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ORIGINAL PAPER - PARTICLES AND NUCLEI

Shell model calculations of the nuclear structure in the "island of inversion" $N=20$ region: ^{31,33,35}Na isotopes

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Abstract
The unmixed intruder configurations (2p–2h) were studied for ^{31,33,35}Na isotopes inside the island of inversion within the SDPF-U Hamiltonian with restrictions. The proton's positions that give a good correlation energy were determined for the first time. The unmixed intruder configurations (2p–2h), in addition of the proton's positions, were found using the restriction technique. The proposed restricted configurations were applied to calculate the low-lying states and probabilities of the transitions $B\left(E2 : 0_1^+ \rightarrow 2_1^+\right)$ with effective charges $e_p=1.3$e and $e_n=0.537$e for the ^{31,33,35} Na isotopes. Moreover, the outcomes of this work compared reasonable well to the available experimental results. The good agreement between the theoretical and the experimental values, especially for the first three positive states 3/2, 5/2 and 7/2, confirmed the domination of unmixed (2p_2h) intruder configurations in the ground states for these nuclei. On the basis of this agreement, two positive states (9/2, 1/2) are suggested in this paper. Furthermore, the theoretical results showed the accuracy of SDPF-U Hamiltonian in calculations for this region ($N=20$).

Keywords Nuclear structure · Shell model · Island of inversion and Na isotopes

1 Introduction

As nuclear shell model provides a basic understanding of an atomic nucleus, it has been regarded as one of the most important frameworks with which to comprehend the structures and the properties of nuclei since the advent of research into atomic nuclei. One of the most significant aspects is the well-recognized "magic numbers", i.e., 2, 8, 20, 28, etc. [1, 2]. Many recent experimental and theoretical studies have suggested that the proton–neutron ratio affects these magic numbers. For instance, when the number of protons and neutrons increases, the well-known closed shell disappears and a new closed shell appears [3]. However, exotic nuclei especially neutron-rich nuclei do not conform to these rules. In these cases, the first unexpected and significant change observed is that the closed shell is far from stable in the ^{31,32}Na isotopes, a region christened the "island of inversion" [4, 5] and recognized as an "archipelago of islands of shell breaking" that are linked to the magic neutron numbers $N=8$, 14, 20, 28, and 40 [6].

The "island of inversion" region around $N=20$ contains Mg, Na, and Ne chains that are linked to the atomic mass number, $A=32$ [4]. The ^{31,33,35}Na isotopes, $Z=11$, and $N=20$, 22, and 24, are nuclei worthy of study given the large deformations in the ground states of these isotopes and intruder effects. For the sodium chain, the ^{31}Na isotope was the first nucleus to exhibit strange behavior far from the drip line in the $N=20$ region of the nuclear chart (Sect. 2.1.1). Pritychenko et al. [7] were the first to determine experimentally the first excited state. Doornenbal et al. [8] later added a third energy level, or the second excited state $7/2^+$, to the level scheme of the ^{31}Na isotope. The existence of the third energy level was later confirmed in their subsequent

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theoretical study [9]. However, Caurier et al. [10] were the first to attempt theoretically to calculate these energy levels by using large-scale shell model calculations. Doornenbal et al. [9] subsequently attempted a theoretical calculation using the SDPF-M interaction with in Monte Carlo shell model (MCSM). For the ^{33}Na isotope, Doornenbal et al. [8] were the first to investigate experimentally its energy levels. Their study suggested that the ground state could be $3/2^+$ with the first excited state being $5/2^+$ or $5/2^+$ with the first excited state being $3/2^+$. Later, Gade et al. [11] confirmed the spin and parity as $3/2^+$ for the ground state and $5/2^+$ for the first excited state they also added a third energy level, or second excited state. Finally, an experimental study by Doornenbal et al. [9] added a fourth energy level, or third excited state $7/2^+$, to the level scheme of the ^{33}Na isotope. As such, only one study has theoretically calculated the energy levels, spin, and parity of the ^{33}Na isotope by using the SDPF-M interaction with MCSM [9]. This same study was also the first and only study to experimentally and theoretically investigate the nuclear structure of the ^{35}Na isotope by calculating the energy, spin, and parity of its ground state, as well as its first and second excited states [9].

Shell model calculations rely on two types of effective interactions in the *sd-pf* model space: (1) the effective SDPF interaction [12], which uses the full *sd-pf* shells for the neutrons and (2) the MCSM of the effective interaction of the SDPF-M [9], which uses the full *sd*-shell with the $f_{7/2}$ and $p_{3/2}$ subshells of the *pf* model space.

