



# Hadron structures from the non-topological soliton

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Received: 29 August 2022 / Accepted: 23 September 2022 / Published online: 13 January 2023  
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## Abstract

We review a series of recent studies on the static properties of SU(3) heavy baryons, based on a non-topological soliton approach, the chiral quark-soliton model. A singly heavy baryon can be thought of as a system composed of  $N_c - 1$  valence quarks bound by the pion mean fields and a heavy quark in the limit of an infinitely heavy quark mass. Using all model parameters fixed in the light-baryon sector, the color factor  $N_c$  is replaced with  $N_c - 1$  to calculate the physical observables of singly heavy baryons. By comparing our results to the experimental data, we show that this pion mean-field approach describes very successfully the isospin mass differences and strong decay widths of the lowest-lying singly heavy baryons, charmed and beauty baryons.

**Keywords** Non-topological soliton · Charm baryons · Beauty baryons · Heavy baryons · Chiral quark-soliton model · Pion mean-field approach

## 1 Introduction

The typical models to describe baryons might be classified into two low-energy effective models today. One is nonrelativistic three-quark models with certain effective potentials between constituent quarks and the other is the chiral models in which bare quarks are surrounded by a cloud of effective meson field and the constituent quark masses are generated dynamically. Undoubtedly, three-quark model successfully described many excited and exotic hadron phenomena since the new baryon state  $\Omega^-$  was predicted by Gell-Mann [1, 2]. The prominent studies done in SU(3) quark model for low-lying hadron states, particularly multiquark states such as pentaquarks, tetraquarks, heptaquarks, and tribaryons can be found in Refs.[3–7]. On the other hand, in the various chiral models with basic features of the low-energy Quantum Chromodynamics (QCD) as the chiral symmetry and its spontaneous breakdown, the representative solitonic approaches are the Skyrme Model [8] and the Chiral Quark-Soliton Model ( $\chi$ QSM) [9–11]. The “soliton” is another expression for the self-consistent meson mean-field in the nucleon in which a nucleon can be viewed as a soliton of

the meson field in a large number of colors ( $N_c$ ) limit [12, 13]. There are many features in common between the two solitonic models, but there are also several main differences for theoretical basis. In the case of the Skyrme model the baryon number is given by the winding number for the topological solitons, namely skyrmions, and its lagrangian need to be supplemented by a Wess-Zumino anomaly term to ensure proper quantization [14–16]. In describing light and heavy baryons to incorporate strangeness or heavy flavors, the original Skyrme model was developed into a “bound-state” picture with a skyrmion to bind a strange or heavy meson carrying the appropriate quantum number [17–23].

The non-topological soliton of the  $\chi$ QSM is treated differently from a skyrmion. Instead of combining a soliton and a strange meson for flavor SU(3) baryons, the  $\chi$ QSM is embedding the flavor SU(2) soliton into the isospin subgroup of the flavor SU(3) by Witten’s suggestion [14]. In addition, the effective low-energy theory of the  $\chi$ QSM is derived from the QCD instanton vacuum which supplies a natural mechanism of the chiral symmetry breakdown [24, 25] and the baryon number of non-topological soliton is given by the quantization constraint from filling  $N_c$  valence quarks in the bound-state level [11]. For small-sized solitons the valence level joins the upper continuum and then the sea level disappears while for large-sized solitons the valence level sinks into the Dirac sea and the valence contribution vanishes. Since the small- and large-sized soliton limits of

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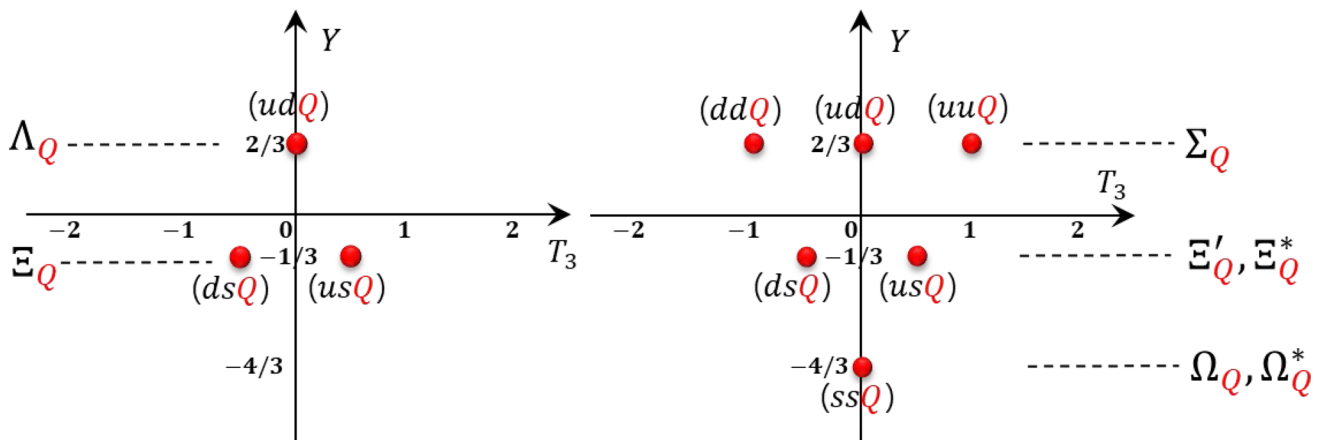
the  $\chi$ QSM coincide with important features and results of the constituent quark model and the Skyrme model, respectively [26], it turns out that the  $\chi$ QSM can play a role of a bridge between two models.

