




# Optimized Capacitor Placement Considering Load and Network Variability

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## Abstract

This paper proposes a methodology for the optimized location and sizing of capacitor banks in distribution networks. The variabilities of system load and grid configuration are both considered. Cluster analysis on daily load curves is employed to model the load variability adequately, while changes in the grid configuration concern feeders' reconfigurations associated with load transfers among them. Voltage profile improvement, energy loss reduction, and minimization of installation costs are pursued objectives, selected here for planning the investment in capacitor banks. The problem is combinatorial in nature and its solution using a genetic algorithm is proposed. Tests with a real distribution system are performed, exploring the knowledge about common network switching operations and employing system loading measurement data. The obtained results show the importance of adequately considering, during the planning phase, loading and topological conditions to which the distribution system will be submitted during operation.

**Keywords** Capacitor placement · Voltage regulation · Optimization · Power system

## 1 Introduction

Distribution systems have experienced fast and significant changes in the last decades, making operation/planning problems more complex. The allocation of capacitor banks in distribution grids aims to reduce power losses and provide voltage profile control. The optimal capacitor placement problem has been the subject of many studies in the technical literature, in which the best locations/ratings of capacitor banks to be installed are determined. Aspects such as power loss reduction, control of voltage profile, and minimization of investment costs are usually sought. Some of these objec-

tives are clearly antagonistic and the optimal solution will be the one in which the best trade-off among them is achieved.

Different approaches have been proposed to solve the optimal capacitor placement problem by employing analytical methods and optimization techniques (Ng et al. 2000; Gallego et al. 2001; Bala et al. 1995; Lee and El-Sharkawi 2002). A constructive heuristic can be found in (Silva et al. 2008). Aiming to deal with the combinatorial nature and complexity of the problem (which is in general multi-objective, multimodal, nonlinear, discontinuous, or non-convex), metaheuristic-based approaches have also been investigated. (El-Fergany and Abdelaziz 2014a, b) proposes a bee colony-based algorithm for the optimal allocation problem. An ant colony search is adopted by Chang for finding optimal capacitor locations and optimal network reconfiguration to reduce power losses (Chang 2008). The use of the plant growth algorithm is proposed in Huang and Liu (2012), in which the reduction in carbon dioxide emissions is modeled.

The use of loss sensitivity analysis for pre-selecting candidate locations for capacitor placement was investigated in (Abul'Wafa 2013; El-Fergany and Abdelaziz 2014a, b; Abou El-Ela et al. 2016). Reference (Xu et al. 2013) assumes that capacitors can only be installed at the low voltage side of

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distribution transformers. The simultaneous optimization of the capacitor placement and grid configuration is addressed in Farahani et al. (2012). Nonlinear loads and the total harmonic distortion (established as the objective of the problem) are considered by Sayadi et al. (2016). The optimization of capacitor placement and conductor resizing has also been subject of investigation (Farahani et al. 2013).

Some works attempted to incorporate load uncertainty (Jannat and Savic 2016; Carpinelli et al. 2012) and distributed generation (Jannat and Savic 2016; Mukherjee and Goswami 2014; Gopiya Naik et al. 2013) into the optimal capacitor placement problem. Multigrid systems with islanded operation capability are considered in the work of Farag and El-Saadany (2015). In Kaur and Sharma (2013) the problem is solved for multiple periods of load growth. The distribution grid reliability and economic aspects are studied (Rahmani-Andebili 2015). A crow search algorithm is proposed for capacitor placement (Askarzadeh 2016) considering a single load and topology scenario. Araújo et al. (2018) presented an interesting approach for capacitor allocation in unbalanced distribution systems. However, the authors adopt a daily load curve characterized by a given number of arbitrarily defined representatives. Besides, the topology variability is not included in that study. In Moradian et al. (2019), a genetic algorithm is employed to allocate switched capacitors aiming to maximize the net saving taking into consideration technical constraints of the distribution network. Montazeri and Askarzadeh (2019) present a study that proposes in which a power loss index to identify high potential busses for capacitor placement. Reference (Melgar-Dominguez et al. 2019) proposes the optimal allocation of capacitor banks and voltage regulators so that the costs of energy supply and carbon emission tax are minimized. A simplified representation of the annual load variability is considered. Capacitor banks and network configuration are jointly optimized in Home-Ortiz et al. (2019). However, capacitor banks are optimally placed for a single network configuration that is also optimally determined. Regarding load variability, three load levels (light, medium and heavy) and corresponding durations were arbitrarily defined.