For the effective SDPF interaction, the SDPF was reformulated to SDPF-NR [13] and SDPF-U [14], the latter of which contained two versions: (1) for $Z \leq 14$ and $N=20$ to 40 with a *(np-nh)* schematic pairing, i.e., $(n\hbar\omega)$ *sd-pf* calculations called SDPF-U, and (2) for $Z>14$, called SDPF-U-Si. As the Hamiltonian schematic pairing has been removed from the monopole interaction, the *(np-nh)* schematic pairing will be absent for nuclei with $Z=15$ to 20, i.e., $(0\ \hbar\omega)$ *sd-pf* calculations for SDPF-U-Si. These two interactions are suitable for describing the nuclear structure of nuclei with $Z=8$–20 and $N=20$–40 [14].

The SDPF-U interaction contained the following seven SPEs for the full *sdpf*-model space: (1) − 3.69900 MeV for $d_{5/2}$, (2) 1.89500 MeV for $s_{1/2}$, and (3) − 2.91500 MeV for $d_{3/2}$ for protons and neutrons in the *sd*-shell; (4) 6.22000 MeV for $f_{7/2}$, (5) 10.95000 MeV for $p_{3/2}$, (6) 6.31400 MeV for $f_{5/2}$, and (7) 6.47900 MeV for $p_{1/2}$ for neutrons in the *pf*-shell. Apart from SPEs, this interaction also contains 768 two-body matrix elements (TBME). Each element is linked to a set of quantum numbers, such as the possible angular momentum J and isospin $T=0$, 1 for two particles interaction in J_1, J_2, J_3, and J_4. The parts that formed the SDPF-U interaction were the USD interaction for protons and neutrons in the *sd*-shell, the Kahana-Lee–Scott interaction for monopole (proton-neutron) interactions, and the KB3 and KB3G interaction for neutrons in the *pf*-shell for $Z \leq 14$ and for $Z>14$, respectively. In the SDPF-U interaction, a pure *sd*-interaction was assumed for $N<20$ and a pure *pf*-interaction for $Z=20$ and $N=20$–40.

There were two situations for $Z=8$–20 and $N=20$–40: (1) for $Z>14$, the schematic Hamiltonian with the pairing interaction was removed from the monopole interaction, meaning that the *(np-nh)* configuration will not exist for $Z=15$–20 nuclei, i.e., $(0\ \hbar\omega)$ *sd-pf* calculations, and (2) that for $Z \leq 14$, the schematic Hamiltonian with pairing interaction will not be removed from the monopole interaction. The SDPF-U interaction assumes a mass dependence of $(18/A)^{1/3}$ for the matrix elements, where A is the nucleus' mass number. A more recent version of the SDPF-U is the SDPF-U-MIX effective interaction [15], a mix between $N=20$ and $N=28$ in some instances.

Many previous theoretical studies used mixed configurations in the full model space, namely (0p–0h), (1p–1h), (2p–2h), and (3p–3h) [14]. However, mixed configurations are rife with the following deficiencies for the SDPF-M and the SDPF-U-MIX interactions: (1) are fail to take into account the presence of intruder states could have an impact at high excitation energies and (2) are fail to take into account that the higher excitation energy of a negative party state might contribute to an over-prediction of the excitation energy for (2p–2h) configurations or an overestimate of the (0p–0h) and (2p–2h) mix, as in the SDPF-M Hamiltonian [16, 17].

Despite the good agreement of the first 0^+, 2^+, and 4^+ with the experimental results, which are very well described by most theoretical models, higher energies are still a challenge for these models. For instance, the second 2^+ state at 3 MeV differed by 0.442 MeV from the 2.558 MeV experimental result of the Caurier et al. [15] study on the SDPF-U-MIX effective interaction. This same study also failed to indicate the third 2^+ or the second 4^+ states, although other experimental studies have referred to these levels. Moreover, similar to the findings of Tripathi [18], the energy of the negative parity states of this study were higher than the experimental results. For ^{33}Mg isotopes, the energies of the positive states 1/2 (0.04 MeV) and 3/2 (0.12 MeV) vastly differed by 0.506 and 0.426 MeV, respectively, from the experimental 0.546 MeV reported by Neyens [19] for the first positive (1/2—7/2) state.

Furthermore, unlike Tripathi [18], Caurier et al. [15] mentions the (3p–3h) intruder configuration but fails to refer to the inversion between the normal (1p–1h) and the intruder (3p–3h) configurations. As such, the aforementioned deficiencies of the mixed technique have prompted researchers to adopt other techniques, such as the truncations of the model space suggested by Mare'chal et al. [20], which truncates the *sd-pf* model space to the *sd* model space and only the $f_{7/2}$ and $p_{3/2}$ shells of the *pf* model space to reduce the dimensionality of the calculations for ^{31}Mg and ^{31}Al. Other studies, such as those of Kimura [21], Kimura [22], and Momiyama et al. [23], used unmixed (0p–0h), (1p–1h), (2p–2h), and (3p–3h) configurations to describe the structure of the ^{31}Mg, ^{33}Mg and ^{35}Mg nuclei using antisymmetric molecular dynamics (AND) with a Gogny interaction.

