

Planning of Distribution Systems Using Mixed-Integer Linear Programming Models Considering Network Reliability

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Abstract In this paper, a two-stage solution methodology for distribution network planning considering reliability indices improvement is proposed. This methodology comprises optimal distribution network expansion and improves network reliability by allocating sectionalizing switches and interconnection circuits (tie line circuits). The optimal expansion problem of radial aerial distribution systems is formulated as a mixed binary linear programming (MILP) problem aiming to reduce the investment and operational costs, subject to physical and operational constraints. The allocation of controlled sectionalizing switches and interconnection circuits is also formulated as a MILP in order to improve the network reliability indices. A pseudo-dynamic planning method is used to solve planning and reliability models through a heuristic technique that first solves the planning model followed by the solution of the reliability model, in each stage of planning horizon. Numerical results are presented for a 54-bus distribution system from literature.

Keywords Distribution systems planning · Reliability indices · Mixed integer linear programming · Mathematical models

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List of Symbols

Sets

NB	Buses in the system
NSE	Existing substations in the system and candidate substations
NNC	New circuits
NTB	Transfer buses (buses without loads or energy sources)
Ω_c	Conductor types
NLS	Load or supply buses
NS	Set of operation scenarios
NC	Set of existing circuits in the system
NTL	Set of interconnection circuits
NC ₀	Existing circuits (circuits in operation in the system)
NCO	Circuits out of operation

Parameters

C_{ij}^0	Energy loss cost
C_i^S	Construction cost of substation i
C_i^O	Operation cost of substation i
$C_{ij,c}^L$	Construction cost of circuit ij with conductor-type c
$C_{ij,ab}^R$	Reconductoring cost of the existing circuit ij from conductor type a to conductor type b
$\gamma_{i,s}$	Permanent fault condition in the bus i in scenario s
C_{ch}	Sectionalizing switch allocation cost
C_N	Construction cost of an interconnection circuit
C_{och}	Operation cost of sectionalizing switch
K_c	Energy loss cost in the planning horizon

K	Energy curtailment cost
M	Degree of freedom of variable $R_{i,s}$
N^E	Number of existing circuits
N_B	Number of system buses
N_S	Number of substations
$R_{ij,c}^E$	Resistance of existing circuit ij , with conductor type c
$R_{ij,c}^N$	Resistance of candidate circuit ij , with conductor type c
\bar{S}_c	Maximum power flow of conductor type c
\bar{S}_i^{SE}	Maximum energy capacity of existing substation i
\bar{S}_i^{SN}	Maximum energy capacity of repowered substation i
$S_{i,s}^S$	Apparent power in substation i in scenario s
\bar{S}_i^S	Maximum apparent power at substation i
S_i^D	Apparent power demand at bus i
\bar{S}_{ij}	Maximum apparent power flow of circuit ij
t_{ij}^O	Conductor type of existing circuit ij
V	Minimum bus voltage magnitude
\bar{V}	Maximum bus voltage magnitude
V^{base}	Base voltage magnitude
V^{nom}	Nominal voltage magnitude
Z_{ij}	Equivalent impedance of circuit ij
$Z_{ij,c}^E$	Impedance of existing circuit ij , with conductor type c
$Z_{ij,c}^N$	Impedance of interconnection circuit ij , with conductor type c
ΔV	Maximum voltage drop
λ_s	Permanent fault rate in scenarios
R	Number of blocks of the piecewise linearization

Variables

$\delta_{ij,c}^L$	Investment binary decision variable for constructing circuit ij , with conductor type c
$\delta_{ij,c}^R$	Investment binary decision variable for reconductoring existing circuit ij , with conductor type c
δ_i^S	Investment binary decision variable for constructing new substation at bus i
S_{ij}	Apparent power flow in circuit ij
S_i^C	Apparent power load at bus i
S_i^S	Apparent power at substation i
\bar{S}_i^S	Maximum apparent power at substation i
$S_{ij,c}^E$	Apparent power flow in existing circuit ij , with conductor type c
$\bar{S}_{ij,c}^E$	Square of $\frac{S_{ij,c}^E}{V^{\text{base}}}$
$S_{ij,c}^N$	Apparent power flow in candidate circuit ij , with conductor type c
$\bar{S}_{ij,c}^N$	Square of $\frac{S_{ij,c}^N}{V^{\text{base}}}$
$R_{i,s}$	Load curtailment in the bus i in scenarios
R_i	Load curtailment in the bus i

V_i	Voltage magnitude at bus i
y_i	Decision variable to use the transfer bus i
$x_{ij,s}^E$	Decision operation variable (open or closed) of switch for existing circuits ij in scenario s
ω_{ij}^E	Investment variable for the allocation of switches in existing circuits ij
$x_{ij,s}^N$	Operation variable (open or closed) of switch in interconnection circuit ij in scenario s
ω_{ij}^N	Investment variable for the allocation of switches in interconnection circuit ij
$S_{ij,s}^E$	Apparent power flow in existing circuit ij in scenario s
$S_{ij,s}^N$	Apparent power flow in interconnection circuit ij in scenario s
$V_{i,s}$	Voltage magnitude at bus i , in scenario s

1 Introduction

The planning of power distribution systems (PPDS) deals with investment analysis required for network expansion due to increasing energy consumption. The network expansion must guarantee a good quality and high reliability service to consumers (Gönem 1986). The PPDS must provide a set of decision-making results in order to expand the distribution networks, e.g., new circuits construction, the replacement of some existing circuits with others of greater capacity, substations construction and/or the expansion of the capacity of existing substations in order to meet the growth of consumer demand with quality and reliability. PPDS includes the minimization of network investment costs and the satisfaction of operational, physical, and financial constraints (Kathor and Leung 1997).

