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Core of the Reinsurance Market with Dependent Risks

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Abstract Baton and Lemaire (Astin Bull 12:57–71, 1981) proved the nonemptiness of the core of a reinsurance market in which the risks of companies are independent. However, cases involving dependent risks have received increasing concerns in modern actuarial science. In this paper, we investigate the nonemptiness of the core of a reinsurance market where the risks of different companies may be dependent. When the exponential utility function is employed, we find an important property on risk premium and show that the core of the market is always nonempty.

Keywords Core · Risk premium · Reinsurance market · Exponential utility · Cooperative game

Mathematics Subject Classification 91A12 · 91B30

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1 Introduction

Traditional insurance theory always assumes independent risks. With the increasing complexity of insurance and reinsurance markets, modern insurance theory raises more concerns on dependent risks [1]. The risks of different companies may actually correlate in many situations. For example, in a car accident, claims of the automobile insurance company and the medical insurance company are clearly dependent [2].

Cooperative game theory provides a natural tool for modeling the reinsurance market [3–5]. To the best of our knowledge, Baton and Lemaire [3] first introduced the collective rationality to a reinsurance market and provided a way to understand reinsurance problem from the viewpoint of cooperative game theory. They found that the core of the market is nonempty when the utility function of each company is of exponential form. This result is important in risk exchange treaties and has often been cited by follow-up researches [6–8]. However, this fundamental result is based on the assumption of independent risks.

In the paper, we study a reinsurance market with a finite set of companies, in which the risks involved by different companies may be dependent. Each company faces a risky situation characterized by a random claim amount. Different from Baton and Lemaire [3], these claim amounts may be dependent. To reduce the risk, any group of the companies could ally and share the risk of the group to enhance their own situation. The objective of the paper is to investigate whether there exists an allocation rule in the grand coalition such that all companies in it feel satisfied and do not secede to form a sub-coalition.

We find the risk premium demonstrates some interesting properties using the exponential utility. We obtain a sufficient and necessary condition for a treaty to be in the core and prove furthermore that there is a nonempty core in the reinsurance market with dependent risks. This implies the cooperation in a reinsurance market could benefit the companies in a more general sense. Our result generalizes the work of Baton and Lemaire [3] and enriches the literature on risk sharing.

For the literature on dependent risks in the insurance market, Albrecher [9] described dependence between claim sizes using copulas. Dependence between claim sizes and claim amounts is discussed in Albrecher and Boxma [10]. Wang and Yuen [2] examined the dependence between different classes of insurance in a company. Arvidsson and Francke [11] discussed several ways for modeling dependence in insurance. For the general actuarial theory of dependent risks, see Denuit et al. [1]. These literature is concerned with the problem that a company faces a sequence of dependent risks, while our study focuses on the risk exchanges among a group of companies in a single-period setting. Besides, there are many papers on the application of game theory to the reinsurance market. For example, Borch [12,13] discussed the Pareto optimality and the individual rationality conditions of the market. Baton and Lemaire [3] extended Borch's work by introducing collective rationality into a reinsurance market and propose a cooperative game framework for the market. Ghica [14] presented a basic game model in a reinsurance market and applies the model with exponential and power utilities. Jaramillo et al. [15] discussed the formation of risk-sharing coalitions among heterogenous individuals and find that heterogeneity in risk may lead to partial risk sharing. The above-mentioned papers are from the perspective of cooperation. For the literature on the reinsurance market from the perspective of competition, please see Aase [6,16,17].

The rest of the paper is arranged as follows. Section 2 provides notations and the proposed model. Section 3 presents some properties of risk premium using the exponential utility and proves the existence of a nonempty core in the reinsurance market. Section 4 concludes the paper.

