CASE STUDY



# Risk Assessment in Underground Coalmines Using Fuzzy Logic in the Presence of Uncertainty

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Received: 17 November 2017 / Accepted: 3 January 2018 / Published online: 30 January 2018 © The Institution of Engineers (India) 2018

Abstract Fatal accidents are occurring every year as regular events in Indian coal mining industry. To increase the safety conditions, it has become a prerequisite to performing a risk assessment of various operations in mines. However, due to uncertain accident data, it is hard to conduct a risk assessment in mines. The object of this study is to present a method to assess safety risks in underground coalmines. The assessment of safety risks is based on the fuzzy reasoning approach. Mamdani fuzzy logic model is developed in the fuzzy logic toolbox of MATLAB. A case study is used to demonstrate the applicability of the developed model. The summary of risk evaluation in case study mine indicated that mine fire has the highest risk level among all the hazard factors. This study could help the mine management to prepare safety measures based on the risk rankings obtained.

Keywords Safety - Mining - Mamdani - Risk ranking - Hazard

# Introduction

In India, from January 2006 to August 2016, there have been 819 fatal accidents in coalmines. The total number since 1901 is 18,200 [[1\]](#page-6-0). These figures revealed that there is a need to lay stress in the area of safety in Indian mining industry. As a solution, Directorate General of Mines Safety (DGMS) have suggested all the mines to develop a Safety Management Plan to improve safety in mines. An effective risk assessment is required to develop a practical safety management plan [\[2](#page-6-0)]. The essential elements of risk assessment are hazard identification, risk analysis, and risk evaluation. Hazard identification is the systematic identification of sources of potential injury. Risk analysis helps in developing an understanding of the risks associated with the identified hazards. Evaluating the risks helps to determine the level of risks related to the identified hazards.

Ashworth et al. [\[3](#page-6-0)] developed a risk model to assist mine management in decision making to improve the management of occupational safety risks. Donoghue [\[4](#page-6-0)] applied qualitative and semi-quantitative risk assessment matrices for ranking occupational health risks in mining and mineral processing. Komljenovic et al. [[5\]](#page-6-0) analysed injuries data of U.S mining operations from 1995 to 2004 and proposed a global risk matrix based on severity and frequency. Shariati [\[6](#page-6-0)] and Kumar [[7\]](#page-6-0) applied Failure Mode and Effects Analysis (FMEA) to assess underground mine risks. Tripathy and Ala [[8\]](#page-6-0) has studied the equipment related fatal accidents in coal mines using FMEA. Kinilakodi and Grayson [\[9](#page-6-0)] developed a Safety Performance Index for assessing the mine safety performance. Kumar and Ghosh [\[10](#page-6-0)] attempted to explore the top and initiating events of the methane explosion in the underground mines using integrated event tree and fault tree analysis. Thompson [[11\]](#page-6-0) applied Workplace Risk Assessment and Control (WRAC) technique to identify the hazards in coalmines. There are many qualitative and quantitative risk assessment techniques for evaluation. However, each technique has its own purpose and outcome [[12](#page-6-0)].

The common quantitative risk assessment techniques applied in the mining industry are FMEA and WRAC [\[13](#page-6-0)]. In both the methods, the risk level (RL) of the identified hazards is calculated as the product of two risk parameters:

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likelihood and consequence  $(C)$  [\[2](#page-6-0)]. For more detailed risk assessment to be carried out, the likelihood can be replaced with probability  $(P)$  and exposure  $(E)$  [[14\]](#page-6-0). However, in the present Indian mining industry, only the number of accident occurred were recorded. The consequence and the exposure data remains unrecorded or unavailable. As the most of the probabilistic risk analysis techniques are dependent on availability and accuracy of the previous data and they fail to evaluate when the data is unavailable [\[15](#page-6-0)]. This attests that it is hard to conduct a probabilistic risk analysis in Indian mining industry. Therefore, it is necessary to develop a new risk assessment method to assess safety risks in underground coalmines

Risk Level (RL) = Probability (P) 
$$
\times
$$
 Exposure (E)  
 $\times$  Consequence (C) (1)

In recent years, the application of risk assessment techniques in fuzzy environment have been proposed to achieve accurate solution when the available data is approximate or uncertain [\[16](#page-6-0)]. Fuzzy logic is applied in mining industry for prediction of roof fall rate [\[17](#page-6-0)], prediction of rock fragmentation due to blasting [\[18](#page-6-0)], and qualitative interpretation of acid mine drainage processes [\[19](#page-6-0)]. All the previous applications revealed that fuzzy set theory could effectually overcome the uncertainty encountered in the practical applications. This study presents a Mamdani fuzzy model for assessing and ranking the safety risks of the underground coalmines with a case study demonstrating the application of the model. The fuzzy model is developed using a Fuzzy Logic Toolbox of MATLAB.

