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Analysis of the effect of medicines over bacteria based on competition graphs with picture fuzzy environment

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Abstract
In this study, the concept of picture fuzzy competition graph along with its two subclasses such as picture fuzzy k-competition graphs and p-competition picture fuzzy graphs are introduced. Picture fuzzy competition graph is one of the generalization of competition graph. Several properties of picture fuzzy competition graphs have been investigated. Some new types of picture fuzzy graphs have been introduced like picture fuzzy open neighborhood graph and picture fuzzy closed neighborhood graph. In addition, the relationship between picture fuzzy k-competition graph and picture fuzzy k-neighborhood graph are established. An application of picture fuzzy competition graph in medical science is presented.

Keywords Picture fuzzy digraphs · Picture fuzzy competition graphs · Picture fuzzy k-competition graphs · p-competition picture fuzzy graphs · Picture fuzzy neighborhood graphs

Mathematics Subject Classification 05C07 · 05C20 · 05C72 · 05E30 · 94C15

1 Introduction

1.1 Research background

In 1968, Cohen (1968) first introduced the concept of competition graph (CG) while studying inflictions of graph theory in relation with an ecological problem. Let $\overrightarrow{G} = (V, \overrightarrow{B})$ be the digraph corresponding to a food cycle . A vertex $r \in V(\overrightarrow{G})$ indicates that a species in the

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food cycle and an edge $(r, x) \in \overrightarrow{B}(\overrightarrow{G})$ indicate that r victims on the species x. The species r and s compete for a victim x if they have a common victim x. Due to this harmony, Cohen (1968) defined graphs representing the relationship of competition within the species of food cycle. An undirected graph $C(\overrightarrow{G})$ of $\overrightarrow{G}$ is the CG with V as vertex set and having an edge (r, s) in $C(\overrightarrow{G})$ if there is a vertex $x \in V$ such that $(r, x), (s, x) \in \overrightarrow{B}$ in $\overrightarrow{G}$.

The concept of fuzzy set (FS) was first introduced by Zadeh (1965) in 1965. In 1986, Atanassov (1986) presented the idea of intuitionistic fuzzy set (IFS) as a generalization of FS to handle uncertainty and incomplete information in real life problems. But they may not be efficient to represent some special types of information. In such cases, a component namely 'neutrality' is needed to represent the information completely. To overcome this issue, picture fuzzy set (PFS) was introduced by Cuong (2014) in 2014 as a generalization of IFS. The new idea of similarity measures for picture hesitant FSs with applications are discussed in Ahmad et al. (2019) and Jan et al. (2019a). The notion of fuzzy graph (FG) theory was explained by Rosenfeld (1975) in 1975 whose basic idea was given by Kauffman (1973) in 1973. Some works also done over interval valued fuzzy graphs in Jan et al. (2019b, c). Later on, Shannon and Atanassov (1994) introduced the notion of intuitionistic fuzzy graph (IFG). Davvaz et al. (2019) studied on nth type IFGs with applications. Al-Hawary et al. (2018) provided the new concept of picture fuzzy graph (PFG) and discussed some operations on it. Samanta and Pal (2013) introduced fuzzy k-CG and p-competition FG. The idea of intuitionistic fuzzy CG was discussed by Sahoo and Pal (2015). As the time goes, a lot of works have also been done over fuzzy CG which can be found in Isaak et al. (1992), Kim et al. (1995), Cho et al. (2000), Samanta et al. (2014, 2015), Pramanik et al. (2016), Akram and Nasir (2017) and Nasir et al. (2017).

1.2 Research challenges and gaps

- The CG and PFG are well known topics. But till now, no work is available in the literature on PFCG.
- The CGs describe in regard to the common victim and related species, but they do not measure the strength of competitions.
- The CGs do not show how much the species depend on a common victim compared to the other species.
- All problems of victim-predator are not modeled using PFCG. For these problems, p-CPFGs may be used effectively.

1.3 Motivation and contribution of this study

In ecological problem, species may be of several types such as lenten and non-lenten. Similarly, victims may be tasteful, digestive, injurious, etc. These terms have no proper meaning. They are fuzzy in nature. So the species and victims may be assumed as PFSs and interrelation between them may be designed with a PFG. Due to uncertainty in description of species and victims, and their relationships, it is necessity to design a PFCG model. The contribution of this article is not only restricted to PFCGs but it introduces the notions of picture fuzzy k-competition graph (PFkCG), p-competition picture fuzzy graph (p-CPFG) and picture fuzzy neighborhood graphs (PFNGs). We also present our developed method by designing a table and a flow-chart for the construction of different types of competition graphs. Finally, we introduce an application of PFCG in medical science to describe the effect of medicines over bacteria based on competition graphs.

1.4 Framework of this study

This work is composed as follows: in Sect. 2, several basic definitions related to PFCGs have been provided. In Sect. 3, the notions of PFCG have been presented and several properties have been studied. Some basic results of PFCG have been studied in Sect. 4. In Sects. 5 and 6, two new subclasses of PFCG, known as PFkCG and p-CPFG, have been introduced. In Sect. 7, the concept of PFNGs have been presented and the relations between PFkCGs and picture fuzzy k-neighborhood graphs (PFkNGs) have been established. In Sect. 8, an application of PFCG in medical science is given. At last, a conclusion is drawn in Sect. 9.

2 Preliminaries

In this section, several basic definitions related to PFCGs are provided. Meantime, we introduce cardinality, support and height of PFSs that will be used for further improvement.

Definition 1 (Jenson and Gutin 2009) A digraph $\overrightarrow{G}$ consists of a non-empty finite vertex set V and a finite set $\overrightarrow{B}$ of edges that are ordered pairs of distinct members of V. For $a_i \in V$, the out-neighborhood and in-neighborhood of a_i are the sets $\aleph^+(a_i) = \{a_j \in V - \{a_i\} : (a_i, a_j) \in \overrightarrow{B}\}$ and $\aleph^-(a_i) = \{a_j \in V - \{a_i\} : (a_j, a_i) \in \overrightarrow{B}\}$, respectively. Also the set $\aleph^+(a_i) \cup \aleph^-(a_i)$ is the open neighborhood of a_i in $\overrightarrow{G}$. A directed walk from a vertex a_i to a_j in $\overrightarrow{G}$ is an alternating sequence of vertices and edges begins with a_i and ends with a_j such that each edge is incident with the vertices preceding and following it. No edge appears more than once but vertex can. A walk is closed if $a_i = a_j$. If all vertices in a walk are distinct, then it is known as path. A path $a_1, a_2, ..., a_l$ $(l \geq 3)$ is a cycle if $a_1 = a_l$.

Definition 2 (Achary and Vartak 1973) The open neighborhood $\aleph(r)$ of r in an undirected graph G is the set of all vertices adjoining to r and the closed neighborhood of r is $\aleph[r] = \aleph(r) \cup \{r\}$. The open-neighborhood graph $\aleph(G)$ and closed-neighborhood graph $\aleph[G]$ of G are the graphs with V as vertex set and having an edge (r, s) in $\aleph(G)$ and $\aleph[G]$ iff $\aleph(r) \cap \aleph(s) \neq \emptyset$ and $\aleph[r] \cap \aleph[s] \neq \emptyset$, respectively in G.

Definition 3 (Brigham and Dutton 1987) The open p-neighborhood graph $\aleph_p(G)$ and closed p-neighborhood graph $\aleph_p[G]$ of G are the graphs with V as vertex and having an edge (r, s) in $\aleph_p(G)$ and $\aleph_p[G]$ iff $|\aleph(r) \cap \aleph(s)| \geq p$ and $|\aleph[r] \cap \aleph[s]| \geq p$, respectively in G.

Definition 4 (Cohen 1968) The CG $C(\overrightarrow{G})$ of $\overrightarrow{G} = (V, \overrightarrow{B})$ is an undirected graph with V as vertex set and having an edge (r, s) in $C(\overrightarrow{G})$ if there is a vertex $x \in V$ such that (r, x), $(s, x) \in \overrightarrow{B}$ in $\overrightarrow{G}$.

There are different variants of CGs. One of them is p-CG which is defined below (Table 1; Fig. 1):

Definition 5 (Kim et al. 1995) Let $p > 0$ is an integer. The p-CG $C_p(\overrightarrow{G})$ of $\overrightarrow{G}$ is an undirected graph with V as vertex set and having an edge (r, s) in $C_p(\overrightarrow{G})$ iff for some distinct

Table 1 Contributions of the authors towards competition graphs with picture fuzzy environment

Authors	Year	Contributions
Euler (1736)	1736	Introduced graphs theory
Cohen (1968)	1968	Introduced competition graphs
Kauffman (1973)	1973	Defined fuzzy graphs
Rosenfeld (1975)	1975	Modified the structure of fuzzy graphs
Shannon and Atanassov (1994)	1994	Introduced intuitionistic fuzzy graphs
Cho et al. (2000)	2000	Defined m-step competition graphs
Samanta and Pal (2013)	2013	Proposed the concept of fuzzy competition graphs
Sahoo and Pal (2015)	2015	Introduced intuitionistic fuzzy competition graphs
Nasir et al. (2017)	2017	Discussed properties of intuitionistic fuzzy competition graphs
Al-Hawary et al. (2018)	2018	Defined picture fuzzy graphs
This paper	–	Competition graphs with picture fuzzy environment

vertices $v_1, v_2, \ldots, v_p \in V$ such that $(r, v_1), (s, v_1), (r, v_2), (s, v_2), \ldots, (r, v_p), (s, v_p) \in \overrightarrow{B}$ in $\overrightarrow{G}$.

Definition 6 (Cuong 2014) Let X be the universe. Then a PFS A is defined on X as $A = \{(r, (\mu_A(r), \eta_A(r), \nu_A(r))) : r \in X\}$, where $\mu_A(r), \eta_A(r), \nu_A(r) \in [0, 1]$ denote the degree of truth membership (DTMS), degree of abstinence membership (DAMS) and degree of false membership (DFMS) of $r \in A$, respectively, with $0 \leq \mu_A(r) + \eta_A(r) + \nu_A(r) \leq 1\ \forall r \in X$. Also $\forall r \in X$, $D_A(r) = 1 - (\mu_A(r) + \eta_A(r) + \nu_A(r))$ represent the denial degree of $r \in A$.

Definition 7 (Al-Hawary et al. 2018) A PFG is a triplet $G = (V, A, B)$ where $A = (\mu_A, \eta_A, \nu_A)$, $B = (\mu_B, \eta_B, \nu_B)$ and (i) $V = \{a_1, a_2, \ldots, a_n\}$ such that $\mu_A, \eta_A, \nu_A : V \to [0, 1]$ denote the DTMS, DAMS and DFMS of $a_i \in V$, respectively, with $0 \leq \mu_A(a_i) + \eta_A(a_i) + \nu_A(a_i) \leq 1\ \forall a_i \in V$, $(i = 1, 2, \ldots, n)$. (ii) $\mu_B, \eta_B, \nu_B : V \times V \to [0, 1]$ denote the DTMS, DAMS and DFMS of edge (a_i, a_j), respectively, such that $\mu_B(a_i, a_j) \leq \min\{\mu_A(a_i), \mu_A(a_j)\}$, $\eta_B(a_i, a_j) \leq \min\{\eta_A(a_i), \eta_A(a_j)\}$ and $\nu_B(a_i, a_j) \leq \max\{\nu_A(a_i), \nu_A(a_j)\}$ with $0 \leq \mu_B(a_i, a_j) + \eta_B(a_i, a_j) + \nu_B(a_i, a_j) \leq 1$ for every (a_i, a_j), $(i, j = 1, 2, \ldots, n)$.

