RESEARCH PAPER



Fitness-for-service assessment of defected welded structural details by experimental evaluation of the fatigue resistance S-N curve

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Abstract During the fabrication of two different structures (a ship and a bridge), made by mild construction steel, some unacceptable anomalies were detected; the structures were almost completely fabricated and a repair of the defected structural details was impractical due to the extent of the defected zones. A fitness-for-service assessment has therefore been carried out, in both cases, by performing fatigue tests on representative samples, cut off from the structure and by evaluating the fatigue resistance curve. A re-assessment of the structural details in question was undertaken, on the basis of the experimental S-N curve, evaluating the fitness for service of the "defected details". The experimental tests showed that the detected anomalies had no influence on the fatigue strength of the welded joints and that the examined details (a butt joint and a cruciform joint) behave in accordance with the relevant FAT classes. The experimental program has also given the opportunity to compare different methodologies for fatigue assessment: the Effective Notch Stress Method, the Fracture Mechanics Method and the Peak Stress Method. The different procedures have been applied to the tested specimens, and the expected fatigue life was compared with the experimental one.

Keywords (IIW Thesaurus) FCA welding · Structural steels · Fatigue strength · Notch effect · Finite element analysis

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1 Introduction

Fitness-for-service assessment is a multi-disciplinary engineering approach that is used to determine whether a certain equipment/structure is fit for operation. The equipment/ structure may contain flaws or have sustained damage. The flaws or damage may have occurred during service or may have been induced during fabrication. In such circumstances, especially when the component is at a late stage of the construction, repair may be costly and difficult to be performed. Specific procedures have been set up in order to take into account all possible damage mechanisms that may be present during service (corrosion, creep, fatigue, etc.).

When a fatigue load is present, the fatigue strength of the defected structure should be evaluated in relation to service conditions. In some cases, where the flaws/anomalies are difficult to model, the theoretical fatigue assessment may be not applicable in a reliable way and the experimental approach seems to be the only practicable option.

In this work, two real cases of defected structures are presented. The two structures (a ship and a bridge) showed some anomalies during fabrication, at a late stage of construction, and the repair of the defected structural details was impractical due to the extent of the defected zones.

A fitness-for-service assessment has therefore been carried out, in both cases, by performing fatigue tests on representative samples, cut off from the structure and by evaluating the fatigue resistance curve. A re-assessment of the structural details in question was undertaken, on the basis of the experimental S-N curve, evaluating the fitness for service of the "defected details".

Due to the availability of experimental data on welded joints prone to be modelled and to be assessed through the different FE-based techniques developed within IIW, a further contribution of this paper is the application of different methodologies for local fatigue assessment (effective notch stress, fracture mechanics, peak stress method) for the two investigated structural weld details.

2 Experimental fatigue assessment

2.1 Case 1—ship structural details

The aim of the first experimental test program was to evaluate the possible fatigue strength reduction of welded joints containing defects (such inclusions, porosities and undercuts) having number/dimensions greater than the acceptance criteria limits of applicable rules [1].

To this purpose, two sets of fatigue specimens have been tested:

- 1 set on defected "not conform" welded joints (N° 10 "NC" specimens) and
- 1 set on acceptable "conform" welded joints (N° 10 "C" specimens).

The fatigue strength reduction of NC specimens was determined through the evaluation of the experimental fatigue S-N curve and by comparison with the results of fatigue tests carried out on C specimens.

The welding defects of NC specimens, detected by radiographic testing (RX), are indicated (see also Fig. 1 in which some paper copies of the RX films are shown):

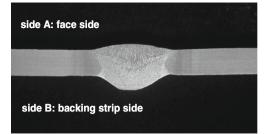


Fig. 2 Macrographic section of welded joint (plate thickness 6 mm)

- Elongated gas inclusions / porosities / slag inclusions for specimens NC1, NC2, NC3
- Shape irregularities and undercuts for specimen NC4
- Several elongated gas inclusions and slag inclusions for specimens NC5, NC6
- Porosities for specimens NC7, NC8, NC9, NC10

The fatigue specimens have been manufactured by welding-shaped plates to strips about 100-mm wide containing the workshop welded joint to be tested. A total number of 20 strips were prepared (10 C and 10 NC), with thicknesses ranging from 5 to 8 mm.

