{\eta}(1 + \nu_\tau^-)^{\psi_\tau}\right). \tag{4}$$

**Definition 9** Let $b_1 = (\mu_1^+, \nu_1^-)$ and $b_2 = (\mu_2^+, \nu_2^-)$ be any two BFNs, then bipolar fuzzy normalized Hamming distance (BFNHD) is defined as

$$d_{\text{BFNHD}}(b_1, b_2) = \frac{1}{2}(|\mu_1^+ - \mu_2^+| + |\nu_1^- - \nu_2^-|). \tag{5}$$

## 5 Original MABAC approach

Let there be a set of $\sigma$ alternatives $\{B_1, B_2, \ldots, B_\sigma\}$, and $\eta$ attributes $\{G_1, G_2, \ldots, G_\eta\}$ with correlated set of weight vectors $\{\psi_1, \psi_2, \ldots, \psi_\eta\}$, $(\tau = 1, 2, \ldots, \eta)$ and $\xi$ experts

$\{\gamma_1, \gamma_2, \ldots, \gamma_\xi\}$ with weighting vector to be $\{\Phi_1, \Phi_2, \ldots, \Phi_\xi\}$, then the form of conventional approach of MABAC model follows the expression:

**Step 1** Evaluation matrix formation $M = [B^{\xi}_{\rho\tau}]_{\sigma\times\eta}$ where $\rho = 1, 2, \ldots, \sigma$, $\tau = 1, 2, \ldots, \eta$ as

$$\tilde{P}_{\rho\times\tau} = M = \begin{matrix} & G_1 G_2 \cdots G_\eta \\ B_1 & \\ B_2 & \\ \vdots & \\ B_\eta & \end{matrix} \begin{bmatrix} B^{\xi}_{11} & B^{\xi}_{12} & \ldots & B^{\xi}_{1\eta} \\ B^{\delta}_{21} & B^{\xi}_{22} & \ldots & B^{\xi}_{2\eta} \\ \vdots & \vdots & \ddots & \vdots \\ B^{\xi}_{\sigma 1} & B^{\xi}_{\sigma 2} & \ldots & B^{\xi}_{\sigma\eta} \end{bmatrix},$$

where $B^{\xi}_{\rho\tau}$ $(\rho = 1, 2, \ldots, \sigma; \tau = 1, 2, \ldots, \eta)$ represents the assessment formula of alternative $B_\rho$ based on the attributes $g_\tau$ $(\tau = 1, 2, \ldots, \eta)$ by the experts $\gamma^{\xi}$.

**Step 2:** Based on some aggregation operators, we can use to aggregate overall $B^{\xi}_{\rho\tau}$ to $B_{\rho\tau}$.

**Step 3:** Normalize the fuse matrix $m = [B_{\rho\tau}]_{\sigma\times\eta}$, $\rho = 1, 2, \ldots, \sigma; \tau = 1, 2, \ldots, \eta$ based on the nature of each attributes as per formula:

For benefit attributes:

$$M_{\rho\tau} = B_{\rho\tau}, (\rho = 1, 2, \ldots, \sigma, \tau = 1, 2, \ldots, \eta). \tag{6}$$

For cost attributes:

$$M_{\rho\tau} = 1 - B_{\rho\tau}, (\rho = 1, 2, \ldots, \sigma, \tau = 1, 2, \ldots, \eta). \tag{7}$$

**Step 4:** For normalized matrix $M_{\rho\tau}$ $(\rho = 1, 2, \ldots, \sigma, \tau = 1, 2, \ldots, \eta)$ and attribute's weight $\psi_\tau$ $(\tau = 1, 2, \ldots, \eta)$, then we computed normalized weighted matrix $\Psi M_{\rho\tau}$ $(\rho = 1, 2, \ldots, \sigma, \tau = 1, 2, \ldots, \eta)$ by the following formula:

$$\Psi M_{\rho\tau} = \psi_\tau M_{\rho\tau}, \quad \rho = 1, 2, \ldots, \sigma, \tau = 1, 2, \ldots, \eta. \tag{8}$$

**Step 5:** Evaluate the values of the border approximation areas (BAA) and for BAA matrix $T = [t_\tau]_{1\times\eta}$ can be computed as

$$t_\tau = \left(\prod_{\rho=1}^{\sigma} \Psi M_{\rho\tau}\right)^{1/\sigma}, (\rho = 1, 2, \ldots, \sigma, \tau = 1, 2, \ldots, \eta). \tag{9}$$