In summary, most of the methods available in the literature so far consider only some specific pre-defined load conditions and a single network configuration when planning capacitor placement in distribution networks. Some facets of the problem, regarding the trade-off among conflicting objectives as well as the representation of network topology variability, still need further modeling and investigation. In this line, the present paper contributes proposing a methodology that employs a genetic algorithm (Glover and Kochenberger 2003; Mitchell 1996) to find optimized locations and ratings of capacitor banks to be placed in power distribution grids. The paper distinguishes itself by considering representative load scenarios and their corresponding

time durations, which are automatically extracted from daily load curves using *k*-means clustering (Pao 1989). System topology variability (grid reconfiguration) is also considered in the optimal capacitor placement. It should be noticed that the optimal reconfiguration of the distribution grid is not among the objectives of this work. The problem addressed here is the allocation of capacitor banks that minimizes power losses and meets operational constraints for different grid configurations, known in advance from the utility's records of usual switching operations. Thus, the attendance of different topology scenarios is considered during the planning phase. The proposed model is tested for a real distribution system, employing measurements of system loading and switching operations. The obtained results corroborate the importance of adequately considering during the planning phase, loading and topological conditions that the distribution system may experience during operation.

## 2 Optimal Allocation of Capacitor Banks

In Brazil and many countries, the electric power regulatory agency penalizes distribution utilities if violations of pre-defined voltage levels occur when supplying power to the consumers. The control of the voltage profile throughout the distribution network usually requires investments in capacitor banks, as well as the determination of their locations in the grid and corresponding ratings. The reduction in power losses in the grid is usually a consequence of voltage control. As a result, the amount of energy imported by the distribution utility to supply its consumers can be reduced, which in turn increases the utility's profit margin.

Objectives such as the minimization of investment costs in capacitor banks—to keep the voltage profile control throughout the grid and the reduction in power losses (with consequent energy savings)—can be formulated as an optimization problem, in which the decision variables are the locations and ratings of the capacitor banks to be installed. In general terms, the objective is to maximize the utility's profit, while the limits imposed on the voltage magnitudes throughout the grid should be respected:

$$\begin{aligned} & \text{Maximize NPR} = \text{ES} - \text{IC} \\ & \text{subject to operational constraints} \end{aligned} \quad (1)$$

where NPR represents the net profit increase achieved in a given time horizon, due to the installation of the capacitor banks; ES represents the income that results from imported energy savings (as a consequence of power loss reduction) in the same time horizon; and IC accounts for the investment costs in capacitor banks.

The operational constraints refer to the requirements dictated by the regulatory agency, to be met by the system loads

and voltage magnitudes throughout the grid. For any given operating scenario, the problem constraints can be checked after running a power flow program. This also enables the computation of the power loss reductions that result from the installation of capacitor banks.

Owing to the combinatorial nature of the optimal capacitor placement problem, a methodology that employs a metaheuristic is proposed in this work. It should also be noted that such a methodology can take advantage of the existing knowledge on the problem to be solved. The search for the optimal solution can be more effective and efficient if an adequate solution encoding is employed and/or when it is possible to intelligently reduce the search space. In the optimal capacitor placement problem, it is reasonable to assume that a limited number of capacitor banks will be present in the optimal solution. Then, the search space can be reduced by assuming that only solutions in which the number of capacitor banks does not exceed a pre-defined quantity (selected according to the experience/expertise of system engineers) are of interest. Adequately modeling of the system load, through the appropriate selection of representative load levels, may also help to enhance the methodology devoted to finding a solution for the optimal capacitor placement problem. Besides accurate load modeling, a methodology for the optimal allocation of capacitor banks should also take into account that the distribution system deals with different network topologies during its operation. Such aspects are addressed herein.

It is not the aim of this paper, from the computational point of view, to select the best metaheuristic to solve the problem of capacitor placement in distribution grids. Instead, the focus is on formulating the allocation of capacitor banks as an optimization problem, in which essential aspects, such as load and topology variability, are adequately considered during the search process. In this perspective, a genetic algorithm was selected due to its flexibility to accommodate almost any type of operation planning criterion (Glover and Kochenberger 2003; Goldberg 1989). The next sections present the proposed methodology and the results obtained when it is applied to a real distribution system.