Two phenomena may result in a reduction of the $N=20$ shell gap: (1) the nucleon–nucleon residual interaction, particularly a monopole proton-neutron interaction, known as a “spin-flip” or spin-isospin interaction and (2) the tensor (non-central) interaction, which is also responsible for shifting energies at a nuclear levels (subshells) [24–31]. Therefore, the two interactions affect the size of the $N=20$ gap depending on the number of protons in the subshell $d_{5/2}$. The first interaction is the attraction between subshells $d_{5/2}$ and $d_{3/2}$, and the second interaction is the repulsion between subshells $d_{5/2}$ and $f_{7/2}$. When the $d_{5/2}$ subshell is full or semi-full, the interactions are very strong; therefore, the shell gap will be wide, preventing the neutrons from moving across this gap. Conversely, when it is empty or semi-empty, the interactions are very weak, and the shell gap will be narrow, therefore, the neutrons will be able to cross this gap (Figs. 1 and 2).

A restricted configuration is a common technique that is often employed to solve island of inversion problems. Ibbotson et al. used restricted configurations of neutrons $(d_{5/2})^6$ $(d_{3/2},s_{1/2})^4$ $(f_{7/2}\,p_{3/2})^2$ for the ^{32}Mg nucleus for the (np-nh) configuration with $n=2$ [32] while Pritychenko et al. used restricted configurations of protons $(d_{5/2})^3$ $(d_{3/2},\, s_{1/2})^0$ and $(d_{5/2})^2$ $(d_{3/2},\, s_{1/2})^1$ for the ^{31}Na nucleus [7]. However, Siiskonen et al. used the restriction method for unmixed (0p–0 h) and (2p–2 h) configurations [33]. Similarly, Yordanov et al. established the ground state of the ^{31}Mg nucleus by using restrictions on the neutron space (sd-$f_{7/2}$, $p_{3/2}$) and the proton space $(d_{5/2})^2$ $(d_{3/2},\, S_{1/2})^2$ with mixing (0,1, and 2) $\hbar\omega$ [34]. However, according to the abovementioned new outputs (spin-flip and tensor interactions effects), none of these restrictions take into account the positions of the protons in unmixed (1p–1h), (2p–2h), and (3p-3h) configurations which would be critically important in producing a high correlation energy to reduce the energy of the $N=20$ shell gap.

Therefore, in this study, the SDPF-U interaction with restricted configurations was used with the OXBASH code [35] to investigate the nuclear structures of ^{31,33,36}Na isotopes. Restricted configurations were used to provide the best correlation energy required for the $N=20$ shell

Fig. 1 Effects of the spin-flip interaction

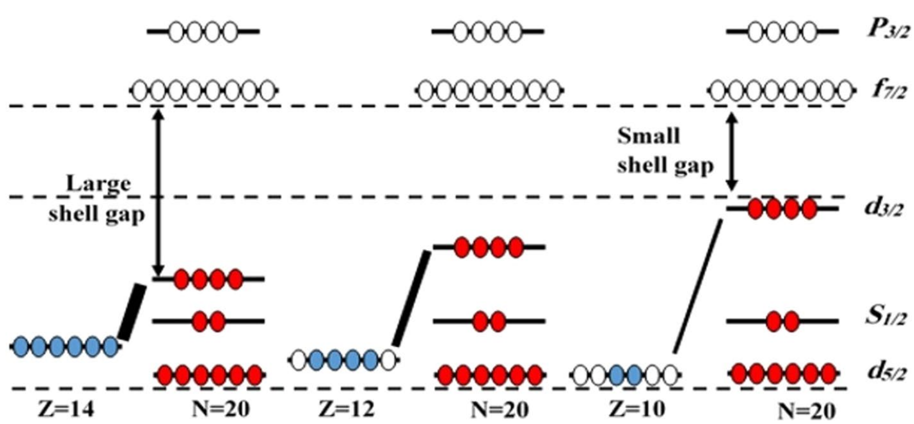
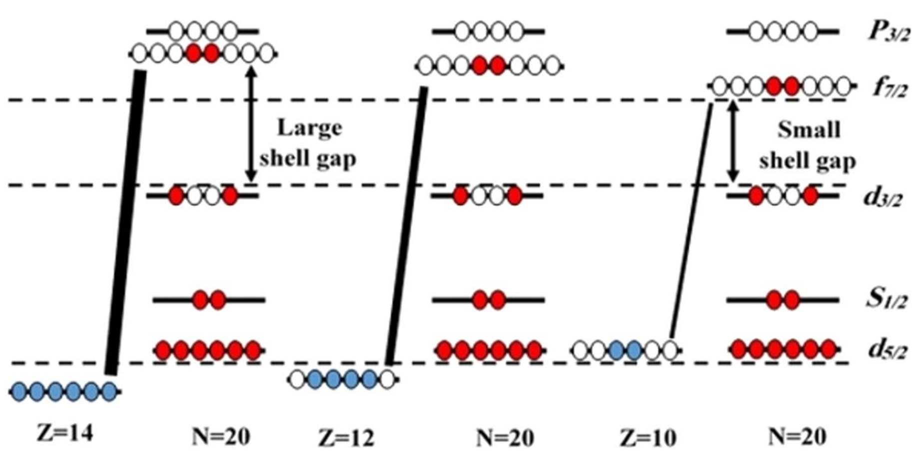


Fig. 2 Effects of the tensor interaction



gap to vanish i.e., the best correlation energy for the inversion phenomenon to occur between the two subshells; $f_{7/2}$ of the *pf*-shell and $d_{3/2}$ of the *sd*-shell. our study also endeavored to investigate the effects of unmixed (2p–2h) intruder configurations and demonstrate the precision of Hamiltonian predictions with this restriction.