A realistic mathematical model of the PPDS problem, that considers the physical and economical characteristics of distribution systems (DS), is a large mixed-integer non-linear programming problem (Kathor and Leung 1997). Several proposals to solve the PPDS problem are presented in the literature, and among them, there are classical optimization techniques such as the Branch-and-Bound (B&B) algorithms (Paiva et al. 2005), heuristic techniques based on the Branch-Exchange algorithms (Miguez et al. 2002) and meta-heuristics such as Genetic Algorithms and Simulated Annealing (Parada et al. 2004; Carvalho et al. 2000; Nahman and Peric 2008; Ramirez-Rosado and Dominguez-Navarro 2004). More complex models for the PPDS problem, considering simultaneous planning of medium- and low-voltage distribution networks (Paiva et al. 2005) and the presence of distributed power generation (Haffner et al. 2008a, b), are also found in the specialized literature.

In general, the optimal planning of distribution systems must include network expansion and reliability but, in a practical sense, it is limited only to new substations allocation and

new primary feeders (Gönem 1986; Kathor and Leung 1997). After defining the network topology with the allocation and specification of substations and primary feeders, a distribution system needs to be designed considering network reliability. The reliability of the system is improved by allocating maneuver sectionalizing switches, interconnection lines between feeders and protection devices in strategic points of the network in order to improve the supply quality and the reliability indices.

The effects on consumers due to the allocation of control and protection devices in the system are usually related to the frequency and duration of the interruption of supply energy. In the literature, there are several works that address the optimized allocation of maneuver switches (Levitin et al. 1994; Billinton and Jonnavithula 1996) and the optimized allocation of protective devices (Silva et al. 2004) to protect and restore the distribution network.

A methodology to solve the planning of distribution networks considering the network reliability is proposed in this work. This methodology is composed of the formulation and solution of two subordinate problems: (1) First, the problem of optimal expansion of radial aerial distribution systems is solved. This problem is formulated as a mixed binary linear programming (MBLP) problem through a linearization technique of quadratic terms shown in detail by Alguacil et al. (2003) which uses a linear disjunctive model. The objective function considers reconductoring investments, construction of new substations or reinforcement of existing substations, and new circuits construction; operation costs related to losses on feeders are also considered. Operational constraints related to voltage magnitude limits, meeting the demand and physical capacity of transformers and feeders are also taken into account; (2) Second an MBLP model to allocate maneuver switches and interconnection lines is solved, thus ensuring the improvement of the network reliability indices. The contribution of this work lies in the proposal of these two linear optimization models, which are solved through classical optimization techniques using commercial solvers (Fourer et al. 2003; CPLEX 2008). The proposed methodology is tested on a 54-bus test system found in literature (Miranda et al. 1994) and the results are analyzed and discussed in detail. The planning horizon adopted is 15 years subdivided into three 5-year periods. A long-term horizon was chosen due to high investment costs of planning elements, requiring a longer period to check the investment return. A pseudo-dynamic planning model is used considering three separate 5-year planning horizons (Kathor and Leung 1997). In this type of planning model, the multi-period problems may be solved as many series of single-period problems; thus, the resulting solution will not be the overall optimum solution because current solutions are not influenced by future decisions during the optimization process.

This paper is structured as follows: In Sect. 2 the models used and constructed to obtain the solutions presented in this paper are introduced. In Sect. 3 the results for the 54-bus system are shown. Finally, in Sect. 4 are presented conclusions of the results achieved.

2 Mathematical Model and Solution Technique

The planning of a distribution network, considering the reliability as dealt in this paper, comprises both the expansion of a distribution system network like the allocation of maneuver switches and interconnection lines. These problems are formulated as mixed-integer linear programming models and computed by commercial solvers. The methodology used to solve the problem of planning medium-voltage distribution networks, considering network reliability, is a heuristic technique that provides the sequential solution for these two models on each period of the planning horizon.

To solve the multi-stages (or dynamic) planning of electric distribution networks is possible to choose between a backward strategy and a forward strategy, both are pseudo-dynamic strategies. The pseudo-dynamic strategy used in this paper is the forward strategy, but it must be emphasized that this choice requires a careful analysis based on the characteristics of the system under study. The growth of the demands between stages is a physical aspect of planning and a very important issue that must be taken into consideration when choosing one strategy. If the demand variation between stages is relatively small, the forward strategy may result a better option. Otherwise, the backward strategy may be a better choice due to large topological changes caused by large demand variations among stages.

2.1 Mathematical Model for the PPDS

The model for the PPDS problem proposed in this work is formulated using the following set of equations:

$$\begin{aligned} \min v = & \sum_{ij \in \text{NNC}} \sum_{c \in \Omega_c} \delta_{ij,c}^L C_{ij,c}^L + \sum_{ij \in \text{NC}} \sum_{c \in \Omega_c} \delta_{ij,c}^R C_{ij,c}^R \\ & + \sum_{i \in \text{NSE}} \delta_i^S C_i^S + K_c \sum_{ij \in \text{NC}} \sum_{c \in \Omega_c} R_{ij,c}^E \bar{S}_{ij,c}^E \\ & + K_c \sum_{ij \in \text{NTL}} \sum_{c \in \Omega_c} R_{ij,c}^N \bar{S}_{ij,c}^N + \sum_{i \in \text{NSE}} S_i^S C_i^O + K \sum_{i \in \text{NB}} R_i \end{aligned} \quad (1)$$