### 3 Proposed Methodology

#### 3.1 Representation of Load Variability

An adequate load representation is crucial for the success of any methodology for capacitor placement in distribution networks. The representation of all demand levels, even on an hourly basis, is not possible due to the huge number of power flow analyses that would be required to assess each proposed solution. For example, considering load variations within 1 year, the adequacy of each proposed solution would

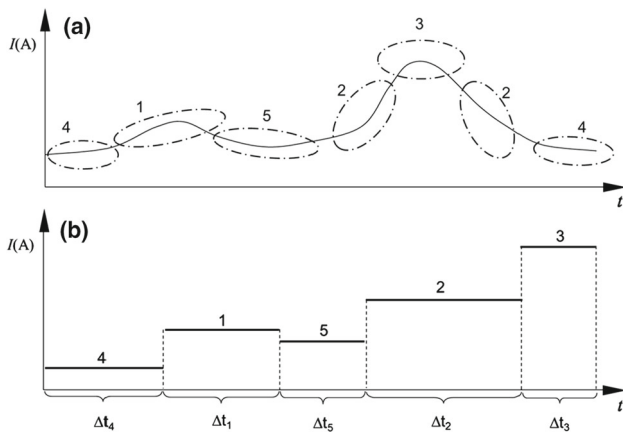
have to be analyzed for 8760 hourly load scenarios. In many cases, aiming to overcome such a computational burden, only two load levels are considered, corresponding to the minimum and maximum system loading observed for the time horizon of interest. In such cases, the main objective is to guarantee that voltage limits will not be violated when the capacitor banks are installed. However, as these load levels are not good representatives of the load variations, it is not possible to correctly evaluate how the investment in capacitor banks will affect power losses and if it will result in energy savings.

In this paper, adequate modeling of load variability is adopted, by extracting a limited number of load levels—good representatives of the different load scenarios that the system may experience. This allows a more accurate assessment of the power loss reduction and voltage control capability yielded by a given solution (represented by locations and ratings of the capacitor banks proposed for installation). It is important to have in mind that selecting many load levels to serve as load representatives may result in prohibitive computation times to obtain the final solution, as each proposed solution must be assessed for each load level during the search process. Then, it is desirable that a reduced number of load representatives be employed and that those are good enough to adequately represent load variability.

Considering a database in which  $nc$  hourly loads are stored, the load representatives can be obtained by executing the following steps:

1. Define the number of clusters to be formed ( $k$  load representatives);
2. Select the first  $k$  load levels of the database as the initial centroids of the  $k$  clusters. The centroid of a cluster will always be a load level that corresponds to the arithmetic mean of the loads currently associated with that cluster;
3. Move each of the  $nc-k$  load levels remaining in the database to the cluster whose centroid is closer to it and recalculate the centroid of that cluster;
4. Check if all load levels in the database are already associated with the clusters whose centroids are closer to them. If a given load level is not associated with the correct cluster, it is moved to that cluster and the corresponding centroid is recalculated. The centroid of the cluster to which that load level was previously associated is also recalculated;
5. Repeat step (iv) until there are no more modifications in the  $k$  clusters, which means that  $k$  groups of loads have been formed based on their similarities.

After executing the above algorithm, it is possible to represent the load variability through  $k$  representative load levels, which are the  $k$  centroids that have been computed. Load representation is much more accurate with these centroids than



**Fig. 1** a load curve, b load representatives

that regarding only the maximum and minimum load levels. However, it is also important to represent such extreme load levels to check if operational constraints will be attended when the system experiences severe operating conditions. In this paper, system loading will be modeled by  $k+2$  load representatives, corresponding to the  $k$  computed centroids together with the maximum and minimum loading conditions.

Figure 1 illustrates how different load levels are clustered, taking five load representatives ( $k=5$ ). Note that not only the magnitude of each load representative (centroid) is determined, but also its corresponding duration. If hourly load levels are stored in the database, the duration of each load representative corresponds, in hours, to the number of loads associated with the cluster being represented. This results in an even more accurate load representation.

The number of load representatives ( $k$ ) to be adopted should be defined having in mind a trade-off between the accuracy of the representation of load variability and the computational effort involved in the search for the optimal solution.

### 3.2 Fitness Function

As indicated in Eq. (1), the objective function of the problem is the maximization of the utility's net profit due to the installation of capacitor banks (and consequent power loss reduction), which is subjected to the limitations imposed on the voltage magnitudes throughout the network. The computation of the net profit should consider the income generated by energy savings (reduction in the energy imported from other utilities) in a given time horizon and the investment cost made in capacitor banks. Note that these two objectives

are mutually opposed and can be assessed by the fitness function presented next, to be maximized.

