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TECHNICAL CORRESPONDENCE

Improved Results on Delay-Dependent Stability of LFC Systems with Multiple Time-Delays

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Abstract In this paper, the problem of delay-dependent robust stability of uncertain load frequency control (LFC) systems with multiple time-delays and exogenous power system disturbance has been considered. Using Lyapunov–Krasosvskii functional method, less conservative delay-dependent stability criteria are proposed in linear matrix inequality formulation to compute the maximum value of the time-delays within which the LFC system under consideration remains asymptotically stable in the sense of Lyapunov. Compared to the existing result in the literature, the proposed result takes into account the effect of unknown exogenous load disturbance into the stability analysis, imparting more applicability and usefulness to the resulting stability criterion in real-time conditions.

Keywords Load frequency control · Delay-dependent stability · Lyapunov stability analysis · Linear matrix inequality · Multiple time-delays

1 Introduction and Problem Formulation

It is well known that load frequency control of multi-area power systems through communication network introduces time-delays in the feedback path Bevrani (2009). In real-time condition, these delays account for the transfer of system information (output or state variables) from power plant (RTUs) to the control center at remote where the control algorithm is embedded and the subsequent transfer of the control effort from the controller back to the plant. The presence of time-delays in a physical system is detrimental to the system performance as well as stability. Excessive time-delays often pave way to system instability Gu et al. (2003). Hence, it becomes necessary to compute the maximum value of the time-delays within which the closed-loop LFC system remains asymptotically stable in the sense of Lyapunov. This is called delay-dependent stability of time-delayed LFC systems Jiang et al. (2012).

Generally, a power system is subjected to sudden unknown load disturbances that perturb the system from its equilibrium point. If system is capable of regaining the same equilibrium point where it was operating at and before the time of perturbation, then it is said to be asymptotically stable (in the sense of Lyapunov). On the other hand, in an unstable system, any perturbation from equilibrium point drifts the system completely away from it. If the system, by itself, regains the equilibrium point upon perturbation from it, then the system is said to be autonomously stable or open-loop stable; else, if it does regain the original equilibrium condition with the aid of a controller, then the system is said to be closed-loop stable.

Time-delays appear in a system either due to the physical characteristics like backlash or dead zone, or from external environment like networked control through open communication channels Bevrani and Hiyama (2009). In either case, in a time-delayed system, the tendency of the system to drift away from the equilibrium point is more when perturbed from it; this, in turn, makes stability of the system susceptible to the size of the time-delay. The delay-dependent stability criteria, which computes the maximum value of the delay within which a system remains asymptotically stable, are basically sufficient conditions that derived using Lyapunov–Krasovskii (LK) functional method Wu et al. (2010). The cri-

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teria are expressed as a set of solvable linear matrix inequalities (LMIs) Yu and Tomsovic (2014). The conservativeness of the delay-dependent stability criteria depends on the choice of the LK functional used in the stability analysis and the techniques employed for bounding the time derivative of the functional. The computational efficiency, on the other hand, depends on the number of decision variables involved in the stability criterion (LMI) Gahinet et al. (1995). Preferably, the delay-dependent stability criterion should both be less conservative as well as computationally less expensive.

In Jiang et al. (2012), a less conservative delay-dependent stability criterion is presented for a class of multi-area LFC systems with PI control closed through a communication channel (that introduces time-delay in the closed-loop system) using LK functional approach in LMI formulation. But in the stability criterion, the effect of exogenous power system disturbance is not taken into consideration, and hence, the criterion can be only employed for ascertaining delay-dependent stability for LFC systems under nominal system conditions. This invariably restricts the applicability of the criterion for ascertaining delay-dependent stability of LFC systems that are subjected to unknown exogenous load disturbances. In addition, the stability criterion presented in Jiang et al. (2012) is taken from the main result of He et al. (2006) that involves more number of decision variables in the LMI. This, in turn, makes the result of Jiang et al. (2012) computationally more expensive. In order to alleviate these drawbacks, in this paper, using LK functional approach and Jenson integral inequality Zhang et al. (2005), a less conservative robust stability criterion is proposed for multi-area LFC systems with load disturbances. In the proposed delay-dependent stability analysis, the effect of unknown exogenous load disturbance onto the system output is minimized in H_∞ sense. Hence, the proposed criterion has more applicability than that of Jiang et al. (2012) in real-time operating conditions. The use of Jenson integral inequality to bound the time derivative of the LK functional in the delay-dependent analysis helps to reduce the total number of decision variables in the LMI yielding a computationally less expensive stability criterion. To present the proposed result in a lucid manner, we have considered a typical two-area LFC system, though an extension to higher area systems is very much possible. The state-space model of two-area PI controlled LFC system with multiple time-delays and exogenous load disturbance Jiang et al. (2012) is given by