2 Shell model calculations

The calculations in this study were based on restrictions to the configurations of nucleons in the model space. These restrictions were implemented to (1) reduce the attraction of the spin-flip interaction between the $\pi d_{5/2}$ and the $\pi d_{3/2}$ subshells and to reduce the repulsion tensor interaction between the $\pi d_{5/2}$ and the $\nu f_{3/2}$ subshells, (2) to reduce mixing between the (0p–0h) normal and the (2p–2h) intruder configurations in the ground and excited states as well as mixing between the (1p–1h) and the (3p–3h) configurations in the excited state only, and (3) to reduce the dimensionality of the calculations due to the difficulty of precisely diagonalizing the Hamiltonian in a wide model space [20].

As seen in Fig. 3, the normal configurations of ^{31,33,35}Na nuclei with $Z=11$ and $N=20$, 22, and 24 are based on the standard shell model. According to the rules of a normal configuration, three protons are meant to be in the $d_{5/2}$ sub-shell. This creates a strong attraction between the $\pi d_{5/2}$ and the $\nu d_{3/2}$ subshells as well as a strong repulsion in the tensor interaction between the $\pi d_{5/2}$ and the $\nu f_{3/2}$ subshells. Both these interactions, subsequently, create a large energy gap between the *sd* and *pf* shells for the neutrons. Under such circumstances, neutrons are not allowed to move up to the *pf*-shell and create (*np-nh*) intruder states. Therefore, the selects restricted configurations are used to provide the best correlation energy required for the $N=20$ shell gap to vanish, i.e., for the best correlation energy for the inversion phenomenon to occur between two subshells [24–31].

3 Odd–even Na isotopes

According to the standard shell model, which has been used for the magnesium chain, the ground state of the ^{31}Na nucleus has a closed ^{16}O core with 15 nucleons, namely three protons and 12 neutrons in the *sd*-shell. Meanwhile, the ground state of the ^{33}Na nucleus has a closed ^{16}O core with 17 nucleons, namely three protons and 12 neutrons in the *sd*-shell in addition to two neutrons in the *pf*-shell. The ground state of the ^{35}Na nucleus has a closed ^{16}O core with 19 nucleons, namely, three protons and 12 neutrons in the *sd*-shell in addition to four neutrons in *pf*-shell. However, as these nuclei are located in the "island of inversion" region, their ground states should have a closed core ^{16}O with three protons in *sd*-shell and $(sd)^{-2}$ $(pf)^{+2}$ neutrons, i.e., have the deformed shape instead of the spherical shape in the standard shell model. These nuclei also have odd atomic mass numbers; odd number of protons and even numbers of neutrons. Therefore, the spin of each level will be half-integer. The *(2p–2h)* intruder configuration is only proposed for use in calculating the energies of the positive parity levels for the sodium chain. This is due to the absence of negative parity, which, experimentally, did not motivates us to move odd neutrons from the *sd*-shell up to the *pf*-shell. The levels marked with " () " refer to the assignment and/or the parity of states that are not well-established through the experiment.

3.1 The case of the ^{31}Na isotope

Figure 4 illustrates the restricted configurations of the ^{31}Na isotope. Two protons in the $\pi d_{5/2}$ subshell and one proton in the $\pi d_{3/2}$ subshell is proposed for the even *(2p-2 h)* intruder configurations of neutrons; i.e., two neutrons in the *pf*-main shell and 10 neutrons in the *sd*-main shell.

As seen in Fig. 5, the ground state was $3/2^+$ which was similar to the findings of previous experimental and theoretical works. The energy of the first excited state $5/2^+$ was predicted to be 0.43 MeV. This was closer to the experimental results of 0.370 MeV reported in ref [12] than the theoretical predictions of 0.197, 0.280, and 0.245 MeV made