$$FF = \left[ \alpha_1 \left( \sum_{i=1}^k \Delta P_i \times h_i \right) \times t_c - \alpha_2 C_{Bcap} \right] - \beta \sum_{i=1}^{np} \left( \sum_{j=1}^{nb} |\Delta V_j| \right)_i \quad (2)$$

where  $nb$ —no. of network buses;  $k$ —no. of load representatives;  $h_i$ —duration of the  $i$ -th load representative;  $t_c$ —price of the energy imported by the distribution utility;  $\Delta P_i$ —power loss reduction obtained for the  $i$ -th load representative;  $np$ —no. of load levels, including the representatives, minimum, and maximum ( $np=k+2$ );  $\Delta V_j$ —magnitude of the voltage violation at bus  $k$  (if any);  $C_{Bcap}$ —total investment cost in capacitor banks;  $\alpha_1$ ,  $\alpha_2$ ,  $\beta$ —weights (penalty factors).

These weights are used to establish a trade-off between the problem objectives. The first two terms in Eq. (2), between square brackets, account for the determination of the net profit increase, considering the economy due to energy savings and the investment costs in capacitor banks. The third term refers to global voltage violations for all load representative scenarios. As distribution utilities are severely penalized by the regulatory agency when voltage violations occur, the penalties  $\alpha_1$ ,  $\alpha_2$  and  $\beta$  should be adjusted so that the third term of Eq. (2) tends to vanish during the search process, while the difference between the first and second terms is maximized.

It should be observed that the time horizon of interest to evaluate the net profit increase obtained with the installation of capacitor banks is automatically considered in the first term of Eq. (2), as it is embedded in the load representation employed, particularly in the duration of each load representative. For example, if the load representatives are extracted from a database in which a load curve of 1 year is stored on an hourly basis, the sum of the duration of all load representatives will be 8760 h. In that case, the net profit obtained in 1 year will be computed.

### 3.3 Solution Encoding

In this work, without loss of generality, it is assumed that fixed capacitor banks—ratings of 300 kvar, 600 kvar, and 1200 kvar—can be installed at buses of a distribution feeder. It is possible to encode a solution for the allocation of capacitor banks in a binary vector of dimension equal to two times the number of the distribution feeder buses. In such a vector, each pair of consecutive elements is associated with a specific bus location and represents the associated capacitor rating

$$n_{max} \leq nb$$

05	12	67	...	110	09	Location Rating			
0	1	1	0	0	...		1	0	0

Fig. 2 Solution encoding in a reduced solution space

according to the following codes: (00)—no capacitor bank proposed; (01)—300 kvar; (10)—600 kvar; (11)—1200 kvar.

In some cases, the allocation of capacitor banks at certain buses of the distribution network may not be possible due to physical or operational restrictions. In such situations, those buses are considered prohibited locations that can be removed from the solution encoding vector. Therefore, the size of the binary solution vector will be two times the number of feasible locations (buses).

It is possible to take advantage of the expertise and experience of distribution engineers in order to devise problem encodings that make the search for the optimal solution more efficient. For example, it is not likely that the optimal solution proposes the allocation of capacitor banks at all buses. Rather, a moderated number of buses are expected. Then, the dimension of the solution vector can be reduced as shown in Fig. 2, where  $n_{max}$  refers to the maximum number of buses at which capacitor banks can be installed. Note that, despite the limitation introduced by  $n_{max}$ , the capacitor banks can still be allocated at any feasible bus of the network. In a population of solutions, each of them proposes the allocation of a certain number of capacitor banks (up to  $n_{max}$ ). A candidate solution is then encoded using two associated vectors, as depicted in Fig. 2. The Location vector indicates the bus at which the capacitor bank will be installed, while the Rating vector indicates the corresponding rating. Therefore, the optimal solution will be the one that maximizes Eq. (2) through the allocation of capacitor banks in no more than  $n_{max}$  buses. This strategy reduces the search space and makes the optimization process more efficient. Besides, if the value of  $n_{max}$  is not underestimated, the global optimal solution will not be constrained by the restricted search space. If violations on operational constraints cannot be eliminated it is possible that  $n_{max}$  is underestimated and its value may be redefined. It should be noted that even if being conservative in the choice of  $n_{max}$ , significant gains can be achieved regarding the efficiency of the search process.