Subject to:

$$\sum_{ji \in \text{NC}} \sum_{c \in \Omega_c} S_{ji,c}^E - \sum_{ij \in \text{NC}} \sum_{c \in \Omega_c} S_{ij,c}^E + \sum_{ji \in \text{NTL}} \sum_{c \in \Omega_c} S_{ji,c}^N$$

$$\begin{aligned}
 & - \sum_{ij \in \text{NTL}} \sum_{c \in \Omega_c} S_{ij,c}^N + S_i^S \\
 & = S_i^C - R_i; \quad \forall i \in \text{NB}
 \end{aligned} \tag{2}$$

$$\begin{aligned}
 & -\Delta V \left(1 - \sum_{c \in \Omega_c} \delta_{ij,c}^R \right) \leq \sum_{c \in \Omega_c} \left(Z_{ij,c}^E \frac{S_{ij,c}^E}{V^{\text{base}}} \right) \\
 & - (V_i - V_j) \leq \Delta V \left(1 - \sum_{c \in \Omega_c} \delta_{ij,c}^R \right); \quad \forall ij \in \text{NTL}
 \end{aligned} \tag{3}$$

$$-\bar{S}_c \delta_{ij,c}^R \leq S_{ij,c}^E \leq \bar{S}_c \delta_{ij,c}^R \quad \forall ij \in \text{NC}, \forall c \in \Omega_c \tag{4}$$

$$\begin{aligned}
 & -\Delta V \left(1 - \sum_{c \in \Omega_c} \delta_{ij,c}^L \right) \leq \sum_{c \in \Omega_c} \left(Z_{ij,c}^N \frac{S_{ij,c}^N}{V^{\text{base}}} \right) \\
 & - (V_i - V_j) \leq \Delta V \left(1 - \sum_{c \in \Omega_c} \delta_{ij,c}^L \right); \quad \forall ij \in \text{NNC}
 \end{aligned} \tag{5}$$

$$\bar{S}_c \delta_{ij,c}^L \leq S_{ij,c}^N \leq \bar{S}_c \delta_{ij,c}^L; \quad \forall ij \in \text{NNC}, \forall c \in \Omega_c \tag{6}$$

$$0 \leq S_i^S \leq S_i^{SE} + S_i^{SN} \delta_i^S; \quad \forall i \in \text{NSE} \tag{7}$$

$$\underline{V} \leq V_i \leq \bar{V}; \quad \forall i \in \text{NB} \tag{8}$$

$$\sum_{c \in \Omega_c} \delta_{ij,c}^R \leq 1; \quad \forall ij \in \text{NC} \tag{9}$$

$$\sum_{c \in \Omega_c} \delta_{ij,c}^L \leq 1; \quad \forall ij \in \text{NTL} \tag{10}$$

$$\delta_{ij,c}^L \leq y_i; \quad \forall ij \in \text{NTL}, \forall c \in \Omega_c, \forall i \in \text{NTB} \tag{11}$$

$$\delta_{ji,c}^L \leq y_i; \quad \forall ji \in \text{NTL}, \forall c \in \Omega_c, \forall i \in \text{NTB} \tag{12}$$

$$\sum_{ji \in \text{NTL}} \sum_{c \in \Omega_c} \delta_{ji,c}^L + \sum_{ij \in \text{NTL}} \sum_{c \in \Omega_c} \delta_{ij,c}^L \geq 2y_i; \quad \forall i \in \text{NTB} \tag{13}$$

$$\begin{aligned}
 & N^E + \sum_{ij \in \text{NNC}} \sum_{c \in \Omega_c} \delta_{ij,c}^L = N_B - N_S \\
 & - \sum_{i \in \text{NTB}} (1 - y_i); \quad \forall ji \in \Omega_N, \forall c \in \Omega_c, \forall i \in \text{NTB}
 \end{aligned} \tag{14}$$

$$\delta_{ij,c}^R, \delta_{ij,c}^L, \delta_i^S, y_i \in \{0, 1\}; \tag{15}$$

$$\frac{S_{ij,c}^E}{V^{\text{base}}} = S_{ij,c}^{E+} - S_{ij,c}^{E-}; \quad \forall ij \in \text{NC}, \forall c \in \Omega_c \tag{16}$$

$$\hat{S}_{ij,c}^E = \sum_{r=1}^R m_{c,r} \Delta_{ij,c,r}^{IE}; \quad \forall ij \in \text{NC}, \forall c \in \Omega_c \tag{17}$$

$$S_{ij,c}^{E+} + S_{ij,c}^{E-} = \sum_{r=1}^R \Delta_{ij,c,r}^{IE}; \quad \forall ij \in \text{NC}, \forall c \in \Omega_c \tag{18}$$

$$0 \leq \Delta_{ij,c,r}^{IE} \leq \bar{\Delta}_c; \quad \forall ij \in \text{NC}, \forall c \in \Omega_c, r = 1 \dots R \tag{19}$$

$$0 \leq S_{ij,c}^{E+}; \quad \forall ij \in \text{NC}, \quad \forall c \in \Omega_c \tag{20}$$

$$0 \leq S_{ij,c}^{E-}; \quad \forall ij \in \text{NC}, \forall c \in \Omega_c \tag{21}$$

$$\frac{S_{ij,c}^N}{V^{\text{base}}} = S_{ij,c}^{N+} - S_{ij,c}^{N-}; \quad \forall ij \in \text{NTL}, \forall c \in \Omega_c \tag{22}$$

$$\hat{S}_{ij,c}^N = \sum_{r=1}^R m_{c,r} \Delta_{ij,c,r}^{IN}; \quad \forall ij \in \text{NTL}, \forall c \in \Omega_c \tag{23}$$

$$S_{ij,c}^{N+} + S_{ij,c}^{N-} = \sum_{r=1}^R \Delta_{ij,c,r}^{IN}; \quad \forall ij \in \text{NTL}, \forall c \in \Omega_c \tag{24}$$

$$0 \leq \Delta_{ij,c,r}^{IN} \leq \bar{\Delta}_c; \quad \forall ij \in \text{NTL}, \forall c \in \Omega_c, r = 1 \dots R \tag{25}$$

$$0 \leq S_{ij,c}^{N+}; \quad \forall ij \in \text{NTL}, \forall c \in \Omega_c \tag{26}$$

$$m_{c,r} = (2r - 1) \bar{\Delta}_c; \quad \forall c \in \Omega_c, r = 1 \dots R \tag{27}$$

$$\bar{\Delta}_c = \frac{\bar{S}_c}{R}; \quad \forall c \in \Omega_c \tag{28}$$

The objective function (1) represents the total investment and operational costs. The first term represents the cost of constructing new circuits using different types of conductors. The second term represents the cost of reconductoring existing circuits. The third term represents the costs of constructing new substations. The fourth and fifth terms are the costs of the energy losses of existing and new circuits, respectively, in the planning horizon. The sixth term is the operational cost of the substation and the last term is the cost of the energy curtailment on the system.