**Step 6:** Compute the distance $D = [d_{\rho\tau}]_{\sigma\times\eta}$ between each alternative and BAA measured by the following equation:

$$d_{\rho\tau} = \begin{cases} d\Big(\Psi M_{\rho\tau}, t_b\Big), & \text{if} \quad \Psi M_{\rho\tau} > t_\tau \\ 0, & \text{if} \quad \Psi M_{\rho\tau} = t_\tau \\ -d\Big(\Psi M_{\rho\tau}, t_\tau\Big), & \text{if} \quad \Psi M_{\rho\tau} < t_\tau, \end{cases} \tag{10}$$

where $d\Big(\Psi M_{\rho\tau}, t_\tau\Big)$ is the mean distance from $\Psi M_{\rho\tau}$ to $t_\tau$. Based on the values of $d_{\rho\tau}$, we can find the following:

- if $d_{\rho\tau} > 0$, which implies that alternatives belong to the upper approximation region $t^{+}(UAA)$
- if $d_{\rho\tau} = 0$, which implies that alternatives belong to the border approximation region $t^{+}(BAA)$

– if $d_{\rho\tau} < 0$, which implies that alternatives belong to the lower approximation region $t^{-}(LAA)$.

It is obvious that best alternatives are belong to $t^{+}(UAA)$, and worst alternatives are belong to $t^{-}(LAA)$.

**Step 7:** Sum the values of each alternative's $d_{\rho\tau}$ by the following equation:

$$S_{\rho} = \sum_{\tau=1}^{\eta} d_{\rho\tau}. \tag{11}$$

## 6 MABAC model with BFNs

Let there be a set of $\sigma$ alternatives $\{B_1, B_2, \ldots, B_\sigma\}$, and $\eta$ attributes $\{G_1, G_2, \ldots, G_\eta\}$ with correlated set of weight vectors $\{\psi_1, \psi_2, \ldots, \psi_\eta\}$, $(\tau = 1, 2, \ldots, \eta)$ and $\xi$ experts $\{\gamma_1, \gamma_2, \ldots, \gamma_\xi\}$ with weighting vector to be $\{\theta_1, \theta_2, \ldots, \theta_\xi\}$, then bipolar fuzzy evaluation matrix $M = [B^{\xi}_{\rho\tau}]_{\sigma\times\eta} = \left(\left(\mu^{+}_{\rho\tau}\right)^{\xi}, \left(\nu^{-}_{\rho\tau}\right)^{\xi}\right)_{\sigma\times\eta}$, $\rho = 1, 2, \ldots, \sigma, \tau = 1, 2, \ldots, \eta$, $(\mu^{+}_{\rho\tau})^{\xi} \in [0, 1]$ represents PMD, and $(\nu^{-}_{\rho\tau})^{\xi} \in [-1, 0]$ represents NMD then the bipolar fuzzy MABAC approach follows the expression:

**Step 1** Evaluation of bipolar fuzzy matrix formulation $P = [B^{\xi}_{\rho\tau}]_{\sigma\times\eta} = \left(\mu^{+\xi}_{\rho\tau}, \nu^{-\xi}_{\rho\tau}\right)_{\sigma\times\eta}$ , $\rho = 1, 2, \ldots, \sigma, \tau = 1, 2, \ldots, \eta$ given as

$$M = [B^{\xi}_{\rho\tau}]_{\sigma\times\eta} = \begin{matrix} & G_1 G_2 \cdots G_\sigma \\ \begin{matrix} B_1 \\ B_2 \\ \vdots \\ B_\eta \end{matrix} & \begin{bmatrix} \left(\mu^{+\xi}_{11}, \nu^{-\xi}_{11}\right) & \left(\mu^{+\xi}_{12}, \nu^{-\xi}_{12}\right) & \cdots & \left(\mu^{+\xi}_{1\eta}, \nu^{-\xi}_{1\eta}\right) \\ \left(\mu^{+\xi}_{21}, \nu^{-\xi}_{21}\right) & \left(\mu^{+\xi}_{22}, \nu^{-\xi}_{22}\right) & \cdots & \left(\mu^{+\xi}_{2\eta}, \nu^{-\xi}_{2\eta}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\mu^{+\xi}_{\sigma1}, \nu^{-\xi}_{\sigma1}\right) & \left(\mu^{+\xi}_{\sigma2}, \nu^{-\xi}_{\sigma2}\right) & \cdots & \left(\mu^{+\xi}_{\sigma\eta}, \nu^{-\xi}_{\sigma\eta}\right) \end{bmatrix} \end{matrix}, \tag{12}$$

where $B^{\xi}_{\rho\tau} = \left(\mu^{+\xi}_{\rho\tau}, \nu^{-\xi}_{\rho\tau}\right)$ $(\rho = 1, 2, \ldots, \sigma; \tau = 1, 2, \ldots, \eta)$ represents formula for bipolar fuzzy information of alternative $B_\rho$ based on the attributes $g_\tau$ $(\tau = 1, 2, \ldots, \eta)$ by the experts $\gamma^{\xi}$.