The ship plates, made in AH36 structural steel (equivalent to S355 grade), 6-mm thick, were welded by FCAW process, in flat position, on a ceramic backing strip, with homologous filler metal. Figure 2 shows a macrographic section carried out on one of the provided strips. A significant weld convexity has been observed on both C and NC strips: the ratio between height of

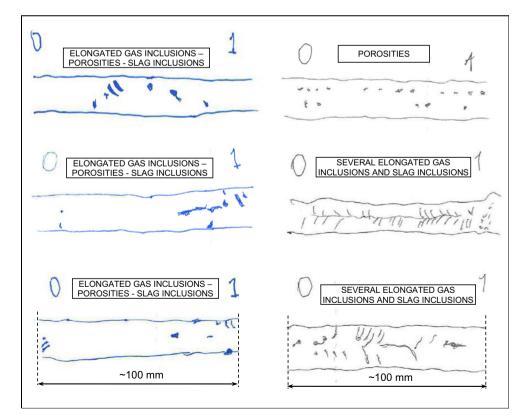


Fig. 1 Some examples of defect size and distribution (from RX testing) on a weld length of about 100 mm (weld width ranging form 12 to 16 mm) specimen

weld convexity (h) and weld width (w) ranges from 0.17 to 0.30(w ranging from 12 to 16 mm, h ranging from 2 to 3.5).

A scheme of the fatigue specimen is provided in Fig. 3. After specimen manufacturing, its profile has been ground and polished lengthwise, in order to avoid fatigue crack initiation from specimen edges, according to recommendations provided in [2]. The welded joints were tested in the as-welded condition.

The obtained results of fatigue tests (performed with fatigue ratio R=0.1) are illustrated in Fig. 4, in which the representative points of fatigue test results are plotted in a log $\Delta\sigma$ -logN diagram. The data have been "normalized" to a single value of $\Delta\sigma$ (205 MPa) in order to help in the comparison and interpretation of test results, considering the relatively low number of tested specimens.

A first important result of the fatigue test program is that the weld defects have not influenced the fatigue strength of the welded joints under examination. All specimens, both C

(conforming) and NC (not conforming) type, broke at the butt weld toe. Some specimens broke from A side, others from B side, and some specimens from both sides.

Such result appears in substantial agreement with provisions given in [3] Par. 3.8.2.

A general remark concerning fatigue test results is that results themselves are very scattered. This is linked both to the weld convexity variability (ratio h/w ranges from 0.17 to 0.30 as previously observed) and to the specimen distortion (angular deflection): Some of the provided strips had significant distortions and therefore the angular deflection of some specimens was not negligible. It was then necessary, for some tests, to measure the specimen deflection and to take into account the bending stresses induced by it, calculating the K_m-misalignment factor according to IIW Recommendations [3].

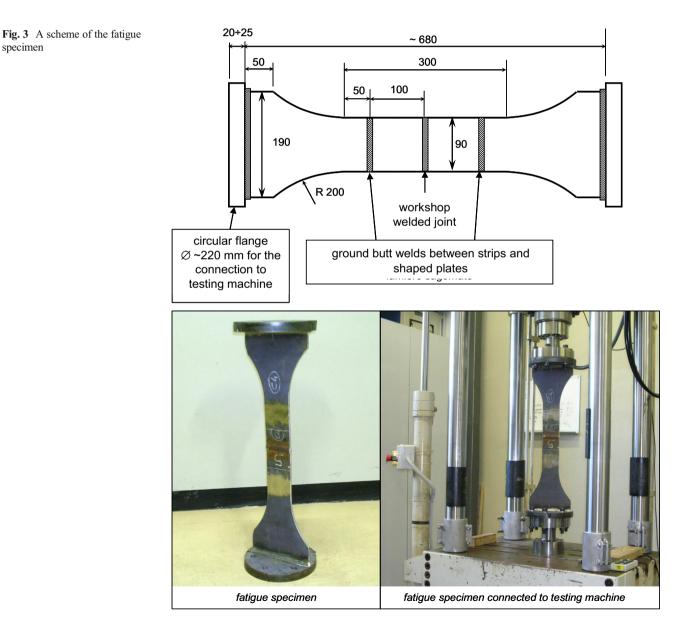
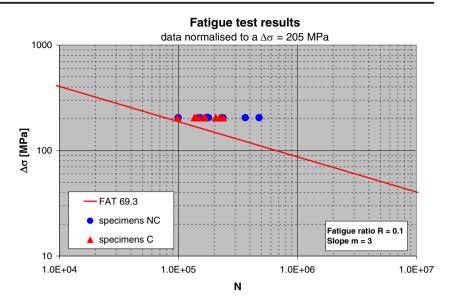


Fig. 4 The obtained results of fatigue tests



Test results have been statistically elaborated according to [3] Par. 3.7, in order to determine the experimental detail category.