Constraint (2) represents the power balance equation at each node of the power distribution system. Equations (3)–(6) are obtained from the application of Kirchhoff’s second law in a circuit. Considering all existing and new circuits of the distribution network, two sets of nonlinear equations are gotten. These sets of nonlinear equations are linearized through the linear disjunctive model to obtain the linear equations (3) and (5) together with Eqs. (4) and (6). Constraint (4) represents the power flow limit constraints of the existing circuits. Constraint (6) represents the apparent power flow limit of the new circuits. The maximum current capacity in the substations is represented by (7). Constraint (8) represents the maximum and minimum voltage magnitude. Equations (11)–(14) describe the behavior of a transfer bus, which is a bus without source or demand but is commonly used in the connection of buses. The transfer bus is modeled by a binary variable, y_i , that is equal to one, $y_i = 1$, when the transfer bus is being used and otherwise, equal to zero, $y_i = 0$. Constraints (11)–(14) avoid the generation of loops due to the presence of the transfer buses in the distribution network and prevent transfer buses to be used as terminal nodes. Constraint (15) represents the binary nature of the following investment variables: construction of new circuits, reconductoring of existing circuits, construction of new substations and repowering of existing substations. The other decision variables represent the operating point of the distribution system. For an investment proposal defined by the values $\delta_{ij,c}^R, \delta_{ij,c}^L, \delta_i^S$ and y_i , several feasible operating points are possible.

Equations (16)–(28) are the linearization of quadratic terms $\left(\frac{S_{ij,c}^E}{V_{base}}\right)^2$ and $\left(\frac{S_{ij,c}^N}{V_{base}}\right)^2$ related to the power losses in existing and new lines, given by $\frac{S_{ij,c}^E}{V_{base}}$ and $\frac{S_{ij,c}^N}{V_{base}}$, respectively. The parameters $m_{c,r}$ and $\bar{\Delta}_c$ are constant values, and the auxiliary variables $S_{ij,c}^{E+}$ and $S_{ij,c}^{E-}$ are non-negative variables used to calculate $|S_{ij,c}^E|$, as shown in (16) and (22). Constraints (17) and (23) are a linear approximation of the square power of S . Equations (18) and (24) state the values of $|S_{ij,c}^E|$ and $|S_{ij,c}^N|$ as the sum of each block linearization. Equations (19) and (25) restrict the minimum and maximum contribution of each linearization block, $|S_{ij,c}^E|$, $|S_{ij,c}^N|$.

In this model, it is assumed that each conductor type has the following characteristics: (a) resistance per unit of length, (b) reactance per unit of length, (c) maximum current capacity, and (d) construction cost per unit of length. The reconductoring of existing circuits is determined by the investment cost, $C_{ij,ba}^R$, in which the investment cost depends on the type of initial conductor (b) and the type of final conductors (a).

2.2 Mathematical Model for the Allocation of Sectionalizing Switches and Interconnection Circuits

The model for optimal allocation of maneuver switches and tie lines in the distribution system is formulated by considering the following assumptions:

- (a) The load demands in the distribution system are represented as constant apparent power.
- (b) The constraints that must be considered in the model include the satisfaction of demands, the compliance of magnitude voltage limits and the meeting of operational limits of transformers and feeders.
- (c) Each fault condition is represented as an operation scenario ($n - 1$ criterion).
- (d) The radial topology of the system must be respected for each operation scenario.
- (e) The system is balanced and represented by its equivalent single-phase circuit.

Thereby, the mixed integer linear programming model for the allocation problem of sectionalizing switches and interconnection circuits in distribution systems is given by (29)–(44).

$$\begin{aligned} \min v = & \sum_{i \in \text{NC}} \sum_{s \in \text{NS}} \lambda_s K_c R_{i,s} + \sum_{ij \in \text{NC}} C_{ch} \omega_{ij}^E \\ & + \sum_{ij \in \text{NTL}} (2C_{ch} + C_N) \omega_{ij}^N \\ & + \sum_{i \in \text{NC}} \sum_{s \in \text{NS}} \lambda_s C_{och} \left(1 - x_{ij,s}^E\right) \\ & + \sum_{i \in \text{NTL}} \sum_{s \in \text{NS}} 2\lambda_s C_{och} x_{ij,s}^N \end{aligned} \tag{29}$$