$$\dot{x}(t) = Ax(t)+A_1x(t-h_1)+A_2x(t-h_2)+B_\omega\omega(t), \quad (1)$$

$$y(t) = Cx(t), \quad (2)$$

where the state vector $x(t) = [\Delta f_1(t) \;\; \Delta P_{m1}(t) \;\; \Delta P_{v1}(t) \;\; \int ACE_1(t)\mathrm{d}t \;\; \Delta P_{12}(t) \;\; \Delta f_2(t) \;\; \Delta P_{m2}(t) \;\; \Delta P_{v2}(t) \;\; \int ACE_2(t)\mathrm{d}t]^T$; the corresponding system matrices are given below:

$$A = \begin{bmatrix} -\frac{D_1}{M_1} & \frac{1}{M_1} & 0 & 0 & -\frac{1}{M_1} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{T_{ch1}} & \frac{1}{T_{ch1}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{R_1T_{g1}} & 0 & -\frac{1}{T_{g1}} & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 2\pi T_{12} & 0 & 0 & 0 & 0 & -2\pi T_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{M_2} & -\frac{D_2}{M_2} & \frac{1}{M_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{ch2}} & \frac{1}{T_{ch2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{R_2T_{g2}} & 0 & -\frac{1}{T_{g2}} & 0 \\ 0 & 0 & 0 & 0 & -1 & \beta_2 & 0 & 0 & 0 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{\beta_1K_{P1}}{T_{g1}} & 0 & 0 & -\frac{K_{I1}}{T_{g1}} & -\frac{K_{P1}}{T_{g1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{K_{P2}}{T_{g2}} & -\frac{\beta_2K_{P2}}{T_{g2}} & 0 & 0 & -\frac{K_{I2}}{T_{g2}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_\omega = \begin{bmatrix} -\frac{1}{M_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{M_2} & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

The initial condition for the delayed system is given by $x(t) = \Phi(t),\ t \in [-\max(h_1, h_2),\ 0]$. The power system load disturbance vector is $\omega(t) = [\Delta P_{D1}(t) \;\; \Delta P_{D2}(t)]^T \in \mathbb{R}^2$, and output vector is $y(t) = [\Delta f_1(t) \;\; \Delta f_2(t)] \in \mathbb{R}^2$. The notations used for the ith area, $i = 1,\ 2$ are given in Table 1.

The objective of this paper was to derive a new delay-dependent stability criterion to compute the maximum allowable bound of the delays h_1 and h_2 such that the two-area LFC system under consideration remains asymptotically stable in the sense of Lyapunov in the presence of unknown exogenous load disturbance. For deriving the proposed result, following lemma (based on Jenson integral inequality Zhang et al. (2005)) is required:

Lemma 1 *For any constant symmetric positive definite matrix* $X \in \mathbb{R}^{n\times n}$, *a scalar* $\gamma > 0$ *and vector function* $\dot{x} : [-\gamma, 0] \mapsto \mathbb{R}^n$ *such that the integration* $\int_{t-\gamma}^{t} \dot{x}^T(s)X\dot{x}(s)ds$ *is well defined, then the following inequality holds:*

$$-h\int_{t-h}^{t} \dot{x}^T(s)X\dot{x}(s)ds \leq \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} -X & X \\ \star & -X \end{bmatrix} \times \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}. \quad (3)$$

Table 1 Notations

$\Delta P_{vi}(t)$	Governor valve position
$\Delta P_{mi}(t)$	Turbine/generator output
$\Delta f_i(t)$	Frequency deviation
$ACE_i(t)$	Area control error
$\Delta P_{12}(t)$	Tie-line power transfer between area 1 and area 2
T_{12}	Tie-line synchronizing coefficient between area 1 and area 2
β_i	Frequency bias factor
K_{Ii}	Integral gain of local PI controller
K_{Pi}	Proportional gain of local PI controller
h_i	Time-delay between controller and power plant in ith area
D_i	Generator damping constant
M_i	Moment of inertia of generator
T_{chi}	Turbine time constant
T_{gi}	Governor time constant
R_i	Speed droop
$\Delta P_{Di}(t)$	Load disturbance

