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AQM Congestion Controller for TCP/IP Networks: Multiclass Traffic

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Abstract

Active queue management (AQM) is a congestion control mechanism for the early notification of the incipient congestion pursued by dropping/marking packets. The novelty in our result is that we are able to consider heterogeneous traffic (i.e., multiclass traffic) where each class has its own packet discarding policy, queue size, and bandwidth share. Then, this is so far the first control theory-based approach for the AQM problem on the TCP/IP routers that allows multiclass AQM. Our proposed technique assumes that each class has already a controller, designed a priori, and focuses on designing a static state feedback controller where the proposed design approach is based on the solution of linear matrix inequalities (LMIs). From the theoretical conditions, a new controller design methodology for discrete time systems with multi-delays, saturated inputs, and disturbances is proposed to overcome the obstacle of bilinearity which leads to a quite simple LMI condition that is numerically tractable with any convex optimization algorithm. A numerical example is provided at the end of this paper to show the effectiveness and performances of the proposed approach in the presence of multiclass traffic.

Keywords AQM · Congestion control · Multiclass traffic · Static state feedback

List of Symbols

$W_1(t)$	Is the average TCP window size (packets)
$q_1(t)$	Is the average queue length (packets)
$t_{r1}(t) = \frac{q_1(t)}{C_{1_0}} + T_{p_1}$	Is the round trip time (s)
T_{p_1}	Is the propagation delay (s)
$C_1(t)$	Is the link capacity (packets/s)
N_1	Is the loading factor (number of TCP sessions)
$p_1 \in [0, 1]$	Is the probability of packet marking/dropping
$W_{1_{\max}}$	Is the maximum window size
$q_{1_{\max}}$	Is the buffer capacity
$\tau_1(t)$	Is the time delay of the system where its values belong to $[-h_{\tau_1}, 0]$
d_1	Is the change rate of the time delay
sat(.)	Is the saturation function
U_i	Is the ith row of U, $i = 1, \ldots, m$
$\overline{\lambda}$	Is the maximal eigenvalue

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1 Introduction

The escalating demands in performance require improved resource management, yet Transport Control Protocol (TCP) based to satisfy new users within existing network resources. Then, AQM arose as an effective way to tune TCP behavior, by discarding selectively packets to smoothly regulate the traffic rate and delay variation, called jitter, which threatens nowadays the multimedia network services (Abharian et al. 2012; Dahmouni et al. 2012; El Fezazi et al. 2019a). AQM mechanisms are introduced to assist the TCP/IP congestion control for its relation with Internet traffic congestion and Quality of Service (QoS) demands of users and applications (Alaoui et al. 2018, 2019; Bigdeli and Haeri 2007; Lamrabet et al. 2017). These techniques can be categorized in various groups such as heuristic methods that are mainly developed by computer scientists and mathematical schemes such as game theory-based algorithms and control theory-based approaches (see the works Xu et al. 2015; Wang et al. 2017; Zhou et al. 2013; Zhu et al. 2016 for details). Control theory techniques are based on the model proposed in Misra et al. (2000), of which the most resounding so far is the proportional–integral (PI) controller proposed in Hollot et al. (2001). Different techniques are also proposed in the literature. Amongst them, in Sabry and Kaittan (2020); Wang et al. (2019) a fuzzy control is used for the AQM control problem on the TCP/IP routers. In the works of Bigdeli and Haeri (2007); Marami et al. (2007); Yazdi and Delavarkhalafi (2018), a model predictive control approach is used for congestion control in data networks. A recursive design method, called backstepping technique, is also applied to the network congestion control by many authors (see Li et al. 2019a, b; Liu et al. 2018). Then, there is a trade-off between computational burden of considered nonlinear TCP/IP dynamics and the result accuracy under real network conditions. For this reason, these approaches need a more effort to reduce this computational burden taking into account the time delays, saturating nature, and disturbances.

Since the control actions in AQM are the discarding probabilities (real numbers bounded between [0, 1]), the AQM approaches that ignore the input saturation are not adequate because saturations might deteriorate the control system's performance and frequently lead to unacceptable losses of stability. Then, to mitigate the saturation effects on stability of such systems, local/global stabilization in a specified region of attraction (Cao et al. 2002; El Fezazi et al. 2017, 2019c; El Fezazi 2019; Naamane et al. 2017) and anti-windup techniques where the emphasis is on the transient performance caused by the saturations (Bender 2013, 2014; Lamrabet et al. 2017, 2018) is studied. The drawback of some proposed approaches is the computing resources demanded to implement them in a real network, which may yield it unfeasible. On the other hand, the works above consider the traffic homogeneous, i.e., they do not differentiate one TCP flow from another. However, some data connections must be primarily secured for allowing financial and e-banking transfers, while others perform better under less varying delays (like Voice Over Internet Protocol (VoIP)), and some have not special specifications (like regular e-mail sending). Hence, a single delay AQM policy for the whole traffic is not likely to address the specifications that telecom carriers and Internet providers hold with high demanding customers nowadays (see Greengrass et al. 2009; Rosen et al. 2000).

In this sense, the present work addresses the AQM problem on the TCP/IP routers by explicitly allowing different time delays for multiple traffic classes where the available link bandwidth is modeled as a time variant disturbance, while formally ensuring the closed-loop stability and a routing performance level under congested traffic conditions. From a formal point of view, the controller synthesis problem for delayed systems subject to input limitations and disturbances is solved in this paper using a new methodology to overcome the obstacle of bilinearity which leads to a quite simple LMI condition that is numerically tractable with any convex optimization algorithm.

2 Modeling and Problem Statement

A detailed description is discussed in this section on the TCP behavior in the TCP/IP networks to establish the linear state space model in order to ensure the control requirements following the formulated problem.

2.1 TCP Behavior Modeling

Consider the scenario that the network consists of n nodes (senders), n nodes (receivers), and a bottleneck router. The bottleneck router sends packets from these senders to the receivers. Then, the model of TCP behavior is described by the following equations (Misra et al. 2000):

$$\begin{aligned}\dot{W}_1(t) &= \frac{1}{t_{r1}(t)} - \frac{W_1(t)W_1(t-t_{r1}(t))}{2t_{r1}(t-t_{r1}(t))}p_1(t-t_{r1}(t))\\ \dot{q}_1(t) &= -C_1(t) + \frac{N_1}{t_{r1}}W_1(t)\end{aligned} \tag{1}$$

The first equation of (1) describes the TCP window control dynamics, whereas the second equation of (1) models the bottleneck queue length as the accumulated difference between the packet arrival rate and the link capacity. The TCP window size and queue length are positive bounded quantities, i.e., $W_1 \in [0, W_{1_{\max}}]$ and $q_1 \in [0, q_{1_{\max}}]$.