**Step 2:** For bipolar fuzzy aggregation operators BFWA or BFWG, we can use these operators to aggregate accumulated $B^{\xi}_{\rho\tau}$ to $B_{\rho\tau}$, then fused BFNs matrix shown below:

$$M = [B_{\rho\tau}]_{\sigma\times\eta} = \begin{matrix} & G_1 G_2 \cdots G_\sigma \\ \begin{matrix} B_1 \\ B_2 \\ \vdots \\ B_\eta \end{matrix} & \begin{bmatrix} \left(\mu^{+}_{11}, \nu^{-}_{11}\right) & \left(\mu^{+}_{12}, \nu^{-}_{12}\right) & \cdots & \left(\mu^{+}_{1\eta}, \nu^{-}_{1\eta}\right) \\ \left(\mu^{+}_{21}, \nu^{-}_{21}\right) & \left(\mu^{+}_{22}, \nu^{-}_{22}\right) & \cdots & \left(\mu^{+}_{2\eta}, \nu^{-}_{2\eta}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\mu^{+}_{\sigma1}, \nu^{-}_{\sigma1}\right) & \left(\mu^{+}_{\sigma2}, \nu^{-}_{\sigma2}\right) & \cdots & \left(\mu^{+}_{\sigma\eta}, \nu^{-}_{\sigma\eta}\right) \end{bmatrix} \end{matrix}, \tag{13}$$

where $B_{\rho\tau} = \left(\mu^{+}_{\rho\tau}, \nu^{-}_{\rho\tau}\right)$ $(\rho = 1, 2, \ldots, \sigma; \tau = 1, 2, \ldots, \eta)$ depicted a formula for bipolar fuzzy information of alternative $B_\rho$ based on the attributes $g_\tau$ $(\tau = 1, 2, \ldots, \eta)$ by the experts $\gamma^{\xi}$.

**Step 3:** Normalize the fuse resultant matrix $P = [B_{\rho\tau}]_{\sigma\times\eta}$, $\rho = 1, 2, \ldots, \sigma; \tau = 1, 2, \ldots, \eta$ based on the nature of each attributes by the following formula:

For benefit attributes:

$$M_{\rho\tau} = A_{\rho\tau} = \Big(\mu^{+}_{\rho\tau}, \nu^{-}_{\rho\tau}\Big),\quad (\rho = 1, 2, \ldots, \sigma, \tau = 1, 2, \ldots, \eta). \tag{14}$$

For cost attributes:

$$M_{\rho\tau} = \Big(B_{\rho\tau}\Big)^{c} = \Big(\nu^{-}_{\rho\tau}, \mu^{+}_{\rho\tau}\Big),\quad (\rho = 1, 2, \ldots, \sigma, \tau = 1, 2, \ldots, \eta). \tag{15}$$

**Step 4:** For normalized matrix $M_{\rho\tau} = \Big(\mu^{+}_{\rho\tau}, \nu^{-}_{\rho\tau}\Big)$ $(\rho = 1, 2, \ldots, \sigma, \tau = 1, 2, \ldots, \eta)$ and attribute's weight $\psi_\tau$, $(\tau = 1, 2, \ldots, \eta)$, then we computed normalized bipolar fuzzy weighting matrix $\Psi M_{\rho\tau} = \Big(\mu^{+'}_{\rho\tau}, \nu^{-'}_{\rho\tau}\Big)$, $\rho = 1, 2, \ldots, \sigma, \tau = 1, 2, \ldots, \eta$ by the following formula:

$$\begin{aligned}\Psi M_{\rho\tau} &= \psi_\tau \oplus M_{\rho\tau},\quad \rho = 1, 2, \ldots, \sigma, \tau = 1, 2, \ldots, \eta \\ &= \Big(1 - \prod_{\tau=1}^{\eta} (1 - \mu^{+}_{\rho\tau})^{\psi_\tau}, - \prod_{\tau=1}^{\eta} \Big(|\nu^{-}_{\rho\tau}|\Big)^{\psi_\tau}\Big).\end{aligned} \tag{16}$$