### 3.4 Representation of Topological Variability

In many cases, a distribution feeder supplies not only its own load but also the load transferred to it. Thus, it is desirable to take into account this possibility during the planning stage, to decide at which bus a capacitor bank will be placed. If such a scenario is not considered, voltage problems may

persist during system operation and power loss reductions may be incorrectly estimated. Hence, the fitness of a given candidate solution should be evaluated by analyzing system performance in different load and topology scenarios. This can be done by reformulating the fitness function of Eq. (2) as:

$$FF = \sum_{top=1}^{N_t} \left( p_{top} \cdot \left[ \alpha_1 \left( \sum_{i=1}^k \Delta P_i \cdot h_i \right) \cdot t_c \right] - \beta \sum_{i=1}^{np} \left( \sum_{k=1}^{nb} |\Delta V_k| \right)_{i, top} \right) - \alpha_2 C_{Bcap} \quad (3)$$

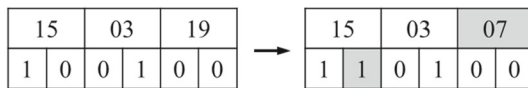
where  $N_t$ —no. of topology scenarios;  $top$ —topology under analysis;  $p_{top}$ —probability of occurrence of the  $top$ -th topology scenario;  $\Delta V_k$ —magnitude of the voltage violation at bus  $k$  (if any).

Note that  $p_{top}$  has been defined as the probability that the  $top$ -th topology scenario occurs, which can be, for example expressed in terms of the relative duration of this topology scenario (e.g., 0.5 = 6 months in 1 year). This information can be extracted from operators' expertise and/or from data recorded in the utility's historical database. It is important to observe that approximate estimates of such probabilities are likely to be good enough to take into account the topology scenarios of interest. Besides, if a given network topology seldom occurs, a very low probability value for such a scenario can be set in Eq. (3), which means that only the evaluation of voltage violations will be of interest for such scenario and the influence of power losses when operating in this condition will be neglected.

### 3.5 Genetic-Based Approach

A Genetic Algorithm (GA) is a population-based metaheuristic inspired in the neo-Darwinian model of evolutionary processes (Mitchell 1996). In addition to the selection and mutation operators widely adopted in evolutionary algorithms, GA makes use of a recombination operator (crossover) during the evolution. Originally intended to the study of self-adaptive systems, it is broadly employed in the solution of very complex problems, due to its intelligence and adaptation capabilities. Although very popular in optimization, GAs have been successfully used in artificial intelligence, pattern recognition, bioinformatics, and other fields.

GAs follow the propositions formulated by Holland (Goldberg 1989). An individual is a solution proposed for a given problem, represented by a chromosome. The chromosome encoding may differ from the original representation of a solution. The quality of the solution is assessed by a fitness function considering the problem objectives. During one



**Fig. 3** Mutation (shaded elements indicate mutated genes)

iteration of the evolutionary process (a generation), selection, and reproduction operations are applied to a set of candidate solutions, called population.

The generational loop can be described as follows:

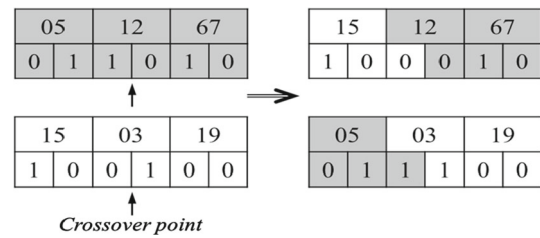
1. Randomly select from the current population individuals for reproduction according to their fitness;
2. Generate offspring by applying the crossover and mutation operators on the selected individuals;
3. Evaluate the offspring fitness;
4. Select survivals from the current population and offspring, and build the population of the next generation;

In general, many generations are necessary until the goals of the evolutionary process are satisfied. More details on traditional GAs can be found in the technical literature (Glover and Kochenberger 2003; Mitchell 1996). The methodology proposed in this paper employs the data structure presented in Sect. 3.3 to represent the chromosomes, in which every element of the location and rating vectors is a gene. Specialized crossover and mutation operators have been developed for the proposed encoding.

Mutation operator introduces changes in a single individual by randomly selecting and modifying its genes. In the mutation operation adopted here, the integer and binary strings are modified independently, with the same probability. When mutation takes place, the new value of the selected gene is sampled from a uniform distribution probability, considering the interval  $[1, nb]$  for the integer string and the values 0 or 1 for the binary one. Figure 3 illustrates an example of mutation.

The proposed mutation operator can perform three distinct actions: resize the capacitor bank, by changing the binary string only; change the capacitor location in the distribution network, by mutating only the integer string; and combine the two previous actions. As a consequence, the mutation implements an effective local search strategy in the solution space.