As the linear model is derived around an operating point and since the obtained model is close enough to the true one, the aim of the stabilization study of the linearized model is to guarantee the nearness between the trajectory of the nonlinear process and the one desired. This leads to reduce the track error and ensure the regulation around the equilibrium point. Then, the probability at an equilibrium point given by the triplet ($W_{1_0} = \frac{t_{r1}C_{1_0}}{N_1}$, $q_{1_0} = C_{1_0}(t_{r1} - T_{p_1})$, $p_{1_0} = \frac{2}{W_{1_0}^2}$) satisfies the condition $p_{1_0} = \frac{2N_1^2}{(q_{1_0}+T_{p_1}C_{1_0})^2}$ that can be derived from Eq. (1).

Defining $\delta\mathfrak{C}_1 = \mathfrak{C}_1 - \mathfrak{C}_{1_0}$ in which $\mathfrak{C} = W, q, p, C$, we can obtain the linearized version of (1) on the equilibrium point as follows:

$$\begin{aligned}\delta\dot{W}_1(t) = &-\frac{N_1}{t_{r1}^2 C_{1_0}}\Big(\delta W_1(t) + \delta W_1(t-t_{r1}(t))\Big)\\ &-\frac{1}{t_{r1}^2 C_{1_0}}\Big(\delta q_1(t)\end{aligned}$$

$$+\delta q_1(t-t_{r1}(t))\Big) - \frac{t_{r1}C_{1_0}^2}{2N_1^2}\delta p_1(t-t_{r1}(t))$$
$$+\frac{t_{r1}-T_{p_1}}{t_{r1}^2 C_{1_0}}\Big(\delta C_1(t)+\delta C_1(t-t_{r1}(t))\Big)$$
$$\delta \dot{q}_1(t) = \frac{N_1}{t_{r1}}\delta W_1(t) - \frac{1}{t_{r1}}\delta q_1(t) - \frac{T_{p_1}}{t_{r1}}\delta C_1(t)$$
$$t_{r1}(t) = \frac{\delta q_1(t)}{C_{1_0}} + t_{r1} \tag{2}$$

2.2 State Representation

State space tools will be used to develop the controller, so (2) is rewritten in the state space form as follows:

$$\begin{aligned}\dot{x}_1(t) &= A_{1_c}x_1(t) + A_{\tau_{1_c}}x_1(t-\tau_1(t)) \\ &\quad + B_{1_c}u_1(t-\tau_1(t)) + D_{w_{1_c}}w_1(t) \\ y_1(t) &= C_{y_{1_c}}x_1(t) \\ z_1(t) &= C_{z_{1_c}}x_1(t)\end{aligned} \tag{3}$$

in which

$$x_1(t) = \begin{bmatrix}\delta W_1(t)\\ \delta q_1(t)\end{bmatrix},$$
$$A_{1_c} = \begin{bmatrix}\frac{-N_1}{t_{r1}^2C_{1_0}} & \frac{-1}{t_{r1}^2C_{1_0}}\\ \frac{N_1}{t_{r1}} & \frac{-1}{t_{r1}}\end{bmatrix},$$
$$A_{\tau_{1_c}} = \begin{bmatrix}\frac{-N_1}{t_{r1}^2C_{1_0}} & \frac{-1}{t_{r1}^2C_{1_0}}\\ 0 & 0\end{bmatrix},$$
$$B_{1_c} = \begin{bmatrix}\frac{-t_{r1}C_{1_0}^2}{2N_1^2}\\ 0\end{bmatrix},$$
$$D_{w_{1_c}} = \begin{bmatrix}\frac{t_{r1}-T_{p_1}}{t_{r1}^2C_{1_0}} & \frac{t_{r1}-T_{p_1}}{t_{r1}^2C_{1_0}}\\ \frac{-T_{p_1}}{t_{r1}} & 0\end{bmatrix},$$
$$w_1(t) = \begin{bmatrix}\delta C_1(t)\\ \delta C_1(t-t_{r1}(t))\end{bmatrix},$$
$$u_1(t) = \delta p_1(t),\ C_{y_{1_c}} = \begin{bmatrix}0 & 1\end{bmatrix},$$
$$y_1(t) = \delta q_1(t),\quad C_{z_{1_c}} = \begin{bmatrix}0 & \frac{1}{C_{1_0}}\end{bmatrix},$$
$$z_1(t) = t_{r1}(t) - t_{r1}.$$

In Eq. (3), the state variables represent the deviation variables of the average TCP window size and the average queue length, respectively, the input represents the deviation variable of the marking/dropping probability, the measured output is the deviation variable of the average queue length, and the regulated output is the deviation of the round trip time compared to the desired value. On the other hand, since the available link bandwidth variations caused by short-term sudden flow are unavoidable, it is more practical to take them as a disturbance. Furthermore, the delay $\tau_1(t)$ satisfies $0 \le \tau_1(t) \le h_{\tau_1}$ and $0 \le \dot{\tau}_1(t) \le d_1 < 1$.

Let several classes of TCP flows pass through the router, we can rewrite the system (3) as the following new general state space model:

$$\begin{bmatrix}\dot{x}_1(t)\\ \vdots\\ \dot{x}_m(t)\end{bmatrix} = \begin{bmatrix}A_{1_c} & \dots & 0\\ \vdots & \ddots & \vdots\\ 0 & \dots & A_{m_c}\end{bmatrix}\begin{bmatrix}x_1(t)\\ \vdots\\ x_m(t)\end{bmatrix} + \begin{bmatrix}A_{\tau_{1_c}} & \dots & 0\\ \vdots & \ddots & \vdots\\ 0 & \dots & A_{\tau_{m_c}}\end{bmatrix}\begin{bmatrix}x_1(t-\tau_1(t))\\ \vdots\\ x_m(t-\tau_m(t))\end{bmatrix}$$
$$+ \begin{bmatrix}B_{1_c} & \dots & 0\\ \vdots & \ddots & \vdots\\ 0 & \dots & B_{m_c}\end{bmatrix}\begin{bmatrix}u_1(t-\tau_1(t))\\ \vdots\\ u_m(t-\tau_m(t))\end{bmatrix} + \begin{bmatrix}D_{w_{1_c}} & \dots & 0\\ \vdots & \ddots & \vdots\\ 0 & \dots & D_{w_{m_c}}\end{bmatrix}\begin{bmatrix}w_1(t)\\ \vdots\\ w_m(t)\end{bmatrix}$$
$$\begin{bmatrix}y_1(t)\\ \vdots\\ y_m(t)\end{bmatrix} = \begin{bmatrix}C_{y_{1_c}} & \dots & 0\\ \vdots & \ddots & \vdots\\ 0 & \dots & C_{y_{m_c}}\end{bmatrix}\begin{bmatrix}x_1(t)\\ \vdots\\ x_m(t)\end{bmatrix}$$
$$\begin{bmatrix}z_1(t)\\ \vdots\\ z_m(t)\end{bmatrix} = \begin{bmatrix}C_{z_{1_c}} & \dots & 0\\ \vdots & \ddots & \vdots\\ 0 & \dots & C_{z_{m_c}}\end{bmatrix}\begin{bmatrix}x_1(t)\\ \vdots\\ x_m(t)\end{bmatrix} \tag{4}$$

