

Flow shipment scheduling in an incomplete hub location-routing network design problem

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Abstract This paper addresses flow shipment scheduling in the hub location-routing problem in which the hub network is not fully interconnected. The aim is to allocate each node to the located hub(s) and to schedule the departure time from the nodes. The proposed integer programming model in this paper, which is inspired by a real-life problem such as postal delivery systems, ensures that the predetermined flow is delivered at the given service time. Moreover, the model solves the problem when the flow arrival demonstrates a piecewise linear pattern. Several classes of preprocessing steps are introduced to solve the model. Computational results obtained from CAB, AP, and a new dataset for Iranian road network (IRN) confirm the efficiency of the proposed preprocessing. In fact, averagely, the run times for CAB, AP, and IRN are reduced by 75.57, 80.41, and 82.96 %, respectively, when the model is solved by preprocessing. Moreover, on average, the number of nodes in the branch and cut tree is decreased by 90.87, 95.67, and 95.86 % for CAB, AP, and IRN, respectively, using the presented preprocessing.

Keywords Hub location · Routing · Incomplete network · Flow shipment scheduling · Preprocessing

Mathematics Subject Classification 90B06

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1 Introduction

Delivering at a desired service level in some systems such as postal and cargo delivery plays a key role in the logistics of commodities. Unlike most previous works which did not consider time-definite delivery service in supply chain networks, it is becoming more and more difficult to ignore it. Moreover, high-speed delivery with no delay affects the competitive environment, hindering those who does not adapt. However, the major difficulty of these applications is to determine the network topology and scheduled departure time from the origins. So, developing a proper model is a must to obtain a high-quality distribution strategy. For the time-sensitive commodities, the distribution topology of the network highly affects the distribution time and cost. Hub location problem (HLP) is a special kind of network design problem, which is applied on the transportation problems (Campbell 2009; Song and Carter 2008; Gelareh and Nickel 2011), delivery systems (Liu et al. 2003; Tan and Kara 2008; Yaman et al. 2007), and telecommunications (Kewcharoenwong and Üster 2014; Carello et al. 2004). Readers interested in the literature dealing with HLP are referred to O'Kelly and Miller (1994), Campbell (1994a, 2002), Klinecicz (1998), Bryan and O'Kelly (1999), Alumur and Kara (2008) and Farahani et al. (2013), and the references cited therein.

The most important task in transshipment network is to pass the flow between origins and destinations. To achieve this goal, there are two alternatives: one is to connect each origin and destination node to each other, which is not efficient and it is very costly. The other way is using hubs to consolidate the flow, which is received from origins and redistribute it to destination nodes directly (if the destination node is allocated to this hub) or other hubs. In the common form of HLP, there are supply or demand nodes. It is assumed that there is a positive flow between nodes according to the real practice. The advantage of using a hub network is that economies of scale can be achieved by combining the flow into an integral whole.

Three assumptions are regularly stated in the classical version of HLP. First of all, the hubs are completely interconnected. Second, the discount factor of using a hub link is between zero and one. Finally, there is no direct connection between non-hubs. The first and last assumptions are relaxed in this paper. An incomplete hub network is designed by the relaxation of the first assumption. For the incomplete hub network topology, as Karimi and Setak (2014) categorized, there are four types, including tree, ring, special, and general topologies. The suggested model in this paper presents the general incomplete hub network. Additionally, other topologies are also discussed to get better interpretation.

In this paper, postal and cargo delivery systems, which are the major applications of hub network design are addressed. In these systems, questions are raised about the location of hubs, routing the flows, designing the network, and scheduling departure time from the origins to reach the destinations on time. However, far too little attention has been paid to this case of HLP, which considers locating, routing, and scheduling problems together in an integrated model.

The flow shipment scheduling in some applications such as postal delivery systems has motivated us to propose a mathematical model for an incomplete hub network. The flow shipment scheduling in an incomplete hub location-routing problem (FSSIHLRP) tries to minimize the routing cost of flows and the fixed costs. Fixed costs include the cost of opening hubs, the cost of establishing hub links (i.e. an edge connecting one hub to another hub), spoke links (i.e. an edge connecting a non-hub to a hub), and non-hub link (i.e. an edge connecting a non-hub to another non-hub). Some preprocessing is used to improve the solution time.

The rest of the paper is organized as follows. Next section illustrates the related background of FSSIHLRP. Section 3 describes the problem and the related model. In Sect. 4, some preprocessing steps are introduced for the model to eliminate some of the variables from the formulation. Characterization and evaluation of the preprocessing steps are studied in Sect. 5. Finally, concluding remarks are given in the last section.

2 Previous related research

To provide a high-level service, FSSIHLRP considers the general topology for hub network design, non-stop or direct shipment, and time-definite delivery system. In this section, after introducing the primary papers in the literature, the general network topology of HLP is reviewed. Next, the literature of the direct shipment in HLP is surveyed. Finally, the commodity time-definite delivery is studied.

The first mathematical formulation for HLP was introduced by O'Kelly (1986). Then, he investigated a quadratic mathematical model for a single allocation hub median location problem (O'Kelly 1987). Campbell (1994b) reported the capacitated, median, covering, and center HLP. The rest of the lengthy and rich literature of HLP includes developing formulations and solution methods.

General hub network topology In the general topology, no network structure other than connectivity is imposed on the hub network. Moreover, non-hub links can be used in this topology. To the best knowledge of the authors, the general topology of HLP was suggested by Yoon et al. (2000) in the telecommunication area. They did not consider the fixed cost of opening the hubs. In addition, they used a hierarchical network, but the direct shipment was not allowed. Nickel et al. (2001) proposed a mathematical model for HLP in the urban public transportation problem, which considers fixed costs of opening the hubs, the cost of establishing hub links, spoke links, and flow shipment. Nevertheless, their model did not take the fixed costs of the non-hub links into account. Our proposed model is an extension of their model. Yoon and Current (2008) subsequently provided another model for the general topology, but the routing costs were not discussed in their model. Gelareh and Nickel (2011) developed Nickel et al. (2001) model and reduced the dimension of the problem by decreasing the number of the variables and constraints. Their model was referred to as HLPPT (i.e. HLP for public transportation). It should be noted that their model which was customized for the transportation problem cannot design a network with only one hub. In addition, they did not address the direct shipment and non-hub link cost in their model. Catanzaro et al. (2011) prepared a mathematical model, which clustered the network into some sub-networks in which at least one hub should be located at least a hub in each of them, and found the routing of the flows to minimize flow shipment cost. A single allocation incomplete hub covering network with fuzzy travel times was suggested by Davari et al. (2013). In their model, the objective function maximized the minimum covered flows on the network. In addition, the direct shipment was not permitted in the model. Setak et al. (2013) suggested a model for hub location-routing problem, in which the general hub network topology was used. To consider the flow shipment scheduling in this network structure, we propose a mathematical formulation in this paper. As another related research, Karimi and Setak (2014) presented a model which minimized the fixed costs of opening hubs, establishing hub links, spoke links, and flow shipment costs in an incomplete network.

Direct shipment As one of the elementary assumptions in this area, the non-stop shipment is not allowed in HLP. However, Aykin (1994) relaxed this assumption for the first time.

[Sung and Jin \(2001\)](#) provided another model of direct shipment using multiple allocation strategy. Furthermore, more researchers such as [Nickel et al. \(2001\)](#), [Yoon and Current \(2008\)](#), [Catanzaro et al. \(2011\)](#), and [Mahmutoğulları et al. \(2015\)](#) have employed the non-stop shipment in their models.

Time-definite delivery system In this delivery system, travel time of each O–D pair is guaranteed with a desired service level. Service time for the commodity shipment is studied in different distribution strategies. [Lin and Chen \(2004\)](#) considered a generalized capacitated hub-and-spoke network and provided two transportation modes, which satisfied a chosen service level. [Tan and Kara \(2008\)](#) presented an integer programming formulation for cargo delivery system and implemented it for the Turkish network. Moreover, their model determined the departure time of the vehicles from hub to hub and hub to non-hub in a single allocation complete hub network topology. [Yaman et al. \(2007\)](#) proposed a mathematical model for the ground transportation cargo delivery systems. They also concentrated on the departure time and the number of stops between each O–D. [Chen et al. \(2008\)](#) considered the routing decision and tree topology over an existing network, while the model minimized the violations of the predefined service time. As another example, [Campbell \(2009\)](#) constructed a model of the time-definite transportation. While guaranteeing a desired service time, he considered the total routing cost, subject to the feasible assignments. [Karimi and Bashiri \(2011\)](#) proposed four mathematical formulations, in which the travel time was limited on each connection. [Ishfaq \(2012\)](#) discussed the design of a less than truck load logistics and offered multiple delivery time services. [Yaman et al. \(2012\)](#) introduced a model, called release time scheduling hub location problem (RSHL), for cargo delivery systems, which decided on the release time of vehicles from each node for the next-day delivery service. [Peker and Kara \(2015\)](#) proposed a p-hub maximal covering problem to find the best locations for the hubs to maximize the satisfied demands within a coverage bound.

