

New Approach of Stabilization Based on New Lyapunov Function for Nonlinear Takagi–Sugeno Discrete-Time Uncertain Systems

Ali Bouyahya¹ · Yassine Manai¹ · Joseph Haggège¹

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Abstract This paper examines the problem of designing a nonlinear state feedback controller (SFC) for Takagi–Sugeno discrete-time parametric uncertain systems via an iterative linear matrix inequalities (LMIs). The objective of this paper is to establish a novel framework of the SFC with conservatism reduction (less restrictive) results by introducing slack variables. These reduced conservative results are demonstrated by a larger feasible areas of stabilization (stabilization domain). Nevertheless, this paper shows that by changing the initial nonquadratic Lyapunov function, a better solution can be reached (less restrictive results). By using simulations, we verify the new condition robustness and a comparison with another approach existing in the literature to demonstrate the effectiveness of this new approach.

Keywords Discrete-time Takagi–Sugeno fuzzy systems · Lyapunov function · Linear Matrix Inequality LMI · Parametric uncertain system · Non-PDC

1 Introduction

In recent years, there has been growing interest in the study of stability and stabilization Takagi–Sugeno (T–S) fuzzy system [Ding \(2009\)](#), [Fang et al. \(2006\)](#), [Liu and Zhang \(2003\)](#),

[Lee et al. \(2001\)](#), [Takagi and Sugeno \(1985\)](#), [Tanaka and Wang \(2001\)](#) due to the fact that it provides a general framework to represent a nonlinear plant by using a set of local linear models which are smoothly connected through nonlinear fuzzy membership functions (MFs). One of the most important issues in the study of T–S fuzzy systems is the analysis of the stability with Lyapunov functions [Lee et al. \(2013\)](#). Via various approaches, a great number of stability/stabilization results for T–S fuzzy systems in both the continuous and discrete time contexts have been reported in the literature [Cao and Frank \(2001\)](#), [Mozelli et al. \(2009\)](#), [Latrach et al. \(2015\)](#).

Two classes of Lyapunov functions are used to analyze these systems: quadratic Lyapunov and nonquadratic Lyapunov functions, the second being less conservative than the first class. Many researches were investigated with nonquadratic Lyapunov functions with T–S nonlinear systems [Lin et al. \(2006\)](#), [Tanaka et al. \(2003\)](#), [Manai and Benrejeb \(2012\)](#), [Hui et al. \(2015\)](#).

Conservatism comes from different sources: the type of T–S fuzzy model, the way the membership functions (MFs) are dropped off to obtain LMI expressions [Lin et al. \(2006\)](#), [Tanaka et al. \(2003\)](#), the integration of MFs information [Manai and Benrejeb \(2012\)](#), [Koo et al. \(2011\)](#), [Fang et al. \(2006\)](#), and the choice of Lyapunov function [Tanaka and Wang \(2001\)](#), [Lee and Kim \(2009\)](#), [Manai and Benrejeb \(2012\)](#), [Kruszewski et al. \(2008\)](#).

In this paper, we deal with the problem of the reduction of conservatism for the discrete-time nonlinear systems based on the choice of the Lyapunov function. Generally, if this conservatism problem cannot be solved by one type of Lyapunov function, it can be solved by another. The conclusion is: If we choose the best Lyapunov function for the appropriate nonlinear systems, then the problem of the conservatism can be solved and better solutions are obtained.

✉ Ali Bouyahya
ali.bouyahya@enit.rnu.tn

Yassine Manai
yacine.manai@enit.rnu.tn

Joseph Haggège
joseph.haggege@enit.rnu.tn

¹ L.A.R.A. Automatic control, National Engineering School of Tunis, University Of Tunis, El Manar BP 37, Belvedere, 1002 Tunis, Tunisia

Several alternative classes of Lyapunov candidates function have been proposed in the literature for this problem, and some approaches have been based on a single Lyapunov function. These methods were basically reduced to the problem of finding a common Lyapunov function for a set of stability conditions. Since a common Lyapunov function is used for all subsystems, they can be quite conservative in some situations. Therefore, piecewise Lyapunov functions are applied to fuzzy control systems to obtain less conservative results and have received increasing attention as they attempt to relax the conservativeness of stability and stabilization problems [Ke et al. \(2011\)](#), [Feng \(2004\)](#), [Zhang and Feng \(2008\)](#), [Qiu et al. \(2010\)](#), [Feng \(2006\)](#), [Johansson et al. \(1999\)](#), [Manaa et al. \(2015\)](#). Others introduce decision variables (slack variables) in order to provide additional degrees of freedom to the LMI problem [Manai and Benrejeb \(2012\)](#), [Manai and Benrejeb \(2012\)](#), [Fang et al. \(2006\)](#), [Sala and Ariño \(2007\)](#). Some recent works investigate a delayed nonquadratic Lyapunov function. They proved that a little modification in the Lyapunov function gives a huge feasible area of stabilization [Manai and Benrejeb \(2012\)](#), [Kruszewski et al. \(2008\)](#), [Daafouz and Bernussou \(2001\)](#), [Lendek et al. \(2015\)](#), [Guerra et al. \(2012\)](#), [Xie et al. \(2014\)](#).