2 Notations and Model

Assume that there are n insurance companies $\{1, 2, \cdots, n\}$, denoted by N, trying to sign an agreement of risk exchanges to improve their situations. A nonempty set $S \subset N$, with cardinality $|S|$, is called a coalition. In particular, N is called the grand coalition. Moreover, let $\mathbb{R}^m$ be the m-dimensional real space. Let the initial situation of company j be $[R_j, F_j(\cdot)]$, where R_j denotes the free reserve and $F_j(\cdot)$ denotes the distribution function of its claim amount ξ_j, for $j \in N$. For $\varnothing \neq S \subset N$, $F_S(\cdot)$ denotes the convolution of $F_j(\cdot)$, for $j \in S$. In this study, we do not assume that the claim amounts of different companies are independent. Moreover, $\{\xi_1, \xi_2, \cdots, \xi_n\}$ may have any kind of dependence structures.

A company j evaluates its situation by the expected exponential utility of

$$U_j([R_j, F_j(\cdot)]) = E\big[u_j(R_j - \xi_j)\big] = \int_0^{\infty} u_j(R_j - x_j)\mathrm{d}F_j(x_j),$$

where

$$u_j(R_j - x_j) = \frac{1}{c_j}\Big(1 - \exp\big(-c_j(R_j - x_j)\big)\Big) \text{ for } x_j \geqslant 0,$$

and $c_j > 0$ is a parameter. Let $\alpha_j = \frac{1}{c_j}$, for $j \in N$ and $\alpha_S = \sum\limits_{j \in S} \alpha_j$, for $\varnothing \neq S \subset N$. For $\varnothing \neq S \subset N$, $(x_l)_{l \in S} = (x_{j_1}, \cdots, x_{j_{|S|}})$, where $j_l \in S$ for $l = 1, \cdots, |S|$. A treaty in $S \subset N$ is a vector $\big(y_j((x_l)_{l \in S})\big)_{j \in S}$, where $y_j((x_l)_{l \in S})$ is the claim that j has to pay when the realized claim of company l is x_l, for $l \in S$. The core of the game is the set of the treaties in N that satisfy the following two conditions [3]:

Condition 1 (Admissibility): $\sum\limits_{j \in N} y_j(x_1, \cdots, x_n) = \sum\limits_{j \in N} x_j.$

Condition 2 (Collective rationality) : No coalition has interest in quitting the grand coalition.

A company j's exponential risk premium is the real number P_j satisfying $E\big[u_j(R_j + P_j - \xi_j)\big] = E\big[u_j(R_j)\big]$, for $j \in N$. Moreover, for a coalition $S \subset N$, let P_j^S be the exponential risk premium of company j when its share in S is $\frac{\alpha_j}{\alpha_S} \sum_{k \in S} \xi_k$, for $j \in S$.

3 The Existence of a Nonempty Core

In this section, in order to obtain the existence of a nonempty core in the market, we need to prove some lemmas first.

Lemma 3.1 *Let $\varnothing \neq S \subset N$. Then, for $j \in S$,*

$$P_j^S = \frac{1}{c_j}\log\left(E\left[\exp\left(\frac{1}{\alpha_S}\xi_S\right)\right]\right), \tag{3.1}$$

where $\xi_S = \sum_{k\in S}\xi_k$ is the total claim amount of the companies in S.

Proof Note that P_j^S is the risk premium of company j when its claim amount is $\frac{\alpha_j}{\alpha_S}\sum_{k\in S}\xi_k$, for $j \in S$. Hence, we have

$$E\left[u_j\left(R_j + P_j^S - \frac{\alpha_j}{\alpha_S}\sum_{k\in S}\xi_k\right)\right] = 0.$$

From Eq. (3.3) in Gerber [18], we know $P_j^S = \frac{1}{c_j}\log\left(E\left[\exp(\frac{1}{\alpha_S}\xi_S)\right]\right)$.