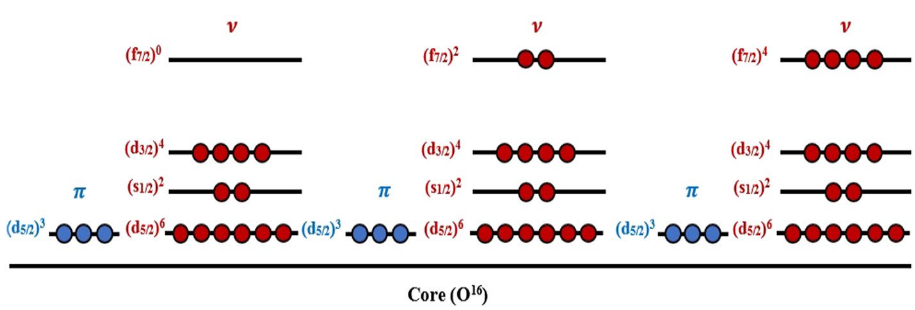


Fig. 3 Normal configurations of the ^{31,33,35}Na isotopes based on the standard shell model

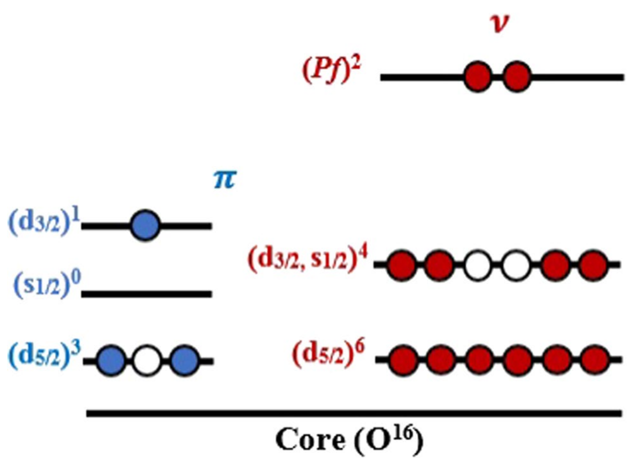


Fig. 4 Restricted configurations used in calculations for the ^{31}Na nucleus

by Pritychenko et al. [7], Caurier et al. [10], and Doornenbal et al. [9], respectively. The energy predicted for the second excited state $7/2^+$ (1.06 MeV) had much better agreement with the experimental results of 1.163 MeV [8] and 1.162 MeV [9] than the theoretical values 1.525 MeV and 1.407 MeV while it was identical with the predicted theoretical value of 1.06 MeV reported in ref [10].

At the time of writing, almost no studies provided the energy of the third excited state $9/2^+$. Only one theoretical study used the Monte Carlo Shell Model (MCSM) and put the energy of the excited state $9/2^+$ at 1.724 MeV [9], which is consistent with the 1.75 MeV of this present study. Similarly, only two theoretical studies put the energy of the last excited state $1/2^+$ at 2.28 MeV [10] and 2.305 MeV [9] while this present study predicted 2.50 MeV.

The reduced transition probabilities ($B(E2 : 3/2^+ \rightarrow 5/2^+)$) of the ^{31}Na isotope showed better agreement with experimental results than theoretical results. As seen in Table 1, the 257 $e^2\ fm^4$ transition probability calculated in this present study was compatible with the experimental results of Pritychenko et al. [7], i.e., 311^{170}_{133} with a 17% error. Nevertheless, it was still within the range of the experimental results; i.e., 481 $e^2\ fm^4$ and 178 $e^2\ fm^4$. The deformation parameter (β_2) calculated in this present study (0.43) was also more compatible with the experimental results of Pritychenko et al. [7], i.e., 0.47^{11}_{12} with an 8.5% error.

3.2 The case of the ^{33}Na isotope

As with the ^{31}Na nucleus, the *(2p-2 h)* configuration was used to calculate the energy levels of the ^{33}Na nucleus (Fig. 6). Similar to the ^{31}Na isotope, two protons in the $\pi d_{5/2}$ subshell and one proton in the $\pi d_{3/2}$ subshell was proposed for an even *(2p-2 h)* intruder configuration of neutrons, i.e., four neutrons in the *pf*-main shell and 10 neutrons in the *sd*-main shell.

Four levels, one ground state and three excited states, were discovered in the ^{33}Na nucleus. The $3/2^+$ predicted for the ground state was in agreement with the findings of previous experimental and theoretical studies. The 0.47 MeV

Table 1 Comparison of the $B(E2 : 3/2^+ \rightarrow 5/2^+)$ for the ^{31}Na isotope, in e^2 fm^4, and β_2 by using the SDPF-U Hamiltonian with restriction with the experimental results reported in refs. [7, 36] and theoretical values reported in Refs [7, 9, 10]

	Present study	EXP	Theoretical		
			Theo$_1$	Theo$_2$	Theo$_3$
B(E2)	257	311^{170}_{133}	216	196	199.7
β_2	0.43	0.47^{11}_{12}	0.39	0.37	0.38