Subject to:

$$\begin{aligned} & \sum_{ij \in \text{NC}} S_{ij,s}^E - \sum_{ij \in \text{NC}} S_{ij,s}^E + \sum_{ji \in \text{NTL}} S_{ij,s}^N \\ & - \sum_{ij \in \text{NTL}} S_{ij,s}^N + S_{i,s}^S \\ & = S_i^D - R_{i,s} \quad \forall i \in \text{NLG}, \forall s \in \text{NS} \end{aligned} \tag{30}$$

$$\begin{aligned} & -\Delta V \left(1 - x_{ij,s}^E\right) \leq \frac{S_{ij,s}^E}{V_{base}} Z_{ij,s}^E - (V_{i,s} - V_{j,s}) \\ & \leq \Delta V \left(1 - x_{ij,s}^E\right) \quad \forall ij \in \Omega_E, \forall s \in \text{NS} \end{aligned} \tag{31}$$

$$-\bar{S}_{ij} x_{ij,s}^E \leq S_{ij,s}^E \leq x_{ij,s}^E \bar{S}_{ij} \quad \forall ij \in \text{NC}, \forall s \in \text{NS} \tag{32}$$

$$\begin{aligned} & -\Delta V \left(1 - x_{ij,s}^N\right) \leq \frac{S_{ij,s}^N}{V_{base}} Z_{ij,s}^N - (V_{i,s} - V_{j,s}) \\ & \leq \Delta V \left(1 - x_{ij,s}^N\right) \quad \forall ij \in \text{NTL}, \forall s \in \text{NS} \end{aligned} \tag{33}$$

$$-\bar{S}_{ij} x_{ij,s}^N \leq S_{ij,s}^N \leq x_{ij,s}^N \bar{S}_{ij} \quad \forall ij \in \text{NC}, \forall s \in \text{NS} \tag{34}$$

$$0 \leq S_{i,s}^S \leq \bar{S}_i^S \quad \forall i \in \text{NLG}, \forall s \in \text{NS} \tag{35}$$

$$\underline{V} \leq V_{i,s} \leq \bar{V} \quad \forall i \in \text{NLG}, \forall s \in \text{NS} \tag{36}$$

$$0 \leq R_{i,s} \leq \gamma_{is} M \quad \forall i \in \text{NLG}, \forall s \in \text{NS} \tag{37}$$

$$\sum_{ij \in \Omega_E} x_{ij,s}^E + \sum_{ij \in \Omega_N} x_{ij,s}^N = N_B - N_S - 1 \quad \forall s \in \text{NS} \tag{38}$$

$$\left(1 - \omega_{ij}^E\right) \leq x_{ij,s}^E \quad \forall ij \in \text{NC} \tag{39}$$

$$x_{ij,s}^N \leq \omega_{ij}^N \quad \forall ij \in \text{NLT} \tag{40}$$

$$x_{ij,s}^E \in \{0, 1\} \quad \forall ij \in \text{NC}, \forall s \in \text{NS} \tag{41}$$

$$x_{ij,s}^N \in \{0, 1\} \quad \forall ij \in \text{NTL}, \forall s \in \text{NS} \tag{42}$$

$$\omega_{ij}^E \in \{0, 1\} \quad \forall ij \in \text{NC} \tag{43}$$

$$\omega_{ij}^N \in \{0, 1\} \quad \forall ij \in \text{NTL} \tag{44}$$

The objective function (29) represents the non-supplied energy and the total investment in the reliability of the distribution network. The first term represents the expected total cost of power load curtailment due to the occurrence of a permanent fault at any section of the network. The second term represents the costs of allocation switches at the existing circuits. The third term represents the cost of allocating switches and the cost of building new interconnection circuits. Note that two switches should be allocated at the interconnection circuit to fully de-energize it. The switches allocated on each interconnection circuit have the same operational logic, which means that they are simultaneously open or closed. The fourth and fifth terms are the operational costs of the switches placed in existing circuits and interconnections circuits, respectively. Note that, in the operating costs, it is considered that switches allocated at the existing circuits are in operating condition (closed switch), while the switches allocated at interconnection circuits are in non-operating conditions (open switch). Constraint (30) represents the appar-

ent power balance equation for each node of the system. Note that the positive variable $R_{i,s}$, is an artificial apparent power generator at node i used to represent the power cut in scenario s .

Equation (31) provides the apparent power flow in the existing circuits for each scenario. Equation (32) limits the apparent power flow in an existing circuit to its maximum capacity. Note that if switch ij in scenario s is in operation ($x_{ij,s}^E = 1$), then, $\frac{S_{ij,s}^E}{V_{base}} = (V_{i,s} - V_{j,s}) / Z_{ij,s}^E$ and $-\bar{S}_{ij} \leq S_{ij,s}^E \leq \bar{S}_{ij}$; otherwise, $\frac{S_{ij,s}^E}{V_{base}} = 0$ and $(V_{i,s} - V_{j,s})$ is free to change. Equation (33) calculates the apparent power flow in the interconnection circuits for each scenario. Equation (34) limits the apparent power flow in the interconnection circuits at their maximum capacity. Note that if switch ij in scenario s is in operation ($x_{ij,s}^N = 1$), then $\frac{S_{ij,s}^N}{V_{base}} = (V_{i,s} - V_{j,s}) / Z_{ij,s}^N$ and $-\bar{S}_{ij} \leq S_{ij,s}^N \leq \bar{S}_{ij}$; otherwise, $\frac{S_{ij,s}^N}{V_{base}} = 0$ and $(V_{i,s} - V_{j,s})$ is free to change.

Constraint (35) ensures the non-violation of the maximum apparent power capacity of substation i in each scenarios, while constraint (36) ensures the maximum and minimum voltage magnitude limits for the system nodes in all scenarios. Equation (37) restricts the load curtailment of node i in scenario s , where M can be the sum of all of the loads in the distribution system and $\gamma_{i,s} = 1$ for a condition of a permanent fault next to bus i in scenario s ; otherwise, $\gamma_{i,s} = 0$. In every scenario s , only one permanent fault condition is allowed.

Constraint (38), along with (30), guarantees the radiality of the distribution networks in each scenario. The binary variables $x_{ij,s}^E$ and $x_{ij,s}^N$ are the operational conditions, opened or closed, of the switches in the existing and interconnection circuits, ij , for scenario s , respectively.