As all the tests (NC and C specimens) have shown an identical failure mode (at the butt weld toe), the data have been statistically treated altogether.

The fatigue strength at 2 millions cycles (FAT) equals 69.3 MPa, with 97.7 % probability of survival. The relevant fatigue curve is plotted in Fig. 4 together with test results. Such category is a bit lower than the detail categories provided for butt welds performed on both sides (or on a ceramic backing strip), FAT 80. Nevertheless, such lower value may be considered as a conservative result, in consideration of the great scatter of test results due both to specimen deformation and to weld convexity variability (in some cases not conforming to standard requirements).

2.2 Case 2-bridge structural details

The second experimental program was aimed to evaluate the influence on fatigue strength of planar flaws in the flange plate of cruciform welded joints.

During the construction of a steel bridge deck, some plate batches, worked and welded in the workshop, showed anomalous ultrasonic responses in their middle plane. A laboratory investigation, on specimens taken out from the bridge structure, pointed out the presence of planar flaws (mid-thickness pearlitic-bainitic segregation band with elongated inclusions and microscopic cracks, with a maximum length measured on the micrographic sections about 2 mm).

As the plates are subjected during service to out-of-plane loads due to fillet welded connections of bracing, an experimental program was carried out in order to compare the fatigue strength of cruciform fillet joints samples, made with production plates, with or without anomalous ultrasonic responses. Preliminary static and fatigue cyclic tests showed basically equivalent behaviour for the samples affected or not affected by the short cracks in the middle plane. A root failure mechanism was observed. Therefore, a full fatigue test program (10 specimens) was carried out in order to confirm that, despite the plate anomalies, the cruciform joints fatigue strength could be assimilated to FAT 36, category considered in the design of the examined structural detail.

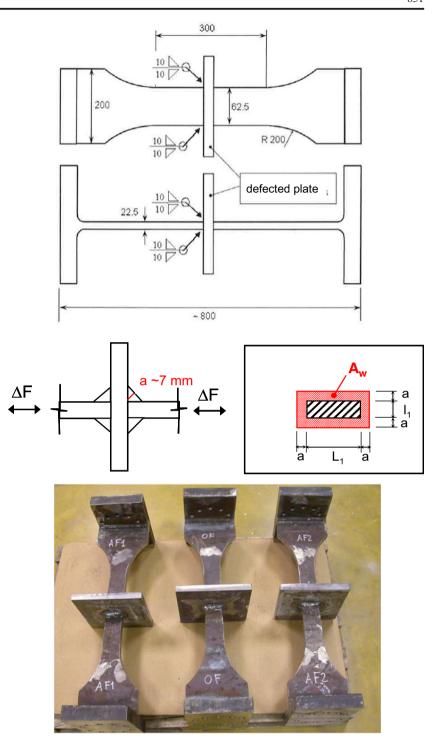
Fatigue tests have been carried out on proper specimens, in order to apply a through-thickness load to the plate, simulating the forces acting on the fillet welded joints connecting transversal ribs and bracing gussets. The fatigue specimen geometry is shown in Fig. 5. The plates are made of S355K2+N steel, thickness 22 mm. The fillet welds have been carried out following the WPS used for construction by the bridge manufacturer workshop, using FCAW welding process and homologous filler material. Specimens were tested in the as welded conditions.

All specimens broke in the throat section with a root failure mechanism. A macrographic section of a broken specimen is shown in Fig. 6.

The fatigue test results have been plotted in the $\log\Delta\sigma$ -logN diagram (see Fig. 7) together with the experimental S-N curve determined in accordance with [3] Par. 3.7.

The fatigue tests have pointed out that the planar defects did not affect the fatigue resistance of such structural details. The experimental fatigue category is a bit lower than category 36, depending on the shortness of the fillet weld length and the consequent high concentration factor induced in the test (the actual fillet welds are longer but the geometrical configuration of the specimens had to be consistent with the testing machine loading capacity). Anyway, the test results may be considered congruent with the FAT 36 design curve, taking also into account the typical large scatterband of fatigue tests performed on fillet welded joints.