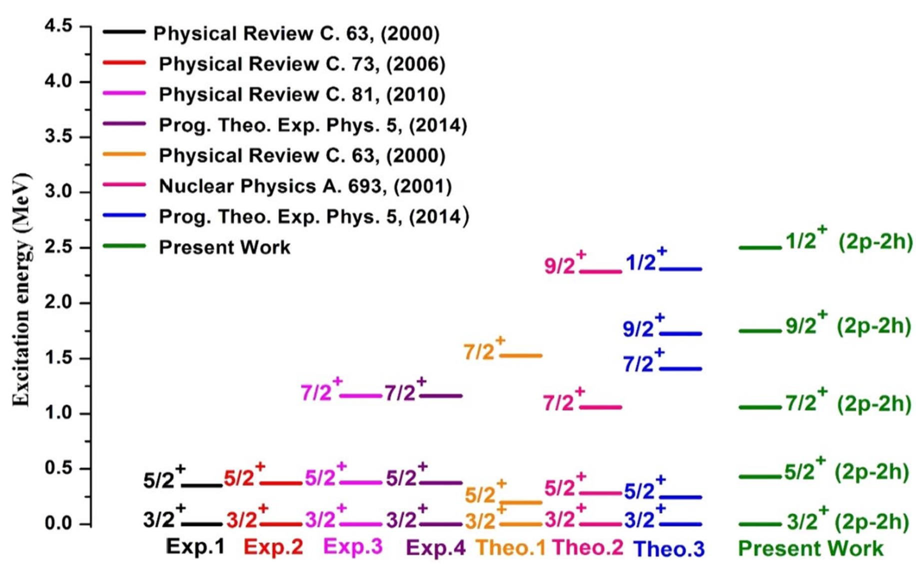


Fig. 5 Comparison of the experimental and theoretical energy states of the ^{31}Na nucleus using the SDPF-U Hamiltonian with restrictions

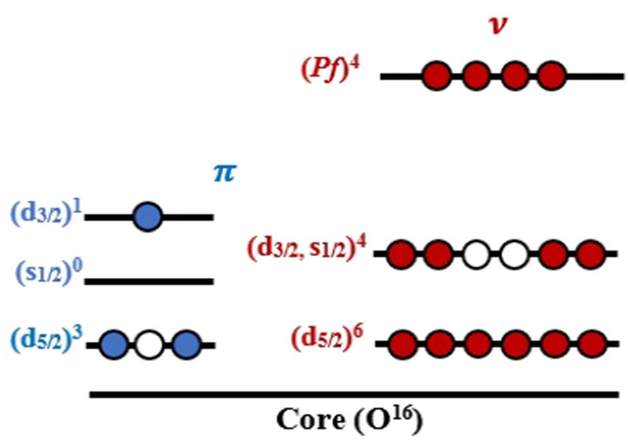


Fig. 6 Restricted configurations used in calculations for the ^{33}Na nucleus

predicted for the first excited state $5/2^+$ agreed better with the 0.467 MeV experimental findings reported in ref [13] than it did with the 0.390 MeV and 0.175 MeV predicted by Gade et al. [11] or the 0.303 MeV predicted by Doornenbal et al. [9]. The 1.08 MeV predicted for the second excited state $7/2^+$ was in very good agreement with the experimental values of 1.117 MeV by Gade et al. [11] and 1.115 MeV by Doornenbal et al. [9], with a 3% error. Theoretically, the 1.08 MeV predicted for the second excited state $7/2^+$ was more compatible with the 1.16 MeV predicted by Gade et al. [11] and the 1.121 MeV predicted by Doornenbal et al. [9], with a 3% error percentage, than it was with the 0.711 predicted by Gade et al. [11] for the same SDPF-U interaction with mixed configurations. The 1.839 MeV predicted for the third excited state $9/2^+$ was closer in value to the only available experimental value of 1.875 MeV than it was to the theoretical predictions of 1.640 MeV and 1.237 MeV by Gade et al. [11] and 1.561 MeV by Doornenbal et al. [9].

As for the last state $1/2^+$, existing experimental studies do not mention the spin. To date, only one theoretical study has mentioned the spin, and it predicted a value of 2.44 MeV [9]. This is consonant with the 2.30 MeV predicted by this present study. Figure 7 provides a clear comparison of the predictions of this study with the results of other theoretical and experimental studies.

At the time of writing, none of the experiment studies mention the $B(E2 : 3/2^+ \rightarrow 5/2^+)$ of the ground state of the ^{33}Na nucleus. Two theoretical studies, the only ones available predict it to be 263 e^2fm^4 [11] and 253.3 e^2fm^4 [9] with deformation parameters of 0.44 and 0.43, respectively (Table 2). The prediction of this present study was 313 e^2 fm^4.

3.3 The case of the ^{35}Na isotope

Only one experimental study mentions the three levels in the spectrum of the ^{35}Na nucleus, one ground state and two excited states. Figure 8 illustrates the distribution of particles that was used in this present study to perform calculations for the ^{35}Na nucleus. Six neutrons in the *pf*-main shell and ten neutrons in the *sd*-main shell in addition to two protons in the $\pi d_{5/2}$ subshell and one proton in $\pi d_{3/2}$ in the *sd*-main shell, was proposed.

