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Residential Demand Response Based on Weighted Load Shifting and Reduction Target

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Abstract

The paper presents a model for demand response of residential consumers including several alternatives for load shifting and for reduction in the electricity bill. The consumer behavior is based on time-of-use tariffs and modeled considering three goals for the end-user: (1) to reduce the electricity bill; (2) to maintain the monthly consumption; and (3) to respond to price signals without load shifting. Using real data from an electrical distribution utility operating in the south of Brazil, we present a practical application that demonstrates the main features and advantages of the proposed model in comparison with available models.

Keywords Demand response · Time-of-use · Price elasticity · Residential consumption

1 Introduction

Although the cost to supply electricity can be extremely variable, in power systems which have undergone a restructuring process, utilities purchase energy at variable prices in the wholesale market and sell it at a fixed price to retailers (Gutiérrez-Alcaraz et al. 2016). However, fixed prices can cause disparities between real costs and the profit generated by flat tariffs; in addition, some typical effects are no longer dealt with in a proper way, such as the concentration of consumption during peak hours, unrealistic prices, and inefficient use of energy resources (Haider et al. 2016).

In a worldwide context, 28% of the global electricity consumption is related to the residential sector; thus, any reduction in the peak demand of this sector can improve the system performance, helping to make it more profitable and more efficient (McKenna and Keane 2016). Since the consumption modification could be encouraged with the introduction of new tariffs, demand response programs (DRPs) are designed to motivate end-users to change their habits by responding to price signals (Kirschen et al. 2000). Energy management, such as price-based energy management schemes or incentive-based approaches, is considered the first option when making decisions on energy policies, due to its benefits from both economic and environmental viewpoints (Rahmani-Andebili and Shen 2017). In fact, the first and most natural step to achieve the goals of peak shaving and load shifting is to motivate consumers to change their habits. Thus, the major challenge of DRPs is to balance opposing objectives of consumers and utilities. On the one hand, end-users seek to minimize the amount paid for energy and at the same time to maximize or keep their level of comfort; in opposition, utilities aim to minimize the cost of the supplied energy.

From a practical perspective, the quantification of the results achieved through DRPs is as important as the implementation of this kind of program. Given that consumers may differ widely in their individual consumption habits, the effect of a given consumer on the system is difficult to assess, since the number of variables involved is large, and the way they relate is very intricate and nonlinear. However, when a large number of consumers are considered, the statistical approach through well-known concepts, such as *diversity*, allows utilities to assess the effects of DRPs through relatively simple models (Moghaddam et al. 2011).

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In the present article, the main subject is the assessment of end-user sensitivity to price variation, defined as price elasticity of demand. Although part of the residential consumers may voluntarily opt to variable tariffs, enabling them to reduce the monthly electricity bill, another part of the consumers may not change their consumption habits. Yet another part of the consumers will modify their load pattern only when a minimum reduction target is established to reduce the monthly electricity bill. Therefore, instead of considering price elasticities as an input parameter of the DRP model, this paper proposes a mathematical model to obtain the elasticities which allow quantifying different consumer behaviors, based on weighted load shifting and including reduction targets.

In the next subsection, an overview of related works is presented in order to highlight the differences and similarities between the proposed method and similar published methods.

1.1 Related Works

Several different methods have been proposed in recent years aiming to evaluate the impacts of DRPs. The majority of these methods explore demand modeling techniques using *price elasticity* of demand, which is considered a key factor in the design of DRPs (Asadinejad et al. 2016), as elasticity quantifies to what extent consumers can and in fact respond to price variations (Lima et al. 2017). The price elasticity and the level of customer participation in DRPs are two critical factors with heavy impact on DR effectiveness (Hajibandeh et al. 2019). If the electricity price varies during different time periods (valley, off-peak, and peak periods), the consumer behavior in terms of demand versus sensitivity can be characterized as (Rahmani-Andebili 2016a): (1) one part of demand of the end-user has single period sensitivity, since it cannot be transferred to other periods, based on electricity prices available in other periods, and it is called *self-elasticity*; (2) another part of demand of the end-user has multi-period sensitivity, since it can be transferred from one period to other periods, based on variable tariffs, and it is called *cross-elasticity*.

In Kirschen et al. (2000), a matrix approach of self and cross-elasticities has been proposed, allowing to evaluate different types of price-based DRPs and customer reactions. However, the authors did not clearly show how they determined the elasticities. In contrast, the determination of elasticities has been the subject of several related works (Rahmani-Andebili and Shen 2017; Hajibandeh et al. 2019; Venkatesan et al. 2012; Aalami et al. 2015; Rahmani-Andebili 2016a, 2013).

The determination of self-elasticities using linear models has been proposed by Moghaddam et al. (2011) in order to express the demand response to a three-part time-of-use (ToU) tariff. Further, price-controlled energy management of end-users has been proposed by Rahmani-Andebili and Shen (2017), considering different mathematical behavioral models, in order to optimize the unit commitment and generation scheduling. Basically, elasticities have decreased or increased to encourage load shifting so as to improve the load factor and thus reduce the overall cost of the generation. However, Rahmani-Andebili and Shen (2017) have adopted elasticities from Kirschen et al. (2000) with some modification; unfortunately, no further detail was provided on the elasticities calculation.

Asadinejad et al. (2016) have proposed clustering of consumer behavior based on the response of households to incentive-based DRPs; price elasticities of each cluster were also presented and discussed. The effect of Tariff Flags on the Brazilian power system was investigated by Lima et al. (2017). Due to the lack of specific available data in the literature, the authors devised an interesting way to determine the price elasticity from historical data of tariffs and residential consumption. However, since the flag mechanism actually represents a monthly adjustment of flat tariffs, aiming to minimize eventual differences between costs and revenues of the utilities, a single elasticity value was used.

The impacts of a comprehensive set of DR programs applied to wind power integration were analyzed by Hajibandeh et al. (2019). Their proposed model represents the optimal amount of demand for customers who participated in DR programs that consider given electricity tariffs, incentive, and penalty. However, the price elasticities of demand were considered as an input parameter; in addition, the authors did not attempt to determine accurate values for elasticities in their study.

Venkatesan et al. (2012) have investigated the validity of assumptions about the consumer rationality to develop demand price elasticity matrices. Even though it was allowed to model different behaviors, a major drawback was that they disregarded the weekly availability for allocating the shifted demand.

Nonlinear models have been investigated by Aalami et al. (2015) and compared with a previously developed linear model, concluding that both models perform almost equally well for small elasticity values as well as for small price changes. Conversely, other authors suggest that costumers react differently and in a nonlinear way to similar types of DRPs; nonlinear models, including social welfare, have been also extensively investigated (Rahmani-Andebili 2016a, b). Further, it has been demonstrated that assuming linear and exponential behavioral models, the improvement in the load factor and the amount of energy saved are similar, with and without implementation of DRPs; moreover, these two types of behaviors result in more saved energy compared with power and logarithmic models.

Finally, a method has been proposed by Salehpour and Tafreshi (2019) to analyze the demand response considering system uncertainties, such as the amount of energy negotiated in energy markets, storage capacity, and reliability. The authors presented a detailed procedure to obtain the elasticity matrix when considering uncertainties. The coefficients of the elasticity matrix are estimated using data analysis concerning historical load and price; these coefficients are thus indicated by lower and upper bounds. However, the matrices proposed to describe the load shifting, occurring during the hours of a given day, disregarded the weekly availability to allocate the shifted loads.

1.2 Main Contributions

Although residential demand response models have already investigated, to our knowledge, no previous work proposed a mathematical model to determine the cross-elasticities based on weighted load shifting, including reduction target. Moreover, the major part of published papers uses the price elasticities taken from Kirschen et al. (2000) as an input parameter, with no details about the modifications adopted.

In the context thus far described, this paper contributes to existing studies in the following aspects:

– we introduce a new formulation for cross-elasticities coefficients, based on weighted load shifting approach, including lossless cases of demand response;
– we present a demand price elasticity matrix, considering the weekly availability for reallocating loads between business days and weekends;
– we define guidelines for demand response design considering different consumer clusters and reduced targets of electricity bill;
– we evaluate the proposed approach with a case study based on data of a real distribution system, and the results are discussed considering consumer and utility gains.

It should be stressed that the major advantage of the method we propose is the systematic way elasticities can be calculated so that the user sensitivity to price variations can be estimated more confidently. To our knowledge, a similar approach has not been attempted up to date.

2 Modeling Demand Response Programs

DRPs are among the most effective ways to promote changes in energy consumption behavior. In practice, to motivate end-users to voluntarily take part in DRPs and therefore change their consumption habits, utilities have to devise strategies to give end-users some advantages, such as lower tariffs and uninterruptible load contracts (Kirschen et al. 2000). The actions typically expected from end-users include: (1) changes in the hours of the day in which they consume energy and (2) reduction in their demand. Further, the actions can also include the replacement of old devices by new, more efficient devices.

The literature often classifies DRPs into two types: incentive-based and price-based type. The first DRP type is frequently promoted by utilities, load-serving entities, or regional grid operators, motivated either by grid reliability problems or by high electricity prices. This type of program normally offers end-users some financial compensation, when they agree to reduce their demand, and includes voluntary and mandatory actions (Rahmani-Andebili 2016a). On the other hand, the price-based DRP is voluntary and gives end-users time-varying energy rates that reflect the electricity cost during different time periods. In the present paper, price-based DRPs are modeled for typical households, considering the possibility of load shifting. The following subsections shortly describe the main concepts related to ToU tariff, elasticity, clustering, and consumer behavior, which are essential to better understand the remaining of the paper.

2.1 Time-of-Use Tariff

Usually, households pay a fixed price for the electricity ($/kWh), which does not reflect the change in electricity cost over time. When consumers adhere to a price-based DRP, they can choose a tariff with hourly differentiation and thus reduce the total amount paid for electricity by consuming during most favorable hours of the day. Since customers can be strongly influenced by price policies, this paper focuses on the impact of ToU tariffs by defining two or more daily periods in which the energy price is directly related to the system load, which means that peak loads are translated into a high price and conversely (Moghaddam et al. 2011).

Furthermore, the energy price may also change along the days of the week or the months of the year. Several reasons can justify the adoption of ToU tariffs, especially the fact that consumers paying flat prices could feel insecure about the volatility of real-time pricing. As a consequence, they may hesitate to adhere to DRPs requiring them to follow the energy price and then adjust their consumption to low-price periods. Nevertheless, the relative simplicity of ToU tariffs makes its acceptance by residential consumers easier.

2.2 Intensity of End-User Demand Response

According to Kirschen et al. (2000), the response of customers to changes in the electricity price depends largely on the time period considered. This time-dependent response can be quantified and characterized by the concept of *elasticity*. Accordingly, when a consumer decides to change his demand at a given time instant as a result of a price change concerning the same instant, the change in the demand is described by the *self-elasticity*. In contrast, when a consumer can change his demand based on prices available for different hours of the day and/or days of the week, the demand change is described by the *cross-elasticity*, since the decision is influenced by tariffs valid for multiple periods.

2.2.1 Demand Price Elasticity Matrix (DPEM)

The end-user sensitivity to price variation can be represented by an elasticity matrix (Kirschen et al. 2000), in which each column represents the load changes throughout the day, corresponding to changes in price at the time instant given by column numbers (Venkatesan et al. 2012). Hence, for a pricing system with n tariff periods, the DPEM is given by (Kirschen et al. 2000):

$$\mathbf{E} = \begin{bmatrix} \varepsilon_{1,1} & \varepsilon_{1,2} & \cdots & \varepsilon_{1,n} \\ \varepsilon_{2,1} & \varepsilon_{2,2} & \cdots & \varepsilon_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon_{n,1} & \varepsilon_{n,2} & \cdots & \varepsilon_{n,n} \end{bmatrix}, \tag{1}$$

where the diagonal elements represent self-elasticities while off-diagonal elements represent cross-elasticities. In matrix form, the total demand variation, $\Delta\mathbf{D}$, is obtained by multiplying elasticities and prices, as given below (Kirschen et al. 2000):

$$\Delta\mathbf{D} = \mathbf{E}\ \Delta\mathbf{P}. \tag{2}$$

The vector $\Delta\mathbf{D}$ has dimension $n \times 1$ and its elements are $\Delta d_i = d_i - d_i^0$, where d_i^0 is the demand for the time period i (peak period, intermediary period, or off-peak period), and where d_i is the corresponding demand after the end-user has subscribed to the ToU tariff. The price variation $\Delta\mathbf{P}$ in (2) is an $n \times 1$ vector whose elements are $\Delta p_i = p_i - p^0$, where p_i is the ToU tariff for the time period i, and p^0 is the flat tariff.

An important feature of DPEM is the ability to mathematically represent the behavior of different types of consumers eligible to join a DRP. The consumer behavior can be classified into two basic types (Venkatesan et al. 2012):

– *long-range consumers*, who can shift their consumption over a wide period of time. In this case, most or all of the cross-elasticities are nonzero;
– *short-range consumers*, who are only concerned with the price during a relatively short time; consequently, the DPEM contains only diagonal elements, with all cross-elasticities being null.

A pure long-range or short-range behavior cannot be assigned to the majority of typical consumers; thus, they actually have to be modeled as having a mixed behavior, since it is not always possible to transfer all of their demand to periods with lower prices. In addition, although DPEM can successfully be used to model the consumer behavior, the construction of this kind of matrix poses some practical difficulties. Therefore, the model we propose includes a procedure to obtain the elements of the DPEM in such a way that they accurately represent the behavior expected from the consumer (see Sect. 3).

2.2.2 Self-Elasticity Elements

These elements are considered a measure of the load curtailment undertaken by the consumer (Venkatesan et al. 2012) to those loads that cannot be moved from one period to another but only turned on/off, as the case of lighting loads (Moghaddam et al. 2011). The self-elasticity is calculated for the electricity price p_i and the demand d_i at the time instant i as follows (Schweppe 1988):

$$\varepsilon_{i,i} = \frac{\partial d_i}{\partial p_i}, \tag{3}$$

where $\varepsilon_{i,i} \leq 0$. The term p_i corresponds to the ToU tariff, whereas the term d_i is replaced by the following affine relation:

$$d_i = a_i p_i + b_i, \tag{4}$$

assuming a linear relationship between demand and price, where the coefficient b_i is the intercept and a_i the line slope at the time i (Moghaddam et al. 2011). The diagonal elements of (1) can now be obtained by inserting (4) into (3) and performing the indicated differentiation, from which results $\varepsilon_{i,i} = a_i$.

2.2.3 Cross-Elasticity Elements

Cross-elasticity elements express load shifting or changes in demand at a given time (Venkatesan et al. 2012), concerning consumption that could be transferred from a peak period to off-peak or valley periods (Moghaddam et al. 2011); mathematically, they are given by:

$$\varepsilon_{i,j} = \frac{\partial d_i}{\partial p_j}, \tag{5}$$

where $i \neq j$ and $\varepsilon_{i,j} \geq 0$. The cross-elasticity $\varepsilon_{i,j}$ indicates the relative change in demand for the ith hour resulting from a change in the electricity price at the jth hour. Unlike the self-elasticity, the demand of the ith hour decreases when the price of jth hour decreases (Aalami et al. 2015). Hence, a column j of DPEM indicates how a change in price during

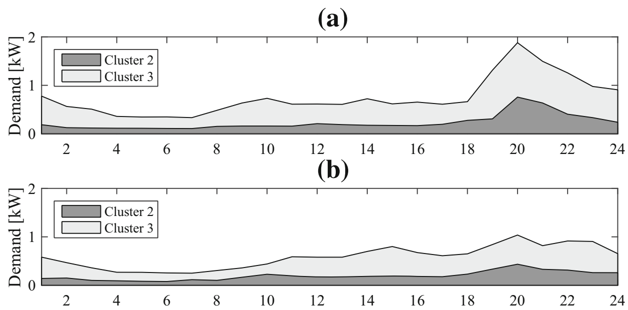

Fig. 1 Typical load profiles during **a** a business day and **b** a Saturday

a single period j affects other periods (Moghaddam et al. 2011).

2.3 Clustering of Consumer Behavior

Historical data of electricity consumption contain significant information which can help utilities to define strategies for demand response programs. Further, the DRP implementation typically requires rules to classify the consumers; these rules can be defined both using clustering methods and considering similarities assessed through load profile analysis (Viegas et al. 2016). On the other hand, the number of clusters can be defined using several criteria, including requirements of regulatory agencies. For instance, the Brazilian Electricity Regulatory Agency (ANEEL 2016) establishes five households clusters, which are based on the annual average consumption, as follows: Cluster 1 (0 to 80 kWh), Cluster 2 (81 to 160 kWh), Cluster 3 (161 to 500 kWh), Cluster 4 (501 to 1000 kWh), and Cluster 5 (above 1001 kWh).

The data required by the clustering process are in general obtained through measurement campaigns promoted by utilities. Based on measured data, similar consumer behavior can then be grouped into clusters; further, load profiles can help understand how electricity is used over a particular period of time, as depicted in Fig. 1.

Given that the measured data are inevitably corrupted by some sort of invalid information (bad data), which can affect the final results, the clustering process is performed in several steps involving aspects such as checking of data consistency and the identification of outliers, to avoid any distortion in the average consumption (Salgado et al. 2016). Outliers may be loosely defined as points that are too far from the expected value calculated from the available data (Pearson 2002); thus, an effective detection of outliers becomes an important step. One of the best approaches to detect outliers is the well-known $3\delta-$*edit rule*, which uses the standard deviation (δ) as a reference value. However, this rule proved ineffective in the presence of multiple outliers due to the *masking effect* of the outliers on δ and on the mean value. Therefore, a more robust reference value is the sample median, as used by the Hampel Filter (Hampel 1985), in which the median absolute deviation (MAD) replaces the standard deviation; MAD is denoted by S and calculated as follows (Salgado et al. 2016):

$$S = \frac{\text{median}_{z \in Z}\left(\left|x_z - x^{\dagger}\right|\right)}{0.6745}, \tag{6}$$

where Z represents all data in the sequence, x_z is the value of the record number z, and $x^{\dagger}$ is the median value for all records in Z (Pearson 2002). Thus, the outlier detection is based on the Hampel Test, which identifies an outlier when $\left|x_k - x^{\dagger}\right|$ is greater than the threshold, defined as $3S$ (Laouafi 2017).

3 Proposed Approach

The approach we propose aims at encouraging households to modify their consumption patterns using price signals through a price-based DRP. Toward this end, it is essential to be able to model and quantify the consumer response to ToU tariffs. Even though individual response to electricity prices is highly nonlinear, when a large number of consumers is considered, those with similar load profile can effectively be clustered and modeled more easily through the aggregate response.

Most of the consumers are in fact interested in the services which electricity can provide, so they primarily agree to ToU tariffs to reduce the electricity bill; however, some consumers, even agreeing to ToU tariffs, are reluctant to change consumption habits. To analyze the impact of different consumer objectives when adhering to price-based DRPs, we propose a novel mathematical model of DPEM. Firstly, the elasticity matrix $\mathbf{E}$ is replaced in (2) by a sum of two matrices:

$$\mathbf{E} = \mathbf{E}_s + f\mathbf{E}_c, \tag{7}$$

where $\mathbf{E}_s$ is a diagonal matrix composed only of self-elasticity elements, which are obtained from real data, and where $\mathbf{E}_c$ is a matrix containing only cross-elasticity elements. Secondly, the off-diagonal elements are calculated considering the availability of the consumer to load shifting. Thirdly, to handle different consumer objectives, DPEM is multiplied by a factor f.

In the following subsection, the calculation of the cross-elasticities is detailed, including the proposed formulation of weighted load shifting. Next, different consumer responses are modeled using the multiplying factor f, with the reduction target being defined.

3.1 Weighted Load Shifting

The problem of load shifting has been already addressed in several papers under some simplifying assumptions regarding how the matrix DPEM is defined. Since the weighted load shifting is modeled by cross-elasticities, Kirschen et al. (2000), for example, assumed that the sum of all cross-elasticities of a column j equals the self-elasticity on the same column, yet with opposite sign (which ensures a lossless situation, since all consumption will be reallocated), as given by:

$$\sum\nolimits_{i=1,i\neq j}^{n}\varepsilon_{i,j}=-\varepsilon_{j,j}. \tag{8}$$

In a more realistic situation, however, not all consumption can be transferred to alternative periods. To overcome this problem, we propose a weighting factor $w_{i,j}$, which takes into account the capacity of load transfer between different tariff periods k, as follows:

$$w_{i,j}=\frac{w_i}{\sum\nolimits_{k=1,k\neq j}^{n}w_k}. \tag{9}$$

Hence, cross-elasticities are calculated as:

$$\varepsilon_{i,j}=-\varepsilon_{j,j}w_{i,j},\ \forall i\neq j. \tag{10}$$

The proposed weighted factor was designed in such a way that it can handle the transfer of consumption from a given tariff period to another, for example, from a peak period to an off-peak period. Thus, when the cross-elasticities are multiplied by the weighting factor, it is possible to avoid infeasible solutions, since, occasionally, there is not enough time to use all the energy shifted from one period to another. This procedure can be exemplified taking into account a typical month with 20 business days and 5 weekends. When a household adheres to a ToU tariff, during business days, the off-peak period is composed of 19 h, the intermediary period of 2 h, and the peak period of the remaining 3 h. Besides, Saturdays and Sundays are assumed with 24 off-peak hours. In this example, 19 h/day multiplied by 20 days/month results in 380 h/month for $w_{i=1}$, the weight for off-peak hours of business days. The weights of intermediary and peak hours for business days are calculated by multiplying 2 h/day and 3 h/day by 20 days/month, resulting in $w_{i=2}=40$ h/month and $w_{i=3}=60$ h/month, respectively. Finally, 24 h per day multiplied by 10 days per month (5 Saturdays and 5 Sundays) results in 240 h/month for $w_{i=4}$, the weight for off-peak hours of weekends. Therefore, the vector of weights assumes the form:

$$\mathbf{W}_i=[w_1\ w_2\ w_3\ w_4]=[380\ 40\ 60\ 240]\ \frac{\text{h}}{\text{month}}, \tag{11}$$

where the weights w_1, w_2, and w_3 apply to off-peak, intermediary, and peak periods of business days, respectively, whereas w_4 applies to the off-peak period of weekends.

3.2 Reduction Target

Another important aspect of the proposed model is the possibility to evaluate different consumer behaviors, based on their objective when opting to ToU tariffs. For those consumers seeking to reduce their electricity bill, we propose to model their demand response as:

$$\sum\nolimits_{i=1}^{n}p_i d_i=(1-r)\,p^0\sum\nolimits_{i=1}^{n}d_i^0, \tag{12}$$

where the reduction target r is given in per unit (pu) and limited to $0\leq r\leq r_{\max}$ and the upper limit $r_{\max}$ depends on the consumer cluster and tariff. The new value of monthly electricity bill is obtained from:

$$\sum\nolimits_{i=1}^{n}p_i d_i=\sum\nolimits_{i=1}^{n}p_i d_i^0+\sum\nolimits_{i=1}^{n}p_i\varepsilon_{i,i}\Delta p_i+\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}p_i f\varepsilon_{i,j}\Delta p_j. \tag{13}$$

To ensure a reduction in the electricity bill, the cross-elasticities of the matrix $\mathbf{E}_{\mathrm{c}}$, appearing in (7), are multiplied by the factor f, obtained from (12) and (13):

$$f=\frac{(1-r)\,p^0\sum_{i=1}^{n}d_i^0-\sum_{i=1}^{n}p_i\left(d_i^0+\varepsilon_{i,i}\Delta p_i\right)}{\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}p_i\varepsilon_{i,j}\Delta p_j}. \tag{14}$$

Moreover, to ensure that cross-elasticities are always positive (as defined in Sect. 2.2.3), the numerator of (14) must yields a positive value, i.e., $(1-r)\,p^0\sum_{i=1}^{n}d_i^0$ must exceed the value of $\sum_{i=1}^{n}p_i\left(d_i^0+\varepsilon_{i,i}\Delta p_i\right)$, leading to $f\geq 0$.

3.3 DPEM Modeling

To demonstrate the flexibility of proposed DPEM, we introduce three hypothetical types of behavior concerning a residential response to ToU tariffs:

- *Behavior type* A: typical of long-range consumers who aim to reduce the monthly electricity bill and are ready to change their monthly consumption;
- *Behavior type* B: common to long-range consumers who seek to reduce the monthly electricity bill while keeping their monthly consumption, thus characterizing a lossless situation;
- *Behavior type* C: expected from short-range consumers who aim to reduce the electricity bill and who at the same time are not ready to shift their electricity consumption over a wide period of time.

Now based on the types of behavior so far defined, the DPEM can be given in the form below for consumers with behavior type A:

$$\mathbf{E}^{\mathrm{A}} = \begin{bmatrix} \varepsilon_{1,1} & f\varepsilon_{1,2} & \cdots & f\varepsilon_{1,n} \\ f\varepsilon_{2,1} & \varepsilon_{2,2} & \cdots & f\varepsilon_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ f\varepsilon_{n,1} & f\varepsilon_{n,2} & \cdots & \varepsilon_{n,n} \end{bmatrix}, \tag{15}$$

where f is given by (14).

For consumers with behavior type B, $f = 1$ can be assumed, as the monthly electricity consumption does not change, i.e., $\sum_{i=1}^{n} d_i$ is equal to $\sum_{i=1}^{n} d_i^o$, even if the consumer adheres to ToU tariffs. As a result, the DPEM takes the form below:

$$\mathbf{E}^{\mathrm{B}} = \begin{bmatrix} \varepsilon_{1,1} & \varepsilon_{1,2} & \cdots & \varepsilon_{1,n} \\ \varepsilon_{2,1} & \varepsilon_{2,2} & \cdots & \varepsilon_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon_{n,1} & \varepsilon_{n,2} & \cdots & \varepsilon_{n,n} \end{bmatrix}. \tag{16}$$

Further, for the behavior type B, we assume a lossless situation, where the monthly consumption is constant and therefore $\sum_{i=1}^{n} \varepsilon_{i,j} = 0,\ \forall j = 1, 2, \cdots, n$.

Finally, for consumers having behavior type C, $f = 0$ can be assumed with all cross-elasticities becoming then zero (Venkatesan et al. 2012). As a result, the DPEM has only diagonal elements, as given below:

$$\mathbf{E}^{\mathrm{C}} = \begin{bmatrix} \varepsilon_{1,1} & 0 & \cdots & 0 \\ 0 & \varepsilon_{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \varepsilon_{n,n} \end{bmatrix}. \tag{17}$$

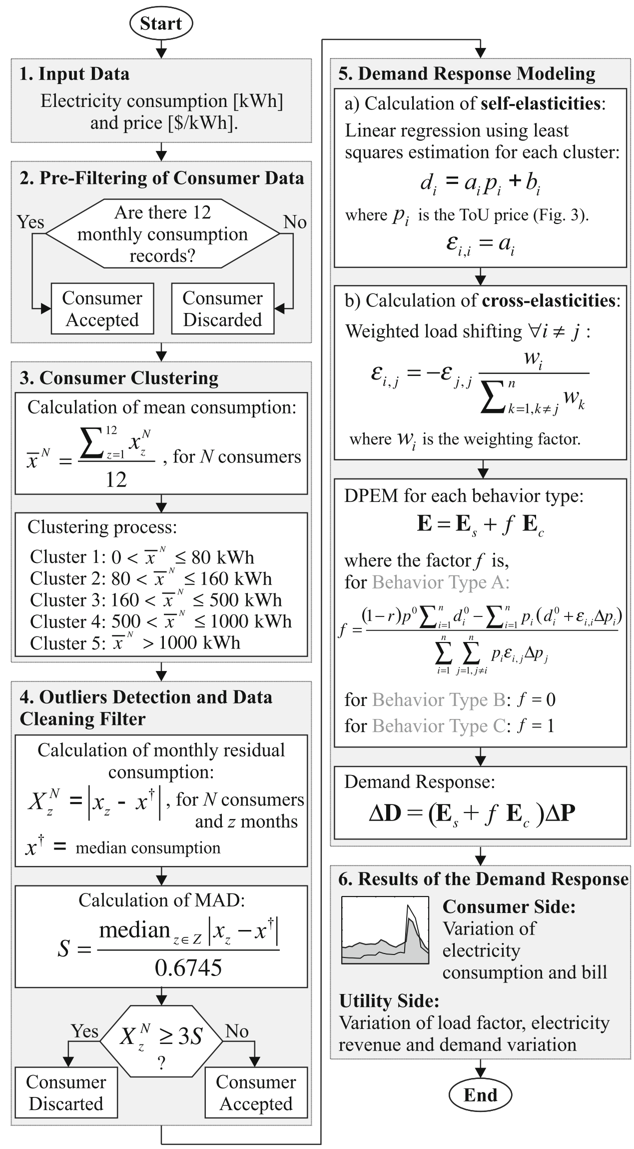

Fig. 2 Overview of the proposed algorithm

4 Case Study

To evaluate the method we proposed thus far, it was implemented in MATLAB® and subsequently applied to a case study based on data of a real distribution system. The algorithm we designed is depicted in Fig. 2 and detailed in what follows.

4.1 Input Data

Given the seasonality of the electricity consumption, historical data must include records over a period of one year. As no specific data for Brazilian low-voltage consumers were available, to model their demand response, we adapted data from the year 2015; this year can be considered representative as the price of electric energy for households increased significantly along 2015. The database we used was provided by a Brazilian electricity utility and contains the monthly consumption of 46,423 consumers (from which 43,300 are households) connected to a 13.8 kV distribution system with 1,796 transformers and 8 feeders.

The ToU tariff chosen (white tariff) was available to low-voltage Brazilian consumers in 2015 (ANEEL 2016b), with a constant price during a tariff cycle (1 year) and 3 daily periods for pricing purposes: peak, intermediary, and off-peak period. Given that each distribution utility serves a different geographical area and that consumers have different consumption habits, the utility can freely change these 3 periods according to its own needs. Figure 3 illustrates the energy prices without taxes for flat and ToU tariffs used in our study, which were converted from Brazilian Real (BRL) to US Dollar (1 US$ = 3.15 BRL).

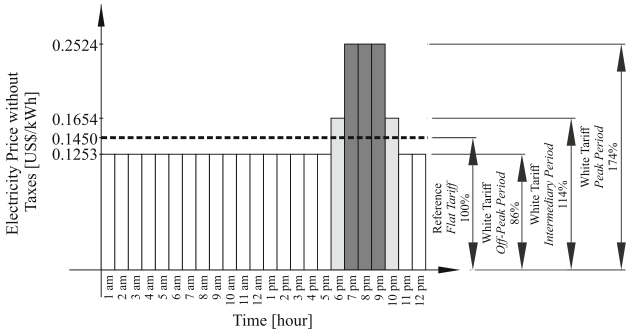

Fig. 3 Flat and white (ToU) tariffs available for the case study

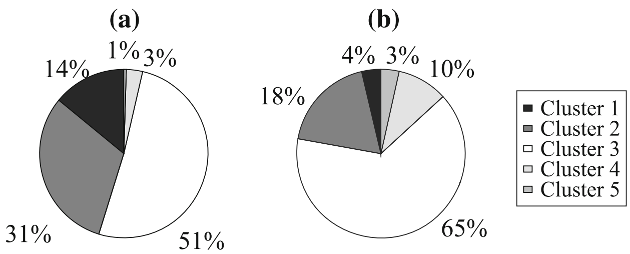

Fig. 4 Percentage share of: **a** consumers per cluster; **b** electricity consumption per cluster

4.2 Pre-filtering of Consumer Data

To avoid errors in the approximation of aggregate demand response, we selected only those households which used electricity throughout all 12 months of 2015. As a result, the pre-filtering process eliminated about 15% of the consumer records.

4.3 Consumer Clustering

Clustering can help simplify the mathematical formulation by representing different consumers with typical load profiles. We classified each consumer by the mean electricity consumption taken from the last 12 records (ANEEL 2016). Figure 4 illustrates selected data in terms of percentage share of consumers per cluster and electricity consumption per cluster.

4.4 Outliers Detection and Data Cleaning Filter

We detected the outliers by using the Hampel filter as described here. Firstly, the median of the data sequence is obtained by rank-ordering the data from the smallest to the largest; then it is taken either the middle value, if the number of data points is odd, or the average of the two values in the middle, if the number of data points is even, as detailed in Pearson (2002). Next, all records of each cluster x_z are compared to its median value $x^\dagger$ and the difference is stored in a residual vector. Finally, the data cleaning process excludes

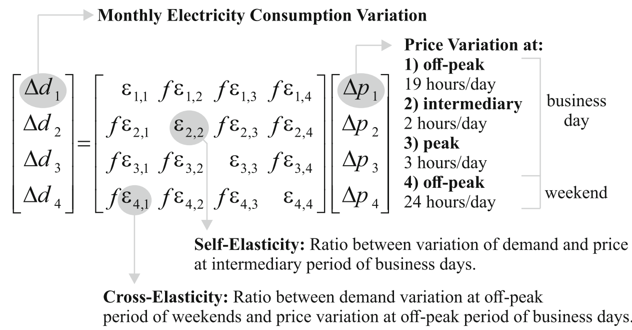

Fig. 5 Details of DPEM adopted for the case study

those customers with monthly residual exceeding three times the MAD ($3S$). In this case study, 20.42% of the customers were eliminated through the pre-filtering process, since at least 12 valid records of monthly consumption are necessary for each consumer (see algorithm in Fig. 2).

4.5 Demand Response Modeling

To the approach we propose here, it is important to predict the preference of consumers along with their response to price changes. Therefore, we propose a model to predict the demand response under all possible scenarios which is based on the habits of each typical consumer belonging to each cluster, as exemplified in Fig. 1. The first step is to define the DPEM dimension according to the type of tariff: a three-part ToU Tariff typically implies 3×3 matrices. Given that we also include the possibility of end-users shifting part of the electricity consumption to weekends, a larger DPEM is required. Therefore, we introduce a DPEM matrix with dimension 4×4, as detailed in Fig. 5, where $\varepsilon_{1,1}$, $\varepsilon_{2,2}$, and $\varepsilon_{3,3}$ are the self-elasticities of off-peak, intermediary, and peak periods on business days, and $\varepsilon_{4,4}$ is the self-elasticity regarding off-peak periods during weekends. The elements along the diagonal of DPEM come from linear regressions calculated for each cluster; the off-diagonal elements depend on the expected behavior of consumers. In addition, the off-diagonal elements of each DPEM column are proportional to self-elasticity on the same column.

A commonly used mathematical means to represent a decreasing price versus demand is the linear function (Moghaddam et al. 2011). However, when the elasticities and price ratios are low, different approximation functions can produce similar results (Aalami et al. 2015). In our case study, we used typical load profiles to estimate the consumption of each cluster over the time. Since the self-elasticities are equal to the linear coefficients a_i, we performed linear regressions using the least squares method for each cluster and each ToU period (off-peak, intermediary, and peak), thus resulting in the values shown in Table 1, where it was assumed

Table 1 Self-elasticities of consumers

Cluster	ToU period			
	Off-peak	Peak	Intermediary	Weekend
1	−0.4734	−0.0789	−0.1085	−0.3255
2	−0.3773	−0.0849	−0.1981	−0.2830
3	−0.5315	−0.0924	−0.2196	−0.3236
4	−0.6458	−0.0942	−0.1884	−0.4171
5	−0.5466	−0.0643	−0.1393	−0.3215

Table 2 Weighting factors

Day	Period	Weighting factor $w_{i,j}$	
Business	Off-peak	$w_{2,1}$	0.1176
		$w_{3,1}$	0.1765
		$w_{4,1}$	0.7059
	Intermediary	$w_{1,2}$	0.5589
		$w_{3,2}$	0.0882
		$w_{4,2}$	0.3529
	Peak	$w_{1,3}$	0.5758
		$w_{2,3}$	0.0606
		$w_{4,3}$	0.3636
Weekend	Off-peak	$w_{1,4}$	0.7917
		$w_{2,4}$	0.0833
		$w_{3,4}$	0.1250

that all prices and quantities have been normalized (Kirschen et al. 2000).

Regarding the weighted load shifting, the weights were calculated by (9) considering the values previously expressed by (11). Table 2 shows the values obtained for each $w_{i,j}$. Note that the biggest weighting factors are related to off-peak periods ($j = 1$ or $j = 4$), which can be explained by the fact that these periods are longer when compared to the peak and intermediary periods.

The determination of DPEM for each type of consumer behavior is described in what follows and numerically exemplified for five clusters.

4.5.1 DPEM for Behavior Type A

To reduce the electricity bill, as stated by (12), the cross-elasticities are multiplied by the factor f. For each cluster we assumed the maximum integer reduction for r. In this case, for Cluster 1, assuming $r = 3\%$ results in $f = 0.6723$ and the matrix $\mathbf{E}_1^A$ becomes:

$$\mathbf{E}_1^A = \begin{bmatrix} -0.4734 & 0.0296 & 0.0420 & 0.1732 \\ 0.0374 & -0.0789 & 0.0044 & 0.0182 \\ 0.0562 & 0.0047 & -0.1085 & 0.0273 \\ 0.2247 & 0.0187 & 0.0265 & -0.3255 \end{bmatrix} \text{pu.} \tag{18}$$

For clusters 2, 3, 4, and 5 we assumed r equal to 5%, 3%, 7% and 9%, respectively, thus resulting in $f = 0.1492$, $f = 0.2276$, $f = 0.7178$, and $f = 0.7981$. In this case, the matrices $\mathbf{E}_2^A$, $\mathbf{E}_3^A$, $\mathbf{E}_4^A$, and $\mathbf{E}_5^A$ become:

$$\mathbf{E}_2^A = \begin{bmatrix} -0.3773 & 0.0071 & 0.0170 & 0.0334 \\ 0.0066 & -0.0849 & 0.0018 & 0.0035 \\ 0.0099 & 0.0011 & -0.1981 & 0.0053 \\ 0.0397 & 0.0045 & 0.0107 & -0.2830 \end{bmatrix} \text{pu,} \tag{19}$$

$$\mathbf{E}_3^A = \begin{bmatrix} -0.5315 & 0.0118 & 0.0288 & 0.0583 \\ 0.0142 & -0.0924 & 0.0030 & 0.0061 \\ 0.0214 & 0.0019 & -0.2196 & 0.0092 \\ 0.0854 & 0.0074 & 0.0182 & -0.3236 \end{bmatrix} \text{pu,} \tag{20}$$

$$\mathbf{E}_4^A = \begin{bmatrix} -0.6458 & 0.0378 & 0.0779 & 0.2370 \\ 0.0545 & -0.0942 & 0.0082 & 0.0250 \\ 0.0818 & 0.0060 & -0.1884 & 0.0374 \\ 0.3272 & 0.0239 & 0.0492 & -0.4171 \end{bmatrix} \text{pu,} \tag{21}$$

$$\mathbf{E}_5^A = \begin{bmatrix} -0.5466 & 0.0287 & 0.0640 & 0.2032 \\ 0.0513 & -0.0643 & 0.0067 & 0.0214 \\ 0.0770 & 0.0045 & -0.1393 & 0.0321 \\ 0.3080 & 0.0181 & 0.0404 & -0.3215 \end{bmatrix} \text{pu.} \tag{22}$$

Since elasticity is a normalized measure of the intensity of the consumers response to tariff variations, we assumed as base values: 0.2215 US$/kWh for the electricity price; 49.95 kWh, 159.10 kWh, 499.05 kWh, 1000.20 kWh, and 1500.00 kWh for the monthly consumption of Clusters 1, 2, 3, 4, and 5, respectively.

4.5.2 DPEM for Behavior Type B

This case expresses the load shifting behavior of those consumers willing to maintain the monthly electricity consumption, before and after they adhere to a DRP. As previously described in Sect. 3, considering $f = 1$, the matrices $\mathbf{E}_1^B$, $\mathbf{E}_2^B$, $\mathbf{E}_3^B$, $\mathbf{E}_4^B$, and $\mathbf{E}_5^B$ become:

$$\mathbf{E}_1^B = \begin{bmatrix} -0.4734 & 0.0441 & 0.0625 & 0.2577 \\ 0.0557 & -0.0789 & 0.0066 & 0.0271 \\ 0.0835 & 0.0070 & -0.1085 & 0.0407 \\ 0.3342 & 0.0278 & 0.0394 & -0.3255 \end{bmatrix} \text{pu,} \tag{23}$$

$$\mathbf{E}_2^B = \begin{bmatrix} -0.3773 & 0.0474 & 0.1141 & 0.2240 \\ 0.0444 & -0.0849 & 0.0120 & 0.0236 \\ 0.0666 & 0.0075 & -0.1981 & 0.0354 \\ 0.2663 & 0.0300 & 0.0720 & -0.2830 \end{bmatrix} \text{pu,} \tag{24}$$

$$\mathbf{E}_3^B = \begin{bmatrix} -0.5315 & 0.0517 & 0.1264 & 0.2562 \\ 0.0625 & -0.0924 & 0.0133 & 0.0270 \\ 0.0938 & 0.0081 & -0.2196 & 0.0404 \\ 0.3752 & 0.0326 & 0.0799 & -0.3236 \end{bmatrix} \text{pu,} \tag{25}$$

$$\mathbf{E}_4^{\mathrm{B}} = \begin{bmatrix} -0.6458 & 0.0526 & 0.1085 & 0.3302 \\ 0.0760 & -0.0942 & 0.0114 & 0.0348 \\ 0.1140 & 0.0083 & -0.1884 & 0.0521 \\ 0.4558 & 0.0333 & 0.0685 & -0.4171 \end{bmatrix} \text{pu}, \tag{26}$$

$$\mathbf{E}_5^{\mathrm{B}} = \begin{bmatrix} -0.5466 & 0.0359 & 0.0802 & 0.2545 \\ 0.0643 & -0.0643 & 0.0084 & 0.0268 \\ 0.0965 & 0.0057 & -0.1393 & 0.0402 \\ 0.3858 & 0.0227 & 0.0507 & -0.3215 \end{bmatrix} \text{pu}. \tag{27}$$

In this case, the sum of the elements in each column is zero, corresponding to the behavior of maintaining the monthly electricity consumption before and after joining a DRP.

4.5.3 DPEM for Behavior Type C

This case describes the behavior of short-range consumers, who aim to reduce the electricity bill without shifting the peak consumption for periods of lower tariff. Since $f = 0$, all cross-elasticities in DPEM are zero and the matrices $\mathbf{E}_1^{\mathrm{C}}$, $\mathbf{E}_2^{\mathrm{C}}$, $\mathbf{E}_3^{\mathrm{C}}$, $\mathbf{E}_4^{\mathrm{C}}$, and $\mathbf{E}_5^{\mathrm{C}}$ have only elements along the diagonal. For example, the matrix $\mathbf{E}_1^{\mathrm{C}}$ becomes:

$$\mathbf{E}_1^{\mathrm{C}} = \begin{bmatrix} -0.4734 & 0 & 0 & 0 \\ 0 & -0.0789 & 0 & 0 \\ 0 & 0 & -0.1085 & 0 \\ 0 & 0 & 0 & -0.3255 \end{bmatrix} \text{pu}. \tag{28}$$

4.6 Results of the Demand Response

Table 3 presents the results of the case study, including the situations in which the consumer adheres to the flat tariff, and to the ToU tariff. The demand response of each behavior type is indicated separately, by the consumer side and by the utility side. The next section is dedicated to the analysis and discussion of the results obtained through the application of the method we propose.

5 Results and Discussion

This section presents a discussion of the impacts produced by the demand response on households that adhere to a ToU tariff; the discussion also includes the impacts to the utility. Regarding the consumer gains, we analyzed the electricity consumption and monthly bill for a single end-user of each cluster. Concerning utility gains, since the exact number of consumers which effectively adhere to ToU tariffs is uncertain, we adopted 50% of implementation potential for DRPs (Rahmani-Andebili 2013). In this case, the load factor, electricity revenue, and demand variation were obtained for the Brazilian distribution system previously described in Sect. 4.1. Table 3 presents the results when residential consumers opt for a flat tariff (*behavior type base*).

With regard to the results considering that the consumers have subscribed to a ToU tariff, we assumed three hypothetical types described earlier (types A, B, and C) and compared them with two models described in the works reviewed in Sect. 1.1 (types I and II). *Behavior type I* was based on Kirschen et al. (2000) and results in the same DPEM for all clusters, which can be expressed by:

$$\mathbf{E}_{\mathrm{I}} = \begin{bmatrix} -0.200 & 0.008 & 0.006 & 0 \\ 0.010 & -0.200 & 0.008 & 0 \\ 0.012 & 0.016 & -0.200 & 0 \\ 0 & 0 & 0 & -0.200 \end{bmatrix} \text{pu}. \tag{29}$$

Similarly, *behavior type II* was based on Rahmani-Andebili (2013) and also gives the same DPEM for all clusters, as follows:

$$\mathbf{E}_{\mathrm{II}} = \begin{bmatrix} -0.230 & 0.016 & 0.049 & 0 \\ 0.040 & -0.020 & 0.010 & 0 \\ 0.033 & 0.010 & -0.160 & 0 \\ 0 & 0 & 0 & -0.230 \end{bmatrix} \text{pu}. \tag{30}$$

Since Kirschen et al. (2000) and Rahmani-Andebili (2013) did not consider the weekly availability for reallocating load between business days and weekends, we modified both original models, including the fourth row and the fourth column of the DPEM given by (29) and (30). In addition, the self-elasticities $\varepsilon_{1,1}$ and $\varepsilon_{3,3}$ of Behavior II corresponds to the maximum value indicated in Rahmani-Andebili (2013).

5.1 Analysis of the Model for Demand Response

Before analyzing consumption and bill variations, it is important to discuss the price elasticity models and their influence on demand response. As described in Sect. 4, we have obtained the self-elasticities by performing linear regressions for each cluster. The diagonal coefficients of each cluster DPEM are equal for behavior types A, B, and C. On the other hand, the off-diagonal coefficients vary according to consumer expectations and habits.

In relation to behaviors A, B, and C, the self-elasticities vary from −0.0643 to −0.6458, and the cross-elasticities vary from zero to 0.4558, based on off-diagonal elements of (18)–(28). Concerning the behaviors I and II, the self-elasticities vary from −0.020 to −0.230, and the cross-elasticities vary from zero to 0.049, although (Kirschen et al. 2000; Rahmani-Andebili 2013) do not distinguish the demand response inside each cluster, the same DPEM being adopted for all residential consumers. Additionally, in the technical literature, self-elasticities of households vary from −0.146 (Lima et al. 2017) to −0.4 (Aalami et al. 2015), while cross-elasticities from 0.001667 (McKenna and Keane 2016)

Table 3 Results of the demand response for the case study

Behavior type	Consumer cluster	Consumer side				Utility side							
		Consumption		Electricity bill		Load factor[c]		Electricity revenue		Demand variation (%)[b]			
										Business days			Weekends
		(kWh)	(%)[b]	$(US\$)$	(%)[b]		(%)[b]	(10^3 US$)	(%)[b]	Off-peak	Intermediary	Peak	Off-peak
Base[a]	1	49.95	–	8.61	–	0.39	–	1094	–	–	–	–	–
	2	159.10	–	32.46	–								
	3	499.05	–	101.81	–								
	4	1000.20	–	204.04	–								
	5	1500.00	–	306.01	-								
A	1	50.17	0.44	8.36	− 3.00	0.79	103.67	1072	− 1.98	4.81	− 4.53	− 21.15	3.89
	2	152.74	− 4.00	30.83	− 5.00								
	3	489.30	− 1.95	98.75	− 3.00								
	4	998.90	− 0.13	189.76	− 7.00								
	5	1501.30	0.09	278.47	− 9.00								
B	1	49.95	0.00	8.29	− 3.80	0.81	108.95	1079	− 1.35	7.26	− 4.98	− 22.65	4.73
	2	159.11	0.00	31.74	− 2.19								
	3	499.05	0.00	99.95	− 1.82								
	4	1000.20	0.00	189.35	− 7.20								
	5	1500.00	0.00	277.40	− 9.35								
C	1	50.63	1.36	8.50	− 1.36	0.79	101.44	1071	− 2.07	3.92	− 4.11	− 20.40	3.81
	2	151.62	− 4.70	30.67	− 5.49								
	3	486.43	− 2.53	98.40	− 3.35								
	4	995.55	− 0.47	190.79	− 6.49								
	5	1506.50	0.43	282.67	− 7.63								
I	1	45.47	− 8.96	7.30	− 15.30	0.70	78.49	1056	− 3.49	1.90	− 7.80	− 20.37	2.37
	2	147.06	− 7.57	29.78	− 8.24								
	3	475.50	− 4.72	96.92	− 4.80								
	4	940.15	− 6.01	179.94	− 11.81								
	5	1369.20	− 8.72	248.20	− 18.89								

Table 3 continued

Behavior type	Consumer cluster	Consumer side				Utility side							
		Consumption		Electricity bill		Load factor[c]		Electricity revenue		Demand variation (%)[b]			
										Business days			Weekends
		(kWh)	(%)[b]	$(US\$)$	(%)[b]		(%)[b]	(10^3 US$)	(%)[b]	Off-peak	Intermediary	Peak	Off-peak
II	1	49.09	−1.73	8.04	−6.71	0.68	74.59	1086	−0.76	4.05	−0.28	−16.79	2.73
	2	156.78	−1.46	32.14	−0.98								
	3	494.51	−0.91	101.52	−0.28								
	4	988.63	−1.16	191.68	−6.06								
	5	1474.80	−1.68	273.77	−10.53								

[a] Results of the case study when consumers opts for the flat tariff
[b] Percent variation resulted from the comparison between flat tariff (base case) and ToU tariff, for each behavior type
[c] Business days

to 0.1675 (Ton et al. 2013). Moreover, no detail was found about the determination of elasticities in the published works so far reviewed.

Empirical studies suggest that demand elasticities for electricity are in general low (Lijesen 2007), even though low price elasticities are essentially the result of insufficient incentives and the inability of consumers to manage their demand in the context of the present electricity market. Moreover, most authors provide little or even no information about the data used to obtain the elasticities, making a more thorough comparison between results very difficult. Besides, elasticities reflect the difference between distinct consumer clusters, which highlights the importance of grouping the consumers according to the similarities of the load profiles, a procedure generally not followed.

5.2 Consumer Side Results

One of the important advantages of the proposed approach is that it can be applied in practice to reduce the monthly electricity bill, which is the highest priority for most consumers since they are mainly interested in minimizing the amount paid for energy (Haider et al. 2016). Based on behavior types A, B, and C, the end-users of Cluster 1 presented a rather modest demand response to the ToU tariff; due to their low monthly consumption, only a reduction no larger than 3.80% can be achieved. By contrast, end-users of Cluster 5 are encouraged to adhere to the ToU tariff, as their higher consumption allows a reduction target of up to 9.35%, which can be achieved with small changes in the consumption habits. In addition, based on the variation of electricity consumption presented in Table 3, end-users of Cluster 2, consuming between 81 and 160 kWh monthly, achieved the greatest reduction (−4.70%). For behavior type I, all clusters presented a significant reduction in the electricity bill (up to 18%) when compared to other types of behavior; however, achieving the reductions indicated requires significant changes in consumption habits.

Our study reveals that households consuming 80 kWh/month respond less to price variations than those consuming 1000 kWh/month. This fact also highlights the importance of the clustering process applied to classify costumer; it also stress the need for using more realistic scenarios when assessing demand response. Additionally, according to our study, any saving depends on the prevailing ToU rates. For example, for the Brazilian ToU tariff, the relationship between the prices for peak and off-peak periods is close to 2, which severely limits the value of the maximum reduction target ($r_{\max}$) to less than 10% for all consumer clusters. On the other hand, when the relationship between the price for peak and off-peak periods is close to 5, as occurs with the tariffs in the UK (Energy 2017), for instance, it is possible to increase the maximum reduction target to up to 25%. The lower tariff in the period from 11 pm to 6 pm contributes to the reduction of the electricity bill, since the off-peak period corresponds to 86% of the time, with a consumption of about 70% of the monthly electricity. Consequently, all clusters reduce the amount spent on electricity for all types of consumer behavior, as detailed in Table 3.

5.3 Utility Side Results

In general, utilities decide on investments taking into account not only technical and economic criteria but also all possible reactions from their consumers. Therefore, it becomes essential to evaluate the impact of customers participation in DRPs from the point of view of the utility. Concerning our case study, we present the utility side results in terms of load factor, electricity revenue, and demand variation. Since price-based energy management schemes are designed to encourage end-users to change their habits, it was found that if 50% of consumers of the real distribution system we considered here adhere to the white tariff, for behavior types A, B, and C, the load factor doubles. Additionally, the improvement on system performance would allow the utility to postpone investments, a revenue reduction of around 2% being then feasible. Further, the lowest reduction was obtained for behavior type B, which represents a lossless situation (customers keep their monthly consumption unchanged). On the other hand, Behavior II presented the lowest revenue reduction, yet with a smaller variation in the load factor, because the DPEM of behavior types I and II is equal for all clusters.