## Methodology

The methodology adopted in this study consists of three steps. The first step is hazard identification and analysis; the second step is the collection of experts' opinion, and the last step is risk quantification using fuzzy logic. Figure 1 represents the graphical view of the complete methodology followed.

## Hazard Identification and Analysis

This phase begins with describing the problem and limits of the study. The next step is to collect accident data from mines and accident statistics from DGMS. The objective of the data collection is to gain information on what incidents and accident occurred in mines over the years. In the hazard identification step, the types of accidents occurred, and the causes of the accidents are studied in detail through evaluating the previous accident or incident data and observations. The common hazard identification techniques are



Fig. 1 Overall risk assessment methodology

checklists, accident or incident reports, brainstorming, and mine inspection reports [\[14](#page-6-0)]. The identified hazards provide information for risk analysis. The risk analysis aims to understand the risks associated with the identified hazards.

## Collection of Experts' Opinion for Risk Parameters

This phase deals with the collection of experts' opinion. Experts' opinions for the values of probability, exposure, and consequence for each identified hazards are sought. If the statistical risk parameter input data is uncertain or unavailable, experts' judgement should be applied. The experts' opinions on risk parameters are collected using linguistic scales and accordingly they are modelled using fuzzy set theory using scales presented in Tables [1,](#page-2-0) [2,](#page-2-0) [3](#page-2-0) and [4](#page-2-0) respectively. The linguistic scales for probability,

<span id="page-2-0"></span>



Table 2 Rating scale for exposure (E)

Linguistic scale	Exposure description	Parameters of MFs
Continuous (E6)	Continuous (several times daily)	(8, 10, 12)
Very frequent (E5)	Frequent (daily)	(6, 8, 10)
Frequent (E4)	Occasional (weekly)	(4, 6, 8)
Low frequent $(E3)$	Unusual (monthly)	(2, 4, 6)
Seldom (E2)	Rare (yearly)	(1, 2, 4)
Unusual $(E1)$	Very rare (more than yearly)	(0, 1, 2)

Table 3 Rating scale for consequence (C)

Linguistic scale Consequence description		Parameters of MFs	
Catastrophic (C6)	Catastrophic (many fatalities, $> 4$ fatalities)	(4, 5, 6)	
Major $(C5)$	Disaster (a few fatalities, 1–4 fatalities)	(3, 4, 5)	
Moderate (C4)	Fatality (one fatality)	(2, 3, 4)	
Minor $(C3)$	Serious (significant chance of fatality, permanent disability)	(1, 2, 3)	
Insignificant $(C2)$	Minor (temporary disability, many lost time injuries)	(0.5, 1, 2)	
Petty $(C1)$	Small injury (minor first aid)	(0, 0.5, 1)	

Table 4 Rating scale for of risk level (RL)

Linguistic scale	Parameters of MFs
High	(200, 500, 700)
Medium	(20, 110, 200)
Low	(0, 10, 20)

exposure, consequence, and risk level are developed by modifying the DGMS risk score [[14\]](#page-6-0).

# Risk Quantification Using Mamdani Fuzzy Logic

Mamdani fuzzy model [\[20](#page-6-0)] is intuitive and well suited for human input. Mamdani fuzzy inference mechanism is based on the compositional rule of inference proposed by Zadeh [\[21](#page-6-0)]. As the in-depth analysis of general fuzzy logic can be found in many works of literature [\[21](#page-6-0), [22](#page-6-0)], this section only provides the brief explanation of Mamdani fuzzy logic system. The principal components of Mamdani fuzzy model are Fuzzification, Knowledge base, Fuzzy Inference System, and Defuzzification.

## <span id="page-3-0"></span>Fuzzification

In fuzzification step, the experts' opinion collected using linguistic scales are translated into fuzzy sets containing linguistic concepts, and the Membership Functions (MFs) are applied to the measurements, and a membership value is determined [[22\]](#page-6-0). A triangular MF converts the linguistic scales in the range of 0–1 using the Eqs. 2 and 3.

$$
\mu(x;a,b,c) = \begin{cases}\n0, & x < a \\
(x-a)/(b-a), & a \le x \le b \\
(c-x)/(c-b), & b \le x \le c \\
0, & c < x\n\end{cases}
$$
\n(2)

$$
\mu(x;a,b,c) = \left(\max\left(\min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right), 0\right)\right) \tag{3}
$$

where a, b, c are the parameters of the linguistic scale and x is the range of the input parameters.