The binary variables ω_{ij}^E and $\omega_{ij,s}^N$ are decision variables for the allocation of switches in the existing circuits and interconnection circuits, respectively. Equation (39) relates the investment variables to the operation variables of the switches

Fig. 1 Initial system

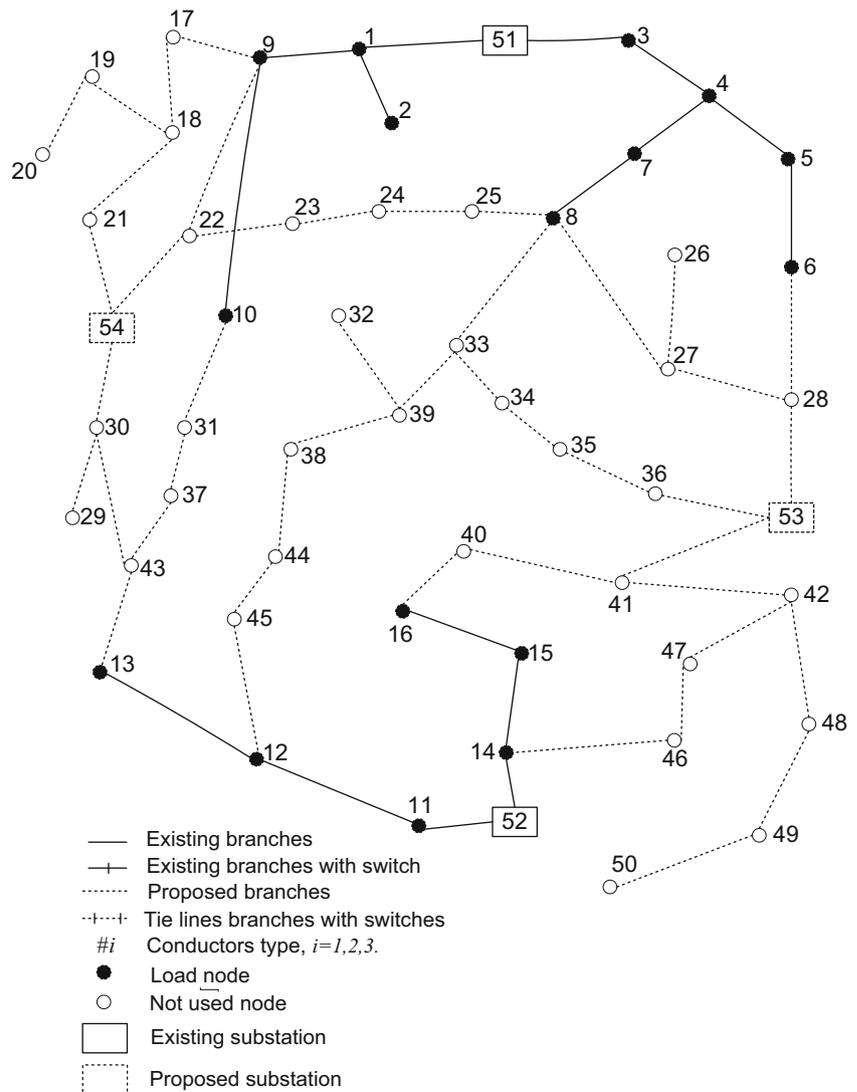
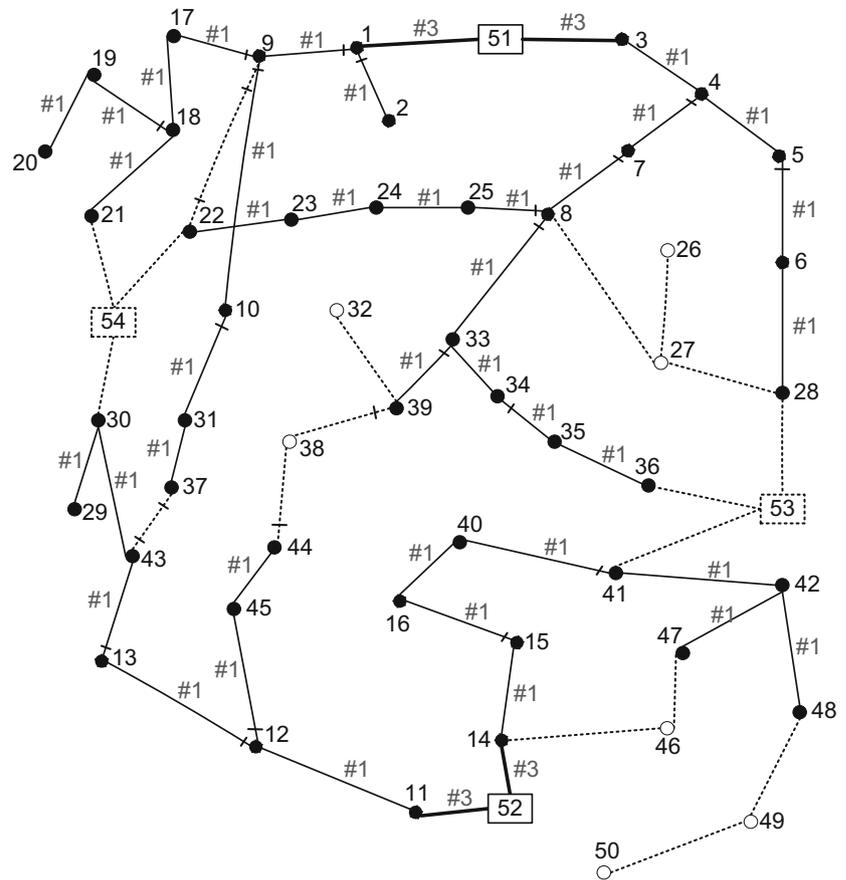