The simplification of the system (4) leads to obtain the following system:

$$\begin{aligned}\dot{x}(t) &= A_c x(t) + \sum_{i=1}^{m}\Big(A_{\tau_{c_i}}x(t-\tau_i(t)) + B_{c_i}u(t-\tau_i(t))\Big) \\ &\quad + D_{w_c}w(t) \\ y(t) &= C_{y_c}x(t) \\ z(t) &= C_{z_c}x(t)\end{aligned} \tag{5}$$

where the plant model considered here to obtain the system (5) is given as: $\psi(t) = [\psi_1^{\mathrm{T}}(t)\ \dots\ \psi_m^{\mathrm{T}}(t)]^{\mathrm{T}}$, $\psi = x, u, w, y, z$, $F_c = \mathrm{diag}\{F_{1_c}, \dots, F_{m_c}\}$, $F = A, D_w, C_y, C_z$, $H_{c_1} = \mathrm{diag}\{H_{1_c}, \dots, 0\}, \dots, H_{c_m} = \mathrm{diag}\{0, \dots, H_{m_c}\}$, and $H = A_\tau, B$. On the other hand, the disturbance $w(t)$ is assumed to be bounded with finite energy, that is, $w(t) \in \mathcal{L}_2$. Hence, for a scalar ω, the disturbance is given by $\|w(t)\|_2^2 = \int_0^\infty w^T(t)w(t)dt \le \omega^{-1} < \infty$.

Assuming periodic sampling, the controller design is based on the following saturated discrete time system with time-varying delay:

$$x(k+1) = Ax(k) + \sum_{i=1}^{m}\Big(A_{d_i}x(k-d_i(k)) + B_i u(k-d_i(k))\Big)$$

$$
\begin{aligned}
&\quad +D_w w(k)\\
y(k) &= C_y x(k)\\
z(k) &= C_z x(k)
\end{aligned} \tag{6}
$$

where

$$
\begin{aligned}
&A{=}e^{A_c T}, \quad B_1 = \int_0^T e^{A_c s} B_{c_1} ds, \ldots,\\
&B_m = \int_0^T e^{A_c s} B_{c_m} ds,\\
&A_{d_1} = \int_0^T e^{A_c s} A_{\tau_{c_1}} ds, \ldots,\\
&A_{d_m} = \int_0^T e^{A_c s} A_{\tau_{c_m}} ds,\\
&D_w = \int_0^T e^{A_c s} D_{w_c} ds,\\
&C_y = C_{y_c}, \quad C_z = C_{z_c},\\
&d_{s_1} \le d_1(k) \le d_{b_1}, \ldots,\\
&d_{s_m} \le d_m(k) \le d_{b_m}.
\end{aligned}
$$

Then, the following assumption on the system (6) is made.

Assumption 2.1 $(A + A_{d_1} + \cdots + A_{d_m}, B_1 + \cdots + B_m)$ and (A, C_y) are controllable and observable.

Assuming that our AQM uses a state feedback controller, we have the following type:

$$
u(k) = Kx(k) = \begin{bmatrix} K_{11}\delta W_1(k) + K_{12}\delta q_1(k) \\ \vdots \\ K_{m1}\delta W_m(k) + K_{m2}\delta q_m(k) \end{bmatrix} \tag{7}
$$

Due to the control bounds where $|u_i(k)| \le u_{0i}$ and $u_{0i} > 0$, the effective control signal applied to the system (6) is given by $u(k) = \mathrm{sat}(Kx(k), u_0)$.

From $u(k - d_i(k)) = \mathrm{sat}(K_i x(k - d_i(k)), u_{i_0})$, the system (6) reads

$$
\begin{aligned}
x(k+1) &= Ax(k) + \sum_{i=1}^{m} \Big(A_{d_i} x(k - d_i(k))\\
&\quad + B_i \mathrm{sat}(K_i x(k - d_i(k)), u_{i_0}) \Big) + D_w w(k)\\
y(k) &= C_y x(k)\\
z(k) &= C_z x(k)
\end{aligned} \tag{8}
$$

2.3 Problem Formulation

The following preliminaries are required to establish the results:

Lemma 1 (Cao et al. 2002) *Let Λ be the set of all diagonal matrices in $\Re^{m\times m}$ with diagonal elements that are either 1 or 0. There are 2^m elements D_j in Λ, $j = 1, \ldots, 2^m$ and denote $D_j^- = I_m - D_j$, which are also elements of Λ. Then, the controller design goal will be mathematically transformed to embed $\mathrm{sat}(Kx(k), u_0)$ within a convex hull of a linear feedbacks group (to avoid the saturation). For this, the set $\mathrm{sat}(Kx(k), u_0) \in \mathcal{C}o\{D_j K + D_j^- H\}x(k)$ is defined for K, H, and $x(k)$ to satisfy $|H_i x(k)| \le u_{0i}$.*

Lemma 2 (Chang et al. 2015) *For matrices Π, U, $\mathbb{M}$, and $\mathbb{N}$ with appropriate dimensions and a scalar ε, the inequality $\Pi + \mathbb{M}\mathbb{N} + \mathbb{N}^T\mathbb{M}^T < 0$ is fulfilled if the following condition holds:*

$$
\begin{bmatrix} \Pi & \varepsilon\mathbb{M} + \mathbb{N}^T U^T \\ * & -\varepsilon U - \varepsilon U^T \end{bmatrix} < 0
$$