A broad categorization of the literature is illustrated in [Table 1](#) by considering seven characteristics, namely hub network structure, network fixed costs, allocation strategy, number of hubs, direct shipment, time-definite delivery system, and flow shipment scheduling. The attributes of FSSIHLRP are also presented in the last row.

To help filling the shown gap in [Table 1](#), in this study, an incomplete hub location-routing problem with a general hub network topology is proposed. There is no capacity restriction for FSSIHLRP which is focused on the scheduling of leaving times from the origins and hubs to destinations. This problem fits in the classes of general hub network topology, non-stop shipment, and time-definite delivery systems.

3 Problem definition and formulation

3.1 Description of the problem

The study of commodity delivery systems plays an important role in the supply chain network. On time delivery is the key factor for the success of such systems. As is known, the transportation systems and delivery of commodities are the applications of HLP. In other words, HLP is one of the most important tools for the transportation network design.

It is supposed that a delivery company wants to meet a desired service time using an appropriate transportation network. The company should know about the components of the transportation network such as location of hubs and non-hubs, commodities routing from origins to destinations, scheduled departing time from origins, and departure time from the hub nodes.

Table 1 Review of the existing literature

Paper	Hub network structure		Network fixed costs			Allocation strategy			Number of hubs	Direct shipment	Time-definite delivery system	Flow shipment scheduling
	Incomplete	Complete	Hubs links	Hub links	Spoke links	Direct links	Multiple					
							Single	Multiple				
General		Not-general		Predefined		Not-predefined						
Aykin (1994)	-	✓	-	-	-	-	✓	-	✓	-	-	-
Yoon et al. (2000)	✓	-	-	✓	✓	-	-	✓	✓	-	-	-
Sung and Jin (2001)	-	✓	✓	-	-	-	✓	-	✓	-	-	-
Nickel et al. (2001)	✓	-	✓	✓	✓	-	-	✓	-	-	-	-
Lin and Chen (2004)	-	✓	✓	-	-	-	✓	-	✓	-	-	-
Tan and Kara (2008)	-	✓	-	-	-	-	✓	-	-	✓	-	-
Yaman et al. (2007)	-	✓	-	-	-	-	✓	-	-	✓	-	-
Chen et al. (2008)	-	✓	-	-	-	-	✓	-	-	✓	-	✓
Yoon and Current (2008)	✓	-	✓	✓	✓	✓	-	✓	-	-	-	-
Campbell (2009)	-	✓	-	-	-	-	✓	-	✓	-	✓	-
Gelareh and Nickel (2011)	✓	-	✓	✓	-	-	-	✓	-	-	-	-
Catanzaro et al. (2011)	✓	-	-	-	-	-	✓	-	✓	-	-	-
Karimi and Bashiri (2011)	-	✓	-	-	-	-	✓	-	✓	-	✓	-
Ishfaq (2012)	-	✓	✓	-	-	-	✓	-	✓	-	✓	-
Yaman et al. (2012)	-	✓	-	-	-	-	✓	-	-	-	✓	✓
Davari et al. (2013)	✓	-	-	-	-	-	✓	-	✓	-	✓	-
Setak et al. (2013)	✓	✓	✓	✓	✓	✓	✓	✓	✓	-	-	-

Table 1 continued

Paper	Hub network structure		Network fixed costs			Allocation strategy		Number of hubs	Direct shipment	Time-definite delivery system	Flow shipment scheduling
	Incomplete	Complete	Hubs links	Hub links	Spoke links	Direct links	Multiple				
							Single	Predefined	Not-predefined		
	General Not-general										
Karimi and Setak (2014)	✓	✓	✓	✓	✓	-	✓	✓	✓	-	-
Peker and Kara (2015)	-	✓	-	-	-	-	✓	✓	-	✓	-
Mahmutogullari et al. (2015)	-	✓	-	-	-	-	✓	✓	-	✓	-
This research	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

In the light of the described problem, in this section, a hub location network is formulated as FSSIHLRP. The resulted network consists of hubs, non-hubs, hub links, spoke links, and non-hub links. As stated before, delivery systems intend to ship the commodity within the qualified service time. In these systems, commodities travel from/to an origin via hub/hubs or directly to a destination. Three important variables (i.e. location of hubs, routing of the flows, and planning the departure times) are determined. The objective of FSSIHLRP is to minimize the fixed cost of designing the network and the routing cost of flows. The advantage of using FSSIHLRP over the related literature is that FSSIHLRP integrates the planning issues with an incomplete hub network design. The following assumptions are considered in FSSIHLRP.

Assumptions

- The locations of customers, their flows and distances, fixed costs of opening hubs, hub links, spoke links, and direct links are given.
- Fixed costs of opening hub links, spoke links, and direct links are symmetric.
- The travel cost graph is non-directional and satisfies the triangle inequality.
- Each node can potentially be a hub.
- There is an advantage for routing streams through hubs.
- The number of hubs is unknown, which means model will decide on that.
- Direct connections between non-hubs are allowed.
- An incomplete hub network is considered.
- A discount factor is used to take advantage of hub links in the network.
- The multiple allocation strategy is employed.
- The direct link between two points yields the non-stop travel between them.
- This model restricts the travel time with a certain service level. Flows departing from a hub need to wait for all the required incoming flows.
- The scheduled departure times from the origins should be no later than the specified times.
- At certain service time, at least predefined units of flow should be delivered to the destinations.

3.2 FSSIHLRP formulation

FSSIHLRP is formulated by the following set, parameters, decision variables, objective function, and constraints.

Set

N Set of all nodes indexed by i, j, k , and m

Parameters

α	Cost discount factor ($0 \leq \alpha \leq 1$); it is used in the travelling cost for hub links,
L	Number of days per year which is equivalent to 365,
T	Maximum coverage service level for travelling time between all pairs,
c_{ij}	Transfer annual cost per unit of flow from nodes i to j ,
t_{ij}	Travel time from nodes i to j ,
F_k	Fixed annual average cost of opening a hub at node k ,
FH_{km}	Fixed annual average cost of establishing a hub link between nodes k and m ,
FS_{ij}	Fixed annual average cost of establishing a spoke link between nodes i and j ,
FD_{ij}	Fixed annual average cost of establishing a direct link between nodes i and j ,
w_{ij}	Daily average amount of flow which is transported between nodes i to j ,
γ_i	The time which the scheduled leaving time from node i should not be later than,

- σ_i Minimum daily flow which is picked from node i ,
- st_k Average service time at hub node k ,
- $f_i(s_i)$ Flow arrival function which calculates the amount of flow accumulated at node i until s_i ; logically, it should be a non-decreasing function.

Decision variables

- a_{ij}^{km} $\begin{cases} 1 & \text{If the flow originated at } i \text{ and destined to } j \text{ is routed via hubs } k \text{ and } m, \\ 0 & \text{Otherwise} \end{cases}$
- e_{ij}^k $\begin{cases} 1 & \text{If the flow originated at non-hub and destined to } j \text{ is routed via hub } k, \\ 0 & \text{Otherwise,} \end{cases}$
- f_{ij}^k $\begin{cases} 1 & \text{If the flow originated at } i \text{ and destined to non-hub } j \text{ is routed via hub } k, \\ 0 & \text{Otherwise,} \end{cases}$
- g_{ij} $\begin{cases} 1 & \text{If } i \text{ is connected and directly routed to } j ; \text{ only one of them is a hub,} \\ 0 & \text{Otherwise,} \end{cases}$
- b_{ij} $\begin{cases} 1 & \text{If } i \text{ is connected and directly routed to } j ; \text{ none of them is a hub,} \\ 0 & \text{Otherwise,} \end{cases}$
- z_{km} $\begin{cases} 1 & \text{If hub nodes } k \text{ and } m \text{ are directly connected,} \\ 0 & \text{Otherwise,} \end{cases}$
- y_k $\begin{cases} 1 & \text{If } k \text{ is hub,} \\ 0 & \text{Otherwise,} \end{cases}$
- s_i Scheduled leaving time from node i ,
- D_{kj} Departure time from hub k to node j ,

To clarify the performances of the binary variables, all of them are depicted in Fig. 1, except y_k . In this figure, the circles and rectangles show the non-hub and hub nodes, respectively. In addition, the line and dash line arrows, respectively, illustrate the direct links and paths between the nodes.