Definition 8 (Mohamedlsmayil and AshaBosely 2019) Let $G = (V, A, B)$ be a PFG. An edge (r, s) is independent strong if $\frac{1}{2}\min\{\mu_A(r), \mu_A(s)\} < \mu_B(r, s)$, $\frac{1}{2}\min\{\eta_A(r), \eta_A(s)\} > \eta_B(r, s)$ and $\frac{1}{2}\max\{\nu_A(r), \nu_A(s)\} > \nu_B(r, s)$. Otherwise, it is weak edge. Strength of (r, s) is given by $\left(\frac{\mu_B(r,s)}{\mu_A(r) \wedge \mu_A(s)}, \frac{\eta_B(r,s)}{\eta_A(r) \wedge \eta_A(s)}, \frac{\nu_B(r,s)}{\nu_A(r) \vee \nu_A(s)}\right)$.

Definition 9 The cardinality of a PFS $A = \{(r, (\mu_A(r), \eta_A(r), \nu_A(r))) : r \in X\}$ is defined as $|A| = (|A|_\mu, |A|_\eta, |A|_\nu)$ where $|A|_\mu$, $|A|_\eta$ and $|A|_\nu$ represent the sum of DTMS, DAMS and DFMS, respectively of all elements of A.

Definition 10 The support of a PFS $A = \{(r, (\mu_A(r), \eta_A(r), \nu_A(r))) : r \in X\}$ is defined as supp(A)=$\{r \in X$: $\mu_A(r) \neq 0, \eta_A(r) \neq 0$ and $\nu_A(r) \neq 0\}$. The height of a PFS A is defined as $h(A) = \left(\sup_{r \in X} \mu_A(r), \sup_{r \in X} \eta_A(r), \inf_{r \in X} \nu_A(r)\right) = (h_\mu(A), h_\eta(A), h_\nu(A))$.

Now, we define picture fuzzy digraph (PFD).

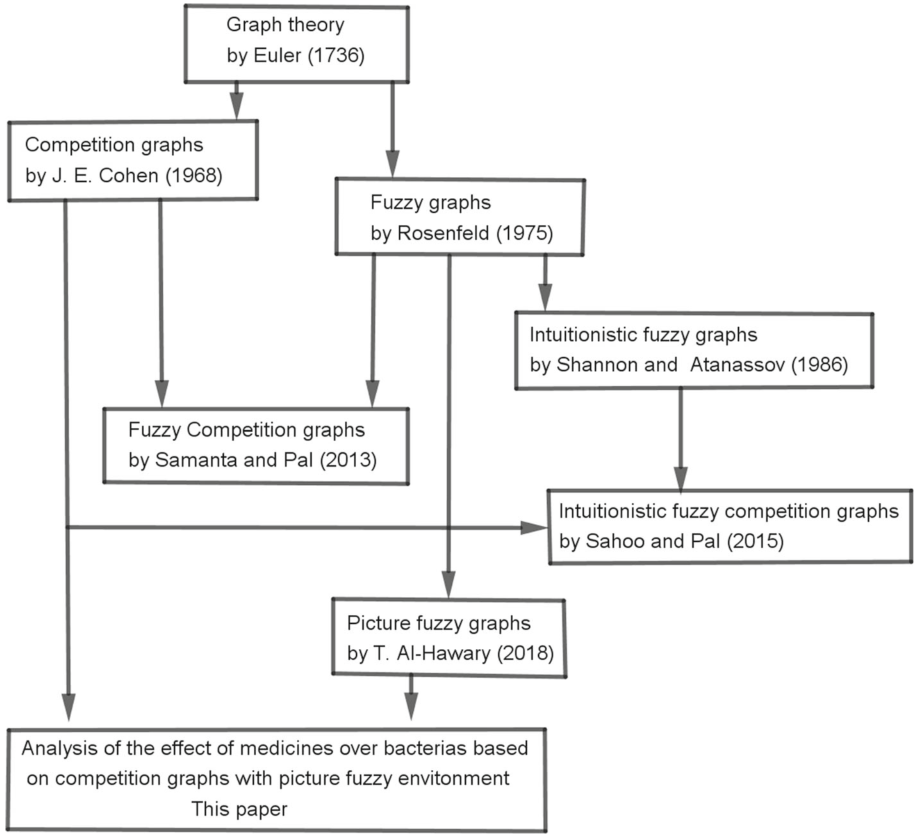


Fig. 1 Flow-chart of the research

Definition 11 A PFD is of the form $\overrightarrow{G} = (V, A, \overrightarrow{B})$ where $A = (\mu_A, \eta_A, \nu_A)$, $\overrightarrow{B} = (\overrightarrow{\mu_B}, \overrightarrow{\eta_B}, \overrightarrow{\nu_B})$ and (i) $V = \{a_1, a_2, \ldots, a_n\}$ such that $\mu_A, \eta_A, \nu_A : V \to [0, 1]$ denote the DTMS, DAMS and DFMS of $a_i \in V$, respectively with $0 \le \mu_A(a_i) + \eta_A(a_i) + \nu_A(a_i) \le 1$ $\forall a_i \in V$, $(i = 1, 2, \ldots, n)$. (ii) $\overrightarrow{\mu_B}, \overrightarrow{\eta_B}, \overrightarrow{\nu_B} : V \times V \to [0, 1]$ denote the DTMS, DAMS and DFMS of edge (a_i, a_j), respectively such that $\overrightarrow{\mu_B}(a_i, a_j) \le \min\{\mu_A(a_i), \mu_A(a_j)\}$, $\overrightarrow{\eta_B}(a_i, a_j) \le \min\{\eta_A(a_i), \eta_A(a_j)\}$ and $\overrightarrow{\nu_B}(a_i, a_j) \le \max\{\nu_A(a_i), \nu_A(a_j)\}$ with $0 \le \overrightarrow{\mu_B}(a_i, a_j) + \overrightarrow{\eta_B}(a_i, a_j) + \overrightarrow{\nu_B}(a_i, a_j) \le 1$ for every (a_i, a_j), $(i, j = 1, 2, \ldots, n)$.

We illustrate it by giving an example.

Example 1 Consider the PFD $\overrightarrow{G} = (V, A, \overrightarrow{B})$ as shown in Fig. 2 where, $V = \{r, s, u, v\}$, $A = \{(r, (0.4, 0.3, 0.2)), (s, (0.5, 0.1, 0.4)), (u, (0.3, 0.4, 0.3)), (v, (0.2, 0.5, 0.3))\}$ and $\overrightarrow{B} = \{((r, s), (0.4, 0.1, 0.15)), ((r, u), (0.2, 0.3, 0.1)), ((r, v), (0.2, 0.3, 0.25)), ((u, s), (0.3, 0.1, 0.1)), ((u, v), (0.2, 0.4, 0.3))\}$. The DTMS, DAMS and DFMS of the vertex r in $\overrightarrow{G}$ are respectively, 0.4, 0.3 and 0.2. Similarly, the DTMS, DAMS and DFMS for other vertices and edges are given.

Definition 12 A PFD $\overrightarrow{G} = (V, A, \overrightarrow{B})$ is complete if $\overrightarrow{\mu_B}(r, s) = \mu_A(r) \wedge \mu_A(s)$, $\overrightarrow{\eta_B}(r, s) = \eta_A(r) \wedge \eta_A(s)$ and $\overrightarrow{\nu_B}(r, s) = \nu_A(r) \vee \nu_A(s)$ $\forall\, r, s \in V$.

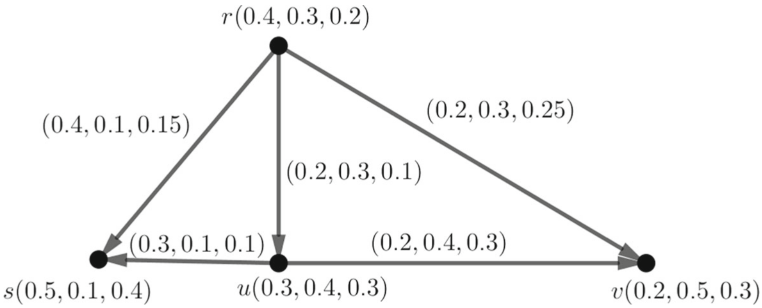


Fig. 2 Example of a PFD

Definition 13 The complement of a PFD $\overrightarrow{G} = (V, A, \overrightarrow{B})$ is also a PFD $\overrightarrow{G}^c = (V, A, \overrightarrow{B^c})$, where $\overrightarrow{\mu_B^c}(r,s) = (\mu_A(r) \wedge \mu_A(s)) - \overrightarrow{\mu_B}(r,s)$, $\overrightarrow{\eta_B^c}(r,s) = (\eta_A(r) \wedge \eta_A(s)) - \overrightarrow{\eta_B}(r,s)$ and $\overrightarrow{\nu_B^c}(r,s) = (\nu_A(r) \vee \nu_A(s)) - \overrightarrow{\nu_B}(r,s)\ \forall\, r, s \in V$.

Definition 14 (Mohamedlsmayil and AshaBosely 2019) A PFD $\overrightarrow{G} = (V, A, \overrightarrow{B})$ is strong if $\overrightarrow{\mu_B}(r,s) = \mu_A(r) \wedge \mu_A(s)$, $\overrightarrow{\eta_B}(r,s) = \eta_A(r) \wedge \eta_A(s)$ and $\overrightarrow{\nu_B}(r,s) = \nu_A(r) \vee \nu_A(s)$ for every $(r,s) \in \overrightarrow{B}$.

Definition 15 A PFD $\overrightarrow{G} = (V, A, \overrightarrow{B})$ is regular if every vertices has the same in degree and out degree and it is irregular if there exist a vertex which is adjoining to at least one vertex with distinct degrees.

3 Picture fuzzy competition graphs (PFCG)

In this section, the definition of PFCG is given as a generalization of fuzzy CG and studied some interesting properties of it. In addition, picture fuzzy out neighborhood (PFON) and picture fuzzy in neighborhood (PFIN) of a vertex of the PFD $\overrightarrow{G}$ are introduced.

Definition 16 PFON of a vertex r in $\overrightarrow{G}$ is $\aleph^+(r) = (X_r^+, (\mu_r^+, \eta_r^+, \nu_r^+))$, where $X_r^+ = \{s : \overrightarrow{\mu_B}(r,s) > 0, \overrightarrow{\eta_B}(r,s) > 0$ and $\overrightarrow{\nu_B}(r,s) > 0\}$ and $\mu_r^+, \eta_r^+, \nu_r^+ : X_r^+ \to [0,1]$ are defined as $\mu_r^+(s) = \overrightarrow{\mu_B}(r,s)$, $\eta_r^+(s) = \overrightarrow{\eta_B}(r,s)$ and $\nu_r^+(s) = \overrightarrow{\nu_B}(r,s)$.

Similarly, we define PFIN of a vertex.

Definition 17 PFIN of a vertex r of $\overrightarrow{G}$ is $\aleph^-(r) = (X_r^-, (\mu_r^-, \eta_r^-, \nu_r^-))$, where $X_r^- = \{s : \overrightarrow{\mu_B}(s,r) > 0, \overrightarrow{\eta_B}(s,r) > 0$ and $\overrightarrow{\nu_B}(s,r) > 0\}$ and $\mu_r^-, \eta_r^-, \nu_r^- : X_r^- \to [0,1]$ are defined as $\mu_r^-(s) = \overrightarrow{\mu_B}(s,r)$, $\eta_r^-(s) = \overrightarrow{\eta_B}(s,r)$ and $\nu_r^-(s) = \overrightarrow{\nu_B}(s,r)$.

The following example illustrates about PFON and PFIN of a vertex.

Example 2 Let $\overrightarrow{G} = (V, A, \overrightarrow{B})$ be a PFD (see Fig. 3) where $V = \{r, s, u, v\}$, $A = \{(r, (0.4, 0.3, 0.2)), (s, (0.6, 0.2, 0.1)), (u, (0.3, 0.4, 0.2)), (v, (0.5, 0.3, 0.2))\}$ and $\overrightarrow{B} = \{((r,s), (0.3, 0.2, 0.2)), ((r,u), (0.3, 0.3, 0.2)), ((r,v), (0.4, 0.3, 0.2)), ((s,u), (0.3, 0.2, 0.1)), ((v,u), (0.3, 0.3, 0.1))\}$. Here, $\aleph^+(r) = \{(s, (0.3, 0.2, 0.2)), (u, (0.3, 0.3, 0.2)), (v, (0.4, 0.3, 0.2))\}$, $\aleph^+(s) = \{(u, (0.3, 0.2, 0.1))\}$ and $\aleph^+(v) = \{(u, (0.3, 0.3, 0.1))\}$. Also, $\aleph^-(s) = \{(r, (0.3, 0.2, 0.2))\}$, $\aleph^-(u) = \{(r, (0.3, 0.3, 0.2)), (s, (0.3, 0.2, 0.1)), (v, (0.3, 0.3, 0.1))\}$ and $\aleph^-(v) = \{(r, (0.4, 0.3, 0.2))\}$.

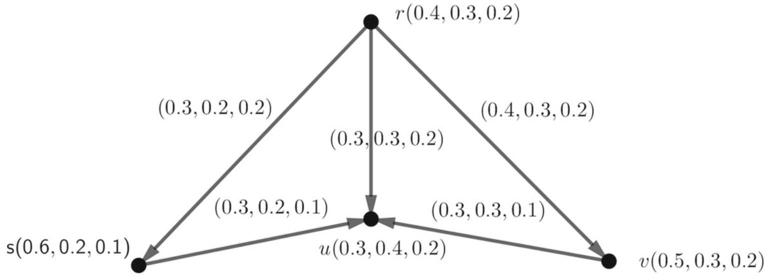


Fig. 3 A PFD with PFIN and PFON of its vertices

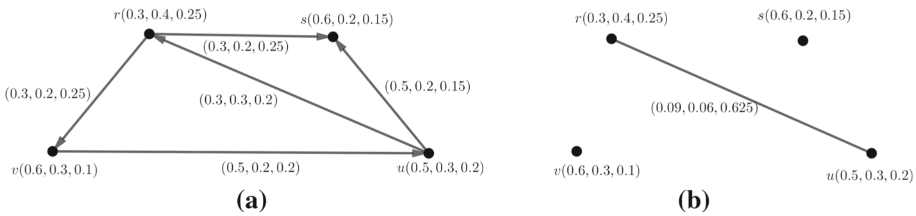


Fig. 4 Example of **a** PFD and **b** its corresponding PFCG

Now, we introduce PFCG.

Definition 18 The PFCG $C(\overrightarrow{G})$ of a PFD $\overrightarrow{G}$ is an undirected graph with V as vertex set and having an edge (r, s) in $C(\overrightarrow{G})$ iff $\aleph^+(r) \cap \aleph^+(s) \neq \emptyset$ in $\overrightarrow{G}$. The DTMS, DAMS and DFMS of (r, s) in $C(\overrightarrow{G})$ are respectively $\mu_B(r, s) = [\mu_A(r) \wedge \mu_A(s)]h_\mu(\aleph^+(r) \cap \aleph^+(s))$, $\eta_B(r, s) = [\eta_A(r) \wedge \eta_A(s)]h_\eta(\aleph^+(r) \cap \aleph^+(s))$ and $\nu_B(r, s) = [\nu_A(r) \vee \nu_A(s)]h_\nu(\aleph^+(r) \cap \aleph^+(s))$.

Example 3 Consider the PFD $\overrightarrow{G}$ given in Fig. 4a. Here $\aleph^+(r) = \{(s, (0.3, 0.2, 0.25)), (v, (0.3, 0.2, 0.25))\}$, $\aleph^+(u) = \{(r, (0.3, 0.3, 0.2)), (s, (0.5, 0.2, 0.15))\}$ and $\aleph^+(v) = \{(u, (0.5, 0.2, 0.2))\}$. Then $\aleph^+(r) \cap \aleph^+(u) = \{(s, (0.3, 0.2, 0.25))\}$. Thus, there exists an edge (r, u) in $C(\overrightarrow{G})$ with DTMS, DAMS and DFMS are (0.09, 0.06, 0.6245) (see Fig. 4b).

Theorem 1 *If $\aleph^+(r) \cap \aleph^+(s)$ contains only a single element of the PFD $\overrightarrow{G}$, then (r, s) is an independent strong edge of $C(\overrightarrow{G})$ iff $|\aleph^+(r) \cap \aleph^+(s)|_\mu > 0.5$, $|\aleph^+(r) \cap \aleph^+(s)|_\eta < 0.5$ and $|\aleph^+(r) \cap \aleph^+(s)|_\nu < 0.5$.*

Proof Let $\overrightarrow{G}$ be a PFD. If $\aleph^+(r) \cap \aleph^+(s) = \{(x, (\theta, \phi, \psi))\}$, where θ, ϕ and ψ are the DTMS, DAMS and DFMS of either edge (r, x) or (s, x). Here, $|\aleph^+(r) \cap \aleph^+(s)|_\mu = \theta = h_\mu(\aleph^+(r) \cap \aleph^+(s))$, $|\aleph^+(r) \cap \aleph^+(s)|_\eta = \phi = h_\eta(\aleph^+(r) \cap \aleph^+(s))$, $|\aleph^+(r) \cap \aleph^+(s)|_\nu = \psi = h_\nu(\aleph^+(r) \cap \aleph^+(s))$. So that, $\mu_B(r, s) = (\mu_A(r) \wedge \mu_A(s)) \times \theta$, $\eta_B(r, s) = (\eta_A(r) \wedge \eta_A(s)) \times \phi$, $\nu_B(r, s) = (\nu_A(r) \vee \nu_A(s)) \times \psi$.

Therefore, (r, s) is an independent strong edge of $C(\overrightarrow{G})$ iff $\theta > 0.5$, $\phi < 0.5$ and $\psi < 0.5$, i.e., iff $|\aleph^+(r) \cap \aleph^+(s)|_\mu > 0.5$, $|\aleph^+(r) \cap \aleph^+(s)|_\eta < 0.5$ and $|\aleph^+(r) \cap \aleph^+(s)|_\nu < 0.5$. □

Theorem 2 *Let $\overrightarrow{G}$ be a PFD. Then its PFCG $C(\overrightarrow{G})$ can not be complete.*

Proof Let $C(\overrightarrow{G})$ be a PFCG with two vertices r and s. Then it cannot be complete because there does not exists any PFD $\overrightarrow{G}$ having two vertices with a common victim. If possible, let a PFD with three vertices be complete. This graph is a order three clique. As there is an edge (r, s), then r and s has a common victim x. So the edges (r, x) and (s, x) must be in $\overrightarrow{G}$. Again since there is an edge (s, x) in $C(\overrightarrow{G})$, then s and x have a common victim and it must be r. Then the edges (r, x) and (x, r) must be in $\overrightarrow{G}$. This is impossible, as two directions of a single edge cannot be allowed in a PFD. Thus, for the PFCG $C(\overrightarrow{G})$ there is no valid PFD $\overrightarrow{G}$. Hence $C(\overrightarrow{G})$ is not complete. □

Theorem 3 *Let $\overrightarrow{G}$ be a complete PFD. If $h_\mu(\aleph^+(r) \cap \aleph^+(s)) = h_\eta(\aleph^+(r) \cap \aleph^+(s)) = h_\nu(\aleph^+(r) \cap \aleph^+(s)) = 1$, then PFCG $C(\overrightarrow{G})$ of $\overrightarrow{G}$ is strong.*

Proof Since $\overrightarrow{G}$ is complete PFD, then $\overrightarrow{\mu_B}(r, s) = \mu_A(r) \wedge \mu_A(s)$, $\overrightarrow{\eta_B}(r, s) = \eta_A(r) \wedge \eta_A(s)$ and $\overrightarrow{\nu_B}(r, s) = \nu_A(r) \vee \nu_A(s)$ $\forall r, s \in V$. If $h_\mu(\aleph^+(r) \cap \aleph^+(s)) = h_\eta(\aleph^+(r) \cap \aleph^+(s)) = h_\nu(\aleph^+(r) \cap \aleph^+(s)) = 1$, the DTMS, DAMS and DFMS of the edge (r, s) in $C(\overrightarrow{G})$ are respectively $\mu_B(r, s) = [\mu_A(r) \wedge \mu_A(s)]h_\mu(\aleph^+(r) \cap \aleph^+(s)) = \mu_A(r) \wedge \mu_A(s)$, $\eta_B(r, s) = [\eta_A(r) \wedge \eta_A(s)]h_\eta(\aleph^+(r) \cap \aleph^+(s)) = \eta_A(r) \wedge \eta_A(s)$ and $\nu_B(r, s) = [\nu_A(r) \vee \nu_A(s)]h_\nu(\aleph^+(r) \cap \aleph^+(s)) = \nu_A(r) \vee \nu_A(s)$.
This proves that $C(\overrightarrow{G})$ is strong. □

Theorem 4 *Let $\overrightarrow{G}$ be a PFD where all edges are strong. Then $\frac{\mu_B(r,s)}{(\mu_A(r) \wedge \mu_A(s))^2} > 0.5$, $\frac{\eta_B(r,s)}{(\eta_A(r) \wedge \eta_A(s))^2} < 0.5$ and $\frac{\nu_B(r,s)}{(\nu_A(r) \vee \nu_A(s))^2} < 0.5$.*

Proof Let all edges of $\overrightarrow{G}$ be strong. Then $\frac{1}{2}[\mu_A(r) \wedge \mu_A(s)] < \overrightarrow{\mu_B}(r, s)$, $\frac{1}{2}[\eta_A(r) \wedge \eta_A(s)] > \overrightarrow{\eta_B}(r, s)$ and $\frac{1}{2}[\nu_A(r) \vee \nu_A(s)] > \overrightarrow{\nu_B}(r, s)$ $\forall (r, s) \in \overrightarrow{G}$.