2 Main Result

The proposed result is stated in the form of following theorem:

Theorem 1 *The system (1) with output equation (2) is asymptotically stable in the sense of Lyapunov satisfying $||y(t)||_2 < \gamma||\omega||_2$ with zero initial condition and a prescribed H_∞ performance level $\gamma > 0$, if there exist real symmetric positive definite matrices P, Q_1, Q_2, R_1, R_2 and R_3 such that the following LMI holds:*

$$\begin{bmatrix} \Phi_{11} & PA_1+R_1 & PA_2+R_2 & PB_\omega & A^TU & C^T \\ \star & -Q_1-R_1-R_3 & R_3 & 0 & A_1^TU & 0 \\ \star & \star & -Q_2-R_2-R_3 & 0 & A_2^TU & 0 \\ \star & \star & \star & -\gamma^2 I & B_\omega^TU & 0 \\ \star & \star & \star & \star & -U & 0 \\ \star & \star & \star & \star & \star & -I \end{bmatrix} < 0, \tag{4}$$

where $\Phi_{11} = A^TP + PA + Q_1 + Q_2 - R_1 - R_2$ and $U = h_1^2R_1 + h_2^2R_2 + (h_1-h_2)^2R_3$.

Proof Consider the LK functional:

$$V(x(t)) = x^T(t)Px(t) + \sum_{i=1}^{2}\int_{t-h_i}^{t} x^T(s)Q_ix(s)\mathrm{d}s + \sum_{i=1}^{2} h_i\int_{-h_i}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)R_i\dot{x}(s)\mathrm{d}s\mathrm{d}\theta + (h_2-h_1)\int_{-h_2}^{-h_1}\int_{t+\theta}^{t}\dot{x}^T(s)R_3\dot{x}(s)\mathrm{d}s\mathrm{d}\theta, \tag{5}$$

where P, Q_1, Q_2, R_1, R_2 and R_3 are symmetric positive definite matrices. The time derivative of (5) along the trajectory of (1) is given by

$$\dot{V}(x(t)) = 2x^T(t)P\dot{x}(t) + \sum_{i=1}^{2}(x^T(t)Q_ix(t) - x^T(t-h_i)Q_ix(t-h_i)) + \dot{x}^T(t)U\dot{x}(t) - \sum_{i=1}^{2} h_i\int_{t-h_i}^{t}\dot{x}^T(s)R_i\dot{x}(s)\mathrm{d}s - (h_2-h_1)\int_{t-h_2}^{t-h_1}\dot{x}^T(s)R_3\dot{x}(s)\mathrm{d}s. \tag{6}$$

Now, by applying Lemma 1 to the integral terms $-\sum_{i=1}^{2} h_i \int_{t-h_i}^{t}\dot{x}^T(s)R_i\dot{x}(s)\mathrm{d}s$ and $-(h_2-h_1)\int_{t-h_2}^{t-h_1}\dot{x}^T(s)\, R_3\dot{x}(s)\mathrm{d}s$, we obtain the following quadratic condition:

$$\dot{V}(x(t)) \le \xi^T(t)(\Pi + \bar{A}^TU\bar{A})\xi(t), \tag{7}$$

where $\xi(t) = [x^T(t)\ \ x^T(t-h_1)\ \ x^T(t-h_2)\ \ \omega^T(t)]$, and

$$\Pi = \begin{bmatrix} \Phi_{11} & PA_1 & PA_2 & PB_\omega \\ \star & \Phi_{22} & R_3 & 0 \\ \star & \star & \Phi_{33} & 0 \\ \star & \star & \star & 0 \end{bmatrix},$$

$$\bar{A} = \begin{bmatrix} A & A_1 & A_2 & B_\omega \end{bmatrix}^T.$$

For a prescribed scalar $\gamma > 0$, define a performance index J as follows:

$$J = \int_0^\infty (y^T(s)y(s) - \gamma^2\omega^T(s)\omega(s))\mathrm{d}s. \tag{8}$$