FSSIHLRP

$$\begin{aligned} \min \sum_i \sum_j \sum_k \sum_m L\alpha c_{km} w_{ij} a_{ij}^{km} + \sum_i \sum_j \sum_k L w_{ij} (c_{ik} e_{ij}^k + c_{kj} f_{ij}^k) + \sum_i \sum_j L c_{ij} w_{ij} g_{ij} \\ + \sum_i \sum_j L c_{ij} w_{ij} b_{ij} + \sum_k F_k y_k + \sum_k \sum_{m>k} F H_{km} z_{km} + \sum_i \sum_{j>i} F S_{ij} g_{ij} + \sum_i \sum_{j>i} F D_{ij} b_{ij} \end{aligned} \tag{1}$$

$$\sum_{m \neq i} a_{ij}^{im} + \sum_{m \neq i} e_{ij}^m + g_{ij} + b_{ij} = 1 \quad \forall i, j, (i \neq j) \tag{2}$$

$$\sum_{m \neq j} a_{ij}^{mj} + \sum_{m \neq j} f_{ij}^m + g_{ij} + b_{ij} = 1 \quad \forall i, j, (i \neq j) \tag{3}$$

$$\sum_{m \neq i,k} a_{ij}^{km} + f_{ij}^k - \sum_{m \neq j,k} a_{ij}^{mk} - e_{ij}^k = 0 \quad \forall i, j, k, (i \neq j, i \neq k, j \neq k) \tag{4}$$

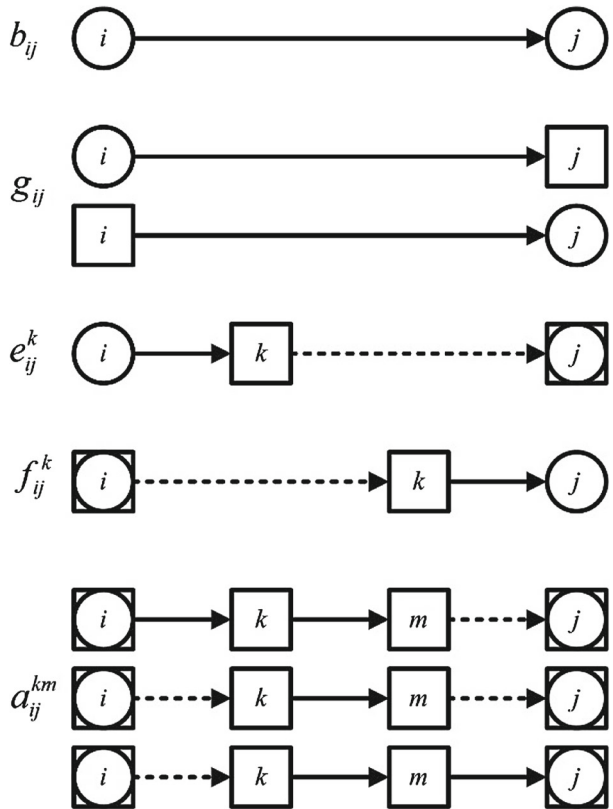
$$e_{ij}^k \leq 1 - y_i \quad \forall i, j, k, (i \neq j) \tag{5}$$

$$f_{ij}^k \leq 1 - y_j \quad \forall i, j, k, (i \neq j) \tag{6}$$

$$e_{ij}^k + \sum_{m \neq j,k} a_{ij}^{mk} \leq y_k \quad \forall i, j, k, (i \neq j, i \neq k, j \neq k) \tag{7}$$

$$f_{ij}^k + \sum_{m \neq i,k} a_{ij}^{km} \leq y_k \quad \forall i, j, k, (i \neq j, i \neq k, j \neq k) \tag{8}$$

Fig. 1 Non-zero binary variables in the path for sending flow between i and j



$$g_{ij} + 2a_{ij}^{ij} + \sum_{m \neq i, j} a_{ij}^{im} + \sum_{m \neq i, j} a_{ij}^{mj} \leq y_i + y_j \quad \forall i, j, (i \neq j) \tag{9}$$

$$g_{ij} \leq 2 - y_i - y_j \quad \forall i, j, (i \neq j) \tag{10}$$

$$a_{ij}^{km} + a_{ij}^{mk} \leq z_{km} \quad \forall i, j, k, m, (i \neq j, k < m) \tag{11}$$

$$b_{ij} \leq 1 - y_i \quad \forall i, j, (i \neq j) \tag{12}$$

$$b_{ij} \leq 1 - y_j \quad \forall i, j, (i \neq j) \tag{13}$$

$$e_{ij}^k \leq g_{ik} \quad \forall i, j, k, (i \neq j) \tag{14}$$

$$f_{ij}^k \leq g_{kj} \quad \forall i, j, k, (i \neq j) \tag{15}$$

$$g_{ij} - g_{ji} = 0 \quad \forall i, j, (i < j) \tag{16}$$

$$b_{ij} - b_{ji} = 0 \quad \forall i, j, (i < j) \tag{17}$$

$$z_{km} - z_{mk} = 0 \quad \forall k, m, (k < m) \tag{18}$$

$$s_i \leq \gamma_i \quad \forall i \tag{19}$$

$$f_i(s_i) \geq \sigma_i \quad \forall i \tag{20}$$

$$(e_{ij}^k + b_{ij}g_{ik})(s_i + st_k y_k + t_{ik}g_{ik}) \leq D_{kj} \quad \forall i, j, k, (i \neq j) \tag{21}$$

$$a_{ij}^{km}(st_m y_m + D_{kj} + t_{km}z_{km}) \leq D_{mj} \quad \forall i, j, k, m, (i \neq j), (k \neq m) \tag{22}$$

$$f_{ij}^m(t_{mj} + D_{mj}) \leq T \quad \forall i, j, m, (i \neq j) \tag{23}$$

$$a_{ij}^{km} \in \{0, 1\} \quad \forall i, j, k, m \quad (24)$$

$$e_{ij}^k \in \{0, 1\} \quad \forall i, j, k \quad (25)$$

$$f_{ij}^k \in \{0, 1\} \quad \forall i, j, k \quad (26)$$

$$b_{ij} \in \{0, 1\} \quad \forall i, j \quad (27)$$

$$g_{ij} \in \{0, 1\} \quad \forall i, j \quad (28)$$

$$z_{km} \in \{0, 1\} \quad \forall k, m \quad (29)$$

$$y_k \in \{0, 1\} \quad \forall k \quad (30)$$

$$s_i \geq 0 \quad \forall i \quad (31)$$

$$D_{kj} \geq 0 \quad \forall i, k \quad (32)$$

Objective function (1) minimizes four cost elements of the problem: (1) Delivery cost (i.e. the 1st, 2nd, 3rd, and 4th parts of the objective function); (2) Fixed cost of opening hubs (i.e. the 5th part of the objective function); (3) Fixed cost of establishing hub links (i.e. the 6th part of the objective function); (4) Fixed cost of establishing direct and spoke links (i.e. the last part of the objective function). Constraints (2), (3), and (4) are the flow conservation constraints. In fact, these constraints hold the flow balance on O–D pairs. Constraints (5) and (6) guarantee that, for e_{ij}^k and f_{ij}^k , i and j should be non-hub nodes, respectively. Constraints (7) and (8) ensure that flows can move through a node to reach the destination when the node is a hub. Constraints (9) illustrate that employing a link for moving flows from an origin to a destination depends on their character (i.e. being hub nodes or not). When only one of i or j is hub, constraints (10) ensure that g_{ij} can be one. It should be noted that, when none of i and j is hub, constraints (9) force g_{ij} to be zero. Therefore, constraints (10) work correctly in this case. The constraints (11) guarantee that a hub link should exist before being used in any flow path. Constraints (12) and (13) demonstrate that b_{ij} can be non-zero when i and j are not hub. Constraints (14) and (15) check that sending or receiving flows between two non-hubs needs a spoke link. Constraints (16)–(18) illustrate that the receiving path is the same as the sending path. Constraints (19) ensure that the flows should not be sent earlier than the latest time. Constraints (20) show that at least the minimum flow, picked from a node, should be delivered to the destination with the desired service time in a day. Constraints (21) and (22) illustrate that, to depart from a hub node toward the destinations, a vehicle should wait for all the vehicles to arrive from the nodes that are routed via this hub to the destinations. Constraints (23) express that the accomplishment time of any delivery service should happen by the predetermined service time. Finally, constraints (24)–(32) show the domain of the variables.

3.3 Linearization

This subsection presents linearization for the non-linear constraints (i.e. (21), (22), and (23)), presented in FSSIHLRP. A Big M-type linearization method is suggested for the mentioned non-linear constraints as follows:

$$s_i + t_{ik}g_{ik} + st_k y_k - (2 - g_{ik} - b_{ij} - e_{ij}^k)BN_1 \leq D_{kj} \quad \forall i, j, k, (i \neq j) \quad (33)$$

$$D_{kj} + st_m y_m + t_{km}z_{km} - (1 - a_{ij}^{km})BN_2 \leq D_{mj} \quad \forall i, j, k, m, (i \neq j), (k \neq m) \quad (34)$$

$$D_{mj} + t_{mj}f_{ij}^m \leq T \quad \forall i, j, m, (i \neq j) \quad (35)$$

New linear constraints (33), (34), and (35) are used instead of (21), (22), and (23), respectively. In these new constraints, BN_1 and BN_2 play the role of big numbers. BN_1 is set

at $\max_{i,j} t_{ij} + \max_k s_k + \max_i \gamma_i$ and BN_2 is set at T . These values are chosen to tighten the constraints.

Theorem 1 Constraints (33), (34), and (35) correctly linearize (21), (22), and (23), respectively.

First, observe that all the delivery times should be within T . Therefore, (35) is a correct linearization for (23). From constraints (2), (3), and (4), for (33), there are three cases if $g_{ik} = 1$.

Case 1 $b_{ij} = 1$ and $e_{ij}^k = 0$. In this case, both (33) and (21) yield the same left-hand sides.

Case 2 $b_{ij} = 0$ and $e_{ij}^k = 1$. This case is similar to case 1.

Case 3 $b_{ij} = 0$ and $e_{ij}^k = 0$. The left-hand side of (21) yields zero and the left-hand side of (33) returns a negative value. Therefore, as D_{kj} is non-negative, the two constraints work similarly in this case.

In addition, there are three cases if $g_{ik} = 0$. These cases are the same as the previous ones. However, the results of these cases when g_{ik} is zero are the same as those of the third case when g_{ik} is one. Therefore, (33) is a correct linearization for (21). Finally, for (34), there are two cases.

Case 1 $a_{ij}^{km} = 1$. In this case, both (34) and (22) yield the same right-hand sides.

Case 2 $a_{ij}^{km} = 0$. The left-hand side of (22) returns zero. Additionally, the left-hand side of (34) yields a negative value. However, as D_{mj} is non-negative, (34) is a true linearization for (22). □

3.4 Piecewise linear flow arrival function

FSSIHLRP is for an arbitrary flow arrival function. This function demonstrates the flows come to origins. As presented before, it should logically be a non-decreasing function. As observed in Fig. 2, the flow arrival function is a piecewise linear and convex function if linear interpolation is used between two points within the time bounds. Moreover, the piecewise linear function can be an approximation of nonlinear arrival pattern, which is more used in the real applications such as postal delivery systems. Clearly, a piecewise linear function includes several parts with a unique constant and coefficient in each part, as shown in Fig. 2. In the approximations of a nonlinear flow arrival function, by growing the number of parts, the error is reduced:

In Fig. 2, the origin contains three parts for piecewise linear function. The notations used in this figure are also defined in the rest of the paper. The piecewise linear flow arrival function is formulated using the following new notations:

Set

M_i Set of all piecewise linear parts for node i indexed by p_i ,

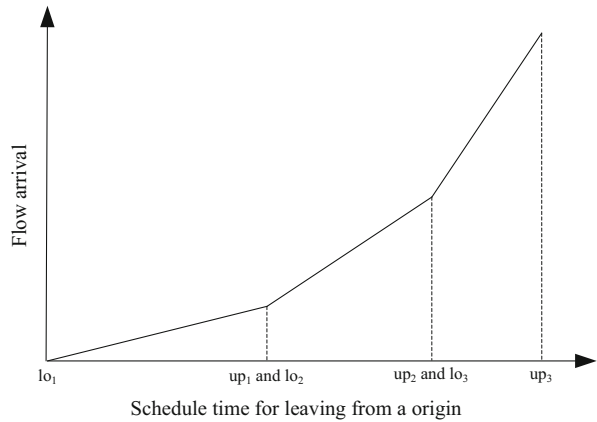
Parameters

- co_{ip_i} Coefficient of part p for node i ,
- con_{ip_i} Constant of part p for node i ,
- up_{ip_i} The upper bound of part p for node i ,
- lo_{ip_i} The lower bound of part p for node i .

Decision variables

s'_{ip_i} Scheduled leaving time from node i at the time related to part p_i ,

Fig. 2 A sample piecewise linear pattern



$$h_{ip_i} \begin{cases} 1 & \text{If part } p_i \text{ is selected for node } i, \\ 0 & \text{Otherwise,} \end{cases}$$

Constraints instead of (19) and (20):

$$\sum_{p_i} s'_{ip_i} \leq \gamma_i \quad \forall i \tag{36}$$

$$\sum_{p_i} (\text{co}_{ip_i} s'_{ip_i} + h_{ip_i} \text{con}_{ip_i}) \sum_j w_{ij} \geq \sigma_i \quad \forall i \tag{37}$$

$$s'_{ip_i} \leq \text{up}_{ip_i} h_{ip_i} \quad \forall i, p_i \tag{38}$$

$$\text{lo}_{ip_i} h_{ip_i} \leq s'_{ip_i} \quad \forall i, p_i \tag{39}$$

$$\sum_{p_i} h_{ip_i} = 1 \quad \forall i \tag{40}$$

$$\sum_{p_i} s'_{ip_i} = s_i \quad \forall i \tag{41}$$

3.5 Extensions to FSSIHLRP

In the previous paragraphs, a basic model called FSSIHLRP was suggested. Now, we introduce other extensions. As is known, in the general topology, the multiple allocation strategy is used for connecting non-hubs to hubs. To employ a single allocation policy, constraints (42) should be added to FSSIHLRP.

$$\sum_{j \neq i} g_{ij} \leq 1 \quad \forall i \tag{42}$$

Another well-known hub network topology is the tree of hubs. This type has been introduced and used in a few articles in this area (Chou 1990; Kim and Tcha 1992; Contreras et al. 2009). In this hub network topology, there is just one route to transport from one hub to the other hubs. Adding constraint (43) to the model changes the hub network to a tree topology. Constraints (18) cause the right side of this constraint to be doubled.

$$\sum_k \sum_{m \neq k} z_{km} = 2 \left(\sum_k y_k - 1 \right) \tag{43}$$

The ring of hub topology is another network design, which is commonly used in telecommunication systems (Lee et al. 1993; Chiu et al. 1995; Contreras et al. 2009). In this design, each hub node is just connected to two hubs. Using constraints (44) helps FSSIHLRP use the ring topology.

$$\sum_{m \neq k} z_{km} = 2y_k \quad \forall k \tag{44}$$

Sometimes, decision makers need to limit the number of stopping in the flow shipment path between each O–D pair. By employing a new parameter, namely q_{ij} , a new constraint (45) should be attached to the model. q_{ij} is the maximum number of stopping hub nodes to shipping flow from origin i to destination j .

$$\sum_k \sum_{m \neq k} a_{km}^{ij} \leq q_{ij} \quad \forall i, j, i \neq j \tag{45}$$

Furthermore, a famous type of hub location is p -HLP. In this type, decision makers or owners of the network decide to use only a predetermined number of hubs, called p . Clearly, adding constraints (46) to FSSIHLRP yields FSSI p -HLP.

$$\sum_k y_k = p \tag{46}$$

4 Preprocessing

Before solving any FSSIHLRP, we perform preprocessing steps to remove some of the variables. These are planned to cut the size of the problem and to tighten the linear relaxation. In addition, they are run by taking advantage of the following properties.

Property 1 *In any optimal solution to the model, for all i and p_i with $\text{up}_{ip_i} \text{co}_{ip_i} + \text{con}_{ip_i} < \frac{\sigma_i}{\sum_j w_{ij}}, h_{ip_i} = 0$.*

Proof Assume that for some i and $p_i, h_{ip_i} = 1$. As stated by constraints (38), the upper bound of s'_{ip_i} is up_{ip_i} . Therefore, when $\sum_j w_{ij} (\text{up}_{ip_i} \text{co}_{ip_i} + \text{con}_{ip_i})$ is less than the minimum flow, which is picked from node i , constraints (37) will be violated, which guarantees that h_{ip_i} should be zero. □

Corollary 1 *It is the corollary of property 1 and constraints (39). For all i, lp_i is defined as the last p_i where $h_{ip_i} = 0$. The minimum feasible value for s_i is lo_{ip_i+1} , which is called slo_i .*

Property 2 *In any optimal solution to the model, for all i, k , and m with $\text{slo}_i + t_{ik} + \text{st}_k + \text{st}_m + t_{km} > T, a_{ij}^{km} = 0$ for all j .*

Proof Without loss of generality, we simply assume that k and m are hub nodes and i is connected to k . Therefore, as shown by constraints (33), D_{kj} will be $\text{slo}_i + t_{ik} + \text{st}_k$. According to constraints (34), if $a_{ij}^{km} = 1$, then D_{mj} will be $\text{slo}_i + t_{ik} + \text{st}_k + \text{st}_m + t_{km}$. If D_{mj} is greater than the maximum coverage service time, constraints (35) will be violated. To capture the feasible solution, in this case, a_{ij}^{km} should be zero. □

Property 3 *In any optimal solution to the model, for all i, j , and k with $\text{slo}_i + t_{ik} + \text{st}_k + t_{kj} > T, e_{ij}^k = 0$ and $f_{ij}^k = 0$.*

Proof Assume that, for some $i, j,$ and $k, e_{ij}^k = 1$ and $f_{ij}^k = 1$. It means that i and j are connected to the same hub (i.e. k) and there is no direct link between them. According to constraints (33), D_{kj} will be $slo_i + t_{ik} + st_k$. Moreover, as demonstrated by constraints (34), $D_{kj} = D_{mj}$, since for this i and $j,$ all a_{ij}^{km} is zero; therefore, the left-hand side of constraints (35) will be $slo_i + t_{ik} + st_k + t_{kj}$. If this left-hand side is greater than the maximum coverage service time, constraints (35) will be violated. To capture the feasible solution, in this case, e_{ij}^k and f_{ij}^k should be zero. □

Property 4 *In any optimal solution to the model, for all $i, j,$ and k with $slo_i + t_{ik} + st_k + \min_m(t_{mj} + t_{km} + st_m) > T, a_{ij}^{km} = 0$ for all $m.$*

Proof Assume that, for some $i, j, k,$ and $m, a_{ij}^{km} = 1$. Clearly, the flow originated at i and delivered to j is routed via hubs k and m . Without loss of generality, we simply suppose that the flow path between i and j is two stops. Therefore, from constraints (33), (34), and (35), if $slo_i + t_{ik} + st_k + \min_m(t_{mj} + t_{km} + st_m)$ as the worst case of routing via hubs k and m is greater than the service time, the model will be infeasible. Hence, a_{ij}^{km} should be zero in this condition. □

5 Computational experiments

5.1 Dataset

For the computational studies, we use two commonly used test beds for HLPs. The first dataset is CAB, which is extracted from Civil Aeronautics Board for airline passengers. This set of problems was introduced by O’kelly (1987). The x -node problem, which is obtained by considering the first x nodes from all the nodes, is employed. The second dataset, called AP, was introduced into the literature by Ernst and Krishnamoorthy (1996). The AP dataset was formed by a real application in a postal delivery system. A characteristic of this dataset was that the flow matrix was not symmetrical.