Using Lemma 1, the system (8) becomes as given in (9) where $\lambda_j \ge 0, \sum_{j=1}^{2^m} \lambda_j = 1$, and $A_{dK_{ij}} = A_{d_i} + B_i(D_{i_j} K_i + D_{i_j}^- H_i)$.

$$
\begin{aligned}
x(k+1) &= Ax(k) + \sum_{j=1}^{2^m} \sum_{i=1}^{m} \lambda_j A_{dK_{ij}} x(k - d_i(k)) + D_w w(k)\\
y(k) &= C_y x(k)\\
z(k) &= C_z x(k)
\end{aligned} \tag{9}
$$

Definition 2.1 The attraction region Φ of the origin for a studied system is defined as follows:

$$
\Phi = \Big\{ x(k) \in \Re^n; \quad \lim_{k\to\infty} x(k) = 0 \Big\} \tag{10}
$$

To obtain a good estimate of the attraction region (10), we are interested in finding at least one region of stability that is analytically well characterized and that can be maximized considering some specific geometric criteria. As indicated below, this will be the case of ellipsoidal and polyhedral regions.

∗ For a scalar β, the ellipsoid is $D_e = \Big\{ x(t) \in \Re^n; \quad x^T(k) P x(k) \le \beta^{-1} \Big\}$;

∗ A polyhedral set is construed as $\Theta = \Big\{ x(k) \in \Re^n; \quad |H_i x(k)| \le u_{0i} \Big\}$.

Problem 2.1 *We aim in this paper to design a controller such that the studied system is stable and satisfies the following requirement:*

$$
\sum_{k=0}^{\infty} \Big(\frac{1}{\gamma} z^T(k) z(k) - w^T(k) w(k) \Big) < 0 \tag{11}
$$

where the specified scalar γ should be minimal as possible.

Remark 2.1 Note that the requirement (11) means that (El Fezazi et al. 2019b)

$$\frac{\|z(k)\|_2^2}{\|w(k)\|_2^2} = \frac{\sum_{k=0}^{\infty} z^T(k)z(k)}{\sum_{k=0}^{\infty} w^T(k)w(k)} < \gamma \tag{12}$$

where the ratio between the norm of the controlled output and that of the disturbance is less than γ.

3 Main Results

Let us now establish sufficient conditions to ensure the H_∞ control considering m different traffic classes traversing the congested router, where each class has its specificities. These conditions are summarized in Theorem 1.

3.1 Stability Results

Theorem 1 *If there exist positive definite symmetric matrices* $P, Q_1, \dots, Q_m, R_1, \dots, R_m$, *appropriately sized matrices* $U, M_1, \dots, M_m, N_1, \dots, N_m, Y_{11}, \dots, Y_{m1}, Y_{12}, \dots, Y_{m2}, Y_{13}, \dots, Y_{m3}$, *and scalars* $\varepsilon_1, \dots, \varepsilon_m$ *satisfying the following conditions:*

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \dots & \Pi_{14} & \Pi_{15} & \dots & \Pi_{17} & \Pi_{18} & \Pi_{19} & 0 & \dots & 0 & \Pi_{113} \\ * & \Pi_{22} & \dots & 0 & \Pi_{25} & \dots & 0 & \Pi_{28} & \Pi_{29} & \Pi_{210} & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & \dots & \Pi_{44} & 0 & \dots & \Pi_{47} & \Pi_{48} & \Pi_{49} & 0 & \dots & \Pi_{412} & 0 \\ * & * & \dots & * & \Pi_{55} & \dots & 0 & \Pi_{58} & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & \dots & * & * & \dots & \Pi_{77} & \Pi_{78} & 0 & 0 & \dots & 0 & 0 \\ * & * & \dots & * & * & \dots & * & \Pi_{88} & \Pi_{89} & 0 & \dots & 0 & 0 \\ * & * & \dots & * & * & \dots & * & * & \Pi_{99} & \Pi_{910} & \dots & \Pi_{912} & 0 \\ * & * & \dots & * & * & \dots & * & * & * & \Pi_{1010} & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & \dots & * & * & \dots & * & * & * & * & \dots & \Pi_{1212} & 0 \\ * & * & \dots & * & * & \dots & * & * & * & * & \dots & * & \Pi_{1313} \end{bmatrix} < 0,$$

$$Q_1 < R_1, \ \dots \ , \ Q_m < R_m, \tag{13}$$

$$\begin{bmatrix} P & N_1^T & \dots & N_m^T & -N_1^T & \dots & -N_m^T \\ * & \beta u_{1_{0i}}^2 & \dots & 0 & -\varepsilon_1 I + \varepsilon_1 U_i & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ * & * & \dots & \beta u_{m_{0i}}^2 & 0 & \dots & -\varepsilon_m I + \varepsilon_m U_i \\ * & * & \dots & * & -\varepsilon_1 U_i - \varepsilon_1 U_i^T & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ * & * & \dots & * & * & \dots & -\varepsilon_m U_i - \varepsilon_m U_i^T \end{bmatrix} \geq 0,$$

$$\beta - \omega \leq 0 \tag{14}$$

where

$$\Pi_{11} = \sum_{i=1}^{m} \Big[Q_i - P + (d_{b_i} - d_{s_i}) R_i \Big],$$
$$\Pi_{12} = -Y_{11}^T, \ \Pi_{22} = Y_{11} + Y_{11}^T - Q_1,$$
$$\Pi_{14} = -Y_{m1}^T, \ \Pi_{44} = Y_{m1} + Y_{m1}^T - Q_m,$$
$$\Pi_{15} = -Y_{12}^T, \ \Pi_{25} = Y_{11} + Y_{12}^T,$$
$$\Pi_{55} = Y_{12} + Y_{12}^T, \ \Pi_{17} = -Y_{m2}^T,$$
$$\Pi_{47} = Y_{m1} + Y_{m2}^T, \ \Pi_{77} = Y_{m2} + Y_{m2}^T,$$
$$\Pi_{18} = \sum_{i=1}^{m} -Y_{m3}^T, \ \Pi_{28} = Y_{13}^T,$$
$$\Pi_{48} = Y_{m3}^T, \ \Pi_{58} = Y_{13}^T, \ \Pi_{78} = Y_{m3}^T,$$
$$\Pi_{88} = -I, \ \Pi_{19} = A^T P,$$
$$\Pi_{29} = A_{d_1}^T P + D_{1_j} M_1^T B_1^T + D_{1_j}^- N_1^T B_1^T,$$
$$\Pi_{49} = A_{d_m}^T P + D_{m_j} M_m^T B_m^T + D_{m_j}^- N_m^T B_m^T,$$
$$\Pi_{89} = D_w^T P, \ \Pi_{99} = -P,$$
$$\Pi_{210} = -D_{1_j} M_1^T - D_{1_j}^- N_1^T,$$
$$\Pi_{910} = -\varepsilon_1 P B_1 + \varepsilon_1 B_1 U,$$
$$\Pi_{1010} = -\varepsilon_1 U - \varepsilon_1 U^T,$$
$$\Pi_{412} = -D_{m_j} M_m^T - D_{m_j}^- N_m^T, \ \Pi_{113} = C_z^T,$$
$$\Pi_{912} = -\varepsilon_m P B_m + \varepsilon_m B_m U,$$
$$\Pi_{1212} = -\varepsilon_m U - \varepsilon_m U^T, \ \Pi_{1313} = -\gamma I,$$

