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Sewage discharge and water self-decay: Streeter and Phelps model application

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Abstract Due to the high waste deposition in superficial water, studies are necessary to emphasize the importance to monitor and apply tools, such as mathematical modelling. In this study, we used the classic Streeter and Phelps model to simulate the travel time necessary to depurate organic matter in the Tatu stream, at Limeira, São Paulo, Brazil, and to simulate the point-to-point depuration of organic matter in comparison to point-to-point empirical analysis. According to the simulations, organic matter would be established to 10 mg/L O_2 in few hours of time course of water without discharges in the stream, having the watercourse self-decay capacity. In addition, the analysis indicates that possible launches are being carried out along the stream, because, at the collection points, the obtained results presented higher biochemical oxygen demand than the expected for organic matter depuration, which denote discharges occurrences. Thus, this study emphasizes the relevance of monitoring actions and puts the model as a suitable tool to identify discharge sources in water.

Keywords Superficial water quality · Dissolved oxygen (DO) · Water management · Water reoxygenation · Water deoxygenation

Mathematics Subject Classification 34-00 · 92B05

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1 Introduction

The superficial water quality of the planet has been changed due to the great exploration of the natural element (Jiang 2017), that is increasingly being used as domestic, industrial, and agricultural discharge source without proper control or authorization in several cases (Ali et al. 2012).

Many Brazilian cities do not threat the wastes collected from dwellings or not even collect them, contributing to the fact that water sources are becoming sewage diluents and reservoirs (Duncan et al. 2014). Around 50% of the Brazilian population do not have access to waste collection services, and around only 43% of the waste collected is treated before being discharged in watercourses (Snis 2017).

Discharges in watersheds result in increased organic matter dissolved and particulate in water (Munster and Haan 1998). The type of waste/effluent determines how much organic material will enter the water and the aspect of it (Scheili et al. 2016). Organic matter decomposition may happen by degradation of particulate organic matter into dissolved organic matter or by oxidative mining of dissolved organic matter into inorganic compounds (Wetzel et al. 1991).

Organic matter decomposition in superficial water directly determines the watercourse self-decay capability, a natural process of physical and biochemical degradation of pollutant compounds in water, which tends to cooperate with the environment balance in regard to water quality along its course (Gray 1992). It depends then mainly on the natural oxygenation of water and the presence of purifying microorganisms to perform organic matter oxidation (Tundisi et al 2008), and so organic material in water tends to its natural balance, while carbonaceous organic matter is oxidized (Von Sperling 2009).

The biochemical oxygen demand (BOD) and dissolved oxygen (DO) are relevant when dealing with the presence of oxygen in water; both are, respectively, associated with deoxygenation and reaeration in aquatic environments. In combination, these parameters are part of the organic matter degradation process through the action of aerobic microorganisms in superficial water, making the water self-decay process possible (Streeter and Phelps 1925).

Studies on self-decay allow researchers to analyze watercourses natural capability to receive certain loads of organic pollutant, and to understand the time to purify these loads, according to the characteristics of each environment (Sibil et al. 2014). Knowledge on the water self-decay capability in watercourses is relevant to understand the dynamics of physical, chemical, and biological processes in these environments (Von Sperling 2008).

Studies involving mathematical modelling to represent scenarios and predictive simulations of water quality is a strategy to reproduce a real situation through simulations that enable understandings and decision-making as well as the study of strategies for remediating and/or predicting contamination status. Therefore, the proposal of this article is to apply the Streeter and Phelps (1925) in a self-decay analysis in a case study in the Tatu stream, in the São Paulo state countryside, Brazil.

2 The model

The model used relates organic matter decomposition with oxygen aeration in superficial water and is classically known as the Streeter and Phelps model (1925), which was the basis for the development of several other models (Yang et al. 2010; Fan et al. 2015). Therefore, while providing oxygen variation in water, the model also helps to study the self-decay capability, predicting scenarios of organic matter concentration in water and its natural efficiency to depurate it (Wang et al. 2013).

Its formulation is based on a differential equation, and its process arises from physical conceptualization of the mass balance phenomenon, from the classical mechanics. In this approach, the reaction phenomenon refers to the deoxygenation process (negative) and reaeration (positive) of the mixture.

The model is given by the following equation (Streeter and Phelps 1925):

$$\frac{\mathrm{d}C}{\mathrm{d}t} = -K_\mathrm{d}L + K_\mathrm{r}(C_\mathrm{s} - C), \quad (1)$$

where C represents the oxygen concentration in water in time t in mg/L; K_d indicates deoxygenation coefficient in day^{-1}, L represents remaining BOD concentration in mg/L, K_r indicates reaeration coefficient in day^{-1}, and C_s represents oxygen concentration for saturation.

The solution is given, expressed in terms of concentration $C = (C_\mathrm{s} - D)$, adopting the initial conditions $L = L_0$ and $D = D_0$, by the following equation:

$$C = C_\mathrm{s} - \left[\left(\frac{K_\mathrm{d}L_0}{K_\mathrm{r} - K_\mathrm{d}}\right)\left(\mathrm{e}^{-K_\mathrm{d}t} - \mathrm{e}^{-K_\mathrm{r}t}\right) + D_0\mathrm{e}^{-K_\mathrm{r}t}\right], \quad (2)$$

where D_0 represents the initial dissolved oxygen saturation of the water in parts per million and D indicates the saturation deficit, in parts per million, after time t.

3 Case study: Tatu stream/Limeira-SP

Tatu stream, located in the countryside of São Paulo state, is a 30-km-long watercourse. Its headwaters is in Cordeirópolis, it flows through Limeira, and then into the Piracicaba river.

In Limeira, the Tatu stream flows through 75% of the urban area and receives water from 14 tributaries (Regattieri 2007). It crosses agricultural and industrial areas, the city downtown, and the region where the galvanic production is concentrated. The stream also receives waste water from a sewage treatment station (70% of the city's waste) (Odebrecht 2015), and intersects with a mineral extraction area and the landfill site of the city.

Thereby, to do this research, we collected superficial water in five points strategically chosen along the stream, according to those activities in the surroundings of the watercourse or that might affect it.

Three collections were made to obtain the organic matter panorama of the stream: one in July and two in October 2014. This was the year with the lowest incidence of rain in decades in the Brazilian Southeast region (MMA 2014), where Limeira is located, which directly influenced the stream's flow rate and its capability to dissolve sewage disposal.

Figure 1 presents the Tatu stream running through Limeira and portrays the collection points (CP) and activities surrounding the stream: CP1 is the first point reachable of the stream already in Limeira, after it crossed the neighbor city of Cordeirpolis; CP2 is near industrial activities; CP3 is located in the city downtown; CP4 is near costume jewelry factories—this point is also located before the waste treatment station that releases primary waste water into the stream, and, finally, CP5 is located after the waste treatment station and also after the landfill site.

BOD and OD analyses from the water samples collected were performed at the Physical Chemical Laboratory of FT-UNICAMP, following the Standard Methods by American Public Health Association (Apha 2012). The results obtained for BOD and OD analyses were used as variables for the Streeter and Phelps model (variables L and C, respectively), to evaluate the self-decay potential of the stream.

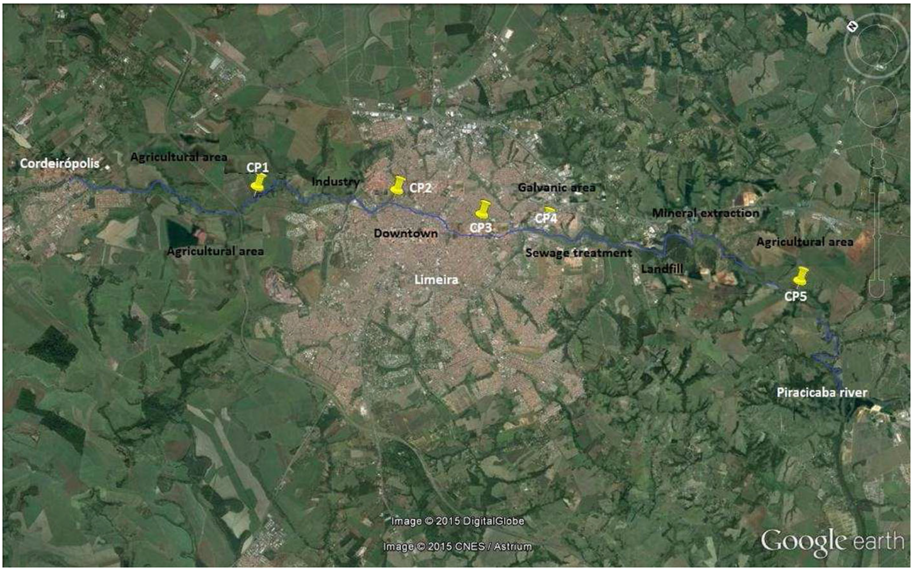

Fig. 1 Collection points of superficial water and details of the area around the Tatu stream, Limeira, São Paulo, Brazil. Source: Adapted from Google Earth®