Meanwhile, some parameters such as fixed costs, travel time, maximum time for leaving from the nodes, minimum flow that is picked, service time at hub nodes, and piecewise linear functions are not a part of the datasets. We produce these parameters in the following ways. The fixed costs are calculated and scaled for CAB and AP by a method derived from Gelareh (2008). We consider $F_k = 2.5 \times 10^7, FH_{km} = 500c_{km},$ and $FS_{ij} = FD_{ij} = 200c_{km}$ as well as $F_k = 5 \times 10^3, FH_{km} = 200c_{km},$ and $FS_{ij} = FD_{ij} = 100c_{km}$ for CAB and AP, respectively. The travel times (hour) are set at $c_{ij}/6000$ and c_{ij} for CAB and AP, respectively. The flow is also divided by L to rescale it to daily flow. The service times are randomly generated between 0 and 1 h for these datasets. The maximum time for leaving from the nodes for all the problems is 8. The minimum flow, which is picked from each node $i,$ randomly generated from a uniform probability distribution with the parameters $(0, 0.4 \sum_j w_{ij}).$ For each node in CAB and AP, the illustrated piecewise linear function in Fig. 3 is used. We take $\alpha \in \{0.5, 0.6, 0.7, 0.8, 0.9\}$ for CAB and AP datasets.

Moreover, in this study, we use a new dataset called Iranian Roads Network (IRN) which contains 20 nodes. The travel time (minute) and the distance between the cities are extracted from Google Map. Moreover, the fixed cost of hubs is calculated by the procedure introduced by Tan and Kara (2008) for the Turkish network. In this procedure, we consider only the population of each city, land price, and industrialization level of each city which are collected

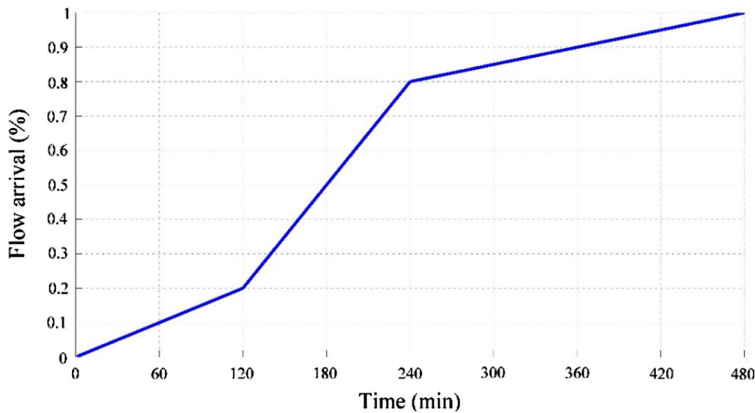


Fig. 3 The employed piecewise linear function for CAB and AP

from Statistical Center of Iran (<http://www.amar.org.ir/>). The travel costs between the nodes are considered as 1% of their distances. The flow matrix is calculated by $w_{ij} = 0.001 \times P_i \frac{P_j}{\sum_{k=1}^{20} P_k - P_i}$ for each pair of cities, in which P_i is the population of the i^{th} city. Moreover, we consider $FH_{km} = 100c_{km}$ and $FS_{ij} = FD_{ij} = 80c_{km}$. Furthermore, the employed piecewise linear function for IRN is shown in Fig. 3. The minimum flow picked from each city is set at 100. In addition, the maximum time for receiving flow for each city is set at 480 min. We take $\alpha \in \{0.5, 0.6, 0.7, 0.8, 0.9\}$ and $T \in \{1620, 1800, 1980, 2160\}$. The IRN dataset is available at <http://wp.kntu.ac.ir/hkarimi/files/IRN.rar>. Additionally, we run the model on a server with a dual core 2.53 GHz Intel processor and 4 GB of RAM and commercial solver CPLEX, version 12.2.

5.2 Preprocessing performances

We put 5400 s time bound on CPLEX. Tables 2, 3, 4, 5, 6, 7 and 8 summarize the performance results. They have four main components. The first three columns describe the problem number, discount factor, and maximum service time level. The second five columns give the optimum solution characteristics such as number of hubs, hub links, spoke links, non-hub links, and optimum objective function value (OFV). In the next two columns, the CPLEX results of the model without our suggested preprocessing are illustrated. This type of run is referred to as type 1. The number of nodes in the branch and cut tree and the solution time are reported in these columns. The final main component of these tables gives the same results which are achieved using the model with preprocessing, called type 2. In AP with $T = 40$ in each discount factor value, Model 1 cannot obtain the optimum solution in the predefined solution time bound. Therefore, the results are illustrated at 5400 s.

As reported in Tables 2, 3, 4, 5, 6, 7 and (8), on average, the solution times for CAB, AP, and IRN are reduced by 75.57, 80.41, and 82.96%, respectively, using all preprocessing. Moreover, averagely, the number of nodes in the branch and cut tree is decreased by 90.87, 95.67, and 95.86% using type 2 for CAB, AP, and IRN, respectively.

The total number of non-fixed variables before preprocessing for CAB and AP with 10 nodes is 12,470. For example, when coverage time bound is 40 for AP, the number of non-fixed variables is decreased by about 60%. To conclude from these Tables 2, 3, 4, 5, 6 and

Table 2 Results for the CAB dataset with type 1 and type 2 (part 1)

Problem number	N	α	T	Optimal solution				Type 1		Type 2		
				No. of hubs	No. of hub links	No. of spoke links	No. of non-hub links	OFV	No. of nodes of branch and cut tree	Solution time (s)	No. of nodes of branch and cut tree	Solution time (s)
1	10	0.5	25	6	10	8	3	542,081,190.957	65	68.571	0	4.062
2	10	0.5	27	6	10	8	3	542,081,190.957	85	34.731	0	3.708
3	10	0.5	30	6	10	8	3	542,081,190.957	48	33.991	0	3.828
4	10	0.5	32	6	10	8	3	542,081,190.957	42	34.743	0	3.016
5	10	0.5	35	6	10	8	3	542,081,190.957	43	35.147	0	2.235
6	10	0.6	25	6	10	8	4	594,227,093.681	87	62.632	0	3.782
7	10	0.6	27	6	10	8	4	594,227,093.681	64	79.428	0	3.718
8	10	0.6	30	6	10	8	4	594,227,093.681	79	58.742	0	3.594
9	10	0.6	32	6	10	8	4	594,227,093.681	73	53.742	0	2.610
10	10	0.6	35	6	10	8	4	594,227,093.681	49	32.531	0	2.078
11	10	0.7	25	5	7	10	7	641,295,940.434	237	31.091	25	15.563
12	10	0.7	27	5	7	10	7	641,295,940.434	215	29.522	21	17.110
13	10	0.7	30	5	7	10	7	641,295,940.434	195	27.628	18	15.516
14	10	0.7	32	5	7	10	9	640,924,329.850	104	29.525	0	12.969
15	10	0.7	35	5	7	10	9	640,924,329.850	98	36.560	0	9.343
16	10	0.8	25	2	1	10	26	661,551,158.748	190	35.648	226	25.765
17	10	0.8	27	2	1	9	26	658,072,364.600	164	33.125	78	22.875
18	10	0.8	30	2	1	10	26	657,374,740.858	85	28.245	0	13.734
19	10	0.8	32	2	1	10	26	657,374,740.858	73	29.548	27	19.500
20	10	0.8	35	2	1	10	26	657,374,740.858	48	28.246	29	14.797
21	10	0.9	25	2	1	16	25	669,078,208.471	182	30.972	106	21.703