## Aggregation of Experts' Opinion

Practically, it is impossible for a single manager or engineer to consider all relevant aspects of an underground mine. Therefore, risk assessment in mines comprises many experts with different background and experience. Each expert may have different opinions on the final judgement. Let us consider the number of experts be 'N,' the number of hazards (E) identified be 'm', and the number of risk factors be 'n'. Let  $e_{ij}$  be the judgement of i hazard for j criteria. Then one gets N matrices of type  $E = [e_{ii}]_{m \times n}$ . Then all experts' opinions on risk parameters of each particular event are aggregated to get an overall quantified value [\[23](#page-6-0)]. The arithmetic mean aggregation [\[24](#page-6-0)] operator defined on fuzzy triangular numbers  $(a_1, b_1, c_1)$ ,  $(a_2, b_2, c_2)$  $\ldots$  (a<sub>n</sub>, b<sub>n</sub>, c<sub>n</sub>) delivers the result as (x, y, z).

Where, 
$$
= 1/n \sum_{k=0}^{n} a_k
$$
,  $y = 1/n \sum_{k=0}^{n} b_k$ ,  $z = 1/n \sum_{k=0}^{n} c_k$  (4)

After aggregating the experts' opinion, defuzzification of risk parameters ought to be done. For defuzzification of triangular fuzzy risk parameters, centroid defuzzification [\[25](#page-6-0)] method is widely used. If the aggregated fuzzified output  $A = (x, y, z)$ , then the formula for centroid method is as follows:

$$
Centroid(A) = \frac{x + y + z}{3}.
$$
 (5)

#### Knowledge Base

Knowledge base consists of both rule base and database. In the database, the MFs of the fuzzy sets used in the fuzzy rules are defined. The rule base includes a number of if– then rules. If–then rules are employed to capture the imprecise modes of reasoning, which plays an essential role in the human ability to make decisions in the environments of uncertainty and imprecision [\[26](#page-6-0)].

## Fuzzy Inference System

A fuzzy inference system maps the fuzzy inputs and rules to outputs using fuzzy set theory. In Mamdani model, the 'MIN' operator is used for combination and implication operations. An implication method states how a fuzzy logic controller scales the MFs of an output linguistic variable based on the rule weight of the corresponding rule. The fuzzy outputs are aggregated by using the 'MAX' operator. Aggregation process is where the outputs of each rule are combined into a single fuzzy set. The MAX–MIN composition used in the model is shown in Fig. 2 [[27](#page-6-0)].

#### Defuzzification

The output generated by the fuzzy inference system will always be fuzzy in nature. Therefore, to convert the fuzzy output to crisp output, defuzzification is needed. Centroid of area defuzzification method [\[28](#page-6-0)] for establishing the output is expressed in Eq. 6.

$$
Centroid of area, z^* = f\mu A(z) \cdot zdz / f\mu A(z) dz
$$
 (6)

where  $z^*$  is the crisp value for the z output, and  $\mu A(z)$  is the aggregated output membership function.

After defuzzification, the fuzzy inference system gives a crisp output value. The crisp value obtained is used to express the risk level of the associated hazard so that remedial actions can be ordered accordingly.

## Case Study

The mine selected is a mechanized underground coalmine of a major public sector coal company located in Odisha, India. Accident data is collected from 2009 to 2015 from the mine and DGMS reports. Hazard factors are identified by analysing



Fig. 2 Mamdani fuzzy inference system using min and max operators

the collected accident data from the mine and DGMS annual reports. DGMS has categorized the identified hazard factors as follows: ground movement; rope haulage; belt conveyor; load haul dumper; explosives; dust, gas and other combustible material; mine fire; fall of persons; and irruption of water.

The collected accident data is defined subjectively, and the descriptive terms are vague and imprecise. Therefore, the judgment of experienced safety experts in the underground coalmine is recorded using a designed survey questionnaire. Safety experts' have rated each of the hazard factors for its probability, exposure, and consequence using linguistic scales shown in Tables [1,](#page-2-0) [2](#page-2-0) and [3.](#page-2-0) The linguistic scores given by the experts' is converted to corresponding fuzzy set numbers. Then the aggregation of fuzzy set numbers for all the hazard factors is done using the Eq. [4,](#page-3-0) and the defuzzification of fuzzy set numbers is done using Eq. [5](#page-3-0). Aggregated and defuzzified fuzzy scores of all the hazard factors for all risk parameters are shown in Table 5. The defuzzified crisp scores of risk parameters are used asinput for fuzzy inference system.

In the developed Mamdani fuzzy model, probability, exposure, consequence are the three input variables and risk level is the output variable. The triangular MFs used in this study to represent the linguistic scales of input and output parameters are shown in Fig. [3](#page-5-0).