Table 2 Comparison of the $B(E2 : 3/2^+ \rightarrow 5/2^+)$ for the ^{33}Na nucleus, in e^2 fm^4, and β_2 using SDPF-U Hamiltonian with restrictions to the theoretical values reported in Refs [9, 11]

	Present work	Theoretical	
		Theo$_1$	Theo$_2$
B(E2)	313	263	253.3
β_2	0.48	0.44	0.43

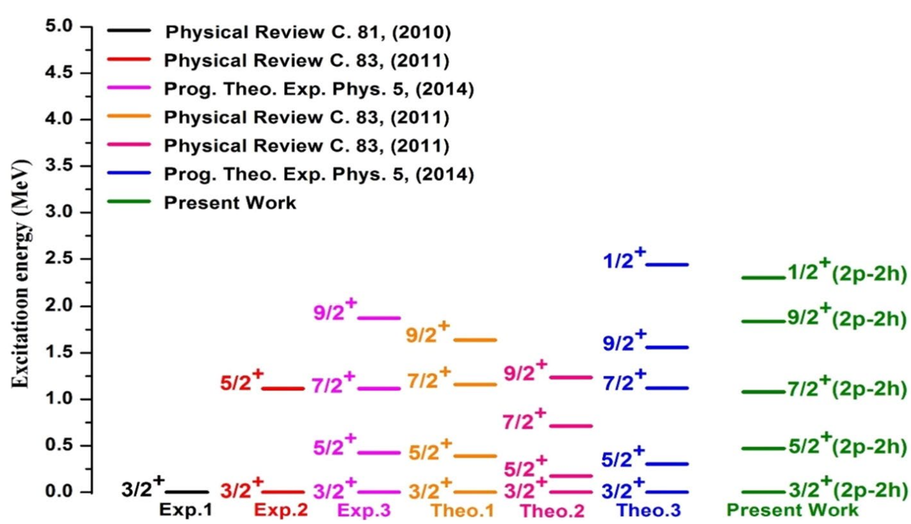


Fig. 7 Comparison of the experimental and theoretical energy states of the ^{33}Na nucleus using the SDPF-U Hamiltonian with restrictions

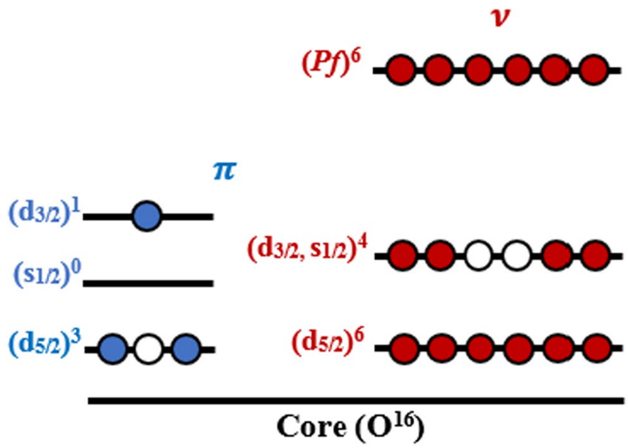


Fig. 8 Restricted configurations used in calculations for the ^{35}Na nucleus

As seen in Fig. 9, the 3/2$^+$ calculated for the ground state is compatible with the findings of previous experimental and theoretical studies. The 0.46 MeV calculated for the first excited state 5/2$^+$ was closer in value to the 0.373 MeV experimental value than it was at the 0.256 MeV and 0.117 MeV theoretical predictions. The 1.01 MeV calculated for the second excited state 7/2$^+$ was in excellent agreement with the 1.014 MeV experimental value, with a difference of only 0.004 MeV while the difference between the previous theoretical value 1.272 MeV reported in ref [9] with a difference 0.258 MeV. The last excited state 9/2$^+$ has been theoretically predicted to be 1.455 MeV. This present study calculated it to be 1.70 MeV. Similar to previous works on the ^{33}Na nucleus, no studies have mentioned the B(E2 : 3/2$^+$ → 5/2$^+$) of the ground state. Only one theoretical study puts its value at 239 e^{2}fm^4 while this present study calculated it to be 283 e^2 fm^4 (Table 3).

Experimental energy levels for the odd atomic mass number sodium isotopes ^{31,33,35}Na, together with the newly predicted levels in this work, are presented in Fig. 10. Remarkably, they show a small variation for the first three states in all sodium isotopes under study. Accordingly, one can consider the behavior of the fourth positive state 9/2 to be similar to the behavior of positive states 3/2, 5/2 and 7/2. Referring to the ratio $R_{4/2}$ which gives the amount of deformation in the ground state, it was 2.5 for the ^{31}Na isotope and 2.3 for ^{33,35}Na isotopes. These values are harmonious with the observed deformation in this area (island of inversion $N=20$).

4 Summary

In this study, the effects of intruder configurations were investigated using the SDPF-U Hamiltonian with a specific nuclei distribution in the island of inversion $N=20$ region.