The  $\chi$ QSM has been describing successfully the properties of lowest-lying flavor SU(3) light baryons for octet, decuplet and higher group representations such as the mass spectra [27–31], the electromagnetic and axial-vector form factors [32–34], the magnetic moments [35–39], hyperon semileptonic decays and strong decay widths [40–43], parton distributions [44–47], transversities of the nucleon [48–50], generalized parton distributions [51], and so on. Recently, Not only the properties of flavor SU(3) baryons in free space but also those surrounded by the various baryonic environments were investigated within the framework of the in-medium modified chiral soliton model by the authors of Ref. [52]. In this modified pion mean-field approach, dynamical model parameters were expressed in terms of the density-dependent functionals by the linear-response approximation and were able to describe the equation of states for various baryonic environments. The results of nuclear binding energy, nuclear symmetry energy, and pressure were in very good agreement with the data extracted from various experiments and astronomical observations.

In recent years, one of the remarkable achievements for hadron structures is an extension of the  $\chi$ QSM into singly heavy baryons from low-lying flavor SU(3) light baryons. One can explain that singly heavy baryons consist of two light quarks and a heavy quark such as a charm or beauty quark in a naive quark model. When the mass of the heavy quark is regarded as  $m_Q \rightarrow \infty$ , there is no need to address the spin-flip of the heavy quark and its spin is conserved [53–55]. The total spin of two light quarks can then be a good quantum number by this heavy-quark spin symmetry.

Since a heavy quark is a common ingredient in heavy baryons, the flavor SU(3) representations of singly heavy baryons being made up of two light quarks and a heavy quark are  $\mathbf{3} \otimes \mathbf{3} = \mathbf{\bar{3}} \oplus \mathbf{6}$  as shown in Fig. 1. For the lowest-lying heavy baryons given by the representations  $\mathbf{3} \otimes \mathbf{3} = \mathbf{\bar{3}} \oplus \mathbf{6}$ , the authors of Ref. [56] investigated a system with the  $N_c - 1$  light quarks inducing the pion mean-field and a heavy quark as a static color source in the large  $N_c$  limit. In this limit,  $N_c - 1$  valence quarks generate the pion mean-field and the system can be characterized as a quark-soliton system. In the case of light baryons, the flavor SU(3) space of the effective Hamiltonian is constrained by the right hypercharge  $Y' = N_c/3$ , which chooses the lowest allowed representations:  $\mathbf{8}$ (octet) and  $\mathbf{10}$ (decuplet). Because of the presence of  $N_c - 1$  valence quarks in the singly heavy baryon case, the constraint is changed by  $Y' = (N_c - 1)/3$  and the lowest allowed representations are  $\mathbf{\bar{3}}$ (anti-triplet) and  $\mathbf{6}$ (sextet). Recent studies have been done within the general framework of this large  $N_c$  mean-field picture for the various physical observables of the lowest-lying singly heavy baryons such as the mass spectra with flavor SU(3) and isospin symmetry breakings [56–58], the magnetic moments and its radiative decays [59–61], quark spin content [62], and the decay widths of strong decays [63].

Interestingly, when we look into the experimental values of the isospin mass differences between antitriplet heavy baryons, the feature of charmed and beauty baryons differ significantly from each other. Although the light-quark component of  $\Delta M_{\frac{2}{3}}(\Xi_c) = \Xi_c^+(usc) - \Xi_c^0(dsc)$  is the same as  $\Delta M_{\frac{2}{3}}(\Xi_b) = \Xi_b^0(usb) - \Xi_b^-(dsb)$  since a heavy quark is the common ingredient for the isospin mass difference in a naive quark model, the experimental value of  $\Delta M_{\frac{2}{3}}(\Xi_b) \simeq -5.9$  MeV are nearly double that for the  $\Delta M_{\frac{2}{3}}(\Xi_c) \simeq -2.98$  MeV [56]. For the sextet case, such



**Fig. 1** The weight diagrams of anti-triplet ( $\bar{\mathbf{3}}$ ) (in the left panel) and sextet ( $\mathbf{6}$ ) (in the right panel) representations of the low-lying singly

heavy baryons with a heavy quark  $Q$  ( $c$  or  $b$ ) and two light quarks, up ( $u$ ) and down ( $d$ ).  $Y$  and  $T_3$  denote the hypercharge and isospin third component, respectively

differences are more noticeable. The experimental values of the isospin mass differences  $\Sigma_c^{*++}(ddc) - \Sigma_c^0(ucc)$  and  $\Sigma_c^{*++}(ddc) - \Sigma_c^{*0}(ucc)$  are  $0.220 \pm 0.013$  MeV and  $0.01 \pm 0.15$  MeV, while those of  $\Sigma_b^+(ddb) - \Sigma_b^-(uub)$  and  $\Sigma_b^{*+}(ddb) - \Sigma_b^{*-}(uub)$  are  $-5.06 \pm 0.18$  MeV and  $-4.37 \pm 0.33$  MeV, respectively. Additionally to the opposing signs, there are also huge differences in the magnitudes between those of charmed and beauty baryons. Therefore we will introduce the additional (Coulomb) interaction to a heavy quark-soliton system and compare our results with the experimental data. The decay widths of heavy baryons are another way to examine how successfully the pion mean-field approach with  $N_c - 1$  modification for the singly heavy baryons works because most of the lowest-lying singly heavy baryons decay into other singly heavy baryons and pseudoscalar mesons, according to well-known experimental data.

The primary discussion points in this paper will be the isospin mass differences and strong decay widths of the lowest-lying singly charmed and beauty baryons to demonstrate how well the  $N_c - 1$  pion mean-field framework is described by comparing with well-known experimental data. We sketch the present work as follows: In Sect. 2, we review the general formalism of the  $\chi$ QSM for singly heavy baryons and describe the isospin mass differences between singly heavy baryons based on the present mean-field approach. In Sect. 3, we discuss the strong decay widths of singly charmed and beauty baryons, based on the same mean-field approach. The final section is devoted to the discussion and conclusion.

## 2 Isospin mass differences of singly heavy baryons

As mentioned previously, the light and heavy baryons are treated on an equal footing within this approach when we consider the change of  $N_c$  to  $N_c - 1$  mean field for the singly heavy baryons. This approach has the remarkable advantage of investigating both light and heavy baryons within the same framework with essentially no free parameters. For example, it was shown in Ref. [28] that all the parameters can be determined from the octet baryons and be used to predict the isospin mass differences between decuplet baryons. In the same way, the values of model dynamical parameters for the flavor SU(3) mass splittings in singly heavy baryons were taken from those already determined in the light octet baryons shown in Ref. [56]. Thus, this well-established mean-field approach from the light to heavy baryon sector will be employed to describe the isospin mass differences of singly charmed and beauty baryons.

The effects of isospin symmetry breaking are attributed to two distinct sources: the electromagnetic (EM) interaction and the hadronic contributions between the masses of up and down

quarks. The EM self-energies to a baryon mass in the  $\chi$ QSM can be written as [28]

$$\begin{aligned} \mathcal{O}^{\text{EM}} &= \bar{q}(x) \gamma_\mu \hat{Q} q(x) \bar{q}(0) \gamma_\mu \hat{Q} q(0) \\ &= - \int \frac{d\omega}{2\pi} \text{tr} \langle x | \frac{1}{\omega + iH(U)} \gamma^\mu \lambda^a \frac{1}{\omega + iH(U)} \gamma^\mu \lambda^b | x \rangle \\ &\quad D_{Qa}^{(8)} D_{Qb}^{(8)}, \end{aligned} \quad (1)$$

where  $q(x)$  denotes the quark fields and the  $D_{Qa}^{(8)}$  are the SU(3) Wigner  $D$ -functions  $D_{(YTT_3)(Y'JJ_3)}^{(\mathcal{R})}$  with the quark charge operator  $\hat{Q}$  in the group representation  $\mathcal{R}$ . The part of trace with Hamiltonian  $H(U)$  and the flavor SU(3) Gell-Mann matrices  $\lambda_{a,b}$  are able to be expanded for the summation indices  $a$  and  $b$  by introducing the projection operators to separate the pure SU(2) part from the SU(3) one and the expression with the parametrized factor  $\alpha_i$  of  $D_{Qa}^{(8)} D_{Qb}^{(8)}$  can be given by

$$\begin{aligned} \mathcal{O}^{\text{EM}} &= \alpha_1 \sum_{i=1}^3 D_{Qi}^{(8)} D_{Qi}^{(8)} \\ &\quad + \alpha_2 \sum_{p=4}^7 D_{Qp}^{(8)} D_{Qp}^{(8)} + \alpha_3 D_{Q8}^{(8)} D_{Q8}^{(8)}. \end{aligned} \quad (2)$$

The expectation values of the EM mass are then obtained by sandwiching the EM self-energies operator in between the model baryon states [11]

$$\begin{aligned} |B\rangle &= \sqrt{\dim(\mathcal{R})} (-1)^{J_3+Y'/2} \\ &\quad D_{(Y,T,T_3)(-Y',J,-J_3)}^{(\mathcal{R})^*}(A). \end{aligned} \quad (3)$$

Here,  $\mathcal{R}$  stands for the allowed irreducible representations of the SU(3)<sub>f</sub> group, *i.e.*,  $\mathcal{R} = 8, 10, \bar{10}, \dots$  and  $Y, T, T_3$  are the corresponding hypercharge, isospin, and its third component, respectively. The right hypercharge  $Y' = 1$  is constrained to be unity for the physical spin states for which  $J$  and  $J_3$  are spin and its third component.

Since the parameterized factor  $\alpha_{1,2,3}$  for the EM masses are already adjusted to the empirical data of octet baryons in Ref. [28], we now turn to the hadronic mass correction from the isospin symmetry breaking by the quark mass difference between up and down quarks. As discussed in Ref. [58], the expression of the collective Hamiltonian with isospin symmetry breaking is

$$\begin{aligned} H_{\text{sb}}^{\text{iso}} &= (m_d - m_u) \\ &\quad \left( \frac{\sqrt{3}}{2} \alpha D_{38}^{(8)}(\mathcal{R}) + \beta \hat{T}_3 + \frac{1}{2} \gamma \sum_{i=1}^3 D_{3i}^{(8)}(\mathcal{R}) \hat{J}_i \right), \end{aligned} \quad (4)$$

here  $\alpha, \beta$ , and  $\gamma$  are model parameters adjusted by experimental data on the baryon octet masses. The  $m_u$  and  $m_d$

represent the up and down current quark masses, respectively. As stated in the Introduction, the presence of  $N_c - 1$  valence quarks instead of  $N_c$  valence quarks alters the pion mean-fields for the singly heavy baryons. While parameters  $\beta$  and  $\gamma$  remain unchanged in the modified mean field, the  $\alpha$  with the order of  $N_c$  should be modified. We use  $\bar{\alpha}$  to distinguish from the original parameter  $\alpha$  which is already determined from the light baryons [29, 56].

The additional contributions to the EM mass for isospin mass differences are the strong hyperfine interaction for the light quarks inside a heavy baryon and the Coulomb interaction between the soliton and a heavy quark. The strong hyperfine interactions between light quarks should be taken into account because the configuration of the light-quark spin will definitely change the mass of each baryon in the baryon antitriplet and sextet [65, 66].

$$H_{\text{hf}} = \delta^{\text{hf}} \mathbf{S}_1 \cdot \mathbf{S}_2, \tag{5}$$

where  $\mathbf{S}_1$  and  $\mathbf{S}_2$  represent the spin operators for the light quarks inside a soliton, which yield the soliton spin  $\mathbf{J}_{\text{sol}} = \mathbf{S}_1 + \mathbf{S}_2$ . The parameter  $\delta^{\text{hf}}$  contains the masses of the up and down quarks, and the strength of the strong hyperfine interaction. Nonetheless, we shall just fit it to the forthcoming experimental data [58]. The isospin symmetry breaking for the masses of the singly heavy baryons should include the EM interactions between the soliton and a heavy quark. While the magnetic contributions are suppressed by the heavy quark mass [67], the Coulomb interactions should be considered because the significant difference in the quark configuration of charmed and beauty baryons is from the electric charges of heavy quarks in the heavy quark symmetry.

$$H_{\text{sol-h}}^{\text{Coul}} = \alpha_{\text{sol-h}} \hat{Q}_{\text{sol}} \hat{Q}_h, \tag{6}$$

where  $\hat{Q}_{\text{sol}}$  and  $\hat{Q}_h$  are the charge operators acting on the soliton and a heavy quark, respectively. The parameter  $\alpha_{\text{sol-h}}$  consists of the expectation value of the inverse distance and the fine structure constant. However, the numerical values of the parameters  $\delta^{\text{hf}}$  and  $\alpha_{\text{sol-h}}$  can be adjusted by the two experimental data marked “input” as listed in Tab. 1. The numerical results of the charmed and beauty baryons are summarized in Table 1 and Figure 2. All isospin mass differences of singly heavy baryons are predicted using only two model parameters, which are fixed by two experimental values and the results of this work agree very well with all experimental data.

### 3 Strong decay widths of the singly heavy baryons

In this section, we briefly introduce recent work on the strong decays of the singly heavy baryons [63]. To calculate the strong decay widths of the singly heavy baryons, one has to sandwich the corresponding decay operator between the model baryon states expressed in Eq. (3). The decay operator with possible rescaling of the coefficients  $G_i$  is given as [68]

$$\hat{G}_\varphi^{(0)} = G_0 D_{\varphi^3}^{(8)} - G_1 d_{3bc} D_{\varphi b}^{(8)} \hat{J}_c - \frac{G_2}{\sqrt{3}} D_{\varphi^8}^{(8)} \hat{J}_3. \tag{7}$$

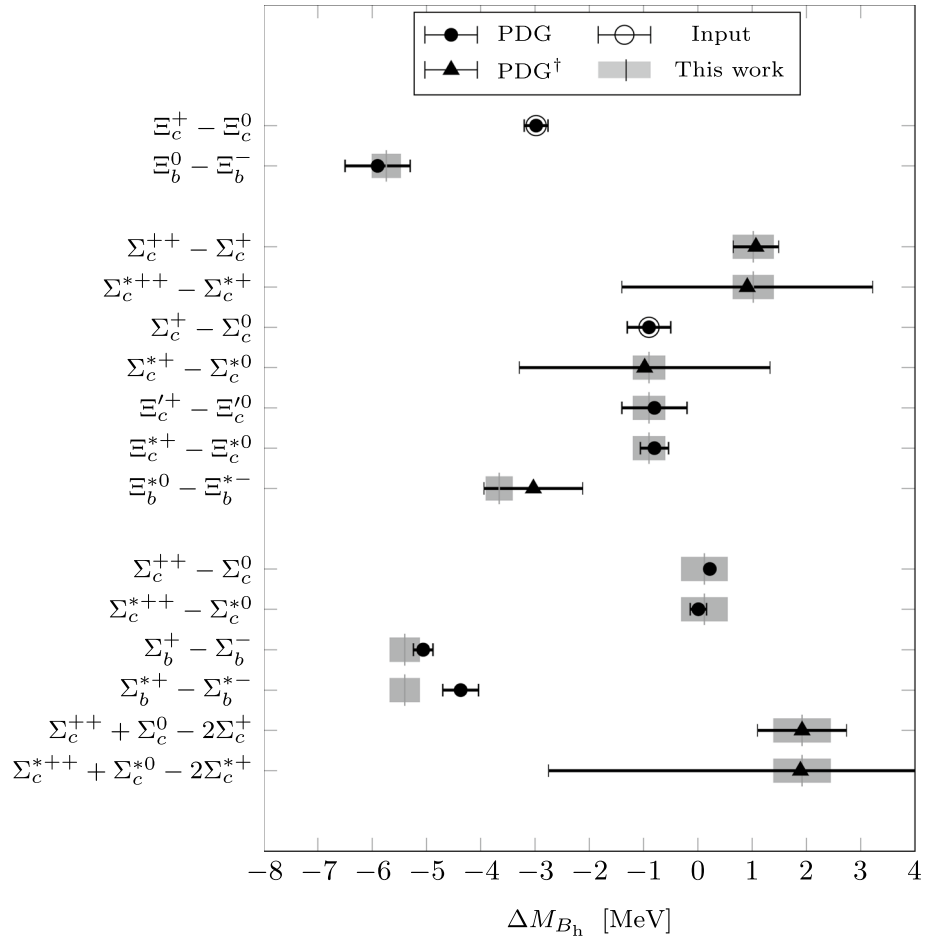
The nonrelativistic formula of the decay widths for  $B \rightarrow B' + \varphi$  can be written as [68, 69]

**Table 1** Isospin mass differences of the charmed and beauty baryons, antitriplet and sextet in units of MeV [58]

$\mathcal{R}_J$	charmed baryons	$\Delta M^{\text{total}}$	PDG [56]	PDG <sup>†</sup>	beauty baryons	$\Delta M^{\text{total}}$	PDG [56]	PDG <sup>†</sup>
$\bar{3}_{1/2}$	$\Xi_c^+ - \Xi_c^0$	<b>input</b>	$-2.98 \pm 0.22$	–	$\Xi_b^0 - \Xi_b^-$	$-5.74 \pm 0.27$	$-5.9 \pm 0.6$	–
	$\Sigma_c^{++} - \Sigma_c^+$	$1.02 \pm 0.38$	–	$1.07 \pm 0.42$	$\Sigma_b^+ - \Sigma_b^0$	$-1.74 \pm 0.34$	–	–
$6_{1/2}$	$\Sigma_c^+ - \Sigma_c^0$	<b>input</b>	$-0.9 \pm 0.4$	–	$\Sigma_b^0 - \Sigma_b^-$	$-3.66 \pm 0.25$	–	–
	$\Xi_c'^+ - \Xi_c'^0$	$-0.90 \pm 0.30$	$-0.8 \pm 0.6$	–	$\Xi_b'^0 - \Xi_b'^-$	$-3.66 \pm 0.25$	–	–
	$\Sigma_c^{++} - \Sigma_c^0$	$0.12 \pm 0.43$	$0.220 \pm 0.013$	–	$\Sigma_b^+ - \Sigma_b^-$	$-5.40 \pm 0.28$	$-5.06 \pm 0.18$	–
	$\Sigma_c^{++} + \Sigma_c^0 - 2\Sigma_c^+$	$1.92 \pm 0.53$	–	$1.92 \pm 0.82$	$\Sigma_b^+ + \Sigma_b^- - 2\Sigma_b^0$	$1.92 \pm 0.53$	–	–
	$\Sigma_c^{*++} - \Sigma_c^{*+}$	$1.02 \pm 0.38$	–	$0.91 \pm 2.31$	$\Sigma_b^{*+} - \Sigma_b^{*0}$	$-1.74 \pm 0.34$	–	–
$6_{3/2}$	$\Sigma_c^{*+} - \Sigma_c^{*0}$	$-0.90 \pm 0.30$	–	$-0.98 \pm 2.31$	$\Sigma_b^{*0} - \Sigma_b^{*-}$	$-3.66 \pm 0.25$	–	–
	$\Xi_c^{*+} - \Xi_c^{*0}$	$-0.90 \pm 0.30$	$-0.80 \pm 0.26$	–	$\Xi_b^{*0} - \Xi_b^{*-}$	$-3.66 \pm 0.25$	–	$-3.03 \pm 0.91$
	$\Sigma_c^{*++} - \Sigma_c^{*0}$	$0.12 \pm 0.43$	$0.01 \pm 0.15$	–	$\Sigma_b^{*+} - \Sigma_b^{*-}$	$-5.40 \pm 0.28$	$-4.37 \pm 0.33$	–
	$\Sigma_c^{*++} + \Sigma_c^{*0} - 2\Sigma_c^{*+}$	$1.92 \pm 0.53$	–	$1.89 \pm 4.64$	$\Sigma_b^{*+} + \Sigma_b^{*-} - 2\Sigma_b^{*0}$	$1.92 \pm 0.53$	–	–

$\Delta M^{\text{total}}$  shows the total theoretical values from the contributions of EM self-energies,  $H_{\text{sb}}^{\text{iso}}$  and  $H_{\text{hf}}$  of a soliton, and the Coulomb interaction  $H_{\text{sol-h}}^{\text{Coul}}$  between the soliton and a heavy quark. The corresponding experimental data on the isospin mass differences taken from the Particle Data Group [56] are listed as PDG. We list the derived values of the isospin mass differences as PDG<sup>†</sup>, using the experimental data on the masses of the corresponding singly heavy baryons

**Fig. 2** Comparison of the present results with the corresponding experimental data [58]. The x-axis denotes the values of the isospin mass differences between singly heavy baryons in units of MeV whereas the y-axis designates the corresponding mass difference of the isospin multiplet. The filled circles stand for the experimental data taken from the PDG [56], the filled triangles represent the data obtained by using the experimental values of the masses of the corresponding heavy baryons [56], and the open circles designate the data taken to be as input. The shaded rectangles represent the present results



**Table 2** Charm sextet baryons decay widths in MeV [63]

#	Decay of $B(\mathcal{R}, J)$	This work	Exp.
1	$\Sigma_c^{++}(\mathbf{6}, 1/2) \rightarrow \Lambda_c^+(\bar{\mathbf{3}}_0, 1/2) + \pi^+$	1.93	$1.89^{+0.09}_{-0.18}$
2	$\Sigma_c^+(\mathbf{6}, 1/2) \rightarrow \Lambda_c^+(\bar{\mathbf{3}}_0, 1/2) + \pi^0$	2.24	$2.3 \pm 0.4$
3	$\Sigma_c^0(\mathbf{6}, 1/2) \rightarrow \Lambda_c^+(\bar{\mathbf{3}}_0, 1/2) + \pi^-$	1.90	$1.83^{+0.11}_{-0.19}$
4	$\Sigma_c^{++}(\mathbf{6}, 3/2) \rightarrow \Lambda_c^+(\bar{\mathbf{3}}_0, 1/2) + \pi^+$	14.47	$14.78^{+0.30}_{-0.19}$
5	$\Sigma_c^+(\mathbf{6}, 3/2) \rightarrow \Lambda_c^+(\bar{\mathbf{3}}_0, 1/2) + \pi^0$	15.02	$17.2^{+4.0}_{-2.2}$
6	$\Sigma_c^0(\mathbf{6}, 3/2) \rightarrow \Lambda_c^+(\bar{\mathbf{3}}_0, 1/2) + \pi^-$	14.49	$15.3^{+0.4}_{-0.5}$
7	$\Xi_c^+(\mathbf{6}, 3/2) \rightarrow \Xi_c(\bar{\mathbf{3}}_0, 1/2) + \pi$	2.35	$2.14 \pm 0.19$
8	$\Xi_c^0(\mathbf{6}, 3/2) \rightarrow \Xi_c(\bar{\mathbf{3}}_0, 1/2) + \pi$	2.53	$2.35 \pm 0.22$

Experimental data are taken from Particle Data Group [56]

$$\Gamma_{B \rightarrow B'+\varphi} = \frac{3^2}{2\pi} \frac{p_\varphi^3}{(M + M')^2} \overline{\mathcal{A}^2}, \quad (8)$$

where  $M$  and  $M'$  are the masses of  $B$  and  $B'$ , respectively, and  $p_\varphi$  denotes the c.m. momentum of the outgoing meson  $\varphi$ . The  $\mathcal{A}$  designates the matrix element for the decay operator in Eq.(7) with the model baryon states in Eq. (3). The

**Table 3** Beauty sextet baryons decay widths in MeV [63]

#	Decay of $B(\mathcal{R}, J)$	This work	Exp.
1	$\Sigma_b^+(\mathbf{6}, 1/2) \rightarrow \Lambda_b^0(\bar{\mathbf{3}}_0, 1/2) + \pi^+$	6.12	$9.7^{+4.0}_{-3.0}$
2	$\Sigma_b^0(\mathbf{6}, 1/2) \rightarrow \Lambda_b^0(\bar{\mathbf{3}}_0, 1/2) + \pi^0$	6.12	$4.9^{+3.3}_{-2.4}$
3	$\Sigma_b^-(\mathbf{6}, 1/2) \rightarrow \Lambda_b^0(\bar{\mathbf{3}}_0, 1/2) + \pi^-$	0.07	$< 0.08$
4	$\Xi_b^+(\mathbf{6}, 1/2) \rightarrow \Xi_c(\bar{\mathbf{3}}_0, 1/2) + \pi$	0.07	$< 0.08$
4	$\Sigma_b^+(\mathbf{6}, 3/2) \rightarrow \Lambda_b^0(\bar{\mathbf{3}}_0, 1/2) + \pi^+$	10.96	$11.5 \pm 2.8$
5	$\Sigma_b^0(\mathbf{6}, 3/2) \rightarrow \Lambda_b^0(\bar{\mathbf{3}}_0, 1/2) + \pi^0$	11.77	$7.5 \pm 2.3$
6	$\Sigma_b^-(\mathbf{6}, 3/2) \rightarrow \Lambda_b^0(\bar{\mathbf{3}}_0, 1/2) + \pi^-$	11.77	$7.5 \pm 2.3$
6	$\Xi_b^0(\mathbf{6}, 3/2) \rightarrow \Xi_b(\bar{\mathbf{3}}_0, 1/2) + \pi$	0.80	$0.90 \pm 0.18$
7	$\Xi_b^-(\mathbf{6}, 3/2) \rightarrow \Xi_b(\bar{\mathbf{3}}_0, 1/2) + \pi$	1.28	$1.65 \pm 0.33$

Experimental data are taken from Particle Data Group [56]

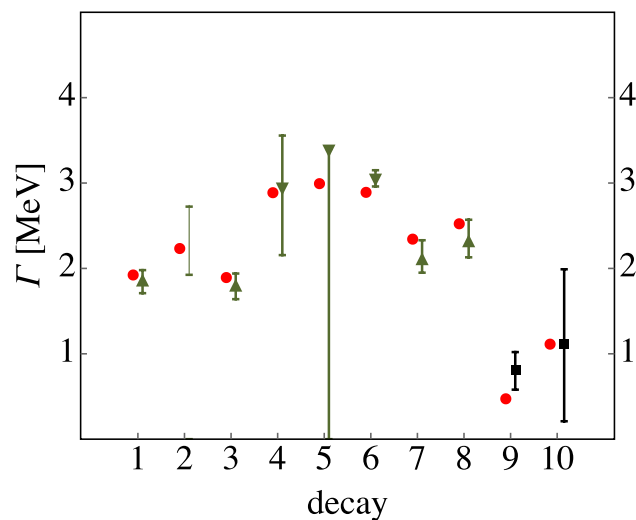
$\overline{\mathcal{A}^2}$  represents the average of  $\mathcal{A}^2$  over the initial and sum over the final spin and isospin states. The coefficients  $G_i$  in the  $\chi$ QSM consist of the inertia parameters of a soliton [70–72] and those should be modified in the singly heavy baryons since all inertia parameters scale as  $N_c$ . In the case of heavy baryons, all inertia parameters should be rescaled by approximately  $(N_c - 1)/N_c$  because the relevant model parameters for the moment of inertia of a soliton are already

determined from the light baryons [29]. Then one can obtain the decay widths of  $B \rightarrow B' + \varphi$  for the singly charmed and beauty baryons in a straightforward manner and immediately compare the results with the experimental data because most decay channels of heavy baryons are the strong decays.

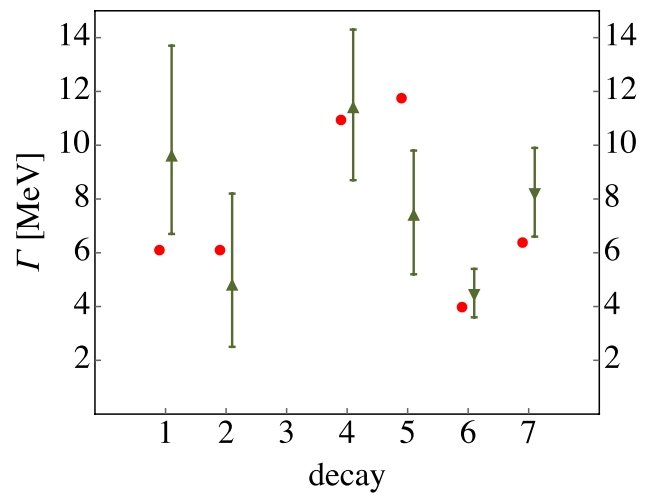
We see the remarkably good agreement of the results from the  $\chi$ QSM with the experimental widths for both charm and beauty baryons (Figs. 3 and 4). Particularly, the experimental value of decay #2,  $\Sigma_c^+(6, 1/2) \rightarrow \Lambda_c^+(\mathbf{3}_0, 1/2) + \pi^0$  was given as the upper limit 4.6 MeV [73] when the theoretical value 2.24 MeV was predicted several years ago. However, it turns out that the experimental value is now  $2.3 \pm 0.4$  MeV [56].

## 4 Discussion and conclusion

In the large  $N_c$  limits, the light and heavy baryons are treated on an almost equal footing within this approach when both light and heavy baryons are described by the pion mean-field approach in the  $\chi$ QSM, namely nontopological soliton picture. Only the change of  $N_c$  to  $N_c - 1$  mean field for the singly heavy baryons is considered since a heavy baryon system is regarded as that consisting of the  $N_c - 1$  light quarks inducing the pion mean-field and a heavy quark as a static color source. Thus the values of model dynamical parameters determined from the light baryon sector are taken as those for the singly heavy baryons with the change of  $N_c$  to  $N_c - 1$  factor and no free parameters. Given excellent agreement of the theoretical predictions for the isospin



**Fig. 3** Decay widths of the charm baryons [63]. Numbers on the horizontal axis label the decay modes as listed in Table 2. Red full circles correspond to our theoretical predictions. Dark green triangles correspond to the experimental data [56]. Data for decays 4–7 of  $\Sigma_c(6, 3/2)$  (down-triangles) have been divided by a factor of 5 to fit within the plot area



**Fig. 4** Decay widths of the bottom baryons [63]. Numbers on the horizontal axis label the decay modes as listed in Table 3. Red full circles correspond to our theoretical predictions. Dark green triangles correspond to the experimental data [56]. Data for decays 6 and 7 of  $\Xi_b(6, 3/2)$  (down-triangles) have been multiplied by a factor of 5 to be better visible on the plot

mass differences and decay widths of singly heavy baryons with the experimentally measured values, the present works have clear physical implications. The pion mean fields play a crucial role in explaining not only the static properties of light baryons but also those of the heavy baryons because light valence quarks govern their structure even in the singly heavy baryons.

**Acknowledgements** I would like to thank H.-Ch. Kim, M. V. Polyakov, M. Praszalowicz for very fruitful collaborations and discussions over the last two decades. I want to express the gratitude to the editors of the Journal of the Korean Physical Society (JKPS) for giving me an opportunity to join the very special 50th anniversary celebration of Division of Nuclear Physics, Korea Physical Society. This research was supported by the Academic Research Fund of Hoseo University in 2021(20210435).

## References

1. Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, (1962)(CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 805; M. Gell-Mann, The Eightfold Way: A Theory of strong interaction symmetry
2. V.E. Barnes, P.L. Connolly, D.J. Crennell, B.B. Culwick, W.C. Delaney, W.B. Fowler, P.E. Hagerty, E.L. Hart, N. Horwitz, P.V.C. Hough et al., Phys. Rev. Lett. **12**, 204–206 (1964)
3. Ys. Oh, Hc. Kim, Phys. Rev. D **70**, 094022 (2004)
4. H. Kim, M.K. Cheoun, Y. Oh, Phys. Rev. D **91**(1), 014021 (2015)
5. H. Kim, K.S. Kim, M. Oka, Phys. Rev. D **102**(7), 074023 (2020)
6. A. Park, W. Park, S.H. Lee, Phys. Rev. D **96**(3), 034029 (2017)
7. A. Park, S.H. Lee, Phys. Rev. D **102**(9), 096024 (2020)
8. G.S. Adkins, C.R. Nappi, E. Witten, Nucl. Phys. B **228**, 552 (1983)

9. D. Diakonov, V.Y. Petrov, P.V. Pobylitsa, Nucl. Phys. B **306**, 809 (1988)
10. D. Diakonov, [arXiv:hep-ph/9802298](https://arxiv.org/abs/hep-ph/9802298) [hep-ph]
11. C.V. Christov, A. Blotz, HCh. Kim, P. Pobylitsa, T. Watabe, T. Meissner, E. Ruiz Arriola, K. Goeke, Prog. Part. Nucl. Phys **37**, 91–191 (1996)
12. W. Pauli, S.M. Dancoff, Phys. Rev. **62**(3–4), 85 (1942)
13. T.H.R. Skyrme, Proc. Roy. Soc. Lond. A **260**, 127–138 (1961)
14. E. Witten, Nucl. Phys. B **223**, 422–432 (1983)
15. E. Witten, Nucl. Phys. B **223**, 433–444 (1983)
16. E. Guadagnini, Nucl. Phys. B **236**, 35–47 (1984)
17. C.G. Callan Jr., I.R. Klebanov, Nucl. Phys. B **262**, 365–382 (1985)
18. U. Magnea, K. Dannbom, D.O. Riska, Nucl. Phys. A **493**, 384–396 (1989)
19. Y. Oh, Phys. Rev. D **75**, 074002 (2007)
20. Ys. Oh, B.Y. Park, D.P. Min, Phys. Rev. D **50**, 3350–3367 (1994)
21. Ys. Oh, D.P. Min, M. Rho, N.N. Scoccola, Nucl. Phys. A **534**, 493–512 (1991)
22. Y.L. Ma, Y. Oh, G.S. Yang, M. Harada, H.K. Lee, B.Y. Park, M. Rho, Phys. Rev. D **86**, 074025 (2012)
23. Y.L. Ma, G.S. Yang, Y. Oh, M. Harada, Phys. Rev. D **87**(3), 034023 (2013)
24. D. Diakonov, V.Y. Petrov, Nucl. Phys. B **245**, 259–292 (1984)
25. D. Diakonov, V.Y. Petrov, Nucl. Phys. B **272**, 457–489 (1986)
26. M. Praszalowicz, A. Blotz, K. Goeke, Phys. Lett. B **354**, 415–422 (1995)
27. A. Blotz, D. Diakonov, K. Goeke, N.W. Park, V. Petrov, P.V. Pobylitsa, Nucl. Phys. A **555**, 765–792 (1993)
28. G.S. Yang, H. Ch, Kim and M. V. Polyakov, Phys. Lett. B **695**, 214–218 (2011)
29. G.S. Yang, HCh. Kim, Prog. Theor. Phys. **128**, 397–413 (2012)
30. G.S. Yang, HCh. Kim, J. Korean Phys. Soc. **61**, 1956–1964 (2012)
31. G.S. Yang, Y. Oh, HCh. Kim, New Phys. Sae Mulli **62**(3), 243–250 (2012)
32. HCh. Kim, A. Blotz, M.V. Polyakov, K. Goeke, Phys. Rev. D **53**, 4013–4029 (1996)
33. T. Affolder et al., CDF. Phys. Rev. D **64**, 012001 (2001). ([**erratum**: Phys. Rev. D **65**, 039902 (2002)])
34. T. Ledwig, A. Silva, HCh. Kim, Phys. Rev. D **82**, 034022 (2010)
35. M. Wakamatsu, N. Kaya, Prog. Theor. Phys. **95**, 767–778 (1996)
36. HCh. Kim, M. Praszalowicz, K. Goeke, Phys. Rev. D **57**, 2859–2870 (1998)
37. G.S. Yang, HCh. Kim, M. Praszalowicz, K. Goeke, Phys. Rev. D **70**, 114002 (2004)
38. K. Goeke, HCh. Kim, M. Praszalowicz, G.S. Yang, Prog. Part. Nucl. Phys. **55**, 350–373 (2005)
39. HCh. Kim, M. Polyakov, M. Praszalowicz, G.S. Yang, K. Goeke, Phys. Rev. D **71**, 094023 (2005)
40. G.S. Yang, H. Ch, Kim and K. Goeke, Phys. Rev. D **75**, 094004 (2007)
41. T. Ledwig, A. Silva, H. Ch, Kim and K. Goeke, JHEP **07**, 132 (2008)
42. G.S. Yang, H.Ch. Kim, PTEP **2013**, 013D01 (2013)
43. G.S. Yang, HCh. Kim, Phys. Rev. C **92**, 035206 (2015)
44. D. Diakonov, V. Petrov, P. Pobylitsa, M.V. Polyakov, C. Weiss, Nucl. Phys. B **480**, 341–380 (1996)
45. M. Wakamatsu, Phys. Rev. D **67**, 034005 (2003)
46. H.D. Son, A. Tandogan, M.V. Polyakov, Phys. Lett. B **808**, 135665 (2020)
47. H.D. Son, [arXiv:2203.17169](https://arxiv.org/abs/2203.17169) [hep-ph]
48. HCh. Kim, M.V. Polyakov, K. Goeke, Phys. Rev. D **53**, 4715–4718 (1996)
49. HCh. Kim, M.V. Polyakov, K. Goeke, Phys. Lett. B **387**, 577–581 (1996)
50. P. Schweitzer, D. Urbano, M.V. Polyakov, C. Weiss, P.V. Pobylitsa, K. Goeke, Phys. Rev. D **64**, 034013 (2001)
51. K. Goeke, M.V. Polyakov, M. Vanderhaeghen, Prog. Part. Nucl. Phys. **47**, 401–515 (2001)
52. N.Y. Ghim, G.S. Yang, H. Ch, Kim and U. Yakhshiev, Phys. Rev. C **103**(6), 064306 (2021)
53. N. Isgur, M.B. Wise, Phys. Lett. B **232**, 113–117 (1989)
54. N. Isgur, M.B. Wise, Phys. Rev. Lett. **66**, 1130–1133 (1991)
55. H. Georgi, Phys. Lett. B **240**, 447–450 (1990)
56. G.S. Yang, HCh. Kim, M.V. Polyakov, M. Praszalowicz, Phys. Rev. D **94**, 071502 (2016)
57. J.Y. Kim, H. Ch, Kim and G. S. Yang, Phys. Rev. D **98**(5), 054004 (2018)
58. G.S. Yang, HCh. Kim, Phys. Lett. B **808**, 135619 (2020)
59. G.S. Yang, HCh. Kim, Phys. Lett. B **781**, 601–606 (2018)
60. G.S. Yang, HCh. Kim, Phys. Lett. B **801**, 135142 (2020)
61. J.Y. Kim, HCh. Kim, G.S. Yang, M. Oka, Phys. Rev. D **103**(7), 074025 (2021)
62. J.M. Suh, J.Y. Kim, G.S. Yang and H.Ch. Kim, [arXiv:2208.04447](https://arxiv.org/abs/2208.04447) [hep-ph]
63. HCh. Kim, M.V. Polyakov, M. Praszalowicz, G.S. Yang, Phys. Rev. D **96**, no.9, 094021 (2017)) [erratum: Phys. Rev. D **97**, no.3, 039901 (2018)]
64. R.L. Workman [Particle Data Group], PTEP **2022**, 083C01 (2022)
65. S. Capstick, Phys. Rev. D **36**, 2800 (1987)
66. M. Karliner, J.L. Rosner, Phys. Rev. D **100**(7), 073006 (2019)
67. M. Oka, Nucl. Phys. A **914**, 447–453 (2013)
68. D. Diakonov, V. Petrov, M.V. Polyakov, Z. Phys. A **359**, 305–314 (1997)
69. M. Praszalowicz, K. Goeke, Phys. Rev. D **76**, 096003 (2007)
70. A. Blotz, M. Praszalowicz, K. Goeke, Rotational corrections to axial currents in semibosonized SU(3) Nambu-Jona-Lasinio model. Phys. Lett. B **317**, 195–199 (1993). [https://doi.org/10.1016/0370-2693\(93\)91592-B](https://doi.org/10.1016/0370-2693(93)91592-B)
71. A. Blotz, M. Praszalowicz and K. Goeke, Axial properties of the nucleon with 1/N(c) corrections in the solitonic SU(3) NJL model. Phys. Rev. D **53**, 485–503 (1996). <https://doi.org/10.1103/PhysRevD.53.485>
72. M. Praszalowicz, T. Watabe, K. Goeke, Nucl. Phys. A **647**, 49–71 (1999)
73. C. Patrignani et al. [Particle Data Group], Chin. Phys. C **40**, no.10, 100001 (2016)

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