Table 1 Circuit data of the 54-bus system

Type of conductor	Resistance (Ω/km)	Impedance (Ω/km)	Capacity (A)	Construction cost (\$/kVA/km)
1	0.3655	0.2520	150	2000
2	0.2921	0.2466	200	3000
3	0.2359	0.2402	250	4000
4	0.1932	0.2279	300	5000

Fig. 2 Solution proposed for the first period



in the existing circuits. If $\omega_{ij}^E = 1$, then $0 \leq x_{ij,s}^E \leq 1$, i.e. the circuit ij is operational or not; otherwise, $x_{ij,s}^E = 1$ and the circuit ij is always operational. Equation (40) relates the investment variables with the operation variables of the switches in the interconnection circuits. If $\omega_{ij}^N = 1$, then $0 \leq x_{ij,s}^N \leq 1$, which means that the interconnection circuit ij , is operating or not; otherwise, $x_{ij,s}^N = 0$ and the interconnection circuit, ij , is always non-operational.

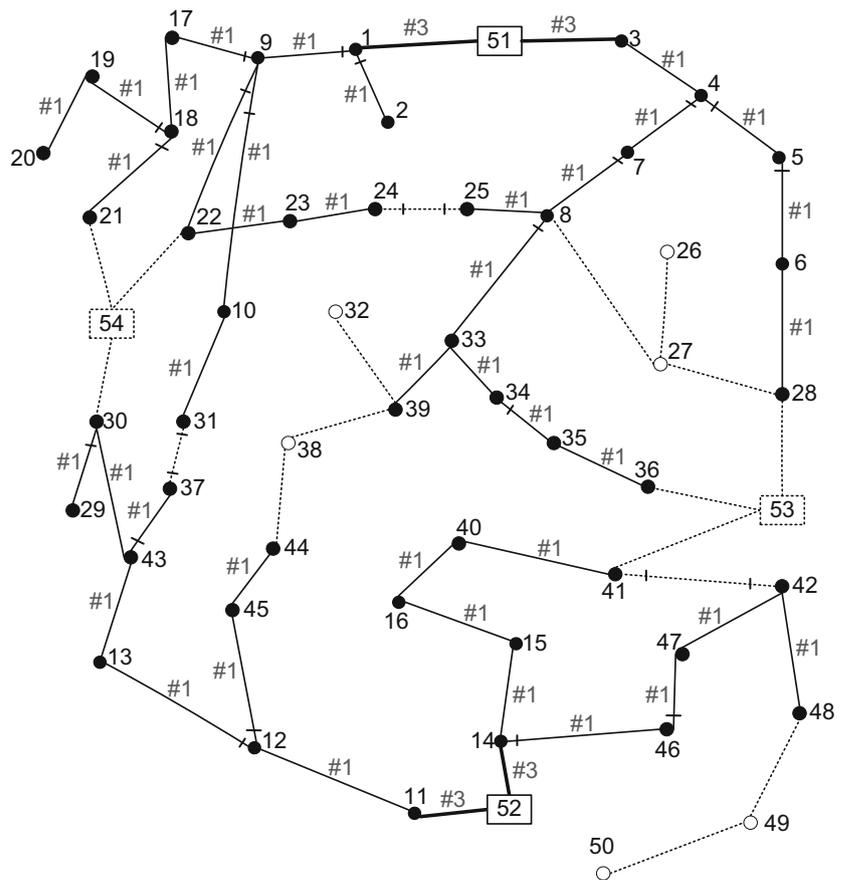
In (41) and (42), $x_{ij,s}^E$ and $x_{ij,s}^N$ are binary variables of the operational features of the switches in the existing circuits and interconnection circuits, respectively, for each scenario. In (43) and (44), ω_{ij}^E and ω_{ij}^N are binary variables for the allocation of sectionalizing switches in the existing circuits and interconnection circuits, respectively. The mathematical model, using Eqs. (29)–(44), is a binary linear programming

problem that can be solved using commercial solvers of integer programming.

3 Test and Results

The models of PPSD and the allocation problem of the sectionalizing switches and interconnection circuits were implemented using the modeling language AMPL (A Mathematical Programming Language) (Fourer et al. 2003) and solved using the CPLEX software (CPLEX 2008). It uses an algorithm based on the Branch-and-Cut optimization technique, which was used through standard options [16]. The CPLEX solver requires the “gap” specification, which is optimally measured between the best integer objective value found and

Fig. 3 Solution proposed for the second period



the value deduced from all of the node subordinate problems already solved (Fourer et al. 2003). The numerical results were obtained for a 54-bus distribution test system (Miranda et al. 1994), its initial topology is illustrated in Fig. 1. This is a 13.5 kV distribution system with 107.8 MVA of capacity that feeds 50 load buses, of which, 10 buses are transfer buses (buses 26, 27, 32, 35, 38, 42, 43, 46, 49 and 50). It has four substations, two existing ones, which can be re-powered, and two candidate substations, 16 existing circuits and 45 circuits that can be constructed during the planning phase. In Fig. 1 is illustrated the initial topology of the system. In the construction of the new circuits and/or reconductoring of existing circuits, four types of conductors are employed, as shown in Table 1. The pseudo-dynamic planning model uses a 15-year planning horizon divided into three 5-year periods.