The linguistic input scales for probability, exposure, and consequence had 6 MFs each. As a result, 216 ( $6 \times 6 \times 6$ ) rules are made in the rule base. After developing MFs and rule base, the aggregated score is entered into the rule viewer of the fuzzy logic toolbox to obtain the risk score (crisp output value) of all the hazard factors as shown in Table [6.](#page-5-0) A sample rule base and rule viewer are shown in Fig. [4.](#page-5-0)

# Results and Discussion

In this study, nine hazard factors are considered based on the DGMS reports and data collected from the underground mine. The best ranking solution for risk factors will be a

Table 5 Aggregated expert's opinion for all hazard factors

circumstance that has a low probability, low exposure, and low consequence and on the contrary, the worst ranking solution for risk factors will be a circumstance whose probability, exposure, and consequence are very high. Thus, in Table [6,](#page-5-0) rank 9 represents those hazards that are having least risk associated with them and rank 1 represents those hazards which are having highest risk associated with them.

From the Table [6,](#page-5-0) one can observe that all the hazard factors have different risk level for the risk parameters used and based on the risk level rating scale; and they form three groups among them. Hazard factors with risk score  $>200$ falls in high RLs group, 20–200 falls in medium RLs group and  $\lt$  20 falls in low RLs group. The order of hazard factors is mine fire  $\ge$  ground movement  $\ge$  belt conveyor  $>$  dust, gas & other combustible materials  $>$  explosives  $>$  rope haulage  $>$  irruption of water  $>$  load haul  $d$ umper  $>$  fall of persons. On performing the in-depth study of the mine records, it was found that the mine fire has occurred three times in the history of the mine and the frequency of roof/side fall accidents are very high. Thus, it is clear to say that the present model can capture the reallife situation of the mine considered. Based on priority risk ranking of all the hazards obtained, the mitigation plan can be prepared accordingly, so that preventive actions can be taken for a riskiest hazard on the priority basis and mine safety can be improved.

## Conclusion

Mining is inherent of hazards, and complete elimination of hazards from mining industry is not possible until today. Therefore, the risk assessment of the mine needs to be studied in detail. In this study, hazard factors related to underground mine are listed and risk ranked using fuzzy logic approach. Mine fire has the highest risk level in the mine followed by ground control. Therefore, resources



<span id="page-5-0"></span>

Fig. 3 Membership function of probability, exposure, consequence and risk level

Table 6 Risk score and ranking of hazard factors

Hazard factors	Fuzzy model	
	Risk score	Ranking
Ground movement	316	$\overline{c}$
Rope haulage	100	6
Belt conveyor	110	3
Load haul dumper	41.7	8
Explosives	103	
Dust, gas and other combustible material	107	4
Mine fire	330	
Fall of persons	10	9
Irruption of water	87.3	



Fig. 4 Sample rule base and rule viewer

should be allotted to control these hazards factors. The fuzzy logic structure helps in capturing the experts' opinion in linguistic terms for the risk parameters and evaluating the risk levels. Both imprecise data and quantitative data can be used in fuzzy logic structure for risk assessment. It can be foreseen that the proposed methodology could be <span id="page-6-0"></span>used for risk assessment by mining management and safety officers, prioritize risks as well as take steps to ameliorate risk in mines. Equipment designers and manufacturers could also use this methodology to focus on specific problem areas on that mining equipment. It can also be used to improve hazard awareness training.

#### References

- 1. DGMS, Standard note 1.1.2016. [http://dgms.gov.in/writeread](http://dgms.gov.in/writereaddata/UploadFile/STDNOTE-1-1-2016636047840119597695.pdf) [data/UploadFile/STDNOTE-1-1-2016636047840119597695.pdf.](http://dgms.gov.in/writereaddata/UploadFile/STDNOTE-1-1-2016636047840119597695.pdf) Accessed 26 Oct 2017
- 2. ISO, I. 31000:2009 Risk Management—Principles and Guidelines (International Organization for Standardization, Geneva, 2009)
- 3. S.G.E Ashworth, A.V. Peake, J.M. Greben, C. Elphinstone, Development and Evaluation of a Prototype Decision Support System for Management of Occupational Safety Risk (Safety in Mines Research Advisory Committee, 1997). [http://www.](http://www.mhsc.org.za/sites/default/files/GEN108.pdf) [mhsc.org.za/sites/default/files/GEN108.pdf](http://www.mhsc.org.za/sites/default/files/GEN108.pdf). Accessed 28 Dec 2017
- 4. A.M. Donoghue, The design of hazard risk assessment matrices for ranking occupational health risks and their applications in mining and minerals processing. Occup. Med. 51(2), 118–123 (2001)
- 5. D. Komljenovic, W.A. Groves, V. