Arab. J. Math. (2015) 4:229–230
DOI 10.1007/s40065-015-0140-y
Arabian Journal of Mathematics

EDITORIAL

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Preface

Published online: 20 November 2015

This is the second part of a special issue of our journal dedicated to the topic of singular perturbation theory and its various aspects, in particular analytic, geometric, and probabilistic. The main concern of perturbation theory in an analytic context is to study unbounded operators acting in infinite dimensional spaces (e.g., differential operators acting on functions belonging to some spaces of square integrable functions) and to treat them by relating them with some simpler operators for which essential properties (e.g., spectral ones) are known. Such problems arise naturally in mathematical physics (quantum mechanics, electromagnetism, hydrodynamics. . .) where singular forces, configurations, and interactions are to be taken into account. Very often these problems are intimately related to those arising in the study of stochastic processes (it suffices to think of diffusion processes as related to elliptic and parabolic partial differential equations). In geometrical contexts, singular perturbation problems arise in the study of natural geometric operators on non-necessarily smooth manifolds or on objects like graphs, which can be looked upon as limits of manifolds. In recent years, new mathematical techniques have been developed to handle such problems. These techniques are coming from different areas, including scattering theory, analysis (PDEs, functional analysis, non-standard analysis, generalized functions. . .), asymptotics, probability theory, statistical mechanics, as well as from the study of manifolds, measured metric spaces, and natural operators on them. The main aim of this issue is to provide an overview of recent developments of singular perturbation theory in these areas and possible synergies between ideas and methods developed there.

The present issue consists of three contributions. The first one by Vieri Benci and Lorenzo Luperi Baglini is titled "Generalized functions beyond distributions." It introduces a modified notion of ultra functions defined on a non-Archimedean field which extends the real line. This leads to the construction of spaces of ultrafunctions having good local properties and having delta bases which are almost orthogonal in a natural sense. Modified ultrafunctions possess several other good properties, e. g., derivatives act on them, and (standard) distributions are associated to them. These properties make the space of modified ultrafunctions very suitable for interesting applications to the theory of singular partial differential equations. The second contribution, by Dmitri Finkelshtein, Yuri Kondratiev and Oleksandr Kutoviy, is titled "Statistical dynamics of continuous systems: perturbative and approximative approaches." It discusses statistical dynamical systems with Markov evolution via an infinite dimensional Fokker–Planck equation in a space of (locally finite) configurations over the d-dimensional Euclidean space. The evolution equation is then reformulated as a hierarchical chain of equations for corresponding correlation functions. The latter is handled by powerful analytic methods (in particular of semigroup theory). The important case of continuous systems performing birth and death

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stochastic processes is treated in details. In the last part of the paper, Glauber type dynamics in the continuum is discussed, as limit of approximations in a bounded volume. Applications include various interesting models of non equilibrium classical mechanics, as well as a central model of plant ecology. The third contribution, by Paolo Giordano and Enxin Wu, is titled "Categorical frameworks for generalized functions." In this paper, a categorical framework for generalized functions is presented. In fact it exhibits a category containing both Schwartz distributions and Colombeau generalized functions as natural objects. For its construction, three spaces which can be looked upon as generalizations of spaces of smooth manifolds are discussed. The largest space is Souriau's diffeological space and the smallest one is Frölicher space. Between the two lie new spaces in which the diffeological structure is determined by a given family of locally defined smooth functionals. They are shown to possess good properties like forming a complete and cocomplete Cartesian closed category, and they constitute suitable frameworks for important distribution spaces (like the ones mentioned above). Some open problems are discussed and some applications to singular problems are mentioned.