Now, if

$$\Pi + \bar{A}^TU\bar{A} + y^T(t)y(t) - \gamma^2\omega^T(t)\omega(t) < 0, \tag{9}$$

then, from (7), following inequality holds good:

$$\dot{V}(x(t)) + y^T(t)y(t) - \gamma^2\omega^T(t)\omega(t) < 0. \tag{10}$$

For $\omega(t) \neq 0$, by integrating (9) from 0 to t and letting $t \to \infty$, with zero initial condition, we get

$$\int_0^\infty y^T(s)y(s)\mathrm{d}s < \gamma^2\int_0^\infty \omega^T(s)\omega(s)\mathrm{d}s, \tag{11}$$

which, in turn, implies that $||y(t)||_2 < \gamma||\omega(t)||_2$. This ensures that $J < 0$. On the other hand, (4) implies that following LMI holds:

$$\begin{bmatrix} \Phi_{11} & PA_1+R_1 & PA_1+R_1 & A^TU \\ \star & -Q_1-R_1-R_3 & R_3 & A_1^TU \\ \star & \star & -Q_2-R_2-R_3 & A_2^TU \\ \star & \star & \star & -U \end{bmatrix} < 0. \tag{12}$$

Thus, $\dot{V}(x(t)) < \lambda||x(t)||^2$ for a sufficiently small scalar $\lambda > 0$. Therefore, the system (1) with $\omega(t) = 0$ is asymptotically stable in the sense of Lyapunov Gu et al. (2003). Hence, the proof is completed. Now, by substituting (2) in (9), and subsequently applying Schur complement Boyd et al. (1994), we deduce the LMI (4) stated in Theorem 1. □

Remark 1 Further reduction in conservatism can be achieved by delay-partitioning technique Han (2009), wherein the delays h_1 and h_2 are partitioned into N segments ($N \geq 2$) of equal width; subsequently, the segmental information is employed while constructing the LK functional. One such result (for $N = 2$) using the following LK functional is presented in Corollary 1.

$$\begin{aligned} V(x(t)) &= x^T(t)Px(t) \\ &+ \int_{t-\frac{h_1}{2}}^{t} \begin{bmatrix} x(s) \\ x\left(s-\frac{h_1}{2}\right) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ \star & Q_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ x\left(s-\frac{h_1}{2}\right) \end{bmatrix} \mathrm{d}s \\ &+ \int_{t-\frac{h_2}{2}}^{t} \begin{bmatrix} x(s) \\ x\left(s-\frac{h_2}{2}\right) \end{bmatrix}^T \begin{bmatrix} R_{11} & R_{12} \\ \star & R_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ x\left(s-\frac{h_2}{2}\right) \end{bmatrix} \mathrm{d}s \\ &+ \sum_{i=1}^{2} \frac{h_i}{2} \int_{-\frac{h_i}{2}}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) Z_i \dot{x}(s) \mathrm{d}s\mathrm{d}\theta \\ &+ (h_2 - h_1) \int_{-h_2}^{-h_1} \int_{t+\theta}^{t} \dot{x}^T(s) Z_3 \dot{x}(s) \mathrm{d}s\mathrm{d}\theta, \end{aligned} \tag{13}$$

where in addition to P, Z_1, Z_2 and Z_3 being symmetric and positive definite, following conditions should also hold good:

$$\begin{bmatrix} Q_{11} & Q_{12} \\ \star & Q_{22} \end{bmatrix} \geq 0, \tag{14}$$

$$\begin{bmatrix} R_{11} & R_{12} \\ \star & R_{22} \end{bmatrix} \geq 0. \tag{15}$$

Corollary 1 *The system (1) with output equation (2) is asymptotically stable in the sense of Lyapunov satisfying $||y(t)||_2 < \gamma||\omega||_2$ with zero initial condition, and a prescribed H_∞ performance level $\gamma > 0$, if there exist real symmetric positive definite matrices P, Z_1, Z_2, and Z_3; $\begin{bmatrix} Q_{11} & Q_{12} \\ \star & Q_{22} \end{bmatrix} \geq 0$ and $\begin{bmatrix} R_{11} & R_{12} \\ \star & R_{22} \end{bmatrix} \geq 0$ such that the following LMI holds:*