When the claim amounts are actually independent, we can deduce from formula (3.1) that

$$\begin{aligned}\frac{1}{c_j}\log\left(E\left[\exp\left(\frac{1}{\alpha_S}\xi_S\right)\right]\right) &= \frac{1}{c_j}\log\left(E\left[\exp\left(\frac{1}{\alpha_S}\sum_{k\in S}\xi_k\right)\right]\right)\\ &= \frac{1}{c_j}\sum_{k\in S}\log\left(E\left[\exp\left(\frac{1}{\alpha_S}\xi_k\right)\right]\right).\end{aligned}$$

This is exactly the result of Lemma (3.1) in Baton and Lemaire [3].

Lemma 3.2 (Hölder Inequality, Finner [19]) *Let X_j, $j = 1, \cdots, m$ with $m \geqslant 2$, be a random variable with limited expectation, and $p_j \in (0, 1)$ with $\sum_{j=1}^m p_j = 1$. Then, for the expectation of random variables, we have*

$$E\Big[\prod_{j=1}^m \mid X_j \mid\Big] \leqslant \prod_{j=1}^m\left(E\left[\mid X_j \mid^{\frac{1}{p_j}}\right]\right)^{p_j}. \tag{3.2}$$

Hölder inequality is necessary for the following property of the exponential risk premium.

Lemma 3.3 *Let ξ_j be the claim amount of company j, for $j \in N$, and $p_j \in [0, 1]$ such that $\sum\limits_{j\in N} p_j = 1$. Then,*

$$\sum_{j\in N} p_j \log\left(E\left[\exp(\xi_j)\right]\right) \geqslant \log\left(E\left[\exp\left(\sum_{j\in N} p_j\xi_j\right)\right]\right). \tag{3.3}$$

Proof When $p_j = 1$ for some $j \in N$, the result becomes true automatically. When $p_j \in [0, 1)$, for $j \in N$, w.l.o.g., we may assume that $p_j \in (0, 1)$, for $j \in N$. By Lemma 3.2, we have

$$\begin{aligned}\sum_{j\in N} p_j \log\Big(E\big[\exp(\xi_j)\big]\Big) &= \log\Big(\prod_{j\in N}\big(E\big[\exp(\xi_j)\big]\big)^{p_j}\Big)\\ &= \log\left(\prod_{j\in N}\left(E\left[\exp\left(\frac{1}{p_j}\cdot p_j\cdot\xi_j\right)\right]\right)^{p_j}\right)\\ &\geqslant \log\Big(E\big[\prod_{j\in N}\exp\left(p_j\xi_j\right)\big]\Big)\\ &= \log\Big(E\big[\exp\big(\sum_{j\in N} p_j\xi_j\big)\big]\Big).\end{aligned}$$

Combining the above two cases, we see inequality (3.3) holds.

Lemma (3.3) shows an important property of the exponential risk premium. When a company faces a set of different risks ξ_j with a weight p_j, for $j \in N$, it should choose to insure jointly rather than to do separately because $\sum_{j\in N} p_j \log\Big(E\big[\exp(\xi_j)\big]\Big)$ measures the risk premium for separate insurance while $\log\Big(E\big[\exp(\sum_{j\in N} p_j\xi_j)\big]\Big)$ measures that for joint insurance.

Example 3.4 In Lemma (3.3), let $N = \{1, 2\}$, $p_1 = p_2 = 0.5$, $\xi_1 \sim U(0, 1)$ and $\xi_2 \sim U(0, 1)$, where $U(0, 1)$ is the uniform distribution on $[0, 1]$. By simple calculation,

(1) when ξ_1 and ξ_2 are independent,
$\sum\limits_{j\in N} p_j \log\Big(E\big[\exp(\xi_j)\big]\Big) = \log\Big(E\big[\exp(\sum\limits_{j\in N} p_j\xi_j)\big]\Big) = 0.54$;
(2) when $\xi_1 = 1 - \xi_2$,
$\sum\limits_{j\in N} p_j \log\Big(E\big[\exp(\xi_j)\big]\Big) = \log(e - 1)$
$= 0.54$ and $\log\Big(E\big[\exp(\sum\limits_{j\in N} p_j\xi_j)\big]\Big)$
$= 0.5$. It is easy to see that $\sum\limits_{j\in N} p_j \log\Big(E\big[\exp(\xi_j)\big]\Big) > \log\Big(E\big[\exp(\sum\limits_{j\in N} p_j\xi_j)\big]\Big)$.