In this paper, a new stabilization condition for Takagi–Sugeno discrete-time parametric uncertain system with the use of a non-PDC controller and new Lyapunov function is discussed. This condition was reformulated with the linear matrix inequality (LMI) technique [Teixeira et al. \(2003\)](#), [Du and Yang \(2010\)](#) which can be efficiently solved by using the convex optimization algorithms. The goal of the proposed approach is to reduce the conservatism of a previous result. The reduction of the conservatism in this field can be shown graphically by increasing the sets of solutions of LMIs or the fast convergence of the systems states variables to their stable equilibrium points during the time, in some cases the reduction of the amplitudes of the control signals. The only way to reduce the conservatism in our study is by increasing the sets of solutions as we treat the asymptotic stabilization conditions where is not important for the fast convergence of the state variables.

The outline of this paper is as follows. First, the T–S system description is discussed. In Sect. 2, materials and mathematics tools are presented. In Sects. 3 and 4, the proposed approach is given, and the main result is proposed in LMI formulation. Robustness conditions to design the feedback controller for parametric uncertain T–S fuzzy models are given using strict LMI constraints. In Section V, examples to show the effectiveness of the proposed approach are proposed. Conclusion completes this paper.

2 System Description and Preliminaries

The discrete-time T–S fuzzy model is described by fuzzy IF–THEN rules, whose collection represents the approximation of the nonlinear system. The i th rule of the T–S fuzzy model is of the following form

$$\begin{aligned} &\text{If } z_1(k) \text{ is } M_{i1} \text{ and } \dots \text{ and } z_p(k) \text{ is } M_{ip} \\ &\text{then } \begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases} \\ &i = 1, \dots, r \end{aligned}$$

where M_{ij} ($i = 1, 2, \dots, r, j = 1, 2, \dots, p$) is the fuzzy set and r is the number of model rules, $x(k) \in \mathfrak{R}^n$ is the states vector, $u(k) \in \mathfrak{R}^m$ is the input vector, $A_i \in \mathfrak{R}^{n \times n}$ is the states matrix, $B_i \in \mathfrak{R}^{n \times m}$ is the control, and $z_1(k), \dots, z_p(k)$ are known premise variables.

The T–S discrete-time parametric uncertain model for a nonlinear system is described under the following form.

$$\begin{aligned} &\text{If } z_1(t) \text{ is } M_{i1} \text{ and } \dots \text{ and } z_p(t) \text{ is } M_{ip} \\ &\text{then } \begin{cases} x(k+1) = (A + \Delta A)x(k) + (B + \Delta B)u(k) \\ y(k+1) = (C + \Delta C)x(k) \end{cases} \quad (1) \\ &i = 1, \dots, r \end{aligned}$$

where $\Delta A_i, \Delta B_i$ are time-varying matrices representing parametric uncertainties in the model. These uncertainties are norm-bounded and structured.

The final outputs of the fuzzy systems are written under the following form.

$$\begin{aligned} x(k+1) = &\sum_{i=1}^r h_i(z(k)) \{(A_i + \Delta A_i)x(k) \\ &+ (B_i + \Delta B_i)u(k)\} \quad (2) \end{aligned}$$

where

$$z(k) = [z_1(k) \ z_2(k) \ \dots \ z_p(k)] \quad (3)$$

$$h_i(z(k)) = \frac{w_i(z(k))}{\sum_{i=1}^r w_i(z(k))} \quad (4)$$

$$w_i(z(k)) = \prod_{j=1}^p M_{ij}(z_j(k)) \quad (5)$$

for all k .

The term $M_{ij}(z_j(k))$ is the membership degree of $z_j(k)$ in M_{ij}

Since

$$\begin{cases} \sum_{i=1}^r w_i(z(k)) > 0 \\ w_i(z(k)) \geq 0 \quad i = 1, \dots, r \end{cases} \quad (6)$$

We have

$$\begin{cases} \sum_{i=1}^r h_i(z(k)) = 1 \\ h_i(z(k)) \geq 0 \quad i = 1, 2, \dots, r \end{cases} \quad (7)$$

for all k

Assumption 1, lemma 1, 2 and 3 present the techniques and tools used through the development of the next theorems.

Assumption 1 is a processing technique of the uncertainty function to a matrix function. Lemmas 1, 2 represent a simplification technique of the quadratic form for some matrix representation, and lemma 3 represents one of some techniques of relaxation (complexity’s reduction) for the stability and stabilization form.

Assumption 1 The matrices of the uncertainties in the system are represented in the following form:

$$[\Delta A_i \quad \Delta B_i] = DF(k) [E_{Ai} \quad E_{Bi}] \quad (8)$$

where D, E_{Bi} and E_{Ai} are known constant matrices and $F(k)$ is an unknown function satisfying :

$$F^T(k) F(k) \leq I \quad (9)$$

where I is the identity matrix.

Lemma 1 Wang and Mendel (1992) Considering X and $Y, Q = Q^T > 0$ matrices of appropriate dimensions, the following inequality holds

$$XY^T + YX^T \leq XQX^T + YQ^{-1}Y^T \quad (10)$$

Lemma 2 (Schur Complément) Boyd et al. (1994) Whether $P \in \mathfrak{R}^{m \times m}$ definite positive matrix, $X \in \mathfrak{R}^{m \times n}$ full-rank matrix in line, and $Q \in \mathfrak{R}^{n \times n}$ any matrix both following inequalities are equivalent

$$1. \quad Q(s) - X^T(s) P^{-1}(s) X(s) > 0, P(s) > 0 \quad (11)$$

$$2. \quad \begin{bmatrix} Q(s) & (*) \\ X(s) & P(s) \end{bmatrix} > 0 \quad (12)$$

Relaxation: Whatever the choice of the Lyapunov Functions, the analysis of the stability and stabilization problem is to find the best conditions of the inequality (13).

$$\sum_{i=1}^r \sum_{i < j}^r h_i h_j x^T \Upsilon_{ij} x < 0 \quad (13)$$

Lemma 3 (Tanaka and Sano 1994) Equation (13) is fulfilled if the following conditions hold:

$$\Upsilon_{ii} < 0 \quad \forall i \in \{1, \dots, r\} \quad (14)$$

$$\Upsilon_{ij} + \Upsilon_{ji} < 0 \quad \forall i, j \in \{1, \dots, r\}^2, i < j \quad (15)$$

The use of these lemmas use is very important for the development of the proposed theorems, and it will appear in the next sections.

3 Stabilization with Non-PDC Controller

This section recalls the technique of stabilization analysis for discrete T–S model based on a nonquadratic Lyapunov function in the discrete case, and the variation of the Lyapunov function is considered for one sample variation. If the final equation of this variation is negative, we obtain a sufficient condition of the T–S stabilization with the state feedback controller. This approach is developed by Guerra et al. (2009).

The following Lyapunov function is used by Guerra and Vermeiren (2004).

$$V(x(k)) = x^T(k) \left(\sum_{i=1}^r h_i(z(k)) G_i \right)^{-T} \left(\sum_{i=1}^r h_i(z(k)) (P_i + \mu R) \right) * \left(\sum_{i=1}^r h_i(z(k)) G_i \right)^{-1} x(k) \quad (16)$$

where P_z is symmetric and definite positive matrix, and G_z is full-rank matrix.

The nonlinearities are expressed by the terms $h_i(z(k)) \geq 0$ with the convex sum property $\sum_{i=1}^r h_i(z(k)) = 1$. In this paper, we consider the pairs $(A_i, B_i), i = \{1, \dots, r\}$ are controllable.

The non-PDC is a state feedback controller, which can be written under the following form in Eq. (17):

$$u(k) = -F_z G_z^{-1} x(k) \quad (17)$$

Using Eqs. (16) and (17), the variation of Lyapunov function leads us to the following inequality

$$\Upsilon_{ij}^k = \begin{bmatrix} -P_i & (*) & (*) & (*) \\ E_{ai} G_j & -\lambda I & 0 & 0 \\ E_{bi} F_j & 0 & -\tau I & 0 \\ A_i G_j - B_i F_j & 0 & 0 & -G_k^T - G_k + P_k + \lambda H_a H_a^T + \tau H_b H_b^T \end{bmatrix} \leq 0 \quad (18)$$

(Guerra et al. 2009) proposes the following theorem

Theorem 1 *Guerra et al. (2009)* Consider the parametric uncertain system (2) and the controller (17) and γ_{ij}^k defined in (18) such that $i, j, k \in \{1, \dots, r\}$. If there exists definite positives matrices P_i , scalars τ, λ and matrices F_i, G_i such as conditions (14) and (15) are satisfied, then the closed-loop fuzzy model is globally asymptotically stable.

Another condition of stabilization for continuous Takagi–Sugeno uncertain systems was investigated by *Manai and Benrejeb (2012)*, *Manai and Benrejeb (2012)* with the use of nonquadratic Lyapunov function based on additional constant and matrix variables (slack variables).

The Lyapunov function used by *Manai and Benrejeb (2012)*, *Manai and Benrejeb (2012)* is written under the following form:

$$V(x(t)) = \sum_{k=1}^r h_k(z(t))V_k(x(t)) \tag{19}$$

$$V_k(x(t)) = x^T(t)(P_k + \mu R)x(t) \tag{20}$$

The authors in *Manai and Benrejeb (2012)* propose the next theorem for the stabilization of T–S continuous parametric uncertain systems.