the system (9) is asymptotically stable. Then, the estimate of the attraction region is given by

$$\mathcal{R} = \Big\{ \phi \in \mathcal{C}[-\overline{d}_b, 0], \max_{[-\overline{d}_b, 0]} \|\phi\| \leq \frac{\varpi}{\iota} \Big\}, \quad \overline{d}_b = \max\{d_{b_1}, \dots, d_{b_m}\} \tag{15}$$

with any ϖ_1 *satisfying* $\varpi \leq \beta^{-1} - \omega^{-1}$ *and*

$$\iota = \overline{\lambda}(P) + \sum_{i=1}^{m} \Big\{ (d_{b_i} + d_{s_i}) \overline{\lambda}(Q_i) + \frac{(d_{b_i} - d_{s_i} + 1)(d_{b_i} + d_{s_i})}{2} \overline{\lambda}(R_i) \Big\}$$

The stabilizing controller gains are $K_1 = U^{-1} M_1, \dots, K_m = U^{-1} M_m$.

Proof Applying the Schur complement and Lemma 2 to the matrix (13), we obtain:

$$\Pi + \mathbb{M}_1 \mathbb{N}_1 + \mathbb{N}_1^T \mathbb{M}_1^T + \cdots + \mathbb{M}_m \mathbb{N}_m + \mathbb{N}_m^T \mathbb{M}_m^T < 0 \tag{16}$$

where

$$\Pi=\begin{bmatrix}\Pi_{11}+\frac{1}{\gamma}C_z^TC_z & \Pi_{12} & \dots & \Pi_{14} & \Pi_{15} & \dots & \Pi_{17} & \Pi_{18} & \Pi_{19}\\ * & \Pi_{22} & \dots & 0 & \Pi_{25} & \dots & 0 & \Pi_{28} & \Pi_{29}\\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\\ * & * & \dots & \Pi_{44} & 0 & \dots & \Pi_{47} & \Pi_{48} & \Pi_{49}\\ * & * & \dots & * & \Pi_{55} & \dots & 0 & \Pi_{58} & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\\ * & * & \dots & * & * & \dots & \Pi_{77} & \Pi_{78} & 0\\ * & * & \dots & * & * & \dots & * & \Pi_{88} & \Pi_{89}\\ * & * & \dots & * & * & \dots & * & * & \Pi_{99}\end{bmatrix},$$

$$\mathbb{M}_1=\left[0\ 0\dots 0\ 0\dots 0\ 0\ -B_1^TP+U^TB_1^T\right]^T,$$
$$\mathbb{M}_m=\left[0\ 0\dots 0\ 0\dots 0\ 0\ -B_m^TP+U^TB_m^T\right]^T,$$
$$\mathbb{N}_1=U^{-1}\left[0\ -D_{1_j}M_1-D_{1_j}^-N_1\ \dots 0\ 0\dots 0\ 0\ 0\right],$$
$$\mathbb{N}_m=U^{-1}\left[0\ 0\ \dots -D_{m_j}M_m-D_{m_j}^-N_m\ 0\dots 0\ 0\ 0\right].$$

Let us now consider the following Lyapunov–Krasovskii (*L*–*K*) functional:

$$\begin{aligned}V(k)&=V_P(k)+\sum_{i=1}^{m}\Big(V_{Q_i}(k)+V_{R_i}(k)\Big)\\&=x^T(k)Px(k)+\sum_{l=k-d_1(k)}^{k-1}x^T(l)Q_1x(l)+\cdots\\&\quad+\sum_{l=k-d_m(k)}^{k-1}x^T(l)Q_mx(l)\\&\quad+\sum_{l=-d_{b_1}+2}^{-d_{s_1}+1}\sum_{\theta=k+l-1}^{k-1}x^T(\theta)R_1x(\theta)+\cdots\\&\quad+\sum_{l=-d_{b_m}+2}^{-d_{sm}+1}\sum_{\theta=k+l-1}^{k-1}x^T(\theta)R_mx(\theta)\end{aligned}\tag{17}$$

The variation in each term of the *L*–*K* functional (17) between consecutive samples is given by

$$\Delta V_P(k)=x^T(k+1)Px(k+1)-x^T(k)Px(k)\tag{18}$$

$$\begin{aligned}\Delta V_{Q_1}(k)&=\sum_{l=k+1-d_1(k+1)}^{k}x^T(l)Q_1x(l)-\sum_{l=k-d_1(k)}^{k-1}x^T(l)Q_1x(l)\\&=x^T(k)Q_1x(k)-x^T(k-d_1(k))Q_1x(k-d_1(k))\\&\quad+\sum_{l=k+1-d_{s_1}}^{k-1}x^T(l)Q_1x(l)-\sum_{l=k+1-d_1(k)}^{k-1}x^T(l)Q_1x(l)\\&\quad+\sum_{l=k+1-d_1(k+1)}^{k-d_{s_1}}x^T(l)Q_1x(l)\end{aligned}\tag{19}$$

$$\begin{aligned}\Delta V_{R_1}(k)&=\sum_{l=-d_{b_1}+2}^{-d_{s_1}+1}\left[\sum_{\theta=k+l}^{k}x^T(\theta)R_1x(\theta)-\sum_{\theta=k+l-1}^{k-1}x^T(\theta)R_1x(\theta)\right]\\&=(d_{b_1}-d_{s_1})x^T(k)R_1x(k)-\sum_{l=k+1-d_{b_1}}^{k-d_{s_1}}x^T(l)R_1x(l)\end{aligned}\tag{20}$$

From $Q_1<R_1$, it is possible to see that

$$\begin{aligned}&-\sum_{l=k+1-d_1(k)}^{k-1}x^T(l)Q_1x(l)\le-\sum_{l=k+1-d_{s_1}}^{k-1}x^T(l)Q_1x(l)\\&\sum_{l=k+1-d_1(k+1)}^{k-d_{s_1}}x^T(l)Q_1x(l)\le\sum_{l=k+1-d_{b_1}}^{k-d_{s_1}}x^T(l)R_1x(l)\end{aligned}\tag{21}$$

In the same way, we can compute the other terms variation in the *L*–*K* functional.