Since distribution systems are designed to supply the peak demand of consumption occurring only for a few hours a day, it is important to evaluate the demand variation of each ToU period, especially during peak periods. Our study demonstrates a high potential for load shifting, around 20% of peak shaving, the double of peak shifting achieved in previous studies with ToU tariffs (Yan et al. 2018). Considering the possibility of reallocating loads between business days and weekends, we obtained 4% of valley filling. Regarding the intermediary and peak periods, according to column twelve and thirteen of Table 3, customers reduced the electricity use independent of the behavior type, since the differences between ToU and Flat tariffs are 0.0204 and 0.1074 US$/kWh, respectively, thus discouraging the use of electricity between 6 and 11 pm. Concerning the off-peak consumption, the results show a more favorable situation for consumers; in this case, a ToU tariff of 0.1253 US$/kWh and a flat tariff of 0.1450 US$/kWh (see Fig. 3) stimulated the use of electricity during this period. As a result, the demand increased during off-peak periods on business days and also on weekends.

6 Conclusions

The potential of demand response programs is related to the extent consumers modify their habits based on price variation. We proposed here a model based on demand price elasticity matrix which allows assessing the load management capability of consumers clusters. Since end-users with similar load profile can be clustered, each group respond differently to time-varying price. The behavior sensitivity was measured by an elasticity matrix, enabling to formulate conditions which reflect the intent of consumers. Different from traditional methods, a weighted load shifting approach was presented, in order to represent those consumers who intend either maintain their electricity consumption or reduce their monthly bill. We also included in our analysis customers only concerned with the current price, e.g., with no intention to shift their loads.

The proposed method was evaluated using data of a real distribution system and the results led to the following conclusions:

- regarding consumer savings: without a significant difference between peak and off-peak prices, time-based tariffs are not attractive. Table 3 demonstrates that this is true independent of the cluster and the behavior type considered, including those behaviors taken from previous works (types I and II);
- to accurately represent the behavior of consumers, it is essential to take into account their consumption habits, which are different for each cluster;
- the method we propose allowed us to determine DPEMs for each cluster considering three types of practical behaviors (consumer objectives);
- the improvement in the load factor and the reduction in the Joule losses can outweigh the consequent reduction of revenue, which is, in any case, estimated as a small reduction;
- although the case study exemplified an application with a ToU tariff, the proposed approach can be applied to any pricing schemes as well.

Finally, as future work, we suggest to model the demand price elasticities under high-level uncertainties of active distribution systems.

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References

Aalami, H. A., Moghaddam, M. P., & Yousefi, G. R. (2015). Evaluation of nonlinear models for time-based rates demand response programs. *International Journal of Electrical Power & Energy Systems*, *65*, 282–290.

ANEEL. (2016). Distribution procedure (Module 2/R.7)

ANEEL. (2016). Homologatory resolution 1858

Asadinejad, A., Varzaneh, M. G., Tomsovic, K., Chen, C. F., & Sawhney, R. (2016). Residential customers elasticity estimation and clustering based on their contribution at incentive based demand response. In *Power and energy society general meeting (PESGM)* (pp 1–5).

Energy, G. (2017). Tide-smart time-of-day tariff.

Gutiérrez-Alcaraz, G., Tovar-Hernández, J., & Lu, C. N. (2016). Effects of demand response programs on distribution system operation. *International Journal of Electrical Power & Energy Systems*, *74*, 230–237.

Haider, H. T., See, O. H., & Elmenreich, W. (2016). A review of residential demand response of smart grid. *Renewable and Sustainable Energy Reviews*, *59*, 166–178.

Hajibandeh, N., Ehsan, M., Soleymani, S., Shafie-Khah, M., & Catalão, J. P. (2019). Prioritizing the effectiveness of a comprehensive set of demand response programs on wind power integration. *International Journal of Electrical Power & Energy Systems*, *107*, 149–158.

Hampel, F. R. (1985). The breakdown points of the mean combined with some rejection rules. *Technometrics*, *27*(2), 95–107.

Kirschen, D. S., Strbac, G., Cumperayot, P., & Mendes, Dd P. (2000). Factoring the elasticity of demand in electricity prices. *IEEE Transactions on Power Systems*, *15*(2), 612–617.

Laouafi, A., et al. (2017). Online electricity demand forecasting based on an effective forecast combination methodology. *Electric Power Systems Research*, *148*, 35–47.

Lijesen, M. G. (2007). The real-time price elasticity of electricity. *Energy Economics*, *29*(2), 249–258.

Lima, D. A., Perez, R. C., & Clemente, G. (2017). A comprehensive analysis of the demand response program proposed in Brazil based on the tariff flags mechanism. *Electric Power Systems Research*, *144*, 1–12.

McKenna, K., & Keane, A. (2016). Residential load modeling of price-based demand response for network impact studies. *IEEE Transactions on Smart Grid*, *7*(5), 2285–2294.

Moghaddam, M. P., Abdollahi, A., & Rashidinejad, M. (2011). Flexible demand response programs modeling in competitive electricity markets. *Applied Energy*, *88*(9), 3257–3269.

Pearson, R. K. (2002). Outliers in process modeling and identification. *IEEE Transactions on Control Systems Technology*, *10*(1), 55–63.

Rahmani-Andebili, M. (2013). Investigating effect of changes in demand response programs on emission level of thermal power plants and market player profit. *International Journal of Electrical and Power Engineering*, *7*(2), 43–51.

Rahmani-Andebili, M. (2016a). Modeling nonlinear incentive-based and price-based demand response programs and implementing on real power markets. *Electric Power Systems Research*, *132*, 115–124.

Rahmani-Andebili, M. (2016b). Nonlinear demand response programs for residential customers with nonlinear behavioral models. *Energy and Buildings*, *119*, 352–362.

Rahmani-Andebili, M., & Shen, H. (2017). Energy management of end users modeling their reaction from a Genco's point of view. In *International conference on computing, networking and communications*. IEEE (pp. 577–581).

Salehpour, M. J., & Tafreshi, S. M. (2019). The effect of price responsive loads uncertainty on the risk-constrained optimal operation of a smart micro-grid. *International Journal of Electrical Power & Energy Systems*, *106*, 546–560.

Salgado, R. M., Machado, T. C., & Ohishi, T. (2016). Intelligent models to identification and treatment of outliers in electrical load data. *IEEE Latin America Transactions*, *14*(10), 4279–4286.

Schweppe, F. C., et al. (1988). *Spot pricing of electricity*. Boston: Kluwer Academic Publishers.

Ton D., et al. (2013). Tool for determining price elasticity of electricity demand and designing dynamic price program. In *IEEE PES innovative smart grid technologies (ISGT)* (pp. 1–6).

Venkatesan, N., Solanki, J., & Solanki, S. K. (2012). Residential demand response model and impact on voltage profile and losses of an electric distribution network. *Applied Energy*, *96*, 84–91.

Viegas, J. L., Vieira, S. M., Melício, R., Mendes, V. M. F., & Sousa, J. M. C. (2016). Classification of new electricity customers based on surveys and smart metering data. *Energy*, *107*, 804–817.

Yan, X., Ozturk, Y., Hu, Z., & Song, Y. (2018). A review on price-driven residential demand response. *Renewable and Sustainable Energy Reviews*, *96*, 411–419.