The crossover operator generates offspring by combining the genes of different individuals. The proposed recombination is based on the one-point crossover strategy (Glover and Kochenberger 2003). For the binary string, the crossover point is randomly chosen, and the binary substrings are swapped accordingly. In that case, the integer genes corresponding to the interchanged substrings are also exchanged, generating one or two offspring, as illustrated in Fig. 4. The proposed crossover performs a substantial modification in



**Fig. 4** Crossover

the capacitor bank configuration, maintaining the population diversity during the search process.

This section presented the main aspects of the proposed methodology. The variability of the load and network topology are modeled and included in the optimization problem. A compact encoding based on an integer-binary representation for the capacitor placement problem has been presented. In the next section, the proposed method will be assessed using the data of a real distribution feeder.

## 4 Test and Results

### 4.1 Description of Simulation

In this section, the proposed method is applied to the distribution network of LIGHT electric company, responsible for supplying energy to the city of Rio de Janeiro, in Brazil. In the simulation studies, data from real feeders are used, as well as historical data of system loading stored in the company's database. It is assumed that commercial (commonly found) fixed capacitor banks of 300 kvar, 600 kvar, and 1200 kvar are available. Their corresponding costs—expressed in the Brazilian currency (BRL, denoted by R\$)—are, respectively: R\$ 4079.00, R\$ 4640.00, and R\$ 7993.00. The price paid for the energy imported by the utility is R\$ 91.91/MWh. The consideration of only fixed capacitor banks makes the optimization problem more challenging to solve, as the search space is more restricted than it would be in the presence of switchable capacitors. LIGHT company employs only fixed capacitor banks in its network, but the proposed methodology can also be applied if switchable capacitor banks are available. The capacitor banks are to be allocated in the 13.8 kV feeders and, without loss of generality, balanced operating conditions are considered. However, the methodology can also be extended to deal with unbalanced distribution systems. The GA is implemented in FORTRAN language. Deterministic tournament selection with two contestants is adopted, as well as an elitist strategy so that the best individual of a given population takes part in the population of the next generation. One-point crossover and uniform mutation are adopted as reproduction operators, with fixed rates

of 50% for crossover and 1% for mutation. Population size was set to 75, and the initial population drawn from a uniform distribution. One hundred generations were carried out in each test.

## 4.2 Pre-processing of Historical Load Data

The load variability is modeled as described in Sect. 3.1. A historical database that contains measurements of each feeder total loading, taken at the distribution substation, is employed to compute the centroids. The load measurements in that database are collected every 15 min. Then, considering a time horizon of 1 year, a total of 35,040 load measurements are stored for each feeder, corresponding to its annual load curve. The time horizon of 1 year is adopted, as it allows capturing seasonal load variability. Based on the measurement of a feeder total loading at a given time instant, together with the knowledge of the demand factors of the loads supplied by that feeder, it is possible to determine the load at each bus for that time instant. Consequently, the annual load curve at each bus of the feeder under study is also obtained. However, besides the normal load variations, the measurement values that are stored in the historical database may also have been affected by two other events: missing measurements or load transfers between feeders. The occurrence of such situations can be easily detected by observing if there are significant variations in the feeder total load from one time instant to another, which is not expected for a measurement cycle of 15 min. Missing measurements are easy to detect and restore, as the missing measurement values drop to zero. On the other hand, a procedure to detect the amount of load transferred to other feeders is necessary.

For planning purposes, the annual load curve of each feeder (without load transfers) is required. Once the loads associated with each feeder are known for the entire time interval of interest, simulations considering load transfers can be performed when planning the allocation of capacitor banks. Then, a simple correction procedure is adopted to detect the occurrence of a load transfer and to correct the loading measurement values, so that they correspond to the ones that would have been measured if no load transfer between feeders has occurred. So, whenever necessary, the correction procedure aims to replace missing measurement values and to restore the values of the feeders' self-loads. Considering the 35,040 loading measurements recorded (current magnitudes) for a given feeder, which compose its annual load curve, a correction is carried out whenever an unexpected load variation  $\Delta I$  (greater than a given threshold value) is observed between two consecutive time instants. In such a situation, the recorded measurement value is incremented (or decremented) by the value  $\Delta I$ . The threshold value adopted here is 0.15, which means that a total load variation of more than 15% within 15 min is not expected. Whenever such a situation

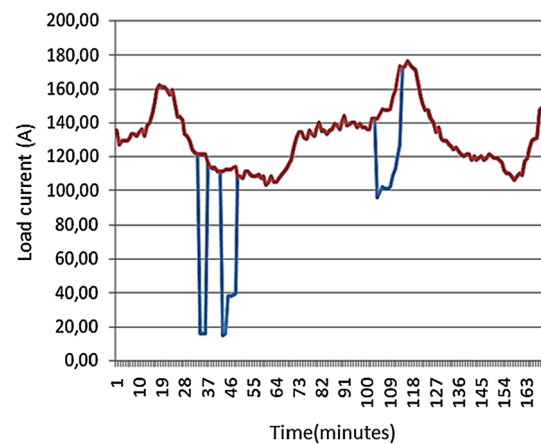


Fig. 5 Load curve correction (blue line measured brown line corrected)

is detected, one assumes that a load transfer occurred, and the correction procedure is triggered. This threshold should be set based on the experience with the system under study. It is important to note that for the purposes of this work, errors inherent to the metering process will be neglected.