**Step 5:** Evaluate the values of the border approximation areas (BAA) and for BAA matrix $T = [t_\tau]_{1\times\eta}$ can be computed as

$$\begin{aligned}t_\tau &= \Big(\prod_{\rho=1}^{\sigma} \Psi M_{\rho\tau}\Big)^{1/\sigma}, (\rho = 1, 2, \ldots, \sigma, \tau = 1, 2, \ldots, \eta) \\ &= \Big\{\Big(\prod_{\rho=1}^{\sigma} \mu^{+}_{\rho\tau}\Big)^{1/\sigma}, -1 + \prod_{\rho=1}^{\sigma} \Big(1 + \nu^{-}_{\rho\tau}\Big)^{1/\sigma}\Big)\Big\}.\end{aligned} \tag{17}$$

**Step 6:** Compute the distance $D = [d_{\rho\tau}]_{\sigma\times\eta}$ between each alternative and BAA measured by the following equation:

$$d_{\rho\tau} = \begin{cases} d\Big(\Psi M_{\rho\tau}, t_\tau\Big), & \text{if} \quad \Psi M_{\rho\tau} > t_\tau \\ 0, & \text{if} \quad \Psi M_{\rho\tau} = t_\tau \\ -d\Big(\Psi M_{\rho\tau}, t_\tau\Big), & \text{if} \quad \Psi M_{\rho\tau} < t_\tau, \end{cases} \tag{18}$$

where $d\Big(\Psi M_{\rho\tau}, t_\tau\Big)$ is the mean distance from $\Psi M_{\rho\tau}$ to $t_\tau$, can calculated by Definition 9.

**Step 7:** Sum the values of each alternative's $d_{\rho\tau}$ by the following equation:

$$S_\rho = \sum_{\tau=1}^{\eta} d_{\rho\tau}. \tag{19}$$

# 7 Numerical example

## 7.1 Numerical MAGDM model for BFNs

In the current time, our civilization is now threatened due to different environmental issues and consumption of fossil fuels. So, the development of new energy power generation projects is now in full swing. More comprehensive evaluation methods are needed to select the proper projects, so we can recognize their strength and weakness, and put forward some new proposal to achieve the goal. Previous studies have recommended directly a single method-oriented technique for renewable energy power generation projects depicted in Zhang et al. (2020). However, a multiple objective comprehensive evaluation method with simple guiding principles has yet not developed. In this part, we provide a procedure for the selection of best renewable energy power generation projects using MABAC approach with BFNs. We consider that there is appointed three experts $e^{\xi}$ $(\xi = 1, 2, 3)$ with associated experts weight vector (0.39, 0.28, 0.33) to select the five possible renewable energy power generation projects $B_{\rho}$ $(\rho = 1, 2, 3, 4, 5)$ which evaluated under four attributes (1) $G_1$ is net present value (2) $G_2$ is static investment return period (3) $G_3$ is return on investment (4) $G_4$ is internal rate of return. The five possible renewable power generation projects $B_{\rho}$ $(\rho = 1, 2, 3, 4, 5)$ are assessed with BFNs using four attributes weights $\psi = (0.16, 0.32, 0.28, 0.24)$ by three experts.

**Step 1** For the bipolar fuzzy evaluated matrix $M = [M^{\delta}_{\rho\tau}]_{\sigma\times\eta} = (\mu^{+\delta}_{\rho\tau}, \nu^{-\delta}_{\rho\tau})$ $\rho = 1, 2, \ldots, \sigma, \tau = 1, 2, \ldots, \eta$.

$$M_1 = \begin{bmatrix} (0.5, -0.3) & (0.2, -0.1) & (0.6, -0.3) & (0.4, -0.2) \\ (0.6, -0.4) & (0.6, -0.4) & (0.4, -0.5) & (0.4, -0.1) \\ (0.4, -0.1) & (0.2, -0.3) & (0.2, -0.6) & (0.5, -0.2) \\ (0.3, -0.4) & (0.7, -0.4) & (0.7, -0.5) & (0.3, -0.4) \\ (0.6, -0.5) & (0.6, -0.2) & (0.3, -0.2) & (0.4, -0.1) \end{bmatrix}$$