Theorem 2 *Manai and Benrejeb (2012)* The T–S continuous uncertain parametric system is stable in closed loop if there exists positive definite symmetric matrices $P_k, k = 1, \dots, r$, and R , matrices F_1, \dots, F_r such that the following LMIs holds:

$$P_k + R > 0, \quad k \in \{1, \dots, r\} \tag{21}$$

$$P_j + \mu R \geq 0, \quad sj = 1, 2, \dots, r \tag{22}$$

$$\begin{bmatrix} \Phi_1 (P_k + \mu R)D_{a_i} (P_k + \mu R)D_{b_i} \\ * & -\lambda I & 0 \\ * & * & -\lambda I \end{bmatrix} < 0$$

$$i, k \in \{1, \dots, r\} \text{ and } 0 < \mu < 1 \tag{23}$$

with

$$\Phi_1 = P_\phi + G_{ii}^T (P_k + \mu R) + (P_k + \mu R) G_{ii} + \lambda (P_k + \mu R) \left[E_{ai} E_{ai}^T + (E_{bi} F_i)^T E_{bi} F_i \right] \tag{24}$$

$$\begin{bmatrix} \Phi_2 (P_k + \mu R) (D_{a_i} + D_{a_j}) (P_k + \mu R) (D_{b_i} + D_{b_j}) \\ * & -\lambda I & 0 \\ * & * & -\lambda I \end{bmatrix} < 0$$

$$\text{for } i, j, k = 1, \dots, r \text{ such that } i < j \tag{25}$$

$$\left\{ \frac{G_{ij} + G_{ji}}{2} \right\}^T (P_k + \mu R) + (P_k + \mu R) \left\{ \frac{G_{ij} + G_{ji}}{2} \right\} < 0 \tag{26}$$

$$\Phi_1 = P_\phi + \left\{ \frac{G_{ij} + G_{ji}}{2} \right\}^T (P_k + \mu R) + (P_k + \mu R) \left\{ \frac{G_{ij} + G_{ji}}{2} \right\} + \lambda (P_k + \mu R) \left[(E_{ai} + E_{aj})^T (E_{ai} + E_{aj}) + (E_{bi} F_j + E_{bj} F_i)^T (E_{bi} F_j + E_{bj} F_i) \right]$$

$$G_{ii} = A_i - B_i F_i, \quad G_{ij} = A_i - B_i F_j, \quad 0 \leq \mu \leq 1 \tag{27}$$

In the following section, we propose a new Lyapunov function based on the Lyapunov function in Eqs. (16) and (19), by adding to the Lyapunov matrices another matrix multiplied by a scalar. The goal from the proposed Lyapunov function is to reduce the conservatism of the theorem 1 by improving the sets of solutions domain.

4 Proposed Approach

Under the condition expressed in theorem 1 and 2, and the Lyapunov function represented by Eqs. (16) and (19), we propose a new nonquadratic Lyapunov function under the following form in Eq. (28). It is considered as an extension of the two last functions in Eqs. (16) and (19).

$$V(x(k)) = x^T(k)G_z^{-T} (P_z + \mu R)G_z^{-1}x(k) \tag{28}$$

The variation of the Lyapunov function $\Delta (V(x(k)))$ is negative if the following inequality is negative:

$$\left(\overline{A}_z - \overline{B}_z F_z G_z^{-1} \right)^T G_z^{-T} (P_z + \mu R)G_z^{-1} \left(\overline{A}_z - \overline{B}_z F_z G_z^{-1} \right) - G_z^{-T} (P_z + \mu R)G_z^{-1} < 0$$

$$\begin{cases} \overline{A}_z = A_z + \Delta A_z \\ \overline{B}_z = B_z + \Delta B_z \end{cases} \tag{29}$$

By multiplying in the left by G_z^T and the right by G_z , Eq. (29) can be written in the next form

$$\left(G_z^T \overline{A}_z^T - F_z^T \overline{B}_z^T \right) G_z^{-T} (P_z + \mu R)G_z^{-1} \left(\overline{A}_z G_z - \overline{B}_z F_z \right) - (P_z + \mu R) < 0 \tag{30}$$

using complement Schur, Eq. (30) becomes

$$\begin{bmatrix} -P_z - \mu R & (*) \\ (A_z + \Delta A_z)G_z - (B_z + \Delta B_z)F_z & -G_z^T - G_z + P_z + \mu R \end{bmatrix} < 0 \tag{31}$$

A superior bound was obtained when we use this lemma 1 (Wang et al 1992)

$$\| \Delta a_z \|_2 \leq 1, \| \Delta b_z \|_2 \leq 1 \tag{32}$$

$$\begin{bmatrix} 0 & (*) \\ \Delta a_z G_z - \Delta b_z F_z & 0 \end{bmatrix} \leq \begin{bmatrix} \lambda^{-1} G_z^T E_{a_z}^T E_{a_z} G_z + \tau^{-1} F_z^T E_{b_z}^T E_{b_z} F_z & 0 \\ 0 & \lambda H_a H_a^T + \tau H_b H_b^T \end{bmatrix} \tag{33}$$

Considering this inequality, Eq. (31) will be in the next form.

$$\begin{bmatrix} -P_z - \mu R + \Omega_1 + \Omega_2 & (*) \\ A_z G_z - B_z F_z & -G_z^T - G_z + P_z + \mu R + \Omega_a + \Omega_b \end{bmatrix} \leq 0 \tag{34}$$

With

$$\begin{aligned} \Omega_1 &= \lambda^{-1} G_z^T E_{a_z}^T E_{a_z} G_z \\ \Omega_2 &= \tau^{-1} F_z^T E_{b_z}^T E_{b_z} F_z \\ \Omega_a &= \lambda H_a H_a^T \\ \Omega_b &= \tau H_b H_b^T \end{aligned}$$