According to the system (9) and Eqs. (18)–(21), we can write:

$$\begin{aligned}\Delta V(k)=\sum_{j=1}^{2^m}\sum_{i=1}^{m}\lambda_j\Big\{&\Big[Ax(k)+A_{dK_{ij}}x(k-d_i(k))+D_ww(k)\Big]^T\\&P\Big[Ax(k)+A_{dK_{ij}}x(k-d_i(k))+D_ww(k)\Big]+x^T(k)\\&\Big[-P+Q_i+(d_{b_i}-d_{s_i})R_i\Big]\\&x(k)-x^T(k-d_i(k))Q_ix(k-d_i(k))\Big\}<0\end{aligned}\tag{22}$$

Using now the Newton–Leibniz formula, the following relations are true:

$$\begin{aligned}&\left[x^T(k-d_1(k))Y_{11}+\sum_{h=k-d_1(k)}^{k-1}y^T(h)Y_{12}+w^T(k)Y_{13}\right]\\&\times\left[-x(k)+x(k-d_1(k))+\sum_{h=k-d_1(k)}^{k-1}y(h)\right]=0\\&\vdots\\&\left[x^T(k-d_m(k))Y_{m1}+\sum_{h=k-d_m(k)}^{k-1}y^T(h)Y_{m2}+w^T(k)Y_{m3}\right]\end{aligned}$$

$$\times\Big[-x(k)+x(k-d_m(k))+\sum_{h=k-d_m(k)}^{k-1}y(h)\Big]=0 \tag{23}$$

where $y(h)=x(h+1)-x(h)$.

Taking account of (22)–(23), we obtain:

$$\Delta V(k)+\frac{1}{\gamma}z^T(k)z(k)-w^T(k)w(k)\leq\sum_{j=1}^{2^m}\lambda_j\eta^T(t)(\Upsilon+L^TP^{-1}L)\eta(t) \tag{24}$$

where

$$\Upsilon=\begin{bmatrix}\Pi_{11}+\frac{1}{\gamma}C_z^TC_z & \Pi_{12} & \dots & \Pi_{14} & \Pi_{15} & \dots & \Pi_{17} & \Pi_{18}\\ * & \Pi_{22} & \dots & 0 & \Pi_{25} & \dots & 0 & \Pi_{28}\\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots\\ * & * & \dots & \Pi_{44} & 0 & \dots & \Pi_{47} & \Pi_{48}\\ * & * & \dots & * & \Pi_{55} & \dots & 0 & \Pi_{58}\\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots\\ * & * & \dots & * & * & \dots & \Pi_{77} & \Pi_{78}\\ * & * & \dots & * & * & \dots & * & \Pi_{88}\end{bmatrix},$$

$$L=\Big[PA\ PA_{dK_{1j}}\ \dots\ PA_{dK_{mj}}\ 0\dots0\ PD_w\Big],$$

$$\eta(k)=\big[x^T(k)\ x^T(k-d_1(k))\ \dots\ x^T(k-d_m(k))\ \textstyle\sum_{h=k-d_1(k)}^{k-1}y^T(h)\ \dots\ \sum_{h=k-d_m(k)}^{k-1}y^T(h)\ w^T(k)\big]^T.$$

From the inequality (24), it is clear that if

$$\Upsilon+L^TP^{-1}L<0 \tag{25}$$

then

$$\Delta V(k)+\frac{1}{\gamma}z^T(k)z(k)-w^T(k)w(k)<0 \tag{26}$$

Applying the Schur complement to the inequality (25), then, replacing $A_{dK_{1j}},\dots,A_{dK_{mj}}$ by $A_{d_1}+B_1(D_{1_j}K_1+D_{1_j}^-H_1),\dots,A_{d_m}+B_m(D_{m_j}K_m+D_{m_j}^-H_m)$, respectively, and introducing nextly new change of variables such that $U^{-1}M_1=K_1,\dots,U^{-1}M_m=K_m,U^{-1}N_1=H_1,\dots,U^{-1}N_m=H_m$, and finally, applying Lemma 2, we obtain Eq. (16). Thus, the matrix (13) is verified. On the other hand, summing up Eq. (26) from 0 to ∞ with respect to k yields:

$$V(\infty)-V(0)+\sum_{k=0}^{\infty}\Big(\frac{1}{\gamma}z^T(k)z(k)-w^T(k)w(k)\Big)<0$$

Then, under the initial condition $V(0)=0$ and since the system is stable ($V(\infty)=0$), we can conclude that the condition (12) is verified.

The verification of the matrix (14) ensures that $|H_ix(k)|\leq u_{0i}$, $\forall x(k)\in D_e$. Moreover, the condition (15) guarantees that $\forall\phi\in\mathcal{R}$, $x(k)$ remains in D_e, and $\Delta V(k)<0$ implies that $x(k)\to0$ if $k\to\infty$. Finally, the inequality (14) ensures that $\beta^{-1}-\omega^{-1}\geq0$ and consequently verifies Eq. (27) and this completes the proof of Theorem 1.

$$V(0)\leq\iota\|\phi\|^2\leq\varpi\leq\beta^{-1}-\omega^{-1} \tag{27}$$

□

Remark 3.1 In this paper, the specified scalar γ can be included as an optimization variable to obtain a lower bound of the guaranteed H_∞ performance. Then, the state feedback controller design is accomplished satisfying the requirement (12) which guarantees the ratio between the norm of the controlled output and that of the disturbance is less than γ.

Remark 3.2 From the above proof, it can be seen that $\Delta V(k)$ remains unaffected by the matrices $Y_{11},\dots,Y_{m1}$, $Y_{12},\dots,Y_{m2}$, $Y_{13},\dots,Y_{m3}$. These matrices provide more degree of freedom and then reduce the conservatism of our result (see the numerical example). On the other hand, to find the optimal values of the parameters $\varepsilon_1,\dots,\varepsilon_m$ we can use a numerical optimization algorithm.