Figure 5 depicts some corrections performed in part of the recorded load curve (approximately 2 days) of a LIGHT feeder. The effectiveness of the corrections can be noted, and a smoother load curve is obtained.

## 4.3 Computation of Load Representatives

The load representatives are obtained through the  $k$ -means algorithm described in Sect. 3.1. Different values for  $k$  are tested, which means that attempts to model the load variability by a different number of representatives are performed. It is important to have in mind that the use of many load representatives (high values of  $k$ ) favors a more detailed representation of the load curve. However, depending on the value of  $k$ , the running time to evaluate each proposed solution for the different load scenarios may be extremely high. Then, it is desirable to have the load curve modeled by a few representatives, provided that those are good enough to allow accurate results.

Figure 6 illustrates the representatives obtained for the load curve of feeder named *Bandeira* of the LIGHT distribution grid, for  $k = 3$ . When adopting such a model, the load variability is represented by three different levels, which correspond to the centroid values computed by the  $k$ -means algorithm. As previously mentioned, the duration of each load representative, expressed in hours, is the number of hourly loads forming the cluster associated with that centroid.

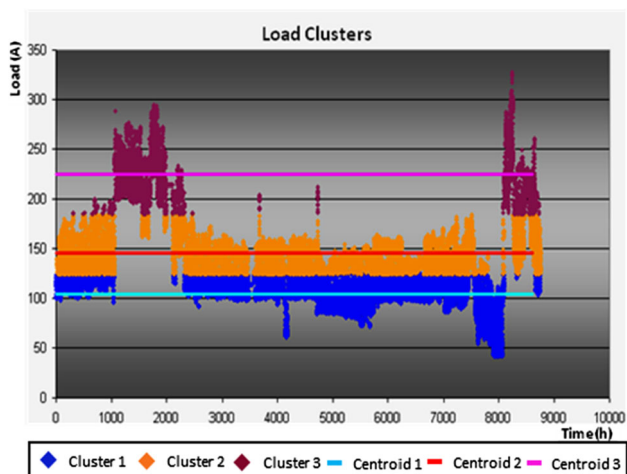


Fig. 6 Load curve of the feeder Bandeira

Table 1 Results for load clusters

No. of clusters	$k = 3$	$k = 5$	$k = 7$
Location (bus no.)	(22, 57)	(22, 57)	(22, 57)
Total power (kvar)	1200	1200	1200
Loss savings (R\$)	44,282	44,165	43,716
Investment (R\$)	9280	9280	9280
Annual savings (R\$)	35,002	34,885	34,436

#### 4.4 Tests Considering Load Variability

Tests considering a load representation that consists of  $k + 2$  load levels are included here. As previously discussed, this corresponds to  $k$  centroids obtained through the  $k$ -means algorithm plus the minimum and maximum load levels obtained from the historical load database. Simulations are performed considering different values for  $k$ . Table 1 shows results obtained with the proposed method, for capacitor allocation in the feeder named Bandeira, when using three different load models.

It can be observed from Table 1 that modeling the system load by three representatives ( $k = 3$ ) is sufficient for an accurate representation of the load variability, as the obtained solution (quantity and location) not changed when more than three load representatives are employed. Note that there is no significant variation in the estimative of loss saving and annual profit when larger values of  $k$  are adopted. Then, one can conclude that an adequate load modeling reduces the computational effort to assess the proposed solutions and preserves the quality of the obtained solution. It is important to stress that the minimum and maximum load levels of the feeder Bandeira are also considered in the evaluation of the FF- $np$  in Eqs. (2) and (3).