After the use of complement, Schur equation (34) becomes:

$$\begin{bmatrix} -P_z - \mu R & (*) & (*) & (*) \\ E_{a_z} G_z & -\lambda I & 0 & 0 \\ E_{b_z} F_z & 0 & -\tau I & 0 \\ A_z G_z - B_z F_z & 0 & 0 & -G_z^T - G_z + P_z + \mu R + \lambda H_a H_a^T + \tau H_b H_b^T \end{bmatrix} \leq 0 \tag{35}$$

The final equation of stabilization of T–S parametric uncertain systems with the use of the new form of Lyapunov function becomes

$$\gamma_{ij}^k = \begin{bmatrix} -P_i - \mu R & (*) & (*) & (*) \\ E_{ai} G_j & -\lambda I & 0 & 0 \\ E_{bi} F_j & 0 & -\tau I & 0 \\ A_i G_j - B_i F_j & 0 & 0 & -G_k^T - G_k + P_k + \mu R + \Xi_b + \Xi_a \end{bmatrix} \leq 0$$

$$\begin{aligned} \Xi_b &= \tau H_b H_b^T \\ \Xi_a &= \lambda H_a H_a^T \end{aligned} \tag{36}$$

where $0 < \mu < 1$

Therefore, the new condition of stabilization of discrete-time Takagi–Sugeno parametric uncertain system is introduced by the next theorem.

Theorem 3 Consider the parametric uncertain system (2) and the control law (17) and γ_{ij}^k defined in (36) such that $i, j, k \in \{1, \dots, r\}$. If there exists symmetric definite positives matrices P_i , scalars $\tau, \lambda, 0 \leq \mu \leq 1$ and matrices R, F_i, G_i such as conditions (14) et (15) are satisfied, then the closed-loop fuzzy model is globally asymptotically stable.

A new Lyapunov function is proposed in this paper, and it represents an extension from two others existing in the literature. In the next section, we present their robustness by showing their influence on the stabilization region.

5 Simulation and Validation of Results

Consider the T–S discrete-time uncertain system with unstable open loop model (System 1). This system is modelled in two sub-systems $r = 2$.

$$\begin{aligned} A_1 &= \begin{bmatrix} -0.5 & 1 \\ -0.1 & 0.5 \end{bmatrix}, A_2 = \begin{bmatrix} -0.9 & 0.5 \\ -0.1 & -0.7 \end{bmatrix} \\ B_1 &= \begin{bmatrix} 3 \\ 4 \end{bmatrix}, B_2 = \begin{bmatrix} 3 \\ 0.1 \end{bmatrix}, H_a = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ H_b &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, E_{a1} = [a \ 0], E_{a2} = [0 \ b] \\ E_{b1} &= E_{b2} = \begin{bmatrix} 3b \\ 0 \end{bmatrix} \end{aligned}$$

with $a, b \in [0, 1]$

For the simulation, we choose the membership functions under the following form.

$$\begin{cases} h_1(x_1) = 0.8216 \left(1 - \frac{\sin(x_1)}{x_1} \right) \\ h_2(x_1) = 1 - h_1(x_1) \end{cases} \tag{37}$$

By applying the theorem 1, with $(a = 1, b = 1)$, the results of LMIs give the next definite positive matrices P_1, P_2 and matrices G_1, G_2, F_1, F_2

$$\begin{aligned} P_1 &= \begin{pmatrix} 0.1799 & 0.0693 \\ 0.0693 & 0.1285 \end{pmatrix}, P_2 = \begin{pmatrix} 0.1352 & 0.0442 \\ 0.0442 & 0.0914 \end{pmatrix} \\ G_1 &= \begin{pmatrix} 0.5054 & 0.1805 \\ 0.1474 & 0.4228 \end{pmatrix}, G_2 = \begin{pmatrix} 0.4228 & 0.1810 \\ 0.1075 & 0.3506 \end{pmatrix} \\ F_1 &= (0.49053.1262), F_2 = (-2.55341.3403) \end{aligned}$$

Figures 1, 2, 3 and 4 illustrate the convergence to the equilibrium point zero of the states variables of subsystems 1 and 2,