Table 1 Distance between collection points (CPs) in kilometers and hours

Collection points	Distance (km)	Travel time (h)
CP 1 to CP 2	4.1	3.0
CP 2 to CP 3	3.7	2.7
CP 3 to CP 4	0.8	0.6
CP 4 to CP 5	8.0	5.7
CP 5 to the stream mouth	4.0	2.9

Table 2 Deoxygenation kinetic constant. Source: Churchill et al. (1962)

Source	k_d (day^{-1})
Concentrated wastewater	0.35–0.45
Low concentration wastewater	0.30–0.40
Primary effluent	0.30–0.40
Secondary effluent	0.12–0.24
Clean water rivers	0.09–0.21
Water for public supply	< 0.12

According to Von Sperling (2009), a fluid takes 7 h and 10 min to travel 10 km in a channel in a lotic system. Based on this statement, we estimate distance and travel times for the five points in kilometers and hours as presented in Table 1.

Regarding the variables used for simulations, besides BOD and OD values obtained through water sample analysis, K_d, K_r and saturation oxygen C_s were obtained by observing the stream hydrology characteristics and analyzing temperature.

Tables 2 and 3 present deoxygenation K_d and reaeration K_r kinetics, respectively.

Table 3 Reaeration kinetic constant. Source: Owens et al. (1964)

Kind	Deep (day^{-1})	Shallow (day^{-1})
Small lakes and swamps	0.12	0.23
Slow courses and large lakes	0.23	0.37
Low speed rivers	0.37	0.46
Medium speed rivers	0.46	0.69
High speed rivers	0.69	1.15
Fast rivers	$>$ 1.15	$>$ 1.61

The K_d adopted was 0.35 day^{-1}, considering that the stream receives primary waste along its course; the K_r used was 0.55 day^{-1}, since the watercourse is shallow and has intermittent speed, between high and low. The saturation oxygen C_s considered was 9.2 mg/L, for hydrous environments with average temperature of 20 °C (Cetesb 2015). We made the simulations using the software Matlab®.

We performed two simulations cycles: in the first case, we checked the self-decay capability of the stream analyzing the travel time needed to depurate the organic matter at CP1, without considering any other release along the path. In the second case, we verified the possibility of discharges at the watercourse simulating point-to-point depuration of organic matter and comparing the simulations to the organic matter load in water found in the laboratory.

Therefore, for the first case, we performed simulations considering only initial BOD flowing through Limeira, which is the BOD load obtained in the analysis of the samples collected at Collection Point 1, considering what would be the journey time until the BOD load reached the permissible value from the resolution CONAMA 357 (Brazil 2005).

This resolution establishes guidelines to be followed by the Brazilian water resources management and, for Class 3 watercourses, the maximum BOD allowed is 10 mg/L O_2. Nowadays, the Tatu stream is classified as a Class 4 watercourse, the least classification possible, in which the waters must be used only for navigation and landscape harmony.

For the second case, we performed organic matter decay simulations from CP1 to the following ones. Thereby, the results of the first collection were adopted to do the simulations. Thus, L_0 inputted in each simulation was the BOD analyzed from each CP, and the final BOD concentration expected according to the simulation, that is, after depuration, should be similar to the L_0 from the simulation of the next section depuration. Different concentrations at the end of the previous simulation curve and the next BOD of the CPs analyzed might indicate unexpected organic matter in the section.

4 Results and discussion

BOD and DO results obtained in the laboratory analysis of the superficial water sample collected at the three collections, from the five points of the Tatu stream, are presented in Table 4.