Fig. 5 Fatigue specimen and throat section for $\Delta \sigma$ calculation

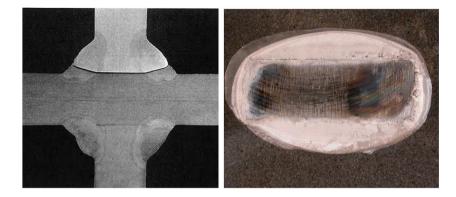


3 Fatigue assessment by means of FE-based techniques

The execution of the previously described fatigue tests has been considered a good opportunity for investigating the suitability of FEM-based approaches to the fatigue assessment of the components under examination. To this purpose, a number of numerical analyses have been carried out, by adopting the ANSYS software. This paragraph briefly summarizes adopted hypotheses and obtained results.

3.1 Ship specimens

Finite element models have been defined by carefully reproducing actual weld profiles (Fig. 8). The 8-nodes element "PLANE183" of the ANSYS library has been adopted, **Fig. 6** Macrographic section of a failed cruciform joint and its fracture surface (plate thickness 22 mm – weld throath ~ 7 mm)



selecting the "plane strain" option. At one end, horizontal displacements have been constrained, while, at the opposite end, maximum test stress (205 MPa) has been imposed. Linear elastic material was assumed, and no local modifications of the material properties due to welding procedures have been accounted for in the calculations.

The evaluation of fatigue strength has been carried out through the effective notch stress approach [3, 4]. In fact, the hot spot stress method has been considered not applicable, being the nominal stress in the captioned specimens clearly identified and stress raising effects exclusively due to the weld profile.

Two different curvature radii at notches (1.0 and 0.05 mm) have been adopted for the assessment. According to [4], the approach with 1.0 mm radius may cause problems particularly in thin structures with plate thickness below 5 mm. Therefore, a small-size notch approach (0.05 mm) has been proposed for welded joints in thin sheet materials, and its application has been comprehensively discussed in [5–7]. The smaller radius has been also considered in this case despite the plate thickness is about 6 mm in order to check the reliability, in relatively thin components, of the approach based on the larger one.

In both specimens, all areas, where crack initiation has been shown to occur (at weld toe on both top and bottom surface), have been investigated.

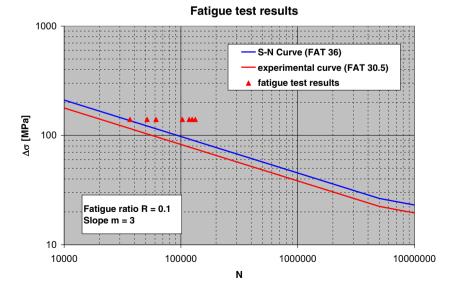
The expected notch stress " σ_{notch} " can be calculated using the following equation:

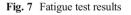
 $\sigma_{\text{notch}} = \sigma^* \text{FAT}(\text{EN}) / \text{FAT}(\text{NS})$

where σ represents the applied stress (205 MPa), "FAT(EN)" the FAT Class to be adopted for the effective notch stress approach and "FAT(NS)" the FAT Class defined by fatigue tests for the nominal stress approach. If a higher notch stress is obtained by numerical analyses, the approach is overly conservative; on the opposite hand, for lower values, obtained results can be considered too optimistic.

In the examined case, assuming 80 for FAT(NS), 225 for FAT(EN) with radius 1.0 mm and 630 for FAT(EN) with radius 0.05 according to [4], the following values should be expected for notch stresses:

- 576 MPa with radius 1.0 mm
- 1614 MPa with radius 0.05 mm





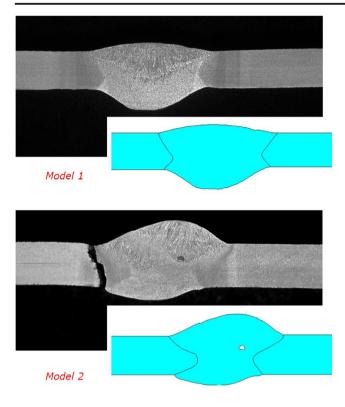


Fig. 8 Finite element models adopted for simulating ship specimens behaviour

FEM results are shown in Table 1; some among calculated stress distributions at notches are shown in Fig. 9. As it can be noted, all positions of potential crack initiation have shown notch stresses which are largely below expected values. These

| results seem in good agreement with considerations in [4] |
|--|
| (Par. 4.2) about "Mild Weld Notches", at least as far as the |
| curvature radius 1.0 mm is concerned. |

It is worth again noting that the geometry of the FE models was basically obtained from digital images. However, the very local notch geometry was created to obtain the desired notch radius (1.0 or 0.05 mm) at weld toe by locally placing a circular arc tangent to the plate and the weld bead.

A further assessment has been carried out by adopting the approach based on fracture mechanics principles, which is aimed to model fatigue crack growth. To this purpose, the following steps have been performed:

- Model 2 has been extruded in order to achieve a 3D configuration, which allows to simulate, through the "submodelling" technique (Fig. 10), the presence of a "small" fatigue crack at the initiation site (crack height a=0.1 mm, crack length 2c=1.0 mm).
- Crack depth was defined as recommended by technical literature for the application of fracture mechanics criteria to fatigue design; crack length was consequently set so that the typical aspect ration for surface-breaking flaw is assumed.
- Through the adoption of the "Virtual Crack Extension" ("VCE") technique, the stress intensity factor has been calculated at the deepest point of crack front and at the intersection between crack front and component surface (Fig. 10).
- Fatigue crack growth has been estimated by adopting BS 7910 [8] stress intensity factor solutions and crack

| Model | Curvature radius (mm) | Notch Stress (MPa) at initiation site | | | |
|-----------------|------------------------------------|---------------------------------------|-------------|------------------------|-----------------|
| | | Top left | Bottom left | Top right | Bottom right |
| 1 | 1.0 | 304.2 | 357.3 | 283.9 | 335.6 |
| 1 | 0.05 | 421.5 | 540.6 | 367.8 | 467.7 |
| 2 | 1.0 | 319.3 | 389.5 | 347.3 | 406.6 |
| 2 | 0.05 | 429.3 | 1.017.9 | 719.1 | 738.4 |
| Initiation site | <u>es</u> "Top left" "Bottom | | | op right" om right" | |

Table 1FEM results

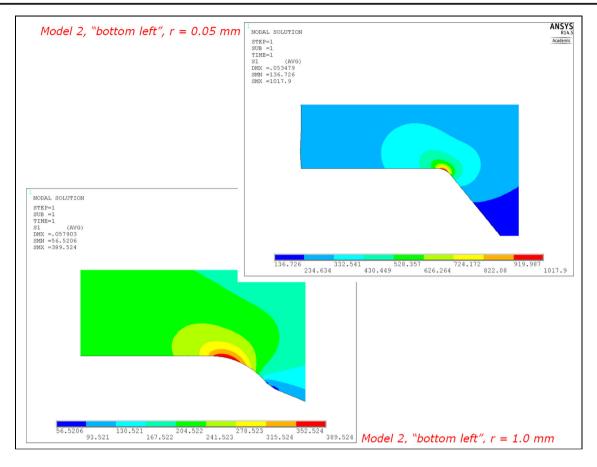
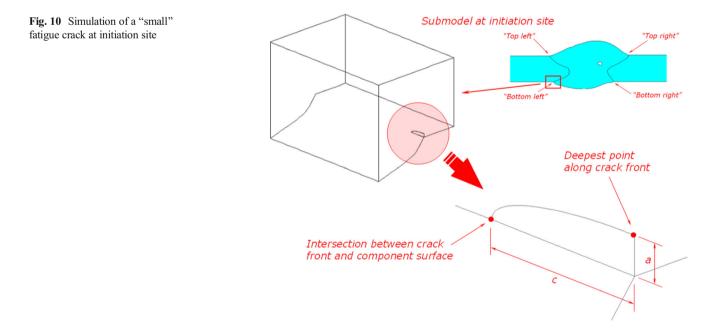


Fig. 9 Maximum principal stress distributions at notches (MPa)

growth laws, using the CRACKWISE software (implemented by TWI). Coefficients Mk and kt ([8], Annex M) have been properly adjusted so that the stress intensity factor calculated by CRACKWISE equals, in both points along crack front, the one obtained by the VCE technique.



It has been assumed that, adopting the "mean plus 2 standard deviations" crack growth law recommended in BS 7910, the same number of fatigue cycles to rupture which can be deduced from the nominal stress design curve (FAT 80) should be obtained.

Taking into account the applied stress (205 MPa), the fatigue life according to the nominal stress approach is about 120'000. Actually, a slope m=3 was assumed even if, since the thickness of investigated structural details ranges from 5 to 8 mm (i.e. they can be considered thin walled thickness details), a higher slope might be more appropriate, as suggested in [6].

Results of the crack propagation approach are in good agreement with such hypothesis, as the crack front reaches the opposite surface after about 115'000 cycles. Crack height and length evolution with time are shown in Fig. 11.

3.2 Bridge specimens

For bridge specimens, the use of 2D models does not look appropriate, as the ratio between width (62.5 mm) and thickness (22.5) of the loaded plate is relatively low. The definition of a 3D model simulating one of tested specimens has then been carried out (Fig. 12), by adopting 20-nodes "SOLID186" elements.

For the bridge specimens too, the effective notch stress approach has been applied by considering both curvature radii mentioned above (1.0 and 0.05 mm).

The model is characterized by three symmetry planes; crack initiation has been shown to occur at the midpoint of fillet welds along loaded plate width (Fig. 12). The effective notch stress approach, therefore, has been implemented again applying the sub-modelling technique, by placing sub-models at the initiation site.

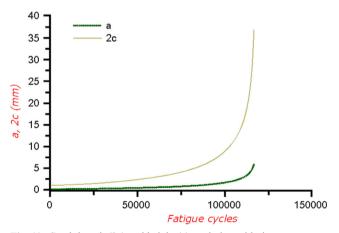


Fig. 11 Crack length (2c) and height (a) evolution with time

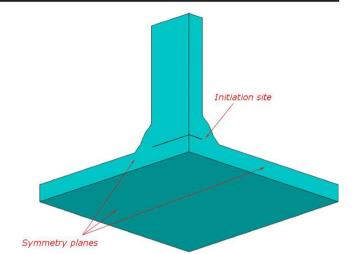


Fig. 12 Model simulating bridge specimen AF2

The specimen simulated in the assessment has been loaded with a force range of 181.8 kN. The total throat section area of fillet welds is about 1.850 mm², resulting in a maximum acting stress of $\sigma = 98.3$ N/mm². According to test results, it seems reasonable to assume that the FAT class of the examined detail is between FAT 30 and FAT 36. Following the same reasoning already illustrated for the ship specimens, the following values should be expected for the notch stresses:

- With radius 1.0 mm: 737 MPa for FAT 30, 614 MPa for FAT 36
- With radius 0.05 mm: 2'064 MPa for FAT 30, 1'720 MPa for FAT 36

It should be noted that, with FAT 30, the expected number of fatigue cycles to failure should be not less than about 56'900, while, with FAT 36, it should be larger than 98'300. The actual number which led to failure during tests is about 76'000.

Obtained results are summarized in Fig. 13. The notch stress equals 695 MPa with r=1.0 mm and 2.542 MPa with r=0.05 mm. The approach, therefore, looks considerably more suitable than for ship specimens, especially when the larger radius is adopted (while the smaller one is definitely too conservative). Again, this is in agreement with comments reported in [4] considering the larger thicknesses of the involved plates and the correspondingly larger weld seams.

In this second case, the evaluation of fatigue crack growth based on fracture mechanics principle is less simple: among stress intensity factor solutions provided in [8]; in fact, the one taking into account cracks at fillet weld root (Fig. 14) is not available in 3D. As noted above, that is likely to affect results reliability. The evaluation

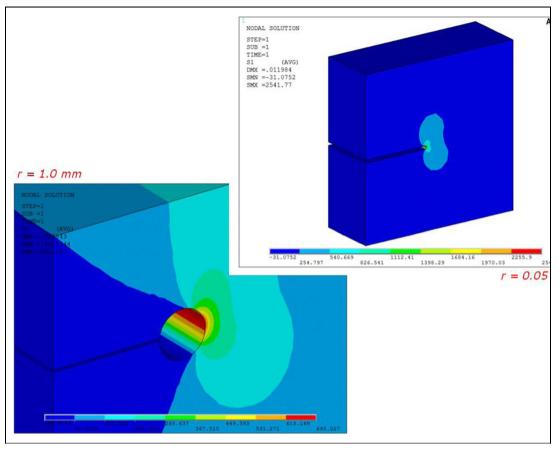


Fig. 13 Maximum principal stress distributions at notches (MPa)

carried out by adopting the 2D solution provided in [8] (with the "mean plus 2 standard deviations" crack growth law), in fact, has given an expected number of cycles to failure of about 22³00. It is then confirmed that, for the considered geometry, the 2D approach is overly conservative. Actually, similar trends were found in [9].