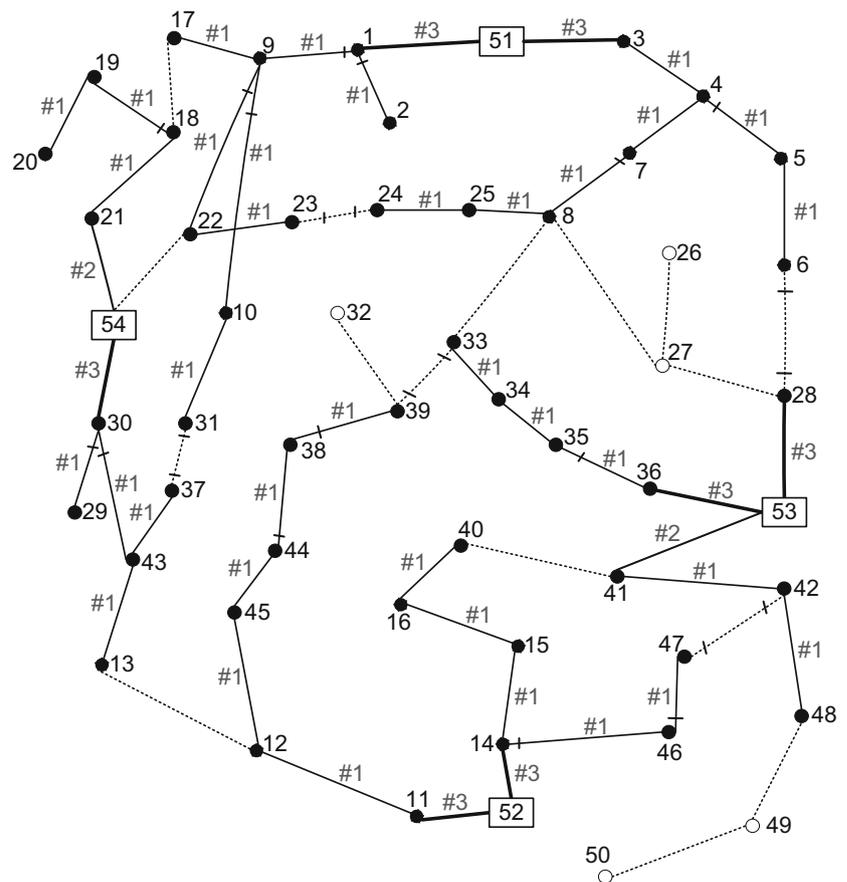
For the allocation of switches and interconnection circuits, it was considered a switch allocation cost of US\$ 1,370.00, an operating cost of US\$ 120.00 and a cost of unserved energy of US\$ 1,000.00 per kW. The circuit breakers, equipped with an overcurrent relay, at the feeder terminations near the substation operate together with the sectionalizing switches and can open or close the circuit in case of permanent faults during some operation stage of the network. The rate of permanent

faults, which is used to allocate the switches, is $\gamma = 0.04$ fault/km/year.

In the first period of the planning horizon, the problem has 488 integer variables and the total cost of the optimal solution is US\$ 276,555.82, where, (US\$ 82,020.00) correspond to the cost of the construction of new circuits, the cost of the repowering of substation #52 is (US\$ 100,000.00) and the cost of circuit losses is (US\$ 94,535.82). The results were obtained with a gap of 0.01 % with a processing time equal to 4.99 s.

Moreover, the problem of allocation and switch operation for this first stage has 2444 integer variables, and the solution was obtained with a computational time of 21.28 s and a gap of 3.17%. This solution has non-supplied energy costs of US\$ 48,805.10, switch allocation costs of US\$ 34,250.00, interconnection circuit costs of US\$ 4,748.00 and switch operation costs of US\$ 9,432.00. For this solution, three interconnection circuits (9-22, 44-39 and 43-37) and six maneuver switches were allocated. Circuit 44-39 should work as a single interconnection network replacing circuits 44-38 and 38-39 (Fig. 2). This condition is used for cases where these situations occur, that is, when there are two interconnection circuits connected to a transfer bus that are replaced by only one circuit without the transfer bus. The solution

Fig. 4 Solution proposed for the third period



of the model also provides a way to operate the switches and circuit breakers for each scenario in eventual contingencies, i.e., informing which switches or circuit breakers must open or close in order to energize as many load buses as possible while respecting the physical and operational constraints of the electric distribution system. In Fig. 2 is shown the topology proposed by the planning model for the first stage.

In the second stage of the planning horizon, the problem has 524 integer variables, the costs for constructing new circuits, repowering substations and losses in existing and new circuits are US\$ 13,730.00, US\$ 100,000.00, US\$ 185,888.00 and US\$ 12,247.50, respectively. The maneuver switch allocation problem has 2754 integer variables, and the optimal solution was found with a gap of 2.44%, with non-supplied energy costs of US\$ 33,076.10, costs of allocation switches of US\$ 39,730.00, costs of building interconnection circuits of US\$ 8,722.40 and a total cost of operation switches of US\$ 5,420.40. In Fig. 3 is shown the proposed planning topology and the allocation of switches.

In the third stage, the problem has 548 integer variables, and the costs of building new circuits, constructing substations and losses in new and existing circuits are US\$ 26,530.00, US\$ 480,000.00, US\$ 551,814.00, and US\$

101,037.00, respectively. The problem of allocating maneuver switches and interconnection circuits has 2872 integer variables, and the optimal solution was found with a gap of 2.44% with non-supplied energy costs of US\$ 31,037.70, switch allocation costs of US\$ 39,730.00, costs of building interconnection circuits of US\$ 8,722.40 and a total cost of operation switches of US\$ 5,420.40. In Fig. 4 is shown the planning topology and the allocation of switches.

4 Conclusions

The proposed MBLP models for the problems of distribution network planning and allocation sectionalizing switches and interconnection circuits in distribution systems were solved using a pseudo-dynamic planning method. Due to the linearity of the proposed models, their convergence to the optimal solution is guaranteed, which allows the use of conventional MBLP solvers to achieve the solution. Furthermore, the models support the main planning actions used by professionals in the electric power energy industry.

Good quality results were obtained in terms of the optimized planning and allocation of switches; they also provided actions to operate the planned network under permanent fault conditions in the most efficient possible manner by reconfiguring the network.

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