Let $C(\overrightarrow{G})$ be the corresponding PFCG of $\overrightarrow{G}$. Case I: If $\aleph^+(r) \cap \aleph^+(s) = \emptyset$ $\forall r, s \in V$, then there is nothing to prove as there does not exists any edge between r, s in $C(\overrightarrow{G})$.
Case:II Let $\aleph^+(r) \cap \aleph^+(s) \neq \emptyset$. Let $\aleph^+(r) \cap \aleph^+(s) = \{(a_1, (\theta_1, \phi_1, \psi_1)), (a_2, (\theta_2, \phi_2, \psi_2)), \ldots, (a_k, (\theta_k, \phi_k, \psi_k))\}$, where θ_i, ϕ_i and ψ_i are the DTMS, DAMS and DFMS of either (r, a_i) or (s, a_i) for $i = 1, 2, \ldots, k$, respectively. So, $\theta_i = \overrightarrow{\mu_B}(r, a_i) \wedge \overrightarrow{\mu_B}(s, a_i)$, $\phi_i = \overrightarrow{\eta_B}(r, a_i) \wedge \overrightarrow{\eta_B}(s, a_i)$ and $\psi_i = \overrightarrow{\nu_B}(r, a_i) \vee \overrightarrow{\nu_B}(s, a_i)$, $i = 1, 2, \ldots, k$. Let $h_\mu(\aleph^+(r) \cap \aleph^+(s)) = \max\{\theta_i : i = 1, 2, \ldots, k\} = \theta_{\max}$, $h_\eta(\aleph^+(r) \cap \aleph^+(s)) = \max\{\phi_i : i = 1, 2, \ldots, k\} = \phi_{\max}$, $h_\nu(\aleph^+(r) \cap \aleph^+(s)) = \min\{\psi_i : i = 1, 2, \ldots, k\} = \psi_{\min}$. Then $\theta_{\max} > \overrightarrow{\mu_B}(r, s)$, $\phi_{\max} < \overrightarrow{\eta_B}(r, s)$ and $\psi_{\min} < \overrightarrow{\nu_B}(r, s)$ for all edges (r, s). So, $\frac{\theta_{\max}}{\mu_A(r) \wedge \mu_A(s)} > \frac{\overrightarrow{\mu_B}(r,s)}{\mu_A(r) \wedge \mu_A(s)} > 0.5$, $\frac{\phi_{\max}}{\eta_A(r) \wedge \eta_A(s)} < \frac{\overrightarrow{\eta_B}(r,s)}{\eta_A(r) \wedge \eta_A(s)} < 0.5$ and $\frac{\psi_{\min}}{\nu_A(r) \vee \nu_A(s)} < \frac{\overrightarrow{\nu_B}(r,s)}{\nu_A(r) \vee \nu_A(s)} < 0.5$. Therefore, $\mu_B(r, s) = [\mu_A(r) \wedge \mu_A(s)]h_\mu(\aleph^+(r) \cap \aleph^+(s)) = [\mu_A(r) \wedge \mu_A(s)] \times \theta_{\max}$, i.e., $\frac{\mu_B(r,s)}{(\mu_A(r) \wedge \mu_A(s))^2} = \frac{\theta_{\max}}{\mu_A(r) \wedge \mu_A(s)} > 0.5$. $\eta_B(r, s) = [\eta_A(r) \wedge \eta_A(s)]h_\eta(\aleph^+(r) \cap \aleph^+(s)) = [\eta_A(r) \wedge \eta_A(s)] \times \phi_{\max}$, i.e., $\frac{\eta_B(r,s)}{(\eta_A(r) \wedge \eta_A(s))^2} = \frac{\phi_{\max}}{\eta_A(r) \wedge \eta_A(s)} < 0.5$ and $\nu_B(r, s) = [\nu_A(r) \vee \nu_A(s)]h_\nu(\aleph^+(r) \cap \aleph^+(s)) = [\nu_A(r) \vee \nu_A(s)] \times \psi_{\min}$, i.e., $\frac{\nu_B(r,s)}{(\nu_A(r) \vee \nu_A(s))^2} = \frac{\psi_{\min}}{\nu_A(r) \vee \nu_A(s)} < 0.5$ $\forall (r, s) \in C(\overrightarrow{G})$. □

Theorem 5 *Let $\overrightarrow{G}$ be a PFD of an underlying undirected crisp graph having cliques $C_1, C_2, \ldots, C_r$ of order 3 with $C_1 \cup C_2 \cup \cdots \cup C_r = V$ and $|C_i \cap C_j| \leq 1$ $\forall i, j = 1, 2, \ldots, r$. Then corresponding PFCG of $\overrightarrow{G}$ can not have any cliques of order ≥ 3.*

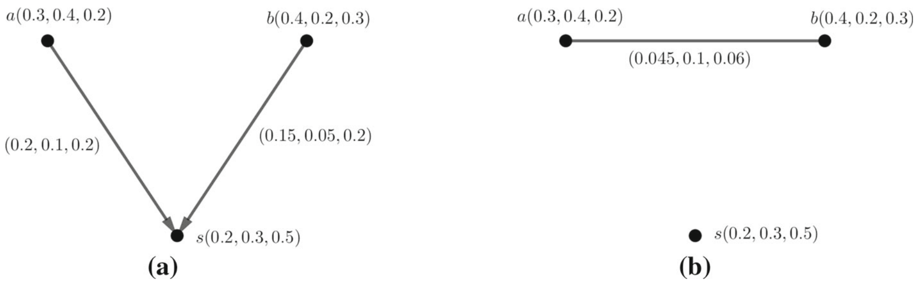


Fig. 5 Example of **a** PFD with strong edges and **b** its corresponding PFCG

Proof Since $C_1 \cup C_2 \cup \ldots \cup C_r = V$ and $|C_i \cap C_j| \leq 1 \; \forall i, j = 1, 2, \ldots, r$, therefore, only triangular orientations consists in PFD also, two triangular orientations have no common edge. In PFCG, each triangular orientation has a single edge and so the graph does not have any cliques of order ≥ 3. □

Theorem 6 *In PFCG of a complete PFD with n vertices has maximum n_{C_3} edges.*

Proof Let $\overrightarrow{G}$ be a complete PFD and $C(\overrightarrow{G})$ be the corresponding PFCG. Then each vertex is adjacent to few vertex in V. Thus $\overrightarrow{G}$ has maximum n_{C_3} orientations. Also for each triangular orientation of $\overrightarrow{G}$, there exists an edge in PFCG $C(\overrightarrow{G})$ and hence maximum number of edges in $C(\overrightarrow{G})$ is n_{C_3}. □

4 Some results on PFCG

In this section, we will study several properties of PFCGs.

Remark 1 If each edges of a PFD are strong, then each edges of the corresponding PFCG need not be strong.

Example 4 Let us consider a PFD $\overrightarrow{G}$ with two vertices $(a, (0.3, 0.4, 0.2))$ and $(b, (0.4, 0.2, 0.3))$ having a common victim $(s, (0.2, 0.3, 0.5))$ and edges are $((a, s), (0.2, 0.1, 0.2))$ and $((b, s), (0.15, 0.05, 0.2))$ (see Fig. 5a). Two edges (a, s) and (b, s) both are strong in $\overrightarrow{G}$. Here, $\aleph^+(a) = \{(s, (0.2, 0.1, 0.2))\}$ and $\aleph^+(b) = \{(s, (0.15, 0.05, 0.2))\}$. Then $\aleph^+(a) \cap \aleph^+(b) = \{(s, (0.15, 0.05, 0.2))\}$. So there is an edge (a, b) in PFCG $C(\overrightarrow{G})$ of $\overrightarrow{G}$ with DTMS, DAMS and DFMS are 0.045, 0.1 and 0.06, respectively. This is not a strong edge as strength of (a, b) is $(0.15, 0.05, 0.2)$ (see Fig. 5b).

Remark 2 Any PFCG $C(\overrightarrow{G})$ of a complete PFD need not be complete.

Example 5 Let us consider a complete PFD $\overrightarrow{G}$ (see Fig. 6a). Here $\aleph^+(r) = \{(s, (0.4, 0.2, 0.3)), (u, (0.4, 0.1, 0.2))\}$, $\aleph^+(s) = \{(v, (0.3, 0.2, 0.3))\}$, $\aleph^+(u) = \{(s, (0.5, 0.1, 0.3)), (v, (0.3, 0.1, 0.3))\}$, $\aleph^+(v) = \{(r, (0.3, 0.3, 0.3))\}$. Also, $\aleph^+(r) \cap \aleph^+(u) = \{(s, (0.4, 0.1, 0.3))\}$ and $\aleph^+(s) \cap \aleph^+(u) = \{(v, (0.3, 0.1, 0.3))\}$.
Therefore, two edges (r, u) and (s, u) exists in $C(\overrightarrow{G})$ of $\overrightarrow{G}$ with DTMS, DAMS and DFMS are $(0.16, 0.01, 0.06)$ and $(0.15, 0.01, 0.09)$, respectively (see Fig. 6b). This graph is not complete as $0.16 \neq \min\{0.4, 0.6\}$, $0.01 \neq \min\{0.3, 0.1\}$ and $0.06 \neq \max\{0.2, 0.2\}$.

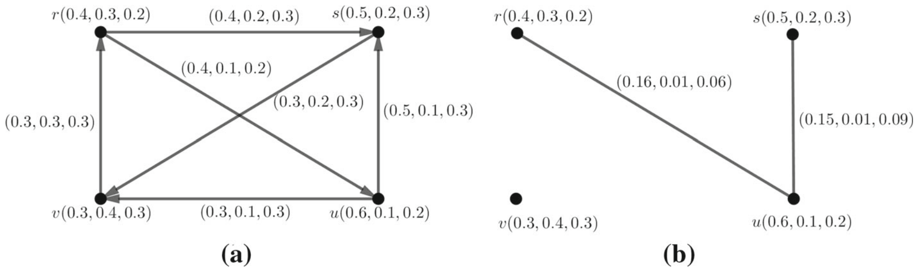
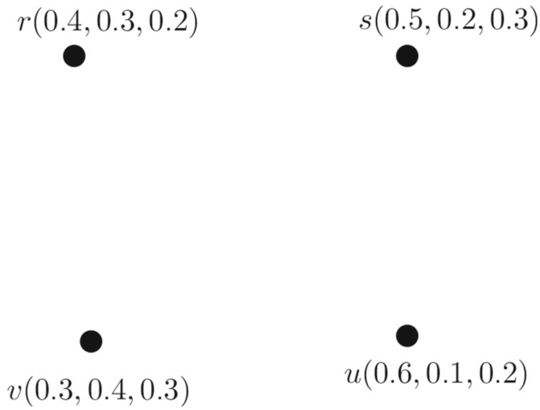


Fig. 6 Example of **a** complete PFD and **b** its corresponding PFCG

Fig. 7 Complement of the complete PFD of Fig. 6a



Remark 3 The complement of a complete PFD $\overrightarrow{G}$ may not have any PFCG $C(\overrightarrow{G})$.

Example 6 Consider the complete PFD $\overrightarrow{G} = (V, A, \overrightarrow{B})$ (see Fig. 6a) and $\overrightarrow{G}^c = (V, A, \overrightarrow{B^c})$ be the complement of $\overrightarrow{G}$. Then the DTMS, DAMS and DAMS of any edge (r, s) are, respectively, $\overrightarrow{\mu^c_B}(r, s) = (\mu_A(r) \wedge \mu_A(s)) - \overrightarrow{\mu_B}(r, s) = 0$, $\overrightarrow{\eta^c_B}(r, s) = (\eta_A(r) \wedge \eta_A(s)) - \overrightarrow{\eta_B}(r, s) = 0$ and $\overrightarrow{\nu^c_B}(r, s) = (\nu_A(r) \vee \nu_A(s)) - \overrightarrow{\nu_B}(r, s) = 0$. So, there does not exists any edge in $\overrightarrow{G}^c$ (see Fig. 7). Hence, $\overrightarrow{G}^c$ do not have any PFCG.

Remark 4 The PFCG $C(\overrightarrow{G})$ of a regular PFD $\overrightarrow{G}$ need not be regular.

Example 7 Consider a regular PFD $\overrightarrow{G}$ (see Fig. 8a). Here $\aleph^+(r) = \{(s, (0.25, 0.2, 0.1)), (v, (0.25, 0.1, 0.1))\}$, $\aleph^+(s) = \{(w, (0.5, 0.3, 0.2))\}$, $\aleph^+(u) = \{(s, (0.25, 0.1, 0.1)), (v, (0.25, 0.2, 0.1))\}$ and $\aleph^+(v) = \{(s, (0.5, 0.3, 0.2))\}$. Also, $\aleph^+(r) \cap \aleph^+(u) = \{(s, (0.25, 0.1, 0.1)), (v, (0.25, 0.1, 0.1))\}$, $\aleph^+(r) \cap \aleph^+(v) = \{(s, (0.25, 0.2, 0.2))\}$ and $\aleph^+(u) \cap \aleph^+(v) = \{(s, (0.25, 0.1, 0.2))\}$. Therefore, three edges (r, u), (r, v) and (u, v) exists in PFCG $C(\overrightarrow{G})$ of $\overrightarrow{G}$ with DTMS, DAMS and DFMS are $(0.075, 0.02, 0.04)$, $(0.075, 0.06, 0.04)$ and $(0.075, 0.02, 0.08)$, respectively. This graph is not regular as $deg(r) = (0.15, 0.08, 0.08) \neq deg(u) = (0.15, 0.04, 0.12) \neq deg(v) = (0.15, 0.08, 0.12)$ (see Fig. 8b).

Remark 5 The PFCG $C(\overrightarrow{G})$ of a irregular PFD $\overrightarrow{G}$ need not be irregular.