Table 2 Results for Recife feeder

Investment (R\$)	30,731
Loss savings (R\$), 1st year	87,226
Payback period (months)	4.5
No. of the bus with a capacitor bank and (rating in kvar)	3 (600), 32 (600), 44 (600), 67 (300), 83 (600), 89 (600)

Table 3 Results for Dafeira feeder

Investment (R\$)	20,781
Loss savings (R\$), 1st year	100,123
Payback period (months)	2.5
No. of the bus with a capacitor bank and (rating in kvar)	34 (600), 69 (600), 78 (600), 111 (600)

Table 4 Results considering topological variability

Investment (R\$)	51,513
Loss savings (R\$), 1st year	105,825
Payback period (months)	6
No. of the bus with a capacitor bank and (rating in kvar), Recife feeder	10 (300), 22 (600), 3 (600), 78 (600), 82 (600), 3 (600)
No. of the bus with a capacitor bank and (rating in kvar), Dafeira feeder	84 (600), 93 (600), 114 (600), 118 (600)

#### 4.5 Tests Considering Topology Variations

Tests are performed with the feeders named Dafeira and Recife of the LIGHT distribution grid, considering the possibility of changes in the feeders' configurations due to network switching operations. This is considered by the situations in which the feeders operate attending only the loads they usually supply, as well as the situations in which there are load transfers between those feeders.

Tables 2 and 3 present results for the allocation of capacitor banks in the feeders Dafeira and Recife, considering that load transfers do not occur, i.e., each feeder supplies its own load during the time horizon of the planning study. On the other hand, Table 4 shows results obtained when, besides the scenarios in which the feeders supply their own load, two more topology scenarios are considered. Those correspond to situations in which part of the load of feeder Bandeira is transferred to feeder Recife, employing different switching operations. All results are obtained considering a load model with five levels, which corresponds to three load representatives ( $k = 3$ ) plus the minimum and maximum peak load scenarios. Note that, besides the financial effects of the capacitor placement, all voltage violations are eliminated. The payback period shown in Tables 2, 3, and 4 refers to the time expressed in months (approximated values) necessary to recover the investment made in the capacitor banks.



When more topology scenarios are taken into account, the capacitor locations are different than those obtained considering that each feeder supplies only its own load. However, in the tested cases, the number of capacitor banks allocated (as well as the corresponding ratings) remained the same. It should be noted that the capacitor bank allocations proposed in Tables 2 and 3 do not prevent voltage violations if the other topology scenarios occur. This result shows that topology variations should be considered during the planning phase so that the capacitor allocation turns effective.

#### 4.6 Comments

The main contribution of this paper is in the way that system load and network configuration variabilities are considered when planning the allocation of the capacitor banks. Regarding load variability, the pre-definition of load level representatives, typically adopted in the literature (e.g., heavy, medium and light loads), is avoided. A clustering algorithm is employed to automatically determine the load level representatives, as well as their corresponding durations within the time window of interest. Such load representatives are extracted from a real historical database containing load measurements taken at intervals of 15 min and corresponding to an annual load curve. Besides the extracted load representatives, the minimum and maximum load levels (observed from the measurements in the historical database) are also considered, as they represent more severe conditions for voltage regulation. It is important to remark that the representation of load variability (number of representatives, as well as their magnitudes and durations) is crucial for a more accurate assessment of the impacts of capacitor placement on energy losses and voltage control when conducting planning studies.

Regarding network configuration variability, load transfers among substation feeders may occur for many different reasons, and it is not likely that network configuration will remain the same over the entire period considered in planning studies. Distribution utilities usually have their transfer schemes and information about common load transfers (as well as how frequently and long they are) can be obtained from the utility's historical database and/or from engineers' experience on system operation. In this paper, information about common load transfers is taken into account when allocating capacitor banks and, as a result, energy losses are minimized considering different configurations that the network may assume.

Finally, it is important to remark that, as in most works found in the technical literature, a constant power load model is adopted in this paper. However, the proposed methodology allows the consideration of any load model, such as a voltage-dependent one, being necessary only to represent them in the load flow program employed to assess the proposed solutions.

The load models to be considered in planning studies will depend on the load characteristics of the distribution system under study.

## 5 Conclusion

This paper presents a methodology for the allocation of capacitor banks in power distribution networks, formulated as a combinatorial optimization problem to be solved by a genetic algorithm. Both capacitor banks' ratings and locations are encoded in a low dimension chromosome vector. The minimization of investment costs, the reduction in power losses, and voltage profile control are among the pursued objectives. The representation of system load and topology variability is incorporated in the proposed model. Tests performed with data obtained from a real distribution system show the importance of adequately modeling load and topology variabilities during the planning studies. The results evidenced that with little or almost no extra investment cost, it is possible to meet specified voltage limits for different topology scenarios. Besides, a more accurate estimation of power loss reduction is obtained.

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