$$M_2 = \begin{bmatrix} (0.4, -0.2) & (0.6, -0.2) & (0.4, -0.1) & (0.5, -0.2) \\ (0.5, -0.2) & (0.3, -0.1) & (0.6, -0.2) & (0.3, -0.1) \\ (0.3, -0.4) & (0.4, -0.2) & (0.3, -0.5) & (0.2, -0.4) \\ (0.6, -0.3) & (0.7, -0.3) & (0.4, -0.2) & (0.7, -0.4) \\ (0.5, -0.4) & (0.5, -0.2) & (0.4, -0.5) & (0.4, -0.2) \end{bmatrix}$$

$$M_3 = \begin{bmatrix} (0.3, -0.5) & (0.6, -0.2) & (0.4, -0.2) & (0.7, -0.2) \\ (0.5, -0.2) & (0.3, -0.1) & (0.7, -0.3) & (0.4, -0.2) \\ (0.2, -0.6) & (0.2, -0.4) & (0.3, -0.1) & (0.2, -0.3) \\ (0.3, -0.2) & (0.4, -0.5) & (0.3, -0.1) & (0.2, -0.1) \\ (0.6, -0.1) & (0.5, -0.2) & (0.4, -0.3) & (0.5, -0.4) \end{bmatrix}.$$

**Step 2:** According to BFWA operator and using the experts weight vector, we obtain $M^{\xi}_{\rho\tau}$ to $M_{\rho\tau}$. Therefore, we computed matrix as follows:

$B_{11} = BFWA((0.5, -0.3), (0.4, -0.2), (0.3, -0.5)) = \Big(1 - (1 - 0.5)^{0.39} \times (1 - 0.4)^{0.28} \times (1 - 0.3)^{0.33}, -((|-0.3|)^{0.39} \times (|-0.2|)^{0.28} \times (|-0.5|)^{0.33}\Big)$

$= (0.4120, -0.3170)$;

similarly, computed matrix M given in Table 2.

**Step 3:** Normalize matrix $M = [M_{\rho\tau}]$ $\rho = 1, 2, \ldots, \sigma, \tau = 1, 2, \ldots, \eta$ based on the character of attributes using formula (12) and (14), here $P_2$ is the cost attribute. As $G_2$ is cost attribute, then $B_12$, $B_{22}$, $B_{32}$, $B_{42}$ and $B_{52}$ are to be normalized by the formula

**Table 2** Aggregated matrix

| Alternatives | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $B_1$ | (0.4120, −0.3170) | (0.4758, −0.1526) | (0.4878, −0.1929) | (0.5464, −0.2000) |
| $B_2$ | (0.5417, −0.2621) | (0.4373, −0.1717) | (0.5739, −0.3268) | (0.3735, −0.1257) |
| $B_3$ | (0.3111, −0.4573) | (0.2619, −0.2945) | (0.2626, −0.3156) | (0.3339, −0.2776) |
| $B_4$ | (0.4015, −0.2936) | (0.6229, −0.3972) | (0.5182, −0.2275) | (0.4230, −0.2532) |
| $B_5$ | (0.5742, −0.2761) | (0.5417, −0.2000) | (0.3628, −0.2955) | (0.4350, −0.1919) |

**Table 3** Normalized matrix

| Alternatives | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $B_1$ | (0.4120, −0.3170) | (0.5242, −0.8474) | (0.4878, −0.1929) | (0.5464, −0.2000) |
| $B_2$ | (0.5417, −0.2621) | (0.5627, −0.8283) | (0.5739, −0.3268) | (0.3735, −0.1257) |
| $B_3$ | (0.3111, −0.4573) | (0.7381, −0.7055) | (0.2626, −0.3156) | (0.3339, −0.2776) |
| $B_4$ | (0.4015, −0.2936) | (0.3771, −0.6028) | (0.5182, −0.2275) | (0.4230, −0.2532) |
| $B_5$ | (0.5742, −0.2761) | (0.4583, −0.8000) | (0.3628, −0.2955) | (0.4350, −0.1919) |

**Table 4** Normalized weighted matrix

| Alternatives | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $B_1$ | (0.0815, −0.8321) | (0.2115, −0.9484) | (0.1708, −0.6308) | (0.1728, −0.6796) |
| $B_2$ | (0.1174, −0.8072) | (0.2326, −0.9415) | (0.2124, −0.7311) | (0.1062, −0.6079) |
| $B_3$ | (0.0579, −0.8823) | (0.3487, −0.8944) | (0.0818, −0.7240) | (0.0929, −0.7352) |
| $B_4$ | (0.0789, −0.8219) | (0.1406, −0.8505) | (0.1849, −0.6606) | (0.1236, −0.7192) |
| $B_5$ | (0.1277, −0.8139) | (0.1781, −0.9311) | (0.1206, −0.7108) | (0.1280, −0.6729) |