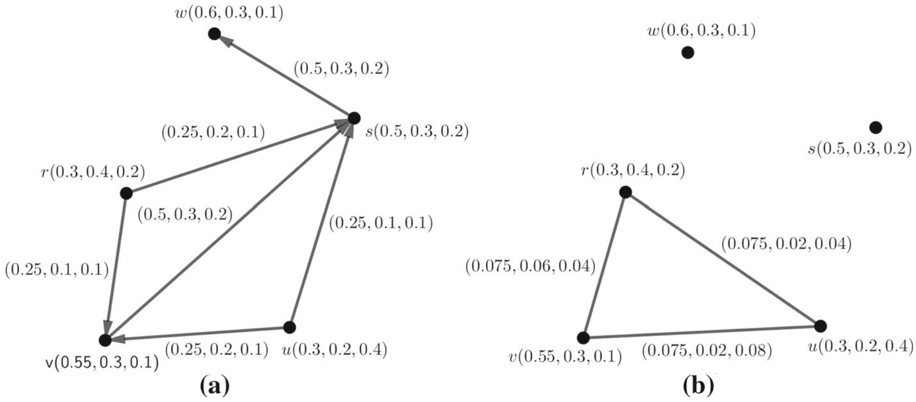


Fig. 8 Example of **a** regular PFD and **b** its corresponding PFCG

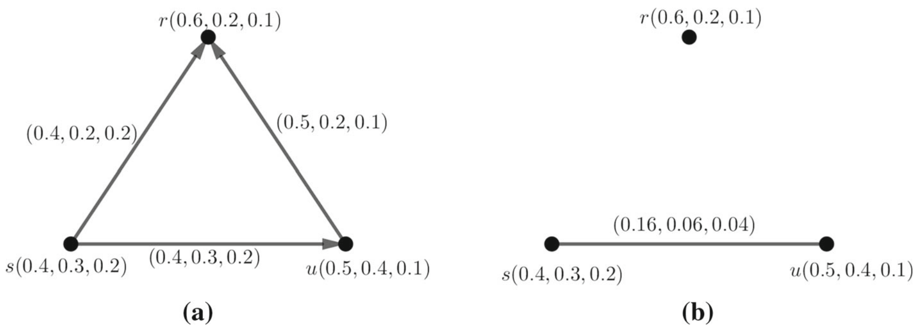


Fig. 9 Example of **a** irregular PFD and **b** its corresponding PFCG

Example 8 Consider a irregular PFD $\overrightarrow{G}$ (see Fig. 9a). Here $\aleph^+(s) = \{(r, (0.4, 0.2, 0.2)), (u, (0.4, 0.3, 0.2))\}$, $\aleph^+(u) = \{(r, (0.5, 0.2, 0.1))\}$. Also, $\aleph^+(s) \cap \aleph^+(u) = \{(r, (0.4, 0.2, 0.2))\}$. Therefore, (s, u) is the only edge in PFCG $C(\overrightarrow{G})$ with DTMS, DAMS and DFMS are (0.16, 0.06, 0.04). This graph is not irregular as degree of two adjacent vertices s and u in $C(\overrightarrow{G})$ are same (see Fig. 9b).

Remark 6 The PFCG $C(\overrightarrow{G})$ of a strong PFD $\overrightarrow{G}$ need not be strong.

Example 9 Consider a strong PFD $\overrightarrow{G}$ of Fig. 10a. Here $\aleph^+(r) = \{(s, (0.4, 0.1, 0.2)), (u, (0.3, 0.3, 0.2))\}$, $\aleph^+(u) = \{(s, (0.3, 0.1, 0.2))\}$ and $\aleph^+(v) = \{(r, (0.4, 0.2, 0.2)), (u, (0.3, 0.2, 0.2))\}$. Also, $\aleph^+(r) \cap \aleph^+(u) = \{(s, (0.3, 0.1, 0.2))\}$ and $\aleph^+(r) \cap \aleph^+(v) = \{(u, (0.3, 0.2, 0.2))\}$. Therefore, (r, u) and (r, v) be two edges in PFCG $C(\overrightarrow{G})$ with DTMS, DAMS and DFMS are (0.09, 0.03, 0.04) and (0.12, 0.04, 0.04), respectively as shown in Fig. 10b. This graph is not strong as $0.12 \neq \min\{0.6, 0.4\}$, $0.04 \neq \min\{0.2, 0.3\}$ and $0.04 \neq \max\{0.2, 0.1\}$, etc.

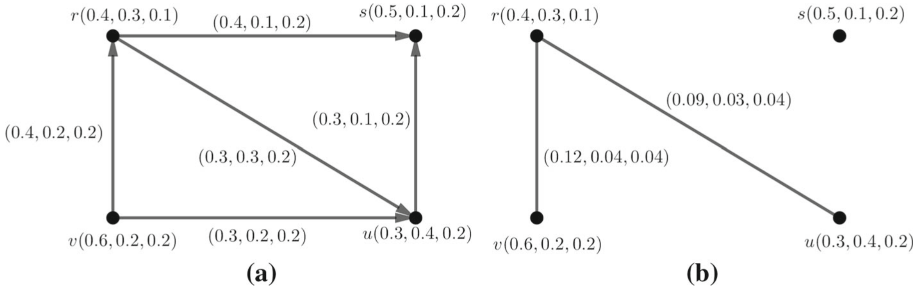


Fig. 10 Example of **a** strong PFD and **b** its PFCG

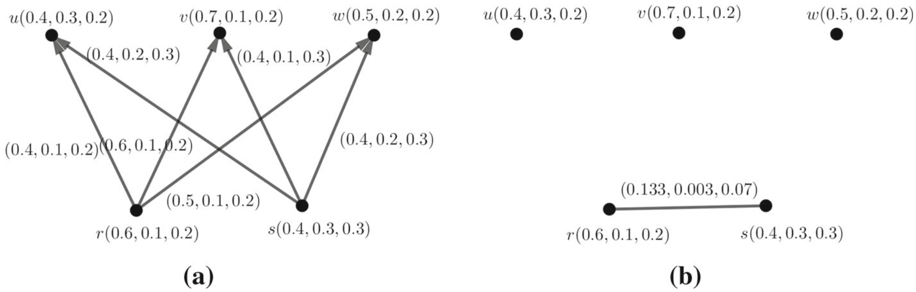


Fig. 11 Example of **a** PFD and **b** its PF0.2-CG

5 Picture fuzzy *k*-competition graphs (PF*k*CG)

In this section, we extends one of the subclasses of PFCG known as PFkCG which is an intersection graph of PFON of vertices in a PFD. The non-negative real number k in PFkCGs measures the strength of competitions of the corresponding PFCGs.

Definition 19 Let $k \geq 0$ be a real number. The PFkCG $C_k(\overrightarrow{G})$ of a PFD $\overrightarrow{G}$ is an undirected PFG G with V as vertex set and having an edge (r, s) in $C_k(\overrightarrow{G})$ iff $|\aleph^+(r) \cap \aleph^+(s)|_\mu > k$, $|\aleph^+(r) \cap \aleph^+(s)|_\eta > k$ and $|\aleph^+(r) \cap \aleph^+(s)|_\nu > k$. Then the DTMS, DAMS and DFMS of (r, s) are, respectively, $\mu_B(r, s) = \frac{k_1 - k}{k_1}[\mu_A(r) \wedge \mu_A(s)]h_\mu(\aleph^+(r) \cap \aleph^+(s))$, $\eta_B(r, s) = \frac{k_2 - k}{k_2}[\eta_A(r) \wedge \eta_A(s)]h_\eta(\aleph^+(r) \cap \aleph^+(s))$ and $\nu_B(r, s) = \frac{k_3 - k}{k_3}[\nu_A(r) \vee \nu_A(s)]h_\nu(\aleph^+(r) \cap \aleph^+(s))$, where $k_1 = |\aleph^+(r) \cap \aleph^+(s)|_\mu$, $k_2 = |\aleph^+(r) \cap \aleph^+(s)|_\eta$ and $k_3 = |\aleph^+(r) \cap \aleph^+(s)|_\nu$. A PF$k$CG is simply a PFCG if $k = 0$.

The following example illustrates picture fuzzy 0.2-competition graphs.

Example 10 Consider a PFD $\overrightarrow{G}$ as given in Fig. 11a. From this digraph, we have $\aleph^+(r) = \{(u, (0.4, 0.1, 0.2)), (v, (0.6, 0.1, 0.2)), (w, (0.5, 0.1, 0.2))\}$ and $\aleph^+(s) = \{(u, (0.4, 0.2, 0.3)), (v, (0.4, 0.1, 0.3)), (w, (0.4, 0.2, 0.3))\}$. Therefore, $\aleph^+(r) \cap \aleph^+(s) = \{(u, (0.4, 0.1, 0.3)), (v, (0.4, 0.1, 0.3)), (w, (0.4, 0.1, 0.3))\}$. So, $k_1 = 1.2$, $k_2 = 0.3$ and $k_3 = 0.9$. If we choose $k = 0.2$, then the DTMS, DAMS and DFMS of the edge $(r, s) \in C_k(\overrightarrow{G})$ are, respectively, $\mu_B(r, s) = 0.133$, $\eta_B(r, s) = 0.003$ and $\mu_B(r, s) = 0.07$. The corresponding picture fuzzy 0.2-competition graphs $C_{0.2}(\overrightarrow{G})$ is shown in Fig. 11b.

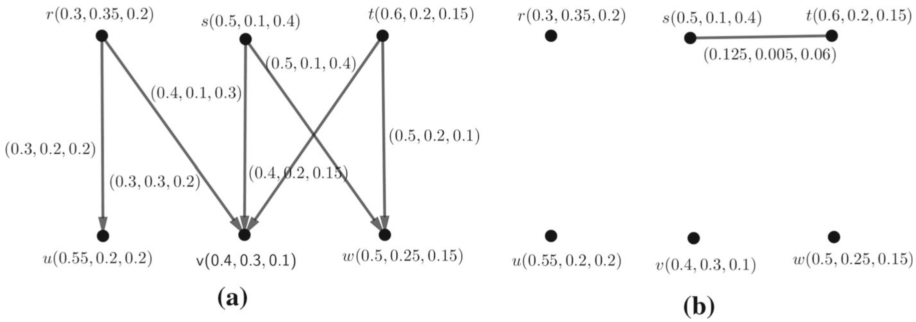


Fig. 12 Example of **a** PFD and **b** its 2-CPFG

Theorem 7 *Let $\overrightarrow{G}$ be a PFD. If $h_\mu(\aleph^+(r) \cap \aleph^+(s)) = 1, h_\eta(\aleph^+(r) \cap \aleph^+(s)) = 1$ and $h_\nu(\aleph^+(r) \cap \aleph^+(s)) = 1$ and if $|\aleph^+(r) \cap \aleph^+(s)|_\mu > 2k$, $|\aleph^+(r) \cap \aleph^+(s)|_\eta < 2k$, $|\aleph^+(r) \cap \aleph^+(s)|_\nu < 2k$, then (r, s) is independent strong edge in $C_k(\overrightarrow{G})$.*

Proof Let $\overrightarrow{G} = (V, A, \overrightarrow{B})$ be a PFD and $C_k(\overrightarrow{G}) = (V, A, B)$ be its corresponding PFkCG. If $h_\mu(\aleph^+(r) \cap \aleph^+(s)) = 1$ and $|\aleph^+(r) \cap \aleph^+(s)|_\mu > 2k$, then $k_1 > 2k$. Therefore, $\mu_B(r, s) = \frac{k_1-k}{k_1}[\mu_A(r) \wedge \mu_A(s)]h_\mu(\aleph^+(r) \cap \aleph^+(s))$ or, $\frac{\mu_B(r,s)}{\mu_A(r)\wedge\mu_A(s)} = \frac{k_1-k}{k_1} > 0.5$. Similarly, $\frac{\eta_B(r,s)}{\eta_A(r)\wedge\eta_A(s)} = \frac{k_2-k}{k_2} < 0.5$ and $\frac{\nu_B(r,s)}{\nu_A(r)\vee\nu_A(s)} = \frac{k_3-k}{k_3} < 0.5$. Then (r, s) is independent strong edge in $C_k(\overrightarrow{G})$. □

6 *p*-Competition picture fuzzy graphs (*p*-CPFG)

Now we discuss other subclass of PFCGs known as p-CPFG which is also, an intersection graph of supports of PFON of vertices in a PFD. The integer $p > 0$ in p-CPFGs measures the strength of competitions of the corresponding PFCGs.