3.2 Optimization Problem

The maximized attraction region can be estimated from the following convex optimization problem considering the closed-loop system (9) ($w(k)=0$):

$$\begin{aligned}&\text{Minimize}\quad \vartheta=\sum_{i=1}^{m}\Big\{\sigma_P+(d_{b_i}+d_{s_i})\sigma_{Q_i}\\&\quad+\frac{(d_{b_i}-d_{s_i}+1)(d_{b_i}+d_{s_i})}{2}\sigma_{R_i}\Big\}\\&\text{subject to}\quad(13),(14),\sigma_PI-P\geq0,\sigma_{Q_1}I-Q_1\geq0,\\&\quad\dots,\sigma_{Q_m}I-Q_m\geq0,\\&\quad\sigma_{R_1}I-R_1\geq0,\dots,\sigma_{R_m}I-R_m\geq0\end{aligned} \tag{28}$$

From Eq. (15) and the technique (28), the maximized estimate of the attraction region is given by $\delta^2\vartheta\leq\beta^{-1}$ where $\delta=\max_{[-\bar{d}_b,0]}\|\phi\|$.

3.3 Implementation Constraints

Since $\delta W_1(k)$ is not available at routers in real networks, we use an approximation as follows (Athuraliya et al. 2001; Sall et al. 2009; Zhang et al. 2007):

$$\delta W_1(k)=\frac{t_{r1}}{N_1}\Big(\frac{N_1}{t_{r1}}W_1(k)-C_1(k)\Big)=\frac{t_{r1}}{N_1}\cdot q_1(k+1)$$

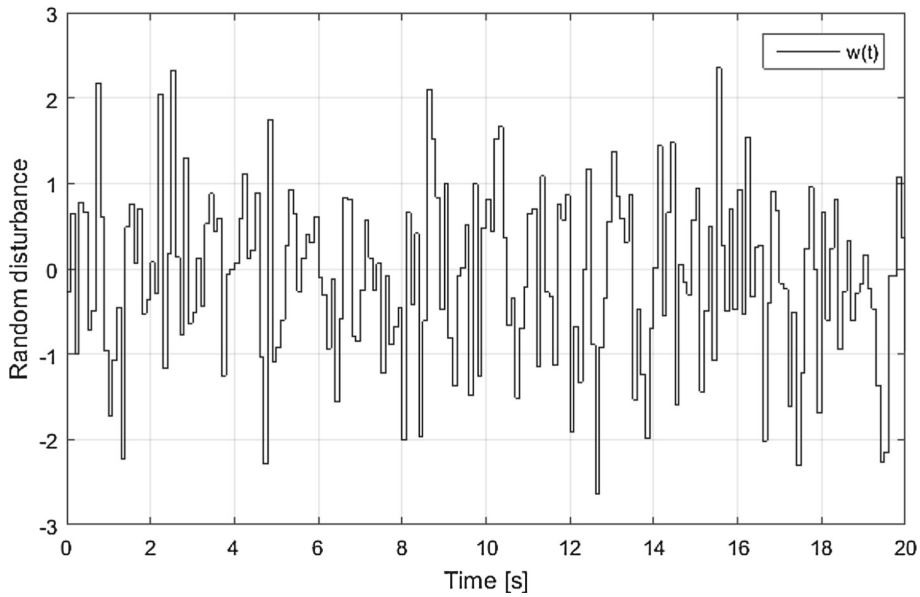

Fig. 1 Disturbance used in the simulations

Hence, the control signal (7) for the network becomes as follows:

$$\delta p(k)=\begin{bmatrix}\begin{bmatrix}K_{11}\frac{t_{r1}}{N_1} & K_{12}\end{bmatrix} & \dots & 0\\ \vdots & \ddots & \vdots\\ 0 & \dots & \begin{bmatrix}K_{m1}\frac{t_{rm}}{N_m} & K_{m2}\end{bmatrix}\end{bmatrix}\begin{bmatrix}\begin{bmatrix}q_1(k+1)\\ \delta q_1(k)\end{bmatrix}\\ \vdots\\ \begin{bmatrix}q_m(k+1)\\ \delta q_m(k)\end{bmatrix}\end{bmatrix}$$

4 Illustrative Example

The objective of this numerical example is the control of the TCP/IP networks when the congested traffic is comprised by three different classes, coping with the saturation of the discharge probability (i.e., the input saturation) according to Theorem 1. The parameters of the three different TCP flows are given by

$$\begin{cases} t_{r1}=0.30, & t_{r2}=0.25, & t_{r3}=0.20\\ C_{1_0}=3500, & C_{2_0}=3600, & C_{3_0}=3700\\ N_1=70, & N_2=75, & N_3=80\\ q_{1_0}=175, & q_{2_0}=165, & q_{3_0}=150\end{cases}$$

The other parameters can be easily calculated and given as follows:

$$\begin{cases} W_{1_0}=15, & W_{2_0}=12, & W_{3_0}=9.25\\ T_{p_1}=0.25, & T_{p_2}=0.2042, & T_{p_3}=0.1595\\ p_{1_0}=0.0089, & p_{2_0}=0.0139, & p_{3_0}=0.0234\end{cases}$$

Taking $T = 0.1$ and $\beta = 1$, applying the stability results presented in Theorem 1 for specific time delays $0.1 \le d_1(k) \le 0.30$, $0.1 \le d_2(k) \le 0.25$, and $0.1 \le d_3(k) \le 0.20$, and using the algorithm proposed in (28), the obtained controller gains are

$$K_1=\begin{bmatrix}-4.0782 & -0.0544 & 0\,0\,0\,0\\ 0 & 0 & 0\,0\,0\,0\\ 0 & 0 & 0\,0\,0\,0\end{bmatrix}\times 10^{-5},$$

$$K_2=\begin{bmatrix}0\,0 & 0 & 0 & 0\,0\\ 0\,0 & -3.7945 & -0.0477 & 0\,0\\ 0\,0 & 0 & 0 & 0\,0\end{bmatrix}\times 10^{-5},$$

$$K_3=\begin{bmatrix}0\,0\,0\,0 & 0 & 0\\ 0\,0\,0\,0 & 0 & 0\\ 0\,0\,0\,0 & -0.0049 & -0.0001\end{bmatrix}\times 10^{-5}$$

where $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-4}$, and $\varepsilon_3 = 10^{-4}$. Then, the obtained stability radius is $\delta = 17$.