$B_{12} = (B_{12})^c = (0.4758, -0.1526)^c = (1-0.4758, |-0.1526|-1)$. Therefore, normalized matrix $M_{\rho\tau}$ given below in Table 3

**Step 4:** For the normalized matrix $M_{\rho\tau}$ $\rho = 1, 2, \ldots, \sigma, \tau = 1, 2, \ldots, \eta$. Now, weighted normalized matrix $\psi M_{\rho\tau} = \psi_\tau \oplus M_{\rho\tau} = \left(1 - \prod_{\tau=1}^{\eta}(1-\mu_{\rho\tau}^{+})^{\psi_\tau}, -\prod_{\tau=1}^{\eta}|\nu_{\rho\tau}^{-}|^{\psi_\tau}\right)$

$\Psi B_{11} = \left(1-(1-0.4120)^{0.16}, -(|-0.3170|)^{0.16}\right) = \left(0.0815, -0.8321\right)$. Computed normalized weighted matrix given in Table 4.

**Step 5:** Evaluate the values of BAA and BAA matrix using equation (15)follows as $t_1 = \left(\prod_{\rho=1}^{\sigma}\Psi M_{\rho\tau}\right)^{1/\sigma} = \left(0.0815 \times 0.1174 \times 0.0579 \times 0.0789 \times 0.1277\right)^{1/5}, -1 + (1-0.8321)^{1/5} \times (1-0.8072)^{1/5} \times (1-0.8823)^{1/5} \times (1-0.8219)^{1/5} \times (1-0.8139)^{1/5}\Big)$ $= \left(0.0890, -0.8339\right)$ and in the same way find the others results as $t_2 = (0.2121, -0.9199)$, $t_3 = (0.1459, -0.6938)$ and $t_4 = (0.1220, -0.6905)$

**Step 6:** Evaluate the distance $d$ using Eq. (16) between alternatives and BAA, for example, $\Delta(\Psi B_{11}) = 0.1247$ and $\Delta(t_1) = 0.1276$. Since $\Delta(\Psi B_{11}) < \Delta(t_1)$ Therefore, $d_{11} = d_{BFNHD}(\Psi B_{11}, t_1) = \frac{1}{2}\left(|0.0815-0.0890| + |-0.8321+0.8339|\right) = -0.0075$ here all the computed distances given in the Table 5.

**Table 5** Distance between alternatives and BAA

| Alternatives | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $B_1$ | − 0.0075 | − 0.0291 | 0.0879 | 0.0617 |
| $B_2$ | 0.0551 | − 0.0011 | 0.0292 | 0.0668 |
| $B_3$ | − 0.0795 | 0.1621 | − 0.0943 | − 0.0738 |
| $B_4$ | 0.0019 | − 0.0021 | 0.0722 | − 0.0271 |
| $B_5$ | − 0.0587 | − 0.0452 | − 0.0423 | 0.0236 |

**Step 7:** Compute sums of the distances $S_\rho$ for each alternatives using equation (19) as follows: $S_1 = \sum_{\tau=1}^{\eta} d_{1\tau} = d_{11} + d_{12} + d_{13} + d_{14} = (-0.0075) + (-0.0291) + 0.0879 + 0.0617 = 0.1130$, for alternative $B_1$.

$S_2 = \sum_{\tau=1}^{\eta} d_{2\tau} = d_{21} + d_{22} + d_{23} + d_{24} = 0.0551 + (-0.0011) + 0.0292 + 0.0668 = 0.1500$, for alternative $B_2$.

$S_3 = \sum_{\tau=1}^{\eta} d_{3\tau} = d_{31} + d_{32} + d_{33} + d_{34} = (-0.0795) + 0.1621 + (-0.0943) + (-0.0738) = -0.0855$, for alternative $B_3$.

$S_4 = \sum_{\tau=1}^{\eta} d_{4\tau} = d_{41} + d_{42} + d_{43} + d_{44} = 0.0019 + (-0.0021) + 0.0722 + (-0.0271) = 0.0449$, for alternative $B_4$.