Definition 20 Let $p > 0$ be an integer. Then p-CPFG $C^p(\overrightarrow{G})$ of a PFD $\overrightarrow{G}$ is an undirected PFG G with V as vertex set and having an edge (r, s) in $C^p(\overrightarrow{G})$ iff $|supp(\aleph^+(r) \cap \aleph^+(s))| \geq p$. Then the DTMS, DAMS and DFMS of $(r, s) \in C^p(\overrightarrow{G})$ are, respectively, $\mu_B(r, s) = \frac{(n-p)+1}{n}[\mu_A(r) \wedge \mu_A(s)]h_\mu(\aleph^+(r) \cap \aleph^+(s)), \eta_B(r, s) = \frac{(n-p)+1}{n}[\eta_A(r) \wedge \eta_A(s)]h_\eta(\aleph^+(r) \cap \aleph^+(s))$ and $\nu_B(r, s) = \frac{(n-p)+1}{n}[\nu_A(r) \vee \nu_A(s)]h_\nu(\aleph^+(r) \cap \aleph^+(s))$, where $n = |supp(\aleph^+(r) \cap \aleph^+(s)|$.

Example 11 Let us consider a PFD $\overrightarrow{G}$ as given in Fig. 12a. Here $\aleph^+(r) = \{(u, (0.3, 0.2, 0.2)), (v, (0.3, 0.3, 0.2))\}$, $\aleph^+(s) = \{(v, (0.4, 0.1, 0.3)), (w, (0.5, 0.1, 0.4))\}$ and $\aleph^+(t) = \{(v, (0.4, 0.2, 0.15)), (w, (0.5, 0.2, 0.1))\}$. Therefore, $\aleph^+(r) \cap \aleph^+(s) = \{(v, (0.3, 0.1, 0.3))\}$, $\aleph^+(r) \cap \aleph^+(t) = \{(v, (0.3, 0.2, 0.2))\}$ and $\aleph^+(s) \cap \aleph^+(t) = \{(v, (0.4, 0.1, 0.3)), (w, (0.5, 0.1, 0.4))\}$. In addition, $n = |supp(\aleph^+(s) \cap \aleph^+(t)| = 2$. If we choose $p = 2$, then the DTMS, DAMS and DFMS of the edge $(s, t) \in C^2(\overrightarrow{G})$ are respectively $\mu_B(s, t) = 0.125$, $\eta_B(s, t) = 0.005$ and $\nu_B(s, t) = 0.06$. The corresponding 2-CPFG $C^2(\overrightarrow{G})$ is shown in Fig. 12b.

Table 2 PFON and PFCN

$x \in V$	$\aleph(x)$	$\aleph[x]$
r	{(s,(0.4,0.2,0.2)),(v,(0.4,0.3,0.2))}	{(r,(0.4,0.3,0.2)),(s,(0.4,0.2,0.2)),(v,(0.4,0.3,0.2))}
s	{(r,(0.4,0.2,0.2)),(u,(0.5,0.1,0.3))}	{(r,(0.4,0.2,0.2)),(s,(0.6,0.2,0.2)),(u,(0.5,0.1,0.3))}
u	{(s,(0.5,0.1,0.3)),(v,(0.4,0.1,0.4))}	{(s,(0.5,0.1,0.3)),(u,(0.5,0.1,0.4)),(v,(0.4,0.1,0.4))}
v	{(r,(0.4,0.3,0.2)),(u,(0.4,0.1,0.4))}	{(r,(0.4,0.3,0.2)),(u,(0.4,0.1,0.4)),(v,(0.45,0.3,0.2))}

Theorem 8 *Let $\overrightarrow{G} = (V, A, \overrightarrow{B})$ be a PFD. If $h_\mu(\aleph^+(r)\cap\aleph^+(s)) = 1$, $h_\eta(\aleph^+(r)\cap\aleph^+(s)) = 0$ and $h_\nu(\aleph^+(r)\cap\aleph^+(s)) = 0$ in $C^{[\frac{n}{2}]}(\overrightarrow{G})$, then (r, s) is independent strong edge, where $n = |supp(\aleph^+(r)\cap\aleph^+(s))|$.*

Proof Let $\overrightarrow{G}$ be a PFD and $G = (V, A, B)$ be its corresponding $[\frac{n}{2}]$-CPFG, where $n = |supp(\aleph^+(r)\cap\aleph^+(s))|$. If $h_\mu(\aleph^+(r)\cap\aleph^+(s)) = 1$, $h_\eta(\aleph^+(r)\cap\aleph^+(s)) = 0$ and $h_\nu(\aleph^+(r)\cap\aleph^+(s)) = 0$. Then, $\mu_B(r, s) = \frac{n-[\frac{n}{2}]+1}{n}[\mu_A(r)\wedge\mu_A(s)]h_\mu(\aleph^+(r)\cap\aleph^+(s))$ or, $\frac{\mu_B(r,s)}{\mu_A(r)\wedge\mu_A(s)} = \frac{n-[\frac{n}{2}]+1}{n} > 0.5$. Similarly, $\frac{\eta_B(r,s)}{\eta_A(r)\wedge\eta_A(s)} = 0 < 0.5$ and $\frac{\nu_B(r,s)}{\nu_A(r)\vee\nu_A(s)} = 0 < 0.5$. This shows that (r, s) is independent strong edge. □

7 Picture fuzzy neighborhood graphs (PFNGs)

The picture fuzzy open neighborhood (PFON) and picture fuzzy closed neighborhood (PFCN) of a vertex in the PFG are defined below.

Definition 21 The PFON of a vertex r of a PFG $G = (V, A, B)$ is $\aleph(r) = (X_r, (\mu_r, \eta_r, \nu_r))$, where $X_r = \{s : \mu_B(r, s) > 0, \eta_B(r, s) > 0$ and $\nu_B(r, s) > 0\}$, $\mu_r, \eta_r, \nu_r : X_r \to [0, 1]$ are defined as $\mu_r(s) = \mu_B(r, s)$, $\eta_r(s) = \eta_B(r, s)$ and $\nu_r(s) = \nu_B(r, s)$. The PFCN of a vertex r is $\aleph[r] = \aleph(r)\cup\{(r, (\mu(r), \eta(r), \nu(r)))\}$.

Now, we define picture fuzzy open neighborhood graphs (PFONG) and picture fuzzy closed neighborhood graphs (PFCNG).

Definition 22 Let $G = (V, A, B)$ be a PFG. The PFONG of G is $\aleph(G) = (V, A, B')$ with V as vertex set and having an edge (r, s) in $\aleph(G)$ iff $\aleph(r)\cap\aleph(s) \neq \emptyset$ in G. The DTMS, DAMS and DFMS of (r, s) in $\aleph(G)$ are, respectively, $\mu'_B(r, s) = [\mu_A(r)\wedge\mu_A(s)]h_\mu(\aleph(r)\cap\aleph(s))$, $\eta'_B(r, s) = [\eta_A(r)\wedge\eta_A(s)]h_\eta(\aleph(r)\cap\aleph(s))$ and $\nu'_B(r, s) = [\nu_A(r)\vee\nu_A(s)]h_\nu(\aleph(r)\cap\aleph(s))$.

Definition 23 Let $G = (V, A, B)$ be a PFG. The PFCNG of G is $\aleph[G] = (V, A, B'')$ with V as vertex set and having an edge (r, s) in $\aleph[G]$ iff $\aleph[r]\cap\aleph[s] \neq \emptyset$ in G. The DTMS, DAMS and DFMS of (r, s) in $\aleph[G]$ are, respectively, $\mu''_B(r, s) = [\mu_A(r)\wedge\mu_A(s)]h_\mu(\aleph[r]\cap\aleph[s])$, $\eta''_B(r, s) = [\eta_A(r)\wedge\eta_A(s)]h_\eta(\aleph[r]\cap\aleph(s))$ and $\nu''_B(r, s) = [\nu_A(r)\vee\nu_A(s)]h_\nu(\aleph[r]\cap\aleph[s])$.

The following example illustrates PFONG and PFCNG (Table 2).

Example 12 Consider the PFG G of Fig. 13a.

Then $\aleph(r)\cap\aleph(u) = \{(s, (0.4, 0.1, 0.3)), (v, (0.4, 0.1, 0.4))\}$ and $\aleph(s)\cap\aleph(v) = \{(r, (0.4, 0.2, 0.2)), (u, (0.4, 0.1, 0.4))\}$. Therefore, there are edges (r, u) and (s, v) in $\aleph(G)$

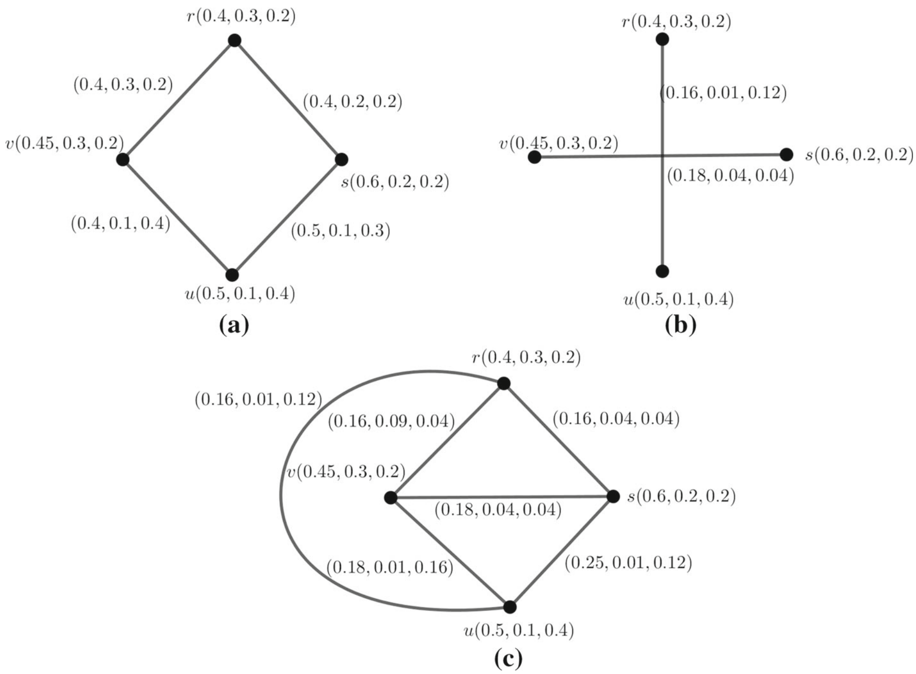


Fig. 13 Example of **a** PFG G, **b** PFONG $\aleph(G)$ and **c** PFCNG $\aleph[G]$

with DTMS, DAMS and DFMS are (0.16, 0.01, 0.12) and (0.18, 0.04, 0.04), respectively, shown in Fig. 13b. Also, $\aleph[r] \cap \aleph[s] = \{(r, (0.4, 0.2, 0.2)), (s, (0.4, 0.2, 0.2))\}$, $\aleph[r] \cap \aleph[u] = \{(s, (0.4, 0.1, 0.3)), (v, (0.4, 0.1, 0.4))\}$, $\aleph[r] \cap \aleph[v] = \{(r, (0.4, 0.3, 0.2)), (v, (0.4, 0.3, 0.2))\}$, $\aleph[s] \cap \aleph[u] = \{(s, (0.5, 0.1, 0.3)), (u, (0.5, 0.1, 0.4))\}$, $\aleph[s] \cap \aleph[v] = \{(r, (0.4, 0.2, 0.2)), (u, (0.4, 0.1, 0.4))\}$, $\aleph[u] \cap \aleph[v] = \{(u, (0.4, 0.1, 0.4)), (v, (0.4, 0.1, 0.4))\}$. Therefore, there are edges (r, s), (r, u), (r, v), (s, u), (s, v) and (u, v) in $\aleph[G]$ with DTMS, DAMS and DFMS are (0.16, 0.04, 0.04), (0.16, 0.01, 0.12), (0.16, 0.09, 0.04), (0.25, 0.01, 0.12), (0.18, 0.04, 0.04) and (0.18, 0.01, 0.16), respectively (see Fig. 13c).