Table 3 A comparison of the $B(E2 : 3/2^+ \rightarrow 5/2^+)$ of the ^{35}Na nucleus, in e^2 fm^4, and β_2 sing the SDPF-U Hamiltonian with restrictions to the theoretical values reported in Ref 9

	Present Work	Theoretical
B(E2)	283	239.6
β_2	0.47	0.40

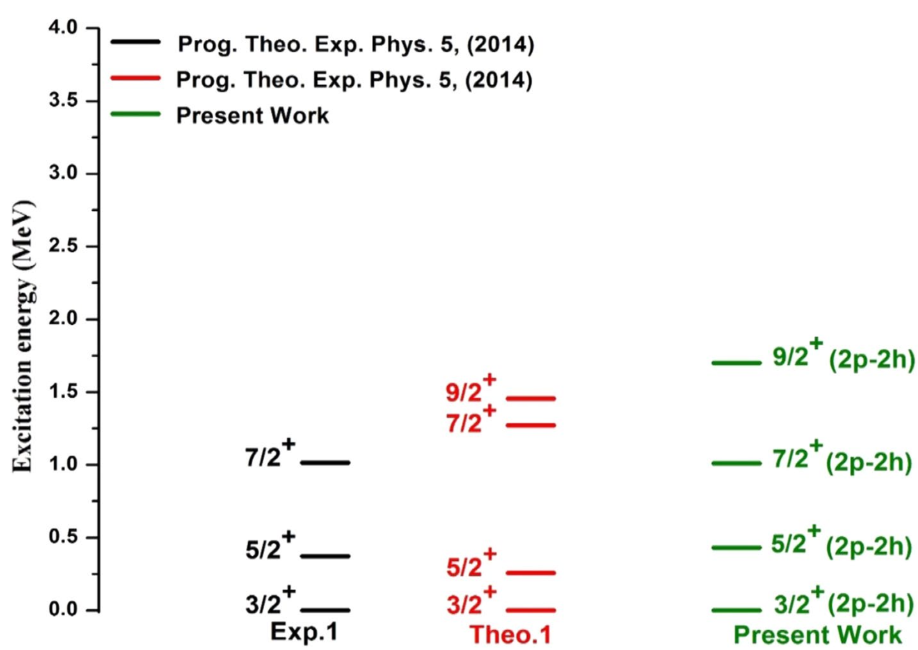


Fig. 9 Comparison of the experimental and theoretical energy states for the ^{35}Na nucleus using the SDPF-U Hamiltonian with restrictions

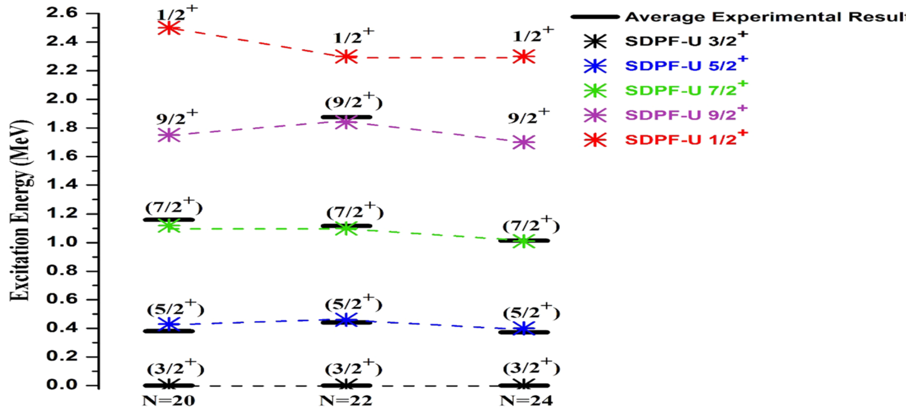


Fig. 10 Energy levels of the odd–even ^{31,33,35}Na isotopes compared to those found by using shell model calculations the SDPF-U Hamiltonian. Experimental data taken from Refs [7–9, 11, 37]

The study was based on calculations of the energy states and the reduced electric quadrupole transition probabilities of the Na chain isotopes to determine the best (protons and neutrons) configurations that provided good correlation with the energy necessary to reduce the $N=20$ shell gap. The results were then compared with recently available experimental data and theoretical predictions. The best agreement between the experimental and the theoretical results for positive ground states and excited states of $^{31.33,35}$Na isotopes was found when the proposed (2p–2h) neutron configurations were $(s_{1/2}, d_{3/2})^{-2}$ and $(f_{7/2}, p_{3/2})^{+2}$ and proton configurations were $(d_{5/2})^{-1}$ and $(s_{1/2})^{+1}$, i.e. two, four and six neutrons in pf-shell for ^{31}Na, ^{33}Na and ^{35}Na and two protons in the $\pi d_{5/2}$ subshell with one proton in the $\pi d_{3/2}$ subshell for all odd–even sodium isotopes considered in this study.

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