From Example 3.4, we can see that if a company faces the risks ξ_1 and x_2, then (1) when the risks are independent, the company has no preference for joint insurance over separate insurance; (2) when the risks having some kind of dependence (negative dependence in Example 3.4), the company may strictly prefers joint insurance.

Lemma 3.5 *There exists a treaty satisfying Conditions* 1 *and* 2 *if and only if there exists* $t_j \in \mathbb{R}^1$, $j \in N$, *such that*

$$\sum_{j \in N} t_j = \sum_{j \in N} P_j^N, \tag{3.4}$$

$$\sum_{j \in S} t_j \leqslant \sum_{j \in S} P_j^S, \text{ for all } \varnothing \neq S \subset N. \tag{3.5}$$

Proof (Necessity) Let $(y_1(x_1, \cdots, x_n), \cdots, y_n(x_1, \cdots, x_n))$ be a treaty that satisfies Conditions 1 and 2. Define

$$t_j = \frac{\log\left(1 - c_j \int_0^\infty u_i(R_j - y_j(x_1, \cdots, x_n)) dF_N(x_1, \cdots, x_n)\right)}{c_j} + R_j, \quad j \in N.$$

Since Conditions 1 and 2 imply Pareto optimality (page 60 of [3]),

$$y_j(x_1, \cdots, x_n) = \frac{1/c_j}{\sum_{k \in N} 1/c_k} \sum_{k \in N} x_k + y_j(\mathbf{0}), \quad j \in N,$$

where $\mathbf{0}$ is a zero vector in $\mathbb{R}^n$. Note that $\sum_{j \in N} y_j(\mathbf{0}) = 0$. Then, from Lemma 3.1,

$$t_j = \frac{1}{c_j} \log\left(E\left[\exp\left(\frac{1}{\alpha_N} \sum_{k \in N} \xi_k\right)\right]\right) + y_j(\mathbf{0}), \quad j \in N.$$

Obviously, we have

$$\sum_{j \in N} t_j = \alpha_N \log\left(E\left[\exp\left(\frac{1}{\alpha_N} \sum_{k \in N} \xi_k\right)\right]\right).$$

If $\sum\limits_{j \in S} t_j > \sum_{j \in S} P_j^S$ for some $\varnothing \neq S \subset N$, then by Lemma (3.5) in Baton and Lemaire [3], there exists $t'_j \in \mathbb{R}^1$, for $j \in S$, such that $t_j > t'_j$, for $j \in S$ and $\sum_{j \in S} t'_j = \sum_{j \in S} P_j^S$. For $j \in S$, let

$$d_j = t'_j - \frac{1}{c_j} \log\left(E\left[\exp\left(\frac{1}{\alpha_S} \sum_{k \in S} \xi_k\right)\right]\right),$$

$$z_j((x_j)_{j \in S}) = \frac{\alpha_j}{\alpha_S} \sum_{k \in S} x_k + d_j.$$

Note that $\sum_{j \in S} d_j = 0$. Thus $(z_j(x_l)_{l \in S})_{l \in S}$ is a treaty in S. By calculation, for $j \in S$, we have

$$\frac{\log\left(1 - \alpha_j \int_0^\infty u_j\left(R_j - z_j((x_l)_{l \in S})\right) \mathrm{d}F_S((x_l)_{l \in S})\right)}{\alpha_j} + R_j = t'_j,$$

$$\frac{\log\left(1 - \alpha_j \int_0^\infty u_j(R_j - y_j(x_1, \cdots, x_n)) \mathrm{d}F_N(x_1, \cdots, x_n)\right)}{\alpha_j} + R_j = t_j.$$