Theorem 9 *For every edge of a PFG G, there exists an edge in PFCNG* $\aleph[G]$.

Proof Let (r, s) is an edge of a PFG $G = (V, A, B)$ and $\aleph[G] = (V, A, B')$ be the corresponding PFCNG of G.
Let $r, s \in \aleph[r]$ and $r, s \in \aleph[s]$, then obviously $r, s \in \aleph[r] \cap \aleph[s]$.
Hence, $h_\mu(\aleph[r] \cap \aleph[s]) \neq 0$, $h_\eta(\aleph[r] \cap \aleph(s)) \neq 0$ and $h_\nu(\aleph[r] \cap \aleph[s]) \neq 0$.
So, $\mu''_B(r, s) = [\mu_A(r) \wedge \mu_A(s)] h_\mu(\aleph[r] \cap \aleph[s]) \neq 0$, $\eta''_B(r, s) = [\eta_A(r) \wedge \eta_A(s)] h_\eta(\aleph[r] \cap \aleph(s)) \neq 0$ and $\nu''_B(r, s) = [\nu_A(r) \vee \nu_A(s)] h_\nu(\aleph[r] \cap \aleph[s]) \neq 0$.
Therefore, For every edge of a PFG G, there exists an edge in PFCNG $\aleph[G]$. □

Based on these PFNGs, open picture fuzzy k-neighborhood graph (OPFkNG) and closed picture fuzzy k-neighborhood graph (CPFkNG) are defined below.

Definition 24 Let $G = (V, A, B)$ be a PFG and $k \geq 0$ be a real number. The OPFkNG of G is $\aleph_k(G) = (V, A, B')$, with V as vertex set and having an edge (r, s) in $\aleph_k(G)$ iff $|\aleph(r) \cap \aleph(s)|_\mu > k$, $|\aleph(r) \cap \aleph(s)|_\eta > k$ and $|\aleph(r) \cap \aleph(s)|_\nu > k$ in G. Then the DTMS, DAMS

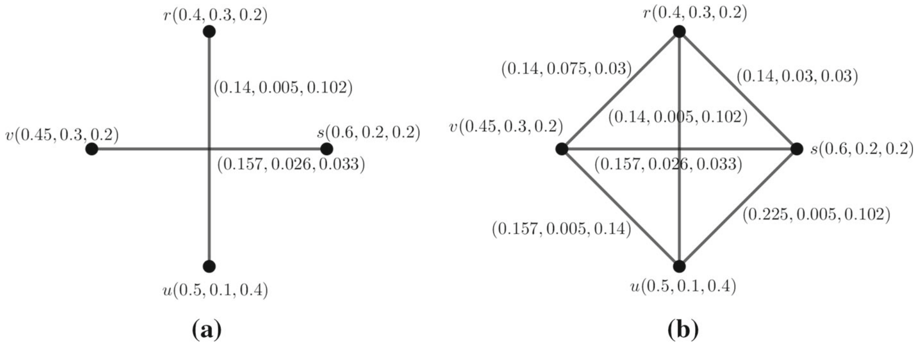


Fig. 14 Example of picture fuzzy **a** open and **b** closed 0.1-neighborhood graph

and DFMS of $(r, s) \in \aleph_k(G)$ are, respectively, $\mu_{B'}(r, s) = \frac{k_1'-k}{k_1'}[\mu_A(r) \wedge \mu_A(s)]h_\mu(\aleph(r) \cap \aleph(s))$, $\eta_{B'}(r, s) = \frac{k_2'-k}{k_2'}[\eta_A(r) \wedge \eta_A(s)]h_\eta(\aleph(r) \cap \aleph(s))$ and $\nu_{B'}(r, s) = \frac{k_3'-k}{k_3'}[\nu_A(r) \vee \nu_A(s)]h_\nu(\aleph(r) \cap \aleph(s))$, where $k_1 = |\aleph(r) \cap \aleph(s)|_\mu$, $k_2 = |\aleph(r) \cap \aleph(s)|_\eta$ and $k_3 = |\aleph(r) \cap \aleph(s)|_\nu$.

Definition 25 Let G be a PFG and $k \geq 0$ be a real number. The CPFkNG of G is $\aleph_k[G] = (V, A, B'')$, with V as vertex set and having an edge (r, s) in $\aleph_k[G]$ iff $|\aleph[r] \cap \aleph[s]|_\mu > k$, $|\aleph[r] \cap \aleph[s]|_\eta > k$ and $|\aleph[r] \cap \aleph[s]|_\nu > k$ in G. Then DTMS, DAMS and DFMS of $(r, s) \in \aleph_k[G]$ are, respectively, $\mu_{B''}(r, s) = \frac{k_1''-k}{k_1''}[\mu_A(r) \wedge \mu_A(s)]h_\mu(\aleph[r] \cap \aleph[s])$, $\eta_{B''}(r, s) = \frac{k_2''-k}{k_2''}[\eta_A(r) \wedge \eta_A(s)]h_\eta(\aleph[r] \cap \aleph[s])$ and $\nu_{B''}(r, s) = \frac{k_3''-k}{k_3''}[\nu_A(r) \vee \nu_A(s)]h_\nu(\aleph[r] \cap \aleph[s])$, where $k_1 = |\aleph[r] \cap \aleph[s]|_\mu$, $k_2 = |\aleph[r] \cap \aleph[s]|_\eta$ and $k_3 = |\aleph[r] \cap \aleph[s]|_\nu$.

The following example illustrates open picture fuzzy 0.1-neighborhood graph and closed picture fuzzy 0.1-neighborhood graph.

Example 13 Consider the PFG $G = (V, A, B)$ as given in Fig. 13a. The DTMS, DAMS and DFMS of the edges (r, u) and (s, v) in $\aleph_{0.1}(G)$ are, respectively, (0.14, 0.005, 0.102) and (0.157, 0.026, 0.033) (see Fig.14a). Also, the DTMS, DAMS and DFMS of the edges (r, s), (r, u), (r, v), (s, u), (s, v) and (u, v) in $\aleph_{0.1}[G]$ are, respectively, (0.14, 0.03, 0.03), (0.14, 0.005, 0.102), (0.14, 0.075, 0.03), (0.225, 0.005, 0.102), (0.157, 0.026, 0.033) and (0.157, 0.005, 0.14) (see Fig. 14b).

Definition 26 Let $\overrightarrow{G} = (V, A, \overrightarrow{B})$ be a PFD. The underlying PFG of $\overrightarrow{G}$ is $U(G) = (V, A, B)$, where $\mu_B(r, s) = \min\{\overrightarrow{\mu_B}(r, s), \overrightarrow{\mu_B}(s, r)\}$, $\eta_B(r, s) = \min\{\overrightarrow{\eta_B}(r, s), \overrightarrow{\eta_B}(s, r)\}$ and $\nu_B(r, s) = \max\{\overrightarrow{\nu_B}(r, s), \overrightarrow{\nu_B}(s, r)\}\ \forall\, r, s \in V$.

Now we establish a relation between PFkCG and OPFkNG.

Theorem 10 *If $\overrightarrow{G}$ is symmetric and loop less PFD, then $C_k(\overrightarrow{G}) = \aleph_k(\mathrm{U}(G))$, where $\mathrm{U}(G)$ is the underlying PFG of G.*

Proof Let $\overrightarrow{G} = (V, A, \overrightarrow{B})$ be a PFD and the corresponding underlying PFG $U(G) = (V, A, B)$.

Let $C_k(\overrightarrow{G}) = (V, A, B')$ and $\aleph_k(U(G)) = (V, A, B'')$. The vertex sets of $C_k(\overrightarrow{G})$ and $\aleph_k(U(G))$ are same with the vertex set of $\overrightarrow{G}$. We have to prove that $\mu_{B'}(r,s) = \mu_{B''}(r,s)$, $\eta_{B'}(r,s) = \eta_{B''}(r,s)$, $\nu_{B'}(r,s) = \nu_{B''}(r,s)$, $\forall r, s \in V$.
Case I: If $\mu_{B'}(r,s) = 0$, $\eta_{B'}(r,s) = 0$, $\nu_{B'}(r,s) = 0$ in $C_k(\overrightarrow{G})$, then there is no edge in $C_k(\overrightarrow{G})$. Also, $|\aleph^+(r) \cap \aleph^+(s)|_\mu \le k$, $|\aleph^+(r) \cap \aleph^+(s)|_\eta \le k$, $|\aleph^+(r) \cap \aleph^+(s)|_\nu \le k$ in $U(G)$. Since $\overrightarrow{G}$ is symmetric, $|\aleph(r) \cap \aleph(s)|_\mu \le k$, $|\aleph(r) \cap \aleph(s)|_\eta \le k$, $|\aleph(r) \cap \aleph(s)|_\nu \le k$ in $U(G)$. Hence, $\mu_{B''}(r,s) = 0$, $\eta_{B''}(r,s) = 0$, $\nu_{B''}(r,s) = 0$ in $\aleph_k(U(G))$.
Case II: If $|\aleph^+(r) \cap \aleph^+(s)|_\mu > k$, $|\aleph^+(r) \cap \aleph^+(s)|_\eta > k$, $|\aleph^+(r) \cap \aleph^+(s)|_\nu > k$ in $U(G)$. Then $\mu_{B'}(r,s) > 0$, $\eta_{B'}(r,s) > 0$, $\nu_{B'}(r,s) > 0$ in $C_k(\overrightarrow{G})$ and there is an edge between r and s in $C_k(\overrightarrow{G})$ with DTMS, DAMS and DFMS are, respectively, $\mu_{B'}(r,s) = \frac{k'-k}{k'}[\mu_A(r) \wedge \mu_A(s)]h_\mu(\aleph^+(r) \cap \aleph^+(s))$, $\eta_{B'}(r,s) = \frac{k'-k}{k'}[\eta_A(r) \wedge \eta_A(s)]h_\eta(\aleph^+(r) \cap \aleph^+(s))$ and $\nu_{B'}(r,s) = \frac{k'-k}{k'}[\nu_A(r) \vee \nu_A(s)]h_\nu(\aleph^+(r) \cap \aleph^+(s))$, where $k' = |\aleph^+(r) \cap \aleph^+(s)|$.
Since $\overrightarrow{G}$ is symmetric, $|\aleph(r) \cap \aleph(s)|_\mu > k$, $|\aleph(r) \cap \aleph(s)|_\eta > k$, $|\aleph(r) \cap \aleph(s)|_\nu > k$ in $U(G)$. So, $\mu_{B''}(r,s) = \frac{k''-k}{k''}[\mu_A(r) \wedge \mu_A(s)]h_\mu(\aleph(r) \cap \aleph(s))$, $\eta_{B''}(r,s) = \frac{k''-k}{k''}[\eta_A(r) \wedge \eta_A(s)]h_\eta(\aleph(r) \cap \aleph(s))$ and $\nu_{B''}(r,s) = \frac{k''-k}{k''}[\nu_A(r) \vee \nu_A(s)]h_\nu(\aleph(r) \cap \aleph(s))$, where $k'' = |\aleph(r) \cap \aleph(s)|$.
Here, $k' = k''$ as $\overrightarrow{G}$ is symmetric PFD.
Thus, $\mu_{B'}(r,s) = \mu_{B''}(r,s)$, $\eta_{B'}(r,s) = \eta_{B''}(r,s)$ and $\nu_{B'}(r,s) = \nu_{B''}(r,s)$ $\forall r, s \in V$. □

In similar way, we can establish a relation between PFkCG and CPFkNG.