$S_5 = \sum_{\tau=1}^{\eta} d_{5\tau} = d_{51} + d_{52} + d_{53} + d_{54} = (-0.0587) + (-0.0452) + (-0.0423) + 0.0236 = -0.1226$, for alternative $B_5$.

From the comprehensive evaluation results of $S_\rho$ for detect the better choice. We get the order list as $B_2 \succ B_1 \succ B_4 \succ B_3 \succ B_5$, and $B_2$ is the favourable solution.

### 7.2 Validity test

Wang and Triantaphyllou (2008) introduced the following testing criteria to evaluate the validity of MADM problems.

- **Test criterion 1:** An effective MADM method does not change the index of the desirable alternative if on replacing a non-optimal alternative with a another worse alternative without changing its relative importance of each decision attribute.
- **Test criterion 2:** An effective MADM method should follow transitive property.
- **Test criterion 3:** If we decompose a MADM problem into some small MADM problems and same method is applied to these MADM problems to rank the alternative, then collective ranking order of the alternatives must be coincide with the raking of decompose decision-making problems.

### 7.3 Validity checked by test criterion 1

For testing the validity of the proposed method under the test criterion 1, we replace a non-optimal alternative $B_5$ with the worse alternative $B_5'$ in original decision matrices to each expert in their rating values is summarized in Table 6.

In this era, normalized weighted modified matrix found as given in Table 7:

For this, evaluate the values of BAA and BAA matrix as follows:

$t_1 = (0.0716, -0.8475)$, $t_2 = (0.2373, -0.9066)$ $t_3 = (0.1442, -0.7377)$, $t_4 = (0.1092, -0.7501)$

Then, $S_\rho$ ($\rho = 1, 2, 3, 4, 5$) values are evaluated to be $S_1 = 0.1138$, $S_2 = 0.1364$, $S_3 = -0.0262$, $S_4 = -0.0237$ and $S_5 = -0.1384$, and the corresponding ranking order of

**Table 6** Rating values of the transferred alternative $B_5'$ by each expert

| Experts | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $e^{(1)}$ | (0.3, −0.5) | (0.4, −0.6) | (0.4, −0.7) | (0.3, −0.8) |
| $e^{(2)}$ | (0.2, −0.6) | (0.3, −0.5) | (0.2, −0.6) | (0.2, −0.6) |
| $e^{(3)}$ | (0.2, −0.3) | (0.2, −0.2) | (0.4, −0.5) | (0.3, −0.5) |

**Table 7** Normalized weighted modified matrix

| Alternatives | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
|---|---|---|---|---|
| $B_1$ | (0.0815, −0.8321) | (0.2115, −0.9484) | (0.1708, −0.6308) | (0.1728, −0.6796) |
| $B_2$ | (0.1174, −0.8072) | (0.2326, −0.9415) | (0.2124, −0.7311) | (0.1062, −0.6079) |
| $B_3$ | (0.0579, −0.8823) | (0.3487, −0.8944) | (0.0818, −0.7240) | (0.0929, −0.7352) |
| $B_4$ | (0.0789, −0.8219) | (0.1406, −0.8505) | (0.1849, −0.6606) | (0.1236, −0.7192) |
| $B_5$ | (0.0431, −0.8784) | (0.3118, −0.8506) | (0.1135, −0.8667) | (0.0738, −0.8957) |

the alternatives is $B_2 \succ B_1 \succ B_4 \succ B_3 \succ B_5'$ which is identical with the proposed approach for the validation of test criterion 1.

### 7.4 Validity checked by test criteria 2 and 3

Under these test, we have decomposed the original decision-making problems into three sub-problems containing alternatives as $\{B_1, B_2, B_4, B_5\}$, $\{B_1, B_3, B_4, B_5\}$ and $\{B_2, B_3, B_4, B_5\}$ and now applying the proposed MABAC method to each of the sub problems. Then, ranking orders of these sub-problems are as $B_2 \succ B_1 \succ B_4 \succ B_5$, and $B_1 \succ B_4 \succ B_3 \succ B_5$, and $B_2 \succ B_4 \succ B_3 \succ B_5$. Therefore, from these we get the final ranking $B_2 \succ B_1 \succ B_4 \succ B_3 \succ B_5$ which is coinciding with the original ranking. Hence, it validates the test criteria 2 and 3.