A further assessment has finally been carried out by adopting the Peak Stress Method ("PSM"), according to indications given by the following technical papers:

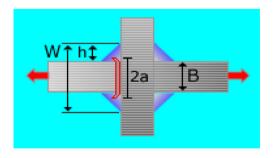


Fig. 14 2D stress intensity factor solution for "cruciform" welded joints ([8], Annex M)

- Document no. IIW-2202 "The Peak Stress Method for fatigue strength assessment of welded joints with weld toe or weld root failures" [10] and
- "The Peak Stress Method combined with 3D finite element models for fatigue assessment of toe and root cracking in steel welded joints subjected to axial or bending loading" [11].

According to documents mentioned above, the model has been defined by adopting the following criteria:

- Size and number of finite elements at weld root are those indicated in [10] (element size 1.0 mm; the root node is shared by four elements only).
- The model is defined through 8-nodes "SOLID185" elements with "keyoption" 2 set to 3 (as recommended in [11]).

Under such hypotheses, the peak stress can be directly read as the maximum principal stress acting at weld root; its value has then to be multiplied by the correction parameter f_w (which, for root initiation, equals 1.41, see [10], Table 1) and used with the FAT 156 design curve ([11], Fig. 3).

Taking into account FAT values deduced from test results for the nominal stress approach, the following peak stress values should be expected:

- For FAT 30 (nominal stress): 511 MPa
- For FAT 36 (nominal stress): 426 MPa.

As shown in Fig. 15, the maximum principal stress at weld root equals 406.3 MPa; by applying the correction parameter, the obtained peak stress is 572.9 MPa, which seems in substantial agreement with experimental results and the assessment carried out through the effective notch stress approach (with r = 1.0 mm).

4 Concluding remarks

A fitness-for-service assessment has been carried out, in two real cases of structures affected by anomalies, by performing fatigue tests on representative samples, which were cut off from the structure, and by evaluating the fatigue resistance curve. A re-assessment of the structural details in question was undertaken, on the basis of the experimental S-N curve. The experiments showed that the detected anomalies had no influence on the fatigue strength of the welded joints and that the examined details (a butt joint, a cruciform joint) behave in accordance with the relevant FAT classes, based on good workmanship crafted structural welded steel joints. Taking advantage of the availability of experimental data on welded joints, in addition, an application study of different local fatigue assessment methodologies, covering effective notch stress, fracture mechanical crack calculation and peak stress method, is applied to these two weld details.

Hereafter, the main results of the assessment are summarized.

- Concerning the Effective Notch Stress Method, its results do not seem to be reliable for the butt welded joint (ship specimens), in agreement with considerations in [4] (Par. 4.2); the same approach with R=1 mm has given very good results in the case of cruciform joints with root failure, while the adoption of R=0.05 has led to fairly conservative results.
- However, a reduced FAT value of about FAT 400 instead of FAT 560 is discussed in [6], based on effective notch stress criteria for thin-walled structures (and failure at shallow-notched weld toe and not at sharp notched weld root): this would lead to fatigue design results more in agreement with experimental ones in case of application of 0.05 mm radius.
- The Fracture Mechanics Method has given in both cases results in good agreement with the experimental findings. As far as the butt weld specimens are concerned, the initial flaw depth of 0.1 mm appears to be appropriate.
- The assessment has confirmed that the Peak Stress Method can be considered appropriate and reliable in evaluating the fatigue strength for failures starting from the root.

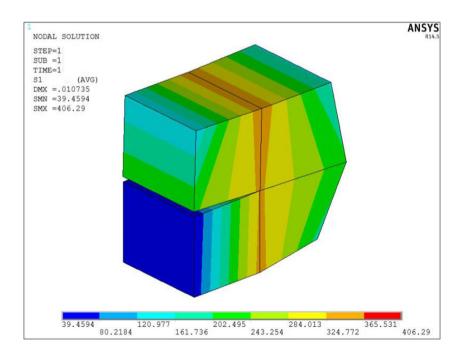


Fig. 15 Maximum principal stress at initiation site (weld root) to be used with the Peak Stress Method (MPa)

It should be finally underlined that the experimental approach is probably the straightforward option for assessing the fatigue strength of details containing defects, especially for non-planar flaws due to difficulties in properly setting numerical analyses.

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