Table 2 continued

Problem number	N	α	T	Optimal solution				Type 1			Type 2		
				No. of hubs	No. of hub links	No. of spoke links	No. of non-hub links	OFV	No. of nodes of branch and cut tree	Solution time (s)	No. of nodes of branch and cut tree	Solution time (s)	
22	10	0.9	27	2	1	10	27	668,244,406.376	152	29,942	75	20,594	
23	10	0.9	30	2	1	10	28	666,975,281.155	96	29,645	56	20,047	
24	10	0.9	32	2	1	11	28	666,293,014.726	79	29,362	19	17,734	
25	10	0.9	35	2	1	11	28	666,293,014.726	79	28,392	45	12,625	

Table 3 Results for the CAB dataset with type 1 and type 2 (part 2)

Problem number	N	α	T	Optimal solution						Type 1			Type 2		
				No. of hubs	No. of hub links	No. of spoke links	No. of non-hub links	OFV	No. of nodes of branch and cut tree	Solution time (s)	No. of nodes of branch and cut tree	Solution time (s)			
26	20	0.5	30	19	66	2	0	3,031,928,290,541	5842	3134.814	0	86.860			
27	20	0.5	32	19	66	2	0	3,031,928,290,541	6214	3458.015	0	101.281			
28	20	0.5	35	19	66	2	0	3,031,928,290,541	4852	2841.941	0	116.015			
29	20	0.5	37	19	67	2	0	3,031,904,691,645	4758	2724.184	0	141.234			
30	20	0.5	40	19	67	2	0	3,031,904,691,645	5	482.214	0	108.031			
31	20	0.6	30	17	58	9	2	3,529,623,928,771	5410	2048.641	0	82.657			
32	20	0.6	32	17	58	9	2	3,529,623,928,771	4821	2081.814	0	95.217			
33	20	0.6	35	17	58	9	2	3,529,623,928,771	6012	2492.015	0	121.391			
34	20	0.6	37	17	59	9	2	3,529,370,119,701	5910	2383.648	0	117.703			
35	20	0.6	40	17	59	9	2	3,529,370,119,701	6	503.281	0	110.312			
36	20	0.7	30	16	58	15	5	4,019,924,614,594	5109	2966.842	0	99.203			
37	20	0.7	32	16	58	15	5	4,019,924,614,594	6189	3483.301	0	112.524			
38	20	0.7	35	16	58	15	5	4,019,924,614,594	4236	2577.090	0	125.328			
39	20	0.7	37	16	59	15	5	4,019,483,347,929	6184	3660.014	0	108.500			
40	20	0.7	40	16	59	15	5	4,019,483,347,929	9	596.984	0	124.469			
41	20	0.8	30	15	54	25	7	4,500,528,869,682	6005	3295.570	0	108.750			
42	20	0.8	32	15	54	25	7	4,500,528,869,682	4219	2228.641	0	128.325			
43	20	0.8	35	15	54	25	7	4,500,528,869,682	6489	3842.281	0	163.422			
44	20	0.8	37	15	55	25	7	4,499,968,549,549	5249	2237.184	0	149.547			
45	20	0.8	40	15	55	25	7	4,499,685,149,549	6	513.004	0	138.125			
46	20	0.9	30	8	19	44	36	4,922,210,917,913	8234	5400.000	993	3119.157			

Table 3 continued

Problem number	N	α	T	Optimal solution				Type 1			Type 2		
				No. of hubs	No. of hub links	No. of spoke links	No. of non-hub links	OFV	No. of nodes of branch and cut tree	Solution time (s)	No. of nodes of branch and cut tree	Solution time (s)	
47	20	0.9	32	8	19	44	37	4,921,991,675.743	5649	5024.578	2926	1434.125	
48	20	0.9	35	8	19	44	36	4,920,907,525.723	4682	5400.000	910	2605.422	
49	20	0.9	37	8	20	44	37	4,919,175,957.088	5492	3514.000	15	259.531	
50	20	0.9	40	8	20	44	37	4,918,890,973.220	10	614.271	0	224.354	

Table 4 Results for the CAB dataset with type 1 and type 2 (part 3)

Problem number	N	α	T	Optimal solution					Type 1			Type 2		
				No. of hubs	No. of hub links	No. of spoke links	No. of non-hub links	OFV	No. of nodes of branch and cut tree	Solution time (s)	No. of nodes of branch and cut tree	Solution time (s)		
51	25	0.5	30	24	99	2	0	4,595,273,390,597	423	4215.084	0	268.157		
52	25	0.5	32	24	98	2	0	4,594,675,387,681	513	5400.000	0	658.251		
53	25	0.5	35	24	98	2	0	4,594,675,387,681	498	5400.000	0	698.215		
54	25	0.5	37	24	99	2	0	4,594,651,788,786	628	5400.000	0	8425.124		
55	25	0.5	40	24	100	2	0	4,593,993,284,335	637	5400.000	0	1007.218		
56	25	0.6	30	24	102	2	0	5,385,926,018,742	381	5014.521	0	700.581		
57	25	0.6	32	24	100	2	0	5,384,907,935,254	628	5400.000	0	763.154		
58	25	0.6	35	24	101	2	0	5,384,899,106,617	518	5400.000	0	851.271		
59	25	0.6	37	24	102	2	0	5,384,657,713,541	681	5400.000	0	1002.284		
60	25	0.6	40	24	103	2	0	5,383,498,547,325	592	5400.000	0	1247.258		
61	25	0.7	30	22	92	10	1	6,170,301,179,139	418	4932.287	0	657.218		
62	25	0.7	32	22	91	10	1	6,170,019,798,879	657	5400.000	0	725.012		
63	25	0.7	35	22	95	10	1	6,169,601,241,633	524	5400.000	0	924.218		
64	25	0.7	37	22	95	10	1	6,169,135,443,723	638	5400.000	0	1125.014		
65	25	0.7	40	22	96	10	1	6,167,926,683,253	618	5400.000	0	1451.025		
66	25	0.8	30	21	91	18	3	6,944,243,179,226	581	5107.257	0	754.218		
67	25	0.8	32	21	90	18	3	6,942,769,142,780	598	5400.000	0	752.958		
68	25	0.8	35	21	90	18	3	6,942,672,702,944	682	5400.000	0	840.812		
69	25	0.8	37	21	91	18	3	6,942,112,382,811	673	5400.000	0	981.257		
70	25	0.8	40	20	88	17	5	6,940,678,789,325	619	5400.000	0	1249.957		
71	25	0.9	30	14	54	52	31	7,666,976,648,284	539	5371.571	0	847.259		

Table 4 continued

Problem number	N	α	T	Optimal solution				Type 1			Type 2		
				No. of hubs	No. of hub links	No. of spoke links	No. of non-hub links	OFV	No. of nodes of branch and cut tree	Solution time (s)	No. of nodes of branch and cut tree	Solution time (s)	
72	25	0.9	32	14	52	31	7,664,025,425,382	628	5400.000	0	861.325		
73	25	0.9	35	14	53	29	7,659,780,000,821	619	5400.000	20	1002.547		
74	25	0.9	37	14	53	28	7,655,814,613,251	600	5400.000	158	1184.284		
75	25	0.9	40	13	44	31	7,655,803,207,193	681	5400.000	210	1521.484		

Table 5 Results for the AP dataset with type 1 and type 2 (part 1)

Problem number	N	α	T	Optimal solution					Type 1				Type 2			
				No. of hubs	No. of hub links	No. of spoke links	No. of non-hub links	OFV	No. of nodes of branch and cut tree	Solution time (s)	No. of nodes of branch and cut tree	Solution time (s)	No. of nodes of branch and cut tree	Solution time (s)		
1	10	0.5	40	5	5	12	0	100,251.634	17,768	5400.000	144	16.442				
2	10	0.5	42	5	4	11	0	95,511.384	23,957	3631.743	292	22.643				
3	10	0.5	45	3	2	10	3	87,232.843	26,679	3039.090	764	48.114				
4	10	0.5	47	3	3	10	2	86,321.218	9	89.532	0	10.432				
5	10	0.5	50	3	3	8	2	83,199.810	5	16.856	0	3.930				
6	10	0.6	40	5	5	12	0	102,993.808	18,034	5400.000	315	22.102				
7	10	0.6	42	4	4	11	2	98,501.280	10,242	3021.245	215	23.444				
8	10	0.6	45	3	2	11	3	89,740.350	5428	562.142	512	32.410				
9	10	0.6	47	3	3	10	2	88,561.462	42	100.245	0	9.024				
10	10	0.6	50	3	3	10	2	88,561.462	23	23.114	0	3.989				
11	10	0.7	40	5	5	12	0	105,723.964	18,245	5400.000	436	27.408				
12	10	0.7	42	4	4	11	2	101,068.350	2062	2952.632	236	25.532				
13	10	0.7	45	2	1	11	3	90,932.064	1462	321.854	150	23.144				
14	10	0.7	47	3	3	10	2	90,801.706	56	168.422	0	9.742				
15	10	0.7	50	3	3	8	2	88,398.583	35	40.233	0	4.213				
16	10	0.8	40	5	5	12	0	108,454.119	18,872	5400.000	812	45.012				
17	10	0.8	42	4	3	12	2	103,234.119	1052	3512.152	301	25.147				
18	10	0.8	45	3	2	11	3	93,443.660	352	124.124	64	18.642				
19	10	0.8	47	2	1	10	5	92,629.500	42	94.278	0	8.674				
20	10	0.8	50	2	1	10	6	92,413.452	28	38.105	0	5.014				
21	10	0.9	40	5	5	12	0	111,184.275	19,428	5400.000	4129	154.891				