## 8 Compare BFN MABAC approach with some BFN operators

In this part, we compare proposed BFN MABAC approach with some existing bipolar fuzzy operators such as bipolar fuzzy weighted aggregation operator BFWA (BFWG) proposed by Gul (2015). We used attribute's weight and result of matrix $M$, and aggregated values of two operators are presented in Table 8. Another, Jana et al. (2019b) proposed Dombi aggregation operators BFDWA (BFDWG) in BFN environment whose aggregated value are given in Table 9. Also, Wei et al. (2018) used Hamacher weighted operators BFHWA (BFHWG) and their corresponding aggregated values are shown in Table 10. Therefore, the existing operators for BFNs with their ranking comparison with the proposed MABAC model is shown in Table 11.

It is shown from the comparison Table 11 that the best alternative is $B_2$ when applied the proposed MABAC method. Although, ranking orders for existing operators are slightly different, but it is noticeable that the best alternative selected by proposed methods is coinciding with some pre-existing techniques. Therefore, the proposed model is stable and reliable.

**Table 8** Aggregated values of BFWA and BFWG operators

| Alternative | BFWA | BFWG |
|---|---|---|
| $B_1$ | (0.5033, −0.3383) | (0.4993, −0.5398) |
| $B_2$ | (0.5232, −0.3378) | (0.5097, −0.5302) |
| $B_3$ | (0.4889, −0.4201) | (0.3979, −0.4899) |
| $B_4$ | (0.4345, −0.3321) | (0.4280, −0.3895) |
| $B_5$ | (0.4490, −0.3625) | (0.4395, −0.5113) |

**Table 9** Aggregated values of BFDWA and BFDWG operators

| Alternative | BFDWA | BFDWG |
|---|---|---|
| $B_1$ | (0.5051, −0.2830) | (0.4970, −0.6642) |
| $B_2$ | (0.5285, −0.2658) | (0.5014, −0.6391) |
| $B_3$ | (0.5442, −0.3914) | (0.3659, −0.5289) |
| $B_4$ | (0.4376, −0.3073) | (0.4247, −0.4172) |
| $B_5$ | (0.4538, −0.3147) | (0.4347, −0.6025) |

**Table 10** Aggregated values of BFHWA and BFHWG operators

| Alternative | BFHWA | BFHWG |
|---|---|---|
| $B_1$ | (0.5023, −0.2264) | (0.5003, −0.4858) |
| $B_2$ | (0.5201, −0.2255) | (0.5133, −0.4811) |
| $B_3$ | (0.4639, −0.2663) | (0.4157, −0.4714) |
| $B_4$ | (0.4327, −0.2275) | (0.4294, −0.3729) |
| $B_5$ | (0.4464, −0.2390) | (0.4416, −0.4708) |

**Table 11** Ranking order of alternatives for some BFN operators

| Methods | Ranking order |
|---|---|
| Gul (2015) BFWA | $B_2 \succ B_1 \succ B_4 \succ B_5 \succ B_3$ |
| Gul (2015) BFWG | $B_4 \succ B_2 \succ B_1 \succ B_5 \succ B_3$ |
| Jana et al. (2019b) BFDWA | $B_2 \succ B_1 \succ B_3 \succ B_5 \succ B_4$ |
| Jana et al. (2019b) BFDWG | $B_4 \succ B_2 \succ B_3 \succ B_1 \succ B_5$ |
| Wei et al. (2018) BFHWA | $B_2 \succ B_1 \succ B_5 \succ B_4 \succ B_3$ |
| Wei et al. (2018) BFHWG | $B_4 \succ B_2 \succ B_1 \succ B_5 \succ B_3$ |
| Proposed BFN MABAC model | $B_2 \succ B_1 \succ B_4 \succ B_3 \succ B_5$ |

## 9 Conclusions

In this article, we study some essential notion of BFNs and conventional MABAC model. The limitations of this decision-making method is that it only focused on defining the “distance" of the criteria function of each observed alternative from the border approximate area. We propose MABAC approach with BFNs for MAGDM. In this study, consider the definition of BFNs and their score function, accuracy function and operational laws. After that, two weighted aggregation operators for BFNs is defined. Next, connect original MABAC approach with BFNs and then develop BFN MABAC model for MAGDM. Also, applying the proposed model for the evaluation of an example for selecting renewable energy power generation project. At last, analyse the propose model for comparative study with some existing BFN operators for showing its efficiency as well as its validation. In the future, the results of the paper can be extended to some other fuzzy and uncertain environments (Biswas 2020; Laha and Biswas 2019; Han et al. 2015; Wei et al. 2017).

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**Conflict of interest** There is no conflict of interest between the authors and the institute where the work has been carried out.

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