$$\begin{bmatrix} \bar{\Phi}_{11} & Q_{12}+Z_1 & PA_1 & R_{12}+Z_2 & PA_2 & PB_\omega & A^T\bar{U} & C^T \\ \star & Q_{22}-Q_{11}-Z_1 & -Q_{12} & 0 & 0 & 0 & 0 & 0 \\ \star & \star & -Q_{22}-Z_3 & 0 & Z_3 & 0 & A_1^T\bar{U} & 0 \\ \star & \star & \star & R_{22}-R_{11}-Z_2 & -R_{12} & 0 & 0 & 0 \\ \star & \star & \star & \star & -R_{22}-Z_3 & 0 & A_2^T\bar{U} & 0 \\ \star & \star & \star & \star & \star & -\gamma^2 I & B_\omega^T\bar{U} & 0 \\ \star & \star & \star & \star & \star & \star & -\bar{U} & 0 \\ \star & \star & \star & \star & \star & \star & \star & -I \end{bmatrix} < 0, \tag{16}$$

with $\bar{\Phi}_{11} = A^TP + PA + Q_{11} + R_{11} - Z_1 - Z_2$ and $\bar{U} = \left(\frac{h_1}{2}\right)^2 Z_1 + \left(\frac{h_2}{2}\right)^2 Z_2 + (h_2-h_1)^2 Z_3$.

Table 2 Parameters of two-area LFC systems

Parameter	$T_{ch}(s)$	$T_g(s)$	R	D	M	$M(s)$
Area 1	0.3	0.1	0.05	1.0	21	10
Area 2	0.4	0.17	0.05	1.5	21.5	12
$T_{12} = 0.1986$						

The case study on a benchmark LFC system to corroborate the effectiveness of the proposed results is presented in the next section.

Remark 2 For a given information on the time-delays, the minimum value of H_∞ attenuation level $\gamma > 0$ can be obtained from Theorem 1 through the following minimization problem:

min γ

subject to $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, $R_2 > 0$ and $R_3 > 0$; and LMI (4).

3 Case Study

In this section, case study is carried out on a typical benchmark two-area LFC system with multiple time-delays. Using Theorem 1 and Corollary 1, the maximum bound of the time-delays is computed. Since Corollary 1 is obtained by delay partition, it is less conservative than Theorem 1. However, Corollary 1 involves more number of decision variables ($6n^2 + 4n$) than Theorem 1 ($3n^2 + 3n$) making it computationally more expensive than Theorem 1. Being sufficient conditions, further reduction in conservatism of the proposed delay-dependent stability criteria can be achieved at the expense of computational efficiency using higher number of delay partitions. The parameters of the benchmark two-area LFC system are taken from Jiang et al. (2012); they are given in Table 2.

The stability margin obtained by the proposed stability criteria stated in Theorem 1 and Corollary 1 is presented

Table 3 Maximum upper delay bound of h

θ	Method	K_I $(K_P = 0.05)$				K_P $(K_I = 0.3)$			
		0.05	0.1	0.2	0.3	0	0.05	0.1	0.2
0^o	Theorem 1	13.954	6.649	3.035	1.761	1.715	1.761	1.694	1.280
	Corollary 1	16.503	7.539	3.272	1.857	1.782	1.857	1.816	1.385
10^o	Theorem 1	14.460	6.785	3.080	1.785	1.741	1.785	1.711	1.127
	Corollary 1	16.964	7.683	3.325	1.880	1.808	1.880	1.826	1.141
20^o	Theorem 1	15.123	7.109	3.228	1.865	1.823	1.865	1.764	0.626
	Corollary 1	17.698	8.045	3.484	1.962	1.892	1.962	1.845	0.629
30^o	Theorem 1	16.322	7.707	3.501	1.913	1.943	1.913	1.454	0.447
	Corollary 1	19.035	8.706	3.779	1.965	2.000	1.965	1.484	0.450
40^o	Theorem 1	18.022	8.673	3.927	1.544	1.612	1.544	1.152	0.353
	Corollary 1	20.684	9.720	4.155	1.579	1.641	1.579	1.177	0.355
45^o	Theorem 1	18.169	8.885	3.593	1.409	1.469	1.409	1.049	0.322
	Corollary 1	20.606	9.753	3.790	1.442	1.497	1.442	1.077	0.324
50^o	Theorem 1	17.888	8.341	3.318	1.303	1.358	1.303	0.975	0.298
	Corollary 1	20.215	9.180	3.503	1.331	1.383	1.331	0.996	0.300
60^o	Theorem 1	16.140	7.396	2.936	1.154	1.203	1.154	0.869	0.265
	Corollary 1	18.256	8.146	3.098	1.179	1.224	1.179	0.887	0.266
70^o	Theorem 1	14.936	6.820	2.706	1.067	1.111	1.067	0.808	0.244
	Corollary 1	16.915	7.516	2.856	1.090	1.130	1.090	0.824	0.246
80^o	Theorem 1	14.273	6.509	2.582	1.022	1.063	1.022	0.775	0.234
	Corollary 1	16.167	7.176	2.726	1.044	1.080	1.044	0.791	0.235
90^o	Theorem 1	13.622	6.390	2.545	1.008	1.047	1.008	0.764	0.230
	Corollary 1	15.620	7.050	2.687	1.031	1.066	1.031	0.781	0.232