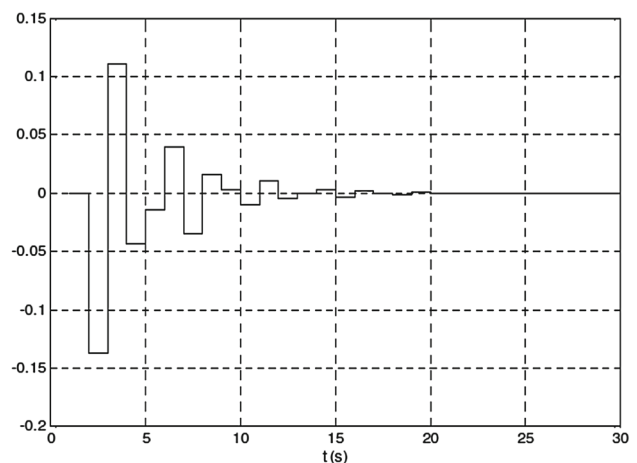


Fig. 1 Non-PDC control signal

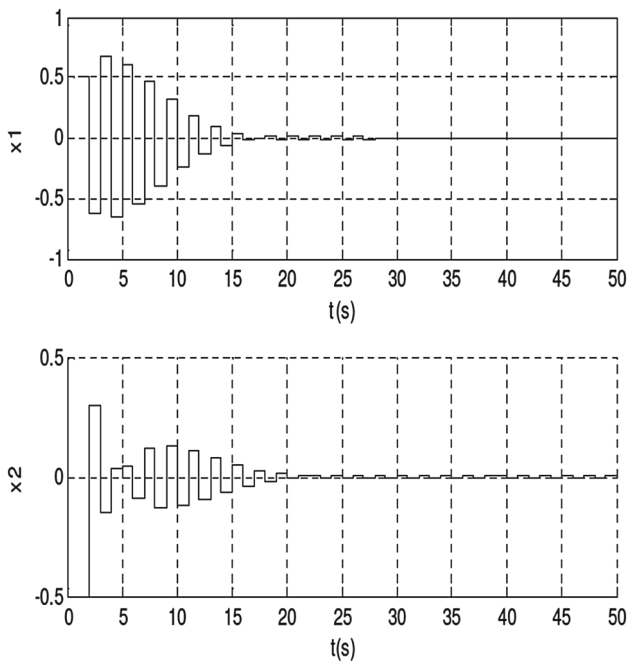


Fig. 2 Evolution of states variables of subsystem 1

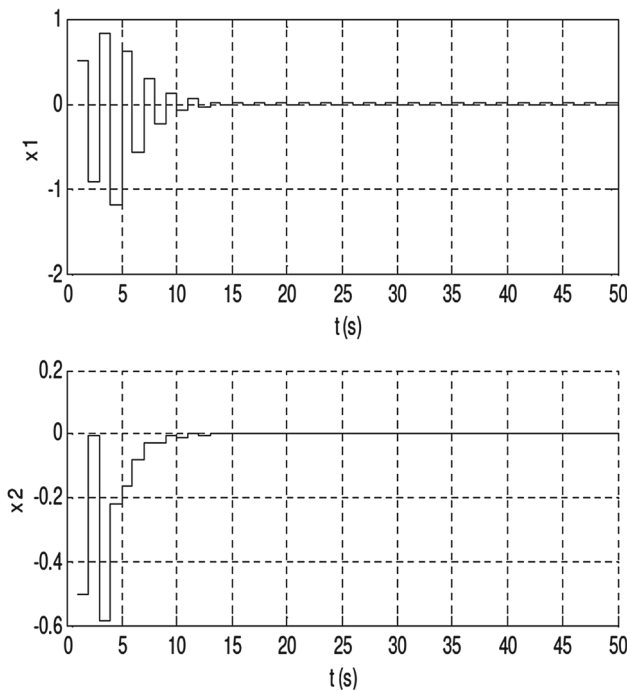


Fig. 3 Evolution of states variables of subsystem 2

the control signal and the final outputs $x(k + 1)$ by using the previous matrices. The initial conditions for the simulation are $x(0)^T = [0.5 \ -0.5]$

In the next, we apply theorem 3 with the nonlinear control law (17) and stabilization conditions of Theorem 3, we choose $(\mu = 0.5)$, and the results of LMIs give the next definite positive matrices P_1, P_2 and matrices G_1, G_2, F_1, F_2, R :