Now, in order to show the proposed controller efficiency to minimize the upper bound of the $\mathcal{L}_2$-gain of $w(k)$ on $z(k)$, we apply the stability results presented in Theorem 1 for $T = 0.1$, $\beta = 1$, $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-4}$, $\varepsilon_3 = 10^{-4}$, $0.1 \le d_1(k) \le 0.30$, $0.1 \le d_2(k) \le 0.25$, and $0.1 \le d_3(k) \le 0.20$. Then, the obtained controller gains are given by

$$K_1=\begin{bmatrix}-0.5998 & -0.0056 & 0\,0\,0\,0\\ 0 & 0 & 0\,0\,0\,0\\ 0 & 0 & 0\,0\,0\,0\end{bmatrix}\times 10^{-3},$$

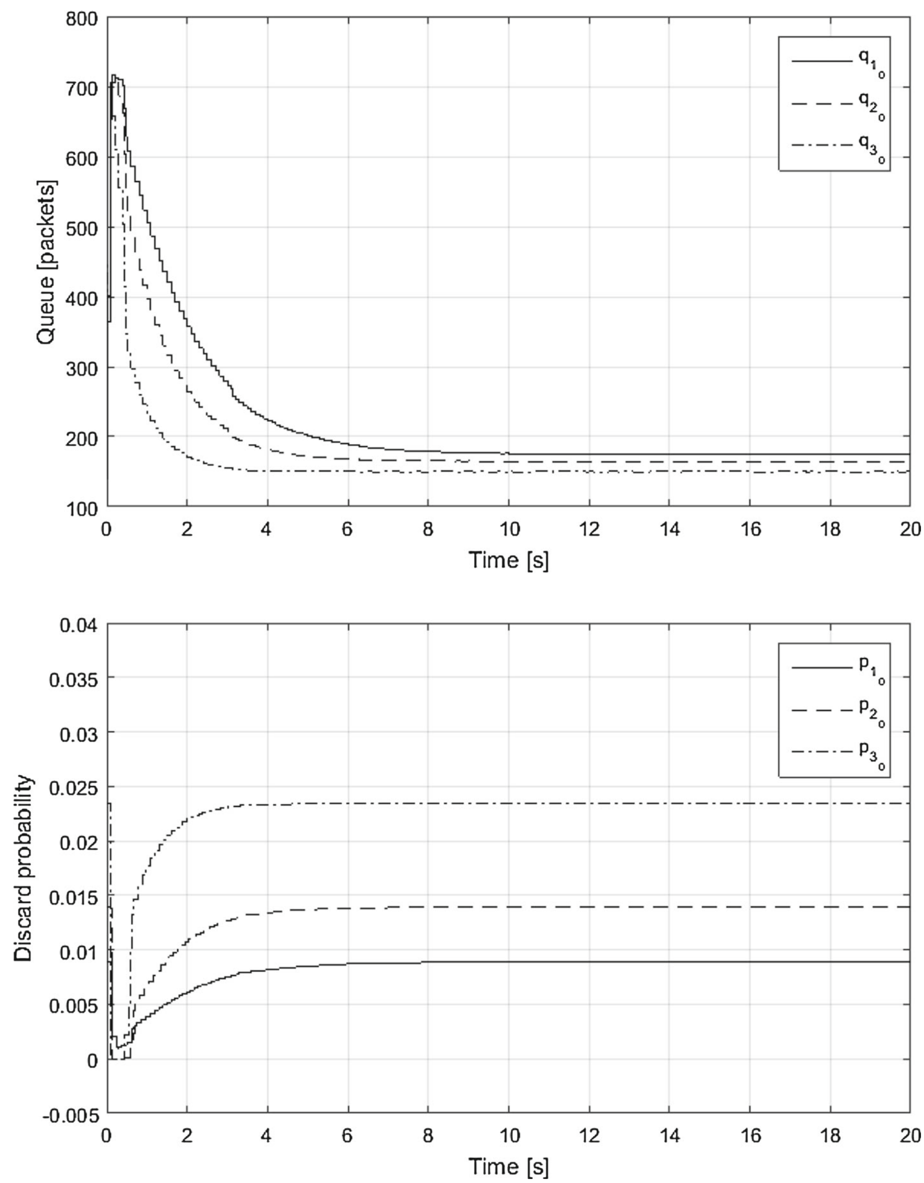

Fig. 2 Variations in queue lengths and discard probabilities over average values

$$K_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1.1463 & -0.0129 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times 10^{-3},$$

$$K_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2.4632 & -0.0297 \end{bmatrix} \times 10^{-3}. \quad (29)$$

On the other hand, the prescribed scalar is $\gamma = 1.8942 \times 10^{-4}$.

The controller is designed to regulate the router queue around a specific predetermined length, so some simulation results are presented in Fig. 2 based on the controller gains obtained in (29). Given the novelty of our approach, we cannot compare it with previous approaches because this is so far the first control theory-based approach for the AQM problem on TCP/IP router that allows the multiclass AQM. The simulation results are based on the initial values of the states $x(0) = [10 \quad -10 \quad 10 \quad -10 \quad 10 \quad -10]^T$ and the Gaussian noise (for a limited time interval) as presented in Fig. 1 to check the effect of random disturbances.

Then, Fig. 2 depicts the overall queue size and discard probability using as references for the queues lengths the nominal values 175, 165, and 150, respectively, and for the probability of packet marking/dropping the nominal values 0.0089, 0.0139, and 0.0234, respectively: the queue length is regulated properly to the desired value, giving low fluctuations in the presence of variations in the network parameters. These good queue length regulation ensures the QoS where the delay in the packets is more controllable when dealing with multiple applications. On the other hand, as shown in Fig. 2, the proposed method requires a minor adaptation of the packet drop probability to achieve the desired performance. Finally, the simulation results confirm the validity of the approach (i.e., multiclass traffic) adopted in this paper.

5 Conclusion

In this work, an H_∞ controller for discrete time multi-delayed systems subject to input saturation and $\mathcal{L}_2$-norm disturbances is designed to ensure the closed loop stability and a certain level of performance, motivated by a multiclass AQM control problem. The gains of this controller are obtained from the LMI conditions: a new approach to overcome the obstacle of the bilinearity without additional restrictive conditions is proposed using free weighting matrices to offer more flexibility to our results. To illustrate the effectiveness of our methodology, a numerical example is presented in which we consider that three different traffic classes pass through a congested router, where each class has its specificities. Our results improve the router queue convergence and motivate the authors to continue their efforts toward achieving a more realistic, implementable and yet rigorous AQM solution for the TCP/IP routers.

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