in Table 3 where $h = \sqrt{h_1^2 + h_2^2}$ and $\theta = \cos^{-1}\left(\frac{h_1}{h}\right)$. In order to minimize the effect of load disturbance onto the system output, the constraint $||y(t)||_2 < \gamma||\omega(t)||_2$ is imposed while deducing the stability criteria; to guarantee robustness against exogenous disturbance signal, a value of $\gamma = 0.1$ has been selected Dey et al. (2012). From the table, it is clear that Corollary 1, being derived using the delay-partitioning approach, is less conservative than Theorem 1. Since the effect of load disturbance is taken into the stability analysis, the delay margin provided by the proposed criteria is more accurate than that of Jiang et al. (2012). The number of decision variables involved in Theorem 1 and Jiang et al. (2012) are compared in Table 4. From the table, it is clear that the proposed criterion involves less number of decision variables than Jiang et al. (2012), and hence, it is computationally less expensive. For time-delays in stable region, the evolution of system frequency $\Delta f_1(t)$ and $\Delta f_2(t)$ when the load is perturbed by a per unit change is shown in Fig. 1. From the figure, it is clear that the asymptotic convergence of the variables illustrates that the delayed LFC system is delay-dependently stable.

Table 4 Number of decision variables

Method	Number of decision variables
Jiang et al. (2012)	$25.5n^2 + 7.5n$
Theorem 1	$3n^2 + 3n$

4 Conclusion

In this paper, robust delay-dependent stability criteria are presented for a class of uncertain LFC systems with multiple time-delays using Lyapunov–Krasovskii functional method in LMI formulation. To reduce conservatism in the delay-dependent stability analysis, delay-partitioning technique is employed, and to reduce the number of decision variables in the LMI, Jenson integral inequality is employed. Since proposed criteria are derived by taking into account the effects of unknown exogenous power system disturbance, they are more effective for ascertaining delay-dependent stability of LFC systems in real-time conditions than a recently reported result in the literature which is deduced without tak-

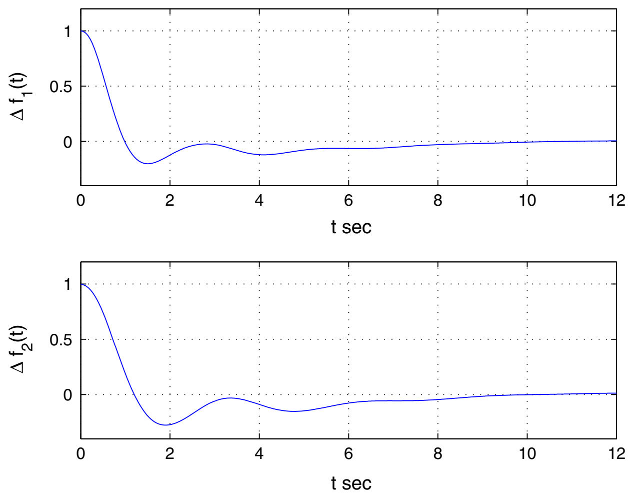

Fig. 1 Evolution of system frequency for perturbed load condition

ing into account the effect of load disturbance. The possibility of analyzing the delay-dependent stability of LFC systems with multiple time-varying delays and exogenous load disturbance will be explored as a future work.

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