In each collection, DO did not present variation: in the first one, we found 8.1 mg/L O_2 at the water samples analysis; in the second, 7.26 mg/L O_2; and in the third, we had 7.52 mg/L O_2, all according to the requirements of Conama 357 legislation, that demands at least 4 mg/L O_2 for Class 3 and 4 watercourses. BOD results, however, are highly above the concentration expected for a Class 3 watercourse, that is until 5 mg/L O_2—there are no specifications for Class 4 water bodies. The BOD results confirm the expected when observing Tatu stream:

Table 4 DO and BOD_5 in superficial water measured for the samples (mg/L)

Collection point (CP)	First collection BOD_5	Second collection BOD_5	Third collection BOD_5
CP 1	74.53	129.87	28.46
CP 2	118.34	84.53	121.38
CP 3	48.50	7.50	137.38
CP 4	23.63	7.04	117.81
CP 5	120.63	143.28	190.94

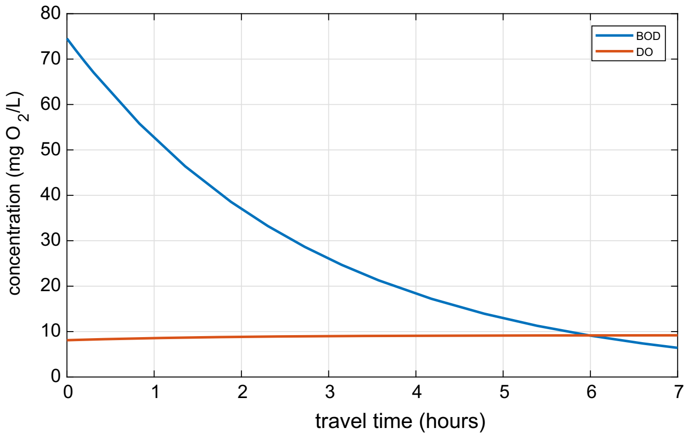

Fig. 2 Self-decay efficiency analysis in Tatu stream from the first collection

in all collection points, it is possible to notice a brown color and a strong odor (Owen et al. 1995 *apud* Teixeira et al. 2011), mainly at CP 5.

Figures 2, 3 and 4 represent scenarios considering the initial BOD value obtained in the sample analysis from CP 1 of the three water collection. They simulate the necessary time for the depuration of organic matter until it reaches around 10 mg/L O_2, that is the quantity required by the current legislation for Class 3 watercourses in Brazil. This classification parameter was used aiming to understand the situation and to evaluate actions to recover the watercourse.

According to Fig. 2, it is necessary approximately 6 h for the watercourse to reach the limit established by CONAMA 357 (Brazil 2005) of 10 mg/L O_2, which equates with about 40 km of water route in the stream, and considering that no organic matter was released in its course.

According to Fig. 3, also considering no organic matter release, it would be necessary approximately 7 h to reach the limit of 10 mg/L O_2.

With a lower initial BOD input L_0, Fig. 4 presents that in 3 h of water course, or 20 km, organic matter would be depurated to 10 mg/L O_2 disregarding any releases into the section.

We verified with these simulations that the self-decay potential of the stream is relatively high, owing to the fact that the water column of the stream is small and the flow speed is intermittent, making sedimentation difficult. Thus, the decrease in BOD may result from