Theorem 11 *If $\overrightarrow{G}$ be a symmetric PFD having loop at each vertex, then $C_k(\overrightarrow{G}) = \aleph_k[\mathrm{U}(G)]$, where $\mathrm{U}(G)$ is the loop less underlying PFG of G.*

Proof Let $U(G) = (V, A, B)$ be an underlying loop less PFG corresponding to a PFD $\overrightarrow{G} = (V, A, \overrightarrow{B})$. Let $C_k(\overrightarrow{G}) = (V, A, B')$ and $\aleph_k[U(G)] = (V, A, B'')$. The vertex sets of $C_k(\overrightarrow{G})$ and $\aleph_k[U(G)]$ are same with the vertex set of $\overrightarrow{G}$. We have to prove that $\mu_{B'}(r,s) = \mu_{B''}(r,s)$, $\eta_{B'}(r,s) = \eta_{B''}(r,s)$, $\nu_{B'}(r,s) = \nu_{B''}(r,s)$, $\forall r, s \in V$.
Since $\overrightarrow{G}$ has a loop at each vertex, the PFON of every vertex contains the vertex itself.
Case I: If $\mu_{B'}(r,s) = 0$, $\eta_{B'}(r,s) = 0$, $\nu_{B'}(r,s) = 0$ in $C_k(\overrightarrow{G})$, then there is no edge between r and s in $C_k(\overrightarrow{G})$. Also, $|\aleph^+(r) \cap \aleph^+(s)|_\mu \le k$, $|\aleph^+(r) \cap \aleph^+(s)|_\eta \le k$, $|\aleph^+(r) \cap \aleph^+(s)|_\nu \le k$ in $U(G)$. Since $\overrightarrow{G}$ is symmetric, $|\aleph[r] \cap \aleph[s]|_\mu \le k$, $|\aleph[r] \cap \aleph[s]|_\eta \le k$, $|\aleph[r] \cap \aleph[s]|_\nu \le k$ in $U(G)$. Hence, $\mu_{B''}(r,s) = 0$, $\eta_{B''}(r,s) = 0$, $\nu_{B''}(r,s) = 0$ in $\aleph_k[U(G)]$.
Case II: If $|\aleph^+(r) \cap \aleph^+(s)|_\mu > k$, $|\aleph^+(r) \cap \aleph^+(s)|_\eta > k$, $|\aleph^+(r) \cap \aleph^+(s)|_\nu > k$ in $U(G)$. Then $\mu_{B'}(r,s) > 0$, $\eta_{B'}(r,s) > 0$, $\nu_{B'}(r,s) > 0$ in $C_k(\overrightarrow{G})$ and there is an edge (r,s) in $C_k(\overrightarrow{G})$ with DTMS, DAMS and DFMS are, respectively, $\mu_{B'}(r,s) = \frac{k'-k}{k'}[\mu_A(r) \wedge \mu_A(s)]h_\mu(\aleph^+(r) \cap \aleph^+(s))$, $\eta_{B'}(r,s) = \frac{k'-k}{k'}[\eta_A(r) \wedge \eta_A(s)]h_\eta(\aleph^+(r) \cap \aleph^+(s))$ and $\nu_{B'}(r,s) = \frac{k'-k}{k'}[\nu_A(r) \vee \nu_A(s)]h_\nu(\aleph^+(r) \cap \aleph^+(s))$, where $k' = |\aleph^+(r) \cap \aleph^+(s)|$.
Since $\overrightarrow{G}$ is symmetric and has a loop at every vertex, then $|\aleph[r] \cap \aleph[s]|_\mu > k$, $|\aleph[r] \cap \aleph[s]|_\eta > k$, $|\aleph[r] \cap \aleph[s]|_\nu > k$ in $U(G)$. So, $\mu_{B''}(r,s) = \frac{k''-k}{k''}[\mu_A(r) \wedge \mu_A(s)]h_\mu(\aleph[r] \cap \aleph[s])$, $\eta_{B''}(r,s) = \frac{k''-k}{k''}[\eta_A(r) \wedge \eta_A(s)]h_\eta(\aleph[r] \cap \aleph[s])$ and $\nu_{B''}(r,s) = \frac{k''-k}{k''}[\nu_A(r) \vee \nu_A(s)]h_\nu(\aleph[r] \cap \aleph[s])$, where $k'' = |\aleph[r] \cap \aleph[s]|$. Here, $k' = k''$ as $\overrightarrow{G}$ is symmetric PFD.
Thus, $\mu_{B'}(r,s) = \mu_{B''}(r,s)$, $\eta_{B'}(r,s) = \eta_{B''}(r,s)$ and $\nu_{B'}(r,s) = \nu_{B''}(r,s)$ $\forall r, s \in V$. □

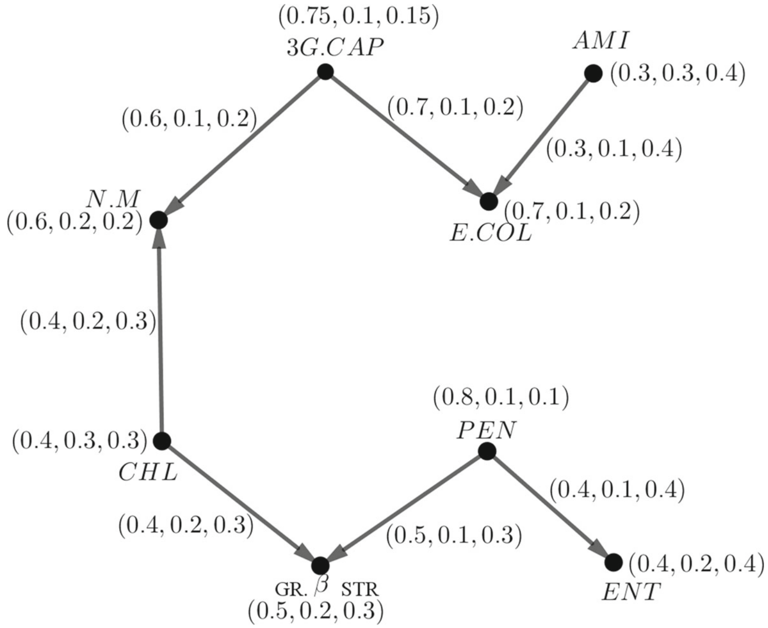


Fig. 15 PFD of medication against bacteria

8 Application of PFCG on the effect of medicines over bacteria

8.1 Model construction

The application of PFCG is very useful in our real life. One of the application is in medical science. Consider a PFD in Fig. 15 representing the competition between certain medicines against some bacteria. Let us consider the set of four medicines {3rd generation cephalosporin, Chloramphenicol, Aminoglycoside, Penicillin-G } used against the set of four bacteria {E. coli, Group β-sterptococci, Neiserria Meningitidis, Enterococcus}. All of these are aching as vertices of the digraph. Suppose the degree of existence in the environment of bacteria *E. coli* is 70%, indeterminacy of existence is 10% and non existence is 20%, i.e., the DTMS, DAMS and DFMS of the *E. coli* are $(0.7, 0.1, 0.2)$. Similarly for the other bacteria. Also assume that the degree of supply of the medicine 3rd generation cephalosporin be 75%, indeterminacy to supply be 10 percent and no supply be 15%, i.e., the DTMS, DAMS and DFMS of the 3rd generation cephalosporin are $(0.75, 0.1, 0.15)$ and similar for the other medicines. The DTMS of each directed edge between medicine and bacteria represents the effectivity, DAMS represents the indeterminacy of the effectivity and DFMS represents the ineffectivity of a medicine for that bacteria (see Fig. 15). It is seen that if there is no supply of the medicine 3rd generation cephalosporin, then both of *E. coli* and Neiserria Meningitidis count will be increased. Thus, we evaluate the effectivity of medicines against the bacteria with the help of PFCG (Table 3).

Table 3 Picture fuzzy out neighborhood of Fig. 15

$v \in V$	$\aleph^+(v)$
3rd generation cephalosporin	{(*E. coli*, (0.7,0.1,0.2)), (Neiserria Meningitidis, (0.6,0.1,0.2))}
Chloramphenicol	{(Neiserria Meningitidis, (0.4,0.2,0.3)), (group β-sterp., (0.4,0.2,0.3))}
Aminoglycoside	{(*E. coli*, 0.3,0.1,0.4)}
Penicillin-G	{(Group β-sterptococci, (0.5,0.1,0.3)), (Enterococcus, (0.4,0.1,0.4))}
Neiserria Meningitidis	Ø
E. coli	Ø
Group β-sterptococci	Ø
Enterococcus	Ø

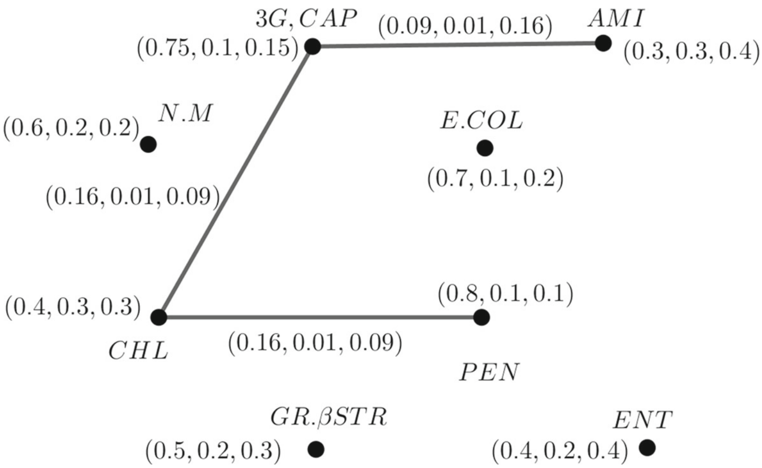


Fig. 16 Corresponding PFCG of Fig. 15

8.2 Decision making

We have, $\aleph^+$(3rd generation cephalosporin)∩$\aleph^+$(Chloramphenicol)={Neiserria Meningitidis,(0.4,0.1,0.3))}, $\aleph^+$(3rd generation cephalosporin)∩$\aleph^+$(Aminoglycoside)={E. coli, (0.3,0.1,0.4))}, $\aleph^+$(Penicillin-G)∩$\aleph^+$(Chloramphenicol)={Group β-sterptococci, (0.4, 0.1,0.3))}.

Thus there are edges between 3rd generation cephalosporin and Chloramphenicol; 3rd generation cephalosporin and Aminoglycoside; Penicillin-G and Chloramphenicol in the PFCG, which indicates the competition in PFCG. The DTMS, DAMS and DFMS of this edges are respectively (0.16, 0.01, 0.09), (0.16, 0.01, 0.09) and (0.09, 0.01, 0.16) (see Fig. 16). Hence there is a competition between 3rd generation cephalosporin and Chloramphenicol; 3rd generation cephalosporin and Aminoglycoside; Penicillin-G and Chloramphenicol in the medical science.

9 Conclusion

In this work, we have described PFCG and its two extended subclasses, one is PFkCG and the other is p-CPFG. To define PFkCG and p-CPFCG, we have taken a real number $k \geq 0$ and an integer $p > 0$. Both measures the strength of competitions of corresponding PFCGs. In addition, we have defined picture fuzzy neighborhood graphs and highlighted some results related to them. This study can be viewed as the generalization of the study on CGs. In addition, an application of PFCG in medical science is given here.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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