Table 5 continued

Problem number	N	α	T	Optimal solution				Type 1		Type 2		
				No. of hubs	No. of hub links	No. of spoke links	No. of non-hub links	OFV	No. of nodes of branch and cut tree	Solution time (s)	No. of nodes of branch and cut tree	Solution time (s)
22	10	0.9	42	4	3	12	2	105,364.427	904	3963.542	342	29.522
23	10	0.9	45	2	1	12	3	93,194.627	285	62.951	44	14.791
24	10	0.9	47	2	1	10	5	93,766.082	32	55.315	0	7.005
25	10	0.9	50	2	1	11	3	91,387.382	20	31.758	0	4.385

Table 6 Results for the AP dataset with type 1 and type 2 (part 2)

Problem number	N	α	T	Optimal solution				Type 1				Type 2			
				No. of hubs	No. of hub links	No. of spoke links	No. of non-hub links	OFV	No. of nodes of branch and cut tree	Solution time (s)	No. of nodes of branch and cut tree	Solution time (s)	No. of nodes of branch and cut tree	Solution time (s)	
26	20	0.5	55	3	2	18	0	108,190,052	845	5400,000	49	2015.215			
27	20	0.5	57	3	3	18	0	105,697,712	924	5400,000	28	1942.286			
28	20	0.5	60	3	3	17	0	103,693,102	948	5400,000	0	201.214			
29	20	0.5	62	3	3	17	0	103,693,102	1002	5400,000	0	184.246			
30	20	0.5	65	3	3	17	0	103,693,102	952	5400,000	0	180.214			
31	20	0.6	55	3	2	18	0	110,788,891	892	5400,000	64	2251.280			
32	20	0.6	57	3	3	18	0	108,374,710	946	5400,000	27	2000.021			
33	20	0.6	60	3	3	17	0	106,481,062	951	5400,000	0	254.128			
34	20	0.6	62	3	3	17	0	106,481,062	1000	5400,000	0	134.289			
35	20	0.6	65	3	2	18	0	105,426,024	934	5400,000	0	152.017			
36	20	0.7	55	3	2	19	0	113,350,765	905	5400,000	72	1987.000			
37	20	0.7	57	3	3	18	0	111,051,714	895	5400,000	42	1057.571			
38	20	0.7	60	3	3	17	0	109,269,022	962	5400,000	0	271.587			
39	20	0.7	62	3	3	17	0	109,269,022	948	5400,000	0	214.571			
40	20	0.7	65	3	2	19	0	108,198,289	1021	5400,000	0	235.715			
41	20	0.8	55	3	2	21	0	115,588,040	951	5400,000	81	1837.257			
42	20	0.8	57	3	3	20	0	113,651,388	984	5400,000	48	1257.579			
43	20	0.8	60	3	3	19	0	111,824,903	965	5400,000	0	266.496			
44	20	0.8	62	3	3	19	0	111,824,903	915	5400,000	0	218.240			
45	20	0.8	65	3	2	19	0	110,889,343	1005	5400,000	0	189.289			
46	20	0.9	55	3	2	21	0	117,615,000	983	5400,000	89	2071.648			

Table 6 continued

Problem number	N	α	T	Optimal solution				Type 1		Type 2		
				No. of hubs	No. of hub links	No. of spoke links	No. of non-hub links	OFV	No. of nodes of branch and cut tree	Solution time (s)	No. of nodes of branch and cut tree	Solution time (s)
47	20	0.9	57	3	3	20	0	115,928,323	986	5400.000	62	1642.751
48	20	0.9	60	3	3	19	0	114,187,443	957	5400.000	0	260.289
49	20	0.9	62	3	3	19	0	114,187,443	995	5400.000	0	228.318
50	20	0.9	65	3	2	20	0	113,420,361	1000	5400.000	0	220.205

Table 7 Results for the AP dataset with type 1 and type 2 (part 3)

Problem number	N	α	T	Optimal solution				Type 1			Type 2		
				No. of hubs	No. of hub links	No. of spoke links	No. of non-hub links	OFV	No. of nodes of branch and cut tree	Solution time (s)	No. of nodes of branch and cut tree	Solution time (s)	
51	25	0.5	60	5	5	25	0	120,124,248	893	5400,000	67	3284,340	
52	25	0.5	62	5	5	26	0	118,021,215	921	5400,000	54	2482,394	
53	25	0.5	65	5	5	26	0	115,915,317	937	5400,000	22	1983,312	
54	25	0.5	67	5	4	27	0	114,315,288	918	5400,000	0	642,218	
55	25	0.5	70	5	4	25	0	114,231,418	999	5400,000	0	587,059	
56	25	0.6	60	5	5	25	0	123,214,021	897	5400,000	77	3950,378	
57	25	0.6	62	5	5	25	0	119,842,017	869	5400,000	62	2948,367	
58	25	0.6	65	5	5	26	0	117,812,472	950	5400,000	28	1875,592	
59	25	0.6	67	5	4	26	0	116,819,041	948	5400,000	0	657,230	
60	25	0.6	70	5	4	25	0	116,728,941	1002	5400,000	0	567,210	
61	25	0.7	60	5	5	24	0	125,301,147	958	5400,000	88	3954,208	
62	25	0.7	62	5	5	25	0	120,984,178	927	5400,000	61	2978,150	
63	25	0.7	65	5	4	26	0	119,840,284	938	5400,000	24	1894,618	
64	25	0.7	67	5	4	25	0	118,215,374	918	5400,000	0	644,201	
65	25	0.7	70	5	4	25	0	118,021,574	892	5400,000	0	597,254	
66	25	0.8	60	5	5	25	0	128,412,424	789	5400,000	98	3199,318	
67	25	0.8	62	5	5	26	0	123,648,157	958	5400,000	71	2942,163	
68	25	0.8	65	5	4	26	0	121,640,281	934	5400,000	35	1876,618	
69	25	0.8	67	5	4	26	0	120,541,281	927	5400,000	0	567,417	
70	25	0.8	70	5	4	25	0	119,952,210	998	5400,000	0	384,510	
71	25	0.9	60	5	5	25	0	132,018,271	847	5400,000	97	3798,974	

Table 7 continued

Problem number	N	α	T	Optimal solution				Type 1		Type 2		
				No. of hubs	No. of hub links	No. of spoke links	No. of non-hub links	OFV	No. of nodes of branch and cut tree	Solution time (s)	No. of nodes of branch and cut tree	Solution time (s)
72	25	0.9	62	5	5	26	0	127,317,051	869	5400.000	49	2978.482
73	25	0.9	65	5	4	26	0	125,701,914	948	5400.000	26	1975.462
74	25	0.9	67	5	4	25	0	123,417,270	912	5400.000	0	670.285
75	25	0.9	70	5	4	25	0	122,928,656	973	5400.000	0	457.284

Table 8 Results for the IRN dataset with Type 1 and Type 2

Problem number	α	T	Optimal solution				Type 1			Type 2		
			No. of hubs	No. of hub links	No. of spoke links	No. of non-hub links	OFV	No. of nodes of branch and cut tree	Solution time (s)	No. of nodes of branch and cut tree	Solution time (s)	
1	0.5	1620	7	9	19	0	151,679.556	856	5400	120	3025.254	
2	0.5	1800	7	9	15	0	146,882.296	735	5400	0	357.324	
3	0.5	1980	7	9	15	0	146,882.296	815	5400	90	2092.401	
4	0.5	2160	7	9	14	0	146,731.184	820	5400	0	215.247	
5	0.6	1620	4	5	25	4	159,103.273	725	5400	115	2987.548	
6	0.6	1800	4	5	22	4	156,272.851	698	5400	0	364.128	
7	0.6	1980	4	5	22	4	156,272.851	814	5400	0	341.987	
8	0.6	2160	4	5	21	4	156,231.667	832	5400	0	254.271	
9	0.7	1620	4	5	24	4	165,427.075	805	5400	118	1972.547	
10	0.7	1800	4	5	22	4	164,002.611	782	5400	0	341.157	
11	0.7	1980	4	5	22	4	164,002.611	789	5400	0	267.021	
12	0.7	2160	4	5	21	4	163,940.832	792	5400	0	194.250	
13	0.8	1620	4	5	25	4	172,531.601	782	5400	116	2578.971	
14	0.8	1800	4	5	21	4	170,927.823	754	5400	0	321.547	
15	0.8	1980	4	5	21	4	170,927.823	803	5400	0	284.581	
16	0.8	2160	4	5	21	4	170,880.022	816	5400	0	154.289	
17	0.9	1620	4	5	26	4	178,464.425	820	5400	102	2015.412	
18	0.9	1800	2	1	22	12	175,075.024	800	5400	0	294.348	
19	0.9	1980	2	1	22	12	175,075.024	782	5400	0	189.284	
20	0.9	2160	2	1	22	12	175,075.024	768	5400	0	152.248	