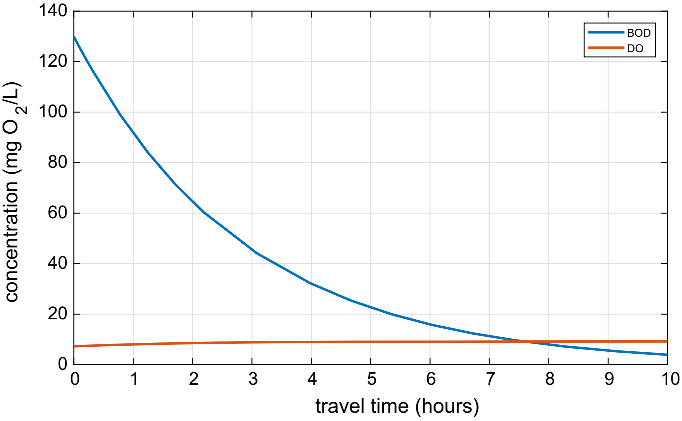

Fig. 3 Self-decay efficiency analysis in Tatu stream from the second collection

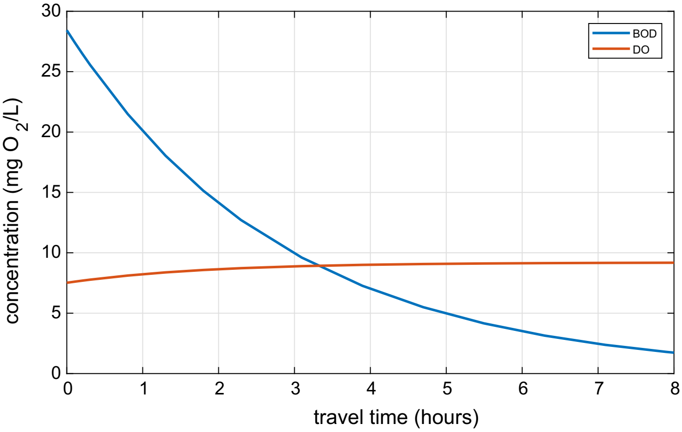

Fig. 4 Self-decay efficiency analysis in Tatu stream from the third collection

waste dissolution in the water flow, as well as from microorganisms' activity (Wetzel et al. 1991).

Simulations of organic matter decay from one Collection Point to the following are presented in Figs. 5, 6, 7, 8, and 9.

Between CPs 1 and 2, the decay rate presented in the simulation differs from the BOD obtained at CP 2, that is, the L_0 in Fig. 6. The BOD expected in CP 2 would be around 26 mg/L, according to the simulation, while the result analyzed from the water sample of the point was near 120 mg/L.

Between CPs 4 and 5, the same happened. In CP 5, the result expected was around 4 mg/L, while the real BOD concentration at the point was around 120 mg/L.

The BOD concentration highly diverges from what is expected in terms of self-decay capability in the stream and what was found concerning organic matter, also above of what is expected by the legislation. This fact suggests the possibility of discharges occurring along the section, and presents the model as suitable to identify stretches where might happen any kind of contamination of watercourses.