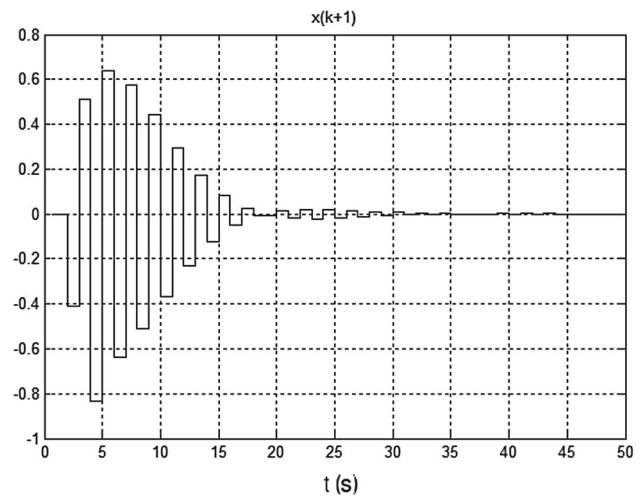


Fig. 4 Evolution of final outputs

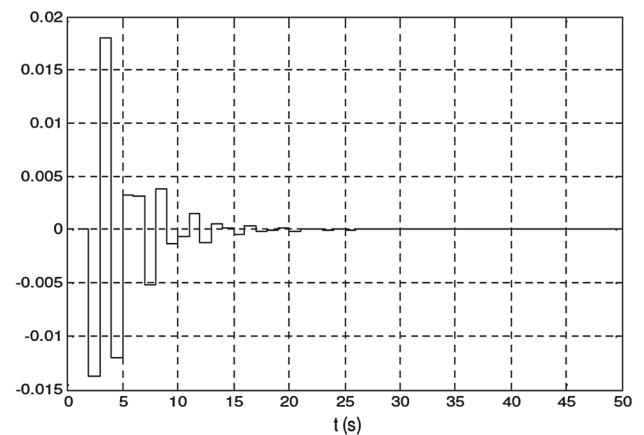


Fig. 5 Non-PDC control law

$$P_1 = \begin{pmatrix} 12.3764 & -0.0057 \\ -0.0057 & 12.3737 \end{pmatrix}, P_2 = \begin{pmatrix} 12.3732 & -0.0032 \\ -0.0032 & 12.37132 \end{pmatrix}$$

$$G_1 = \begin{pmatrix} 0.1863 & 0.0895 \\ 0.0539 & 0.2205 \end{pmatrix}, G_2 = \begin{pmatrix} 0.0756 & 0.0756 \\ 0.0302 & 0.1332 \end{pmatrix}$$

$$R = \begin{pmatrix} -26.5969 & 0.0265 \\ 0.1963 & -26.5400 \end{pmatrix}$$

$$F_1 = [0.6392 \ 0.9413], F_2 = [0.3253 \ 0.5623]$$

Figures 5, 6 and 7 illustrate the convergences of control signal and the states variables of sub-systems 1 and 2 to the equilibrium point zero by using the previous matrices. The initial conditions for the simulation are

$$x(0)^T = [0.5 \ -0.5]$$

Figures 8 and 9 present the feasible area of stabilization or the sets of solutions of LMI for the theorem 1, 3 with system 1. The mark (o) represents the feasible area of stabilization of theorem 3 and mark (*) represents that of theorem 1.

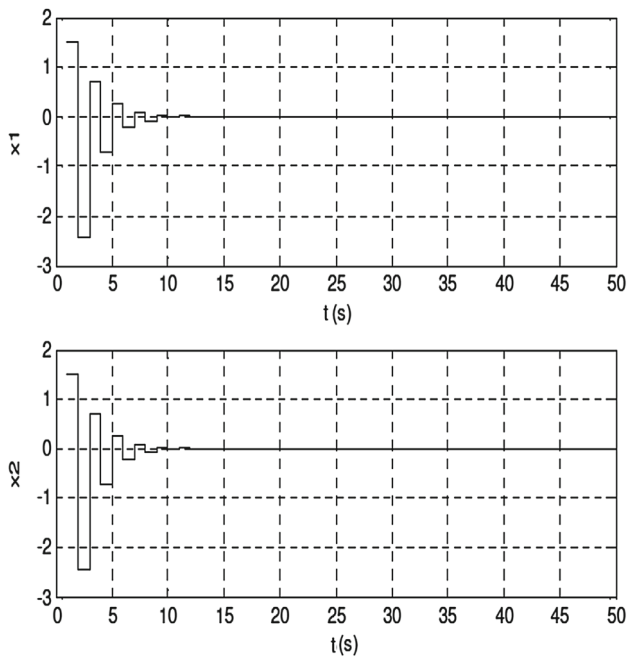


Fig. 6 Evolution of states variables of subsystem 1

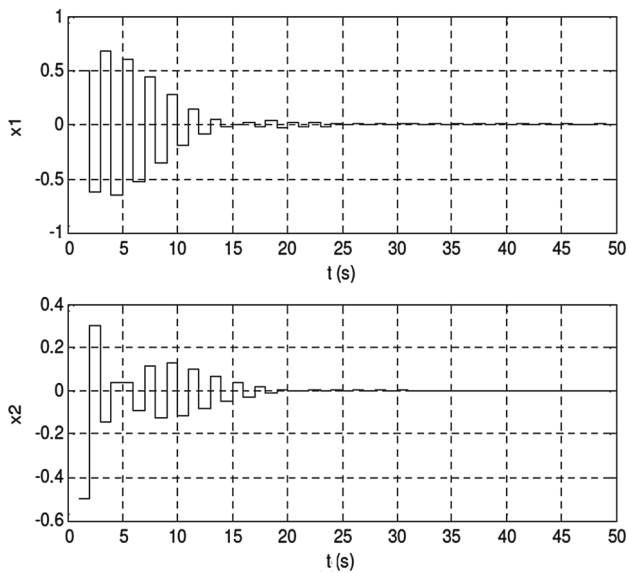


Fig. 7 Evolution of states variables of subsystem 2

For theorem 3, the value $\mu = 0.5$ is selected. The subsequent figures demonstrate the robustness of the proposed approach and then the first condition of stabilization in theorem 1. Our approach gives a full feasible area of stabilization between $[a, b] = [-1, 1]$. This area is considered as reduction of conservatism. The more we find a larger area of solutions, the more we reduce the conservatism.