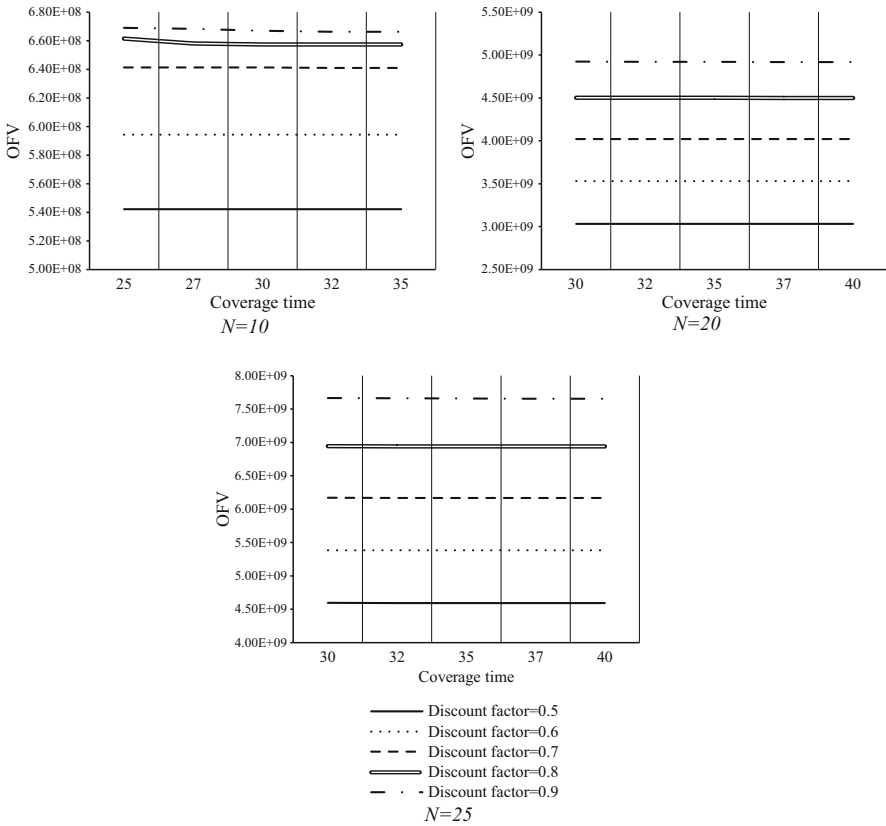


Fig. 4 OFV performances in terms of discount factor and coverage time for CAB

7, in the tight problems (i.e. the problem including less service time), the solution time and number of nodes in the branch and cut tree are significantly improved by type 2. Therefore, we can also conclude that the linear relaxation is improved significantly with the inclusion of the preprocessing.

5.3 Impact of maximum coverage service time and discount factor

Even though the primary emphasis of the FSSIHLRP is the total cost minimization, other interesting insights can be gained for by a comparison to the coverage service time measure. In general, a solution yielding the minimum total cost is not obtained by the minimum coverage service time. In fact, these two parameters are in conflict. A loss in one causes a gain in the other. Therefore, in the light of this conflict, in Figs. 4, 5, and 6, for CAB, AP with different sizes, and IRN, the effect of service time and also the discount factor on the OFV is described. In Fig. 6, the lines with 1800 and 1980 coverage time are the same, but the line with 2160 coverage time is different. However, due to the resolution of the chart, these three lines lay on each other.

The direct impact of higher coverage time is a reduction in the OFV of a network. Therefore, decision makers should consider this conflict in their problems. Moreover, it is interesting to note that a higher discount factor yields a higher OFV. This impact has been

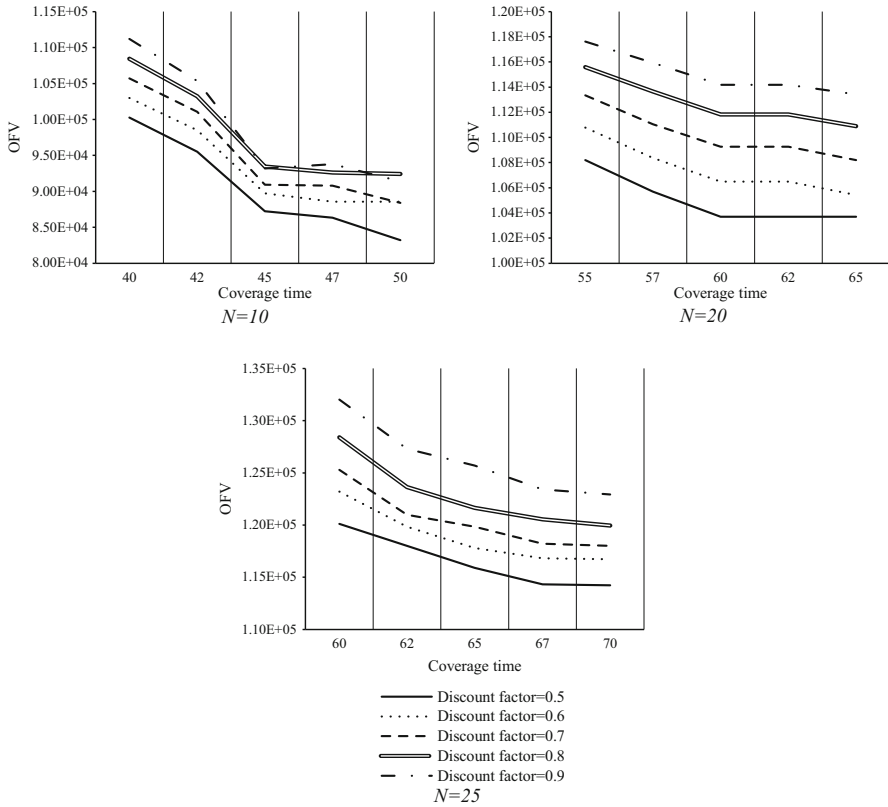


Fig. 5 OFV performances in terms of discount factor and coverage time for AP

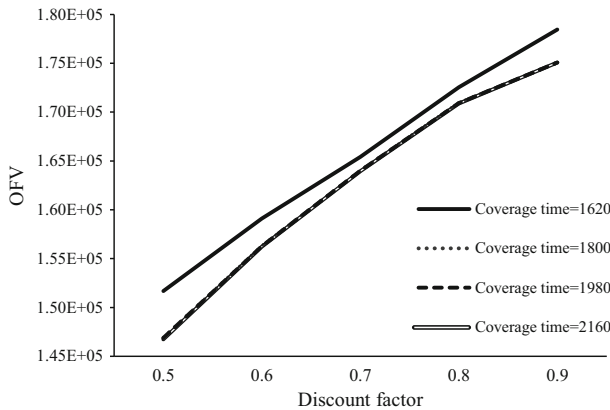


Fig. 6 OFV performances in terms of discount factor and coverage time for IRN

discussed more in the hub location literature by O’Kelly and Bryan (1998). In addition, as an interesting observation for all problems of IRN, the scheduled leaving time from the cities close to the border such as Bandar Abbas, Tabriz, and Zahedan is much more than that in

the cities close to the center of Iran such as Arak, Isfahan, and Yazd. Furthermore, in this dataset, by increasing the discount factor, the number of hubs is decreased. As another additional insight, which is gained from the study of IRN, number of non-hub links is increased by increasing discount factor. Further analysis showed that by increasing the service time, the number of spoke links is decreased. Consequently, there is a significant negative relation between the number of spoke links and non-hub links for IRN. As a surprising observation, in all problems of IRN, the hub location at Tehran survives, while the remaining hub locations switch from a city to another city by changing the discount factor. In addition, as a striking observation, for all problems of IRN except for $\alpha = 0.5$, the hub networks topology are tree. Furthermore, in IRN, with the exception of $\alpha = 0.9$, no change in hub network topology is detected. As another interesting observation, the created network in $\alpha = 0.6$, $T = 1620$ is the same as $\alpha = 0.8$, $T = 1620$. The difference in the OFVs is due to the difference in the discount factors.

6 Conclusion

The purpose of the current study was to introduce a mathematical formulation that decides on the locations of hubs in an incomplete hub network topology, the allocations of non-hubs to hubs, non-hubs to non-hubs and the departure time from nodes simultaneously to improve service quality at minimum cost. Additionally, some preprocessing was introduced based on the new assumptions. In FSSIHLRP, the scheduled leaving time from the origin node and the departure time from a hub to a node were determined. In this study, a model and some tools were provided for decision makers to consider the maximum service time in their companies. To evaluate the proposed preprocessing, the well-known CAB and AP datasets and a new dataset called IRN were used. The results showed the addition of these tools was better than solving the model without them both in terms of reducing the CPU time and the used node in the branch and cut tree. In conclusion, using these tools caused a faster and efficient branch and cut algorithm.

The generalizability of these results is subject to certain limitations. For instance, no consideration of the effect of the OFV on the service time and vice versa and not including the effects of time zone in the problem, since this network is a large one like a global network. Overcoming these restrictions will lead to more real-world and challenging future works.

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