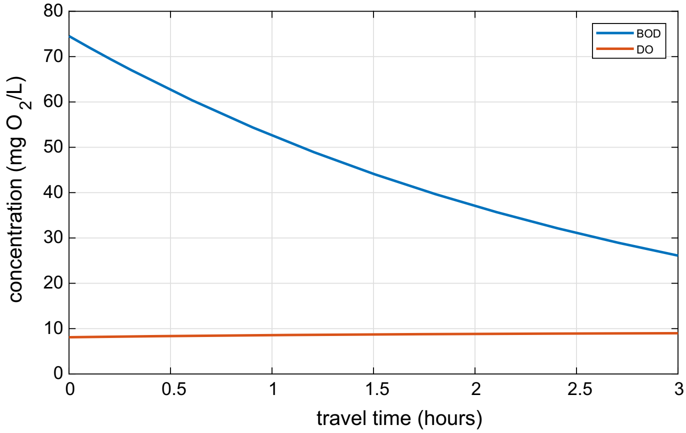

Fig. 5 Organic matter depuration at Collection Point 1

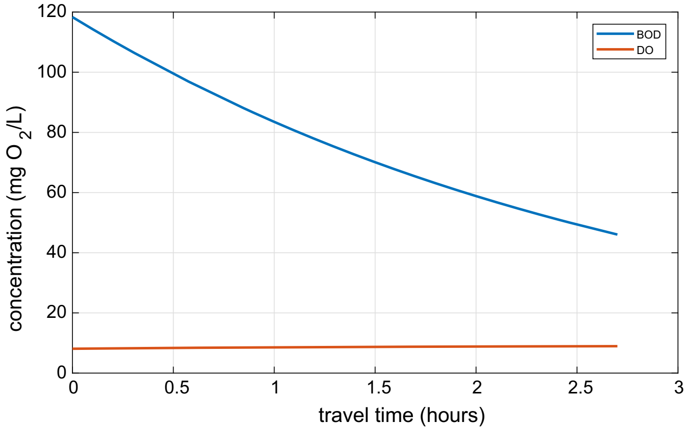

Fig. 6 Organic matter depuration at Collection Point 2

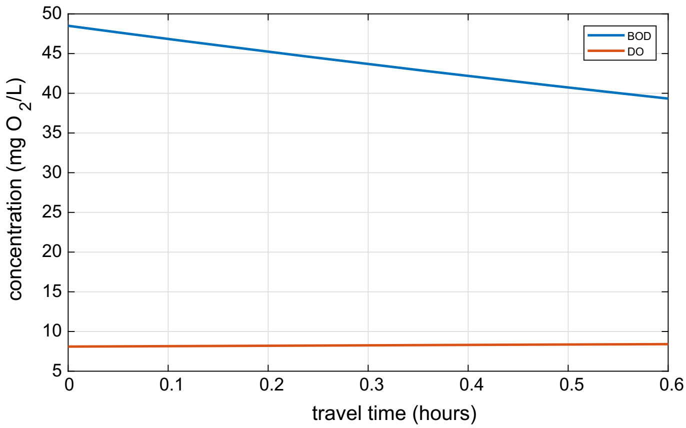

Fig. 7 Organic matter depuration at Collection Point 3

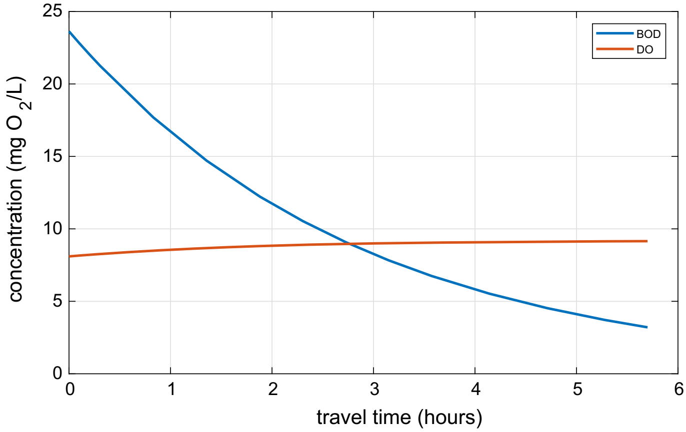

Fig. 8 Organic matter depuration at Collection Point 4

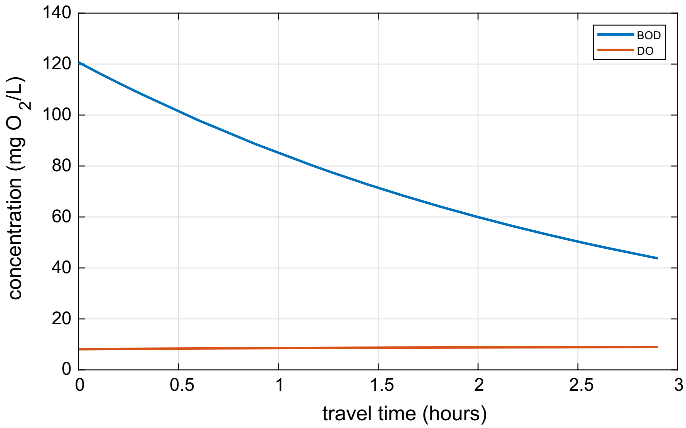

Fig. 9 Organic matter depuration at Collection Point 5

Although we found high incidence of BOD in all the collection points, in comparison to the required by the Brazilian water legislation, the simulations presented high self-decay capability for the stream. Since its watercourse is shallow and the velocity is intermittent, the water aeration is higher and microbiological depuration speed increases.

The simulations represent self-decay between two CPs, which means that the organic matter represented by the curve in the graph was supposed to be similar to the BOD found in the lab analysis of the next point.

At some collection points, there are evidences of discharges along the stream, mainly from the sewage treatment station after performing only primary treatment in the waste from the city.

These evidences suggest Streeter and Phelps model as a possible tool to be applied for water resources management not only for self-decay capability monitoring in surface water or organic matter prediction scenarios, but also to investigate illegal contamination sources along a stream.

5 Conclusions

The results obtained in this study indicate the self-decay capability of the Tatu stream. Without considering any discharges into the watercourse, in a few hours, organic matter would be established in accordance to the legislation.

The simulations evidence discharges along the stream. Since almost all of the results analyzed were up to the required by the legislation, accounting on its natural capability to oxidize organic matter is not the best alternative to improve the watercourse quality. It is necessary to invest in discharge reductions and waste treatment to transform the current scenario of the stream.

The model studied has the potential to be applied to study oxygen disposition, organic matter behavior, and self-decay capability in water, and also to identify discharge sources in watercourses.

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