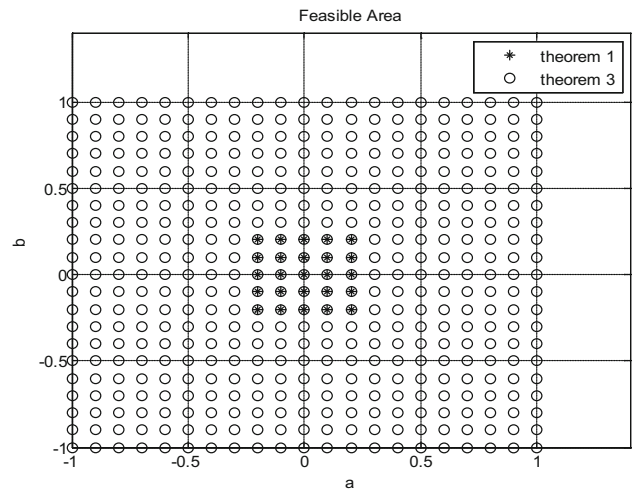


Fig. 8 Comparison between theorems 1 and 3

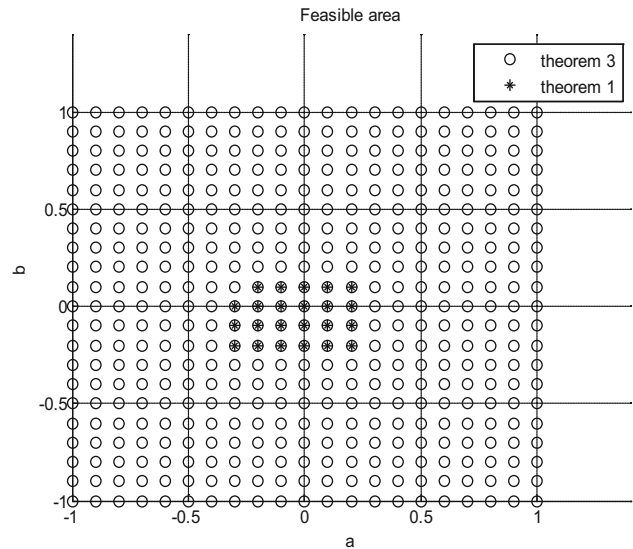


Fig. 9 Comparison between theorems 1 and 3

Consider another model with $r = 2$ (System 2)

$$A_1 = \begin{bmatrix} a & -0.5 \\ 0 & -0.86 \end{bmatrix}, A_2 = \begin{bmatrix} 0.87 & 0 \\ -0.5 & b \end{bmatrix}, B_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 3 \\ 0.1 \end{bmatrix}$$

$$E_{a2} = \begin{bmatrix} 0 & b \end{bmatrix}, E_{b1} = E_{b2} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$H_a = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, H_b = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, E_{a1} = \begin{bmatrix} a & 0 \end{bmatrix}$$

The next figure gives the feasible area of stabilization that belongs to System 2 by applying theorem 1 and 3 with $[a, b] = [-1, 1]$.

Figures 8 and 9 present a comparison between the proposed condition of stabilization presented in theorem 3 and the condition of stabilization in theorem 1. The proposed condition gives larger sets of solution in terms of linear matrix inequalities; with the addition of new matrix variable multiplied by positive constant in the Lyapunov function, the proposed Lyapunov function and the new condition of stabilization give less conservative results than the ones given by theorem 1. This result demonstrate the interest of approach.

6 Conclusion

This paper develops a new condition of stabilization for Takagi–Sugeno discrete-time parametric uncertain systems in terms of combination of the LMI techniques and new non-quadratic Lyapunov function, which permits a reduction of the conservatism of some previous results existing in the literature and significant increase the solution sets for non-linear models which represent the goal of this work.

The main feature of our contribution is the increase in the solutions sets into one sample variation of the Lyapunov function. In the case of k -samples variation, it has no significant effect to consider. The interest of the proposed approach has been shown through examples and simulations.

Further works include the development of design methods for nonlinear discrete-time nonparametric, and mixed uncertain systems include the generalization of these results to k -sample variation investing another form of nonquadratic Lyapunov functions.

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of Heterogeneous Platforms. His research interests are embedded systems, stability and stabilization of Takagi–Sugeno fuzzy systems and its applications.



Joseph Haggège was born in 1975 in Tunis, Tunisia. He graduated from “Ecole Nationale d’Ingénieurs de Tunis” in 1998, and he received the Ph.D. degree in Electrical Engineering 2003 and the Habilitation in 2010. He is currently full professor at the “Ecole Nationale d’Ingénieurs de Tunis.” His research interests are in the area of heuristic optimisation, embedded systems and robust digital control.



Ali Bouyahya was born in Tunisia on March 1986, and he received the B.S. degree in Electrical Engineering from Ecole Supérieure de Technologie et d’informatique (ESTI) in 2010 and Master degree in Automatic and Signal Processing and the Doctorate degree in Electrical Engineering from the “Ecole Nationale d’Ingénieurs de Tunis” (ENIT) Tunisia in 2012 and 2016, respectively. His doctorate thesis is prepared within the framework of unit research

“Laboratoire de Recherche en Automatique” (L.A.R.A) about stability and stabilization of Takagi–Sugeno fuzzy systems and its applications.