Then $\int_0^\infty u_j(R_i - z_j((x_l)_{l\in S}))\mathrm{d}F_S((x_l)_{l\in S})) > \int_0^\infty u_j\left(R_j - y_j(x_1, \cdots, x_n)\right)$ $\mathrm{d}F_N(x_1, \cdots, x_n)$, for $j \in S$. In this case, the companies in S will secede from the grand coalition, which leads to a contradiction. Thus $\sum_{j\in S} t_j \leqslant \sum_{j\in S} P_j^S$, for all $\varnothing \neq S \subset N$.

(Sufficiency) Let t_j, $j \in N$, be a real number that satisfies (3.4) and (3.5). For $j \in N$, define

$$y_j(\mathbf{0}) = t_j - \frac{1}{c_j}\log\left(E\left[\exp\left(\alpha_N\sum_{k\in N}\xi_k\right)\right]\right),$$
$$y_j(x_1, \cdots, x_n) = \frac{\alpha_j}{\alpha_N}\sum_{k\in N} x_k + y_j(\mathbf{0}).$$

Then, $(y_1(x_1, \cdots, x_n), \cdots, y_n(x_1, \cdots, x_n))$ satisfies Condition 1.

If $(y_1(x_1, \cdots, x_n), \cdots, y_n(x_1, \cdots, x_n))$ does not satisfy Condition 2, then there is some $S \subset N$ and a treaty $(z_j((x_l)_{l\in S}))_{l\in S}$ in S satisfying Pareto optimality [3] such that for $j \in S$,

$$\int_0^\infty u_j(R_j - z_j((x_l)_{l\in S}))\mathrm{d}F_S((x_l)_{l\in S}))$$
$$> \int_0^\infty u_j(R_j - y_j(x_1, \cdots, x_n))\mathrm{d}F_N(x_1, \cdots, x_n).$$

Let $t_j' = \frac{\log\left(1-\alpha_j\int_0^\infty u_j(R_j - z_j((x_l)_{l\in S}))\mathrm{d}F_S((x_l)_{l\in S}))\right)}{\alpha_j} + R_j$, for $j \in S$. Then, $t_j' < t_j$, for $j \in S$ and $\sum_{j\in S} t_j' < \sum_{j\in S} t_j$. Moreover, for $(z_j(x_l)_{l\in S})_{j\in S}$, we have $\sum_{j\in S} t_j' = \frac{1}{\alpha_S}\log\left(E[\exp(\alpha_S\sum_{k\in S}\xi_k)]\right)$. Thus

$$\alpha_S\log\left(E\left[\exp\left(\frac{1}{\alpha_S}\sum_{k\in S}\xi_k\right)\right]\right) < \sum_{j\in S} t_j,$$

which leads to a contradiction. Hence, $(y_1(x_1, \cdots, x_n), \cdots, y_n(x_1, \cdots, x_n))$ satisfies Conditions 1 and 2.

Lemma 3.5 is actually intuitive. We consider t_j as the premium allocated to company j in some treaty $(y_1(x_1, \cdots, x_n), \cdots, y_n(x_1, \cdots, x_n))$, for $j \in N$. Condition (3.4) says that the sum of the premiums of all companies, $\sum_{j\in N} t_j$, equals the premium of the grand coalition, $\sum_{j\in N} P_j^N$. Condition (3.5) says that the sum of the allocated premiums of the companies in any group cannot exceed the premium of the group. Then the treaty $(y_1(x_1, \cdots, x_n), \cdots, y_n(x_1, \cdots, x_n))$ satisfies Conditions 1 and 2. From Lemma 3.5, to judge if the core of the game is nonempty, it is equivalent to find real numbers t_j, $j \in N$, that satisfy (3.4) and (3.5). Based on this, we can see that the game has a nonempty core when the exponential utility is employed.

Theorem 3.6 *The core of the reinsurance market with exponential utility is nonempty.*

Proof From Lemma 3.5, for the reinsurance market with any kind of dependent risks, the core is nonempty if and only if there exist real numbers t_j, for $j \in N$, that satisfy (3.4) and (3.5).

From Bondareva–Shapley theorem [20,21], the system of (3.4) and (3.5) has a solution if and only if

$$\sum_{\varnothing \neq S \subset N} \lambda(S) \sum_{j \in S} P_j^S - \sum_{j \in N} P_j^N \geqslant 0 \tag{3.6}$$

holds for all possible function $\lambda : 2^N \setminus \varnothing \to [0, 1]$ with $\sum_{\varnothing \neq S \subset N : j \in S} \lambda(S) = 1$, for $j \in N$, where 2^N is the set of all subsets of N.

In fact, we have

$$\sum_{j \in N} P_j^N = \alpha_N \log \left(E\left[\exp(\frac{1}{\alpha_N} \xi_N) \right] \right).$$

Hence, for the reinsurance market with any kind of dependent risks, by Lemma 3.3,

$$\begin{aligned}
\sum_{\varnothing \neq S \subset N} \lambda(S) \sum_{j \in S} P_j^S &= \sum_{\varnothing \neq S \subset N} \lambda(S) \left(\alpha_S \log \left(E\left[\exp\left(\frac{1}{\alpha_S} \xi_S \right) \right] \right) \right) \\
&= \sum_{\varnothing \neq S \subset N} \lambda(S) \sum_{j \in S} \alpha_j \log \left(E\left[\exp\left(\frac{1}{\alpha_S} \xi_S \right) \right] \right) \\
&= \sum_{j \in N} \alpha_j \sum_{S \subset N : j \in S} \lambda(S) \log \left(E\left[\exp\left(\frac{1}{\alpha_S} \xi_S \right) \right] \right) \\
&\geqslant \sum_{j \in N} \alpha_j \log \left(E\left[\exp\left(\sum_{S \subset N : j \in S} \frac{1}{\alpha_S} \xi_S \lambda(S) \right) \right] \right) \\
&= \alpha_N \sum_{j \in N} \frac{\alpha_j}{\alpha_N} \log \left(E\left[\exp\left(\sum_{S \subset N : j \in S} \frac{1}{\alpha_S} \lambda(S) \xi_S \right) \right] \right) \\
&\geqslant \alpha_N \log \left(E\left[\exp\left(\sum_{j \in N} \frac{\alpha_j}{\alpha_N} \sum_{S \subset N : j \in S} \frac{1}{\alpha_S} \lambda(S) \xi_S \right) \right] \right) \\
&= \alpha_N \log \left(E\left[\exp\left(\frac{1}{\alpha_N} \sum_{\varnothing \neq S \subset N} \sum_{j \in S} \frac{\alpha_j}{\alpha_S} \lambda(S) \xi_S \right) \right] \right) \\
&= \alpha_N \log \left(E\left[\exp\left(\frac{1}{\alpha_N} \xi_N \right) \right] \right).
\end{aligned}$$

Thus inequality (3.6) holds and the theorem follows.

4 Conclusion

In this paper, we study a reinsurance market consisting of risk-dependent companies. In the case of commonly used exponential utilities, we show that the risk premium exhibits an important property (Lemma 3.3) that indicates the joint insurance is always better than separate insurance for a company. Lemma 3.3 also presents an inequality on the exponential risk premium and contributes to the literature on risk exchanges from the technical perspective. We then prove the reinsurance market with exponential utilities has a nonempty core even if the risks of different companies were dependent. The result implies that the cooperation in the reinsurance market benefits all companies in a more general and practical setting. This finding significantly strengthens Baton and Lemaire's result and contributes to the literature on risk exchange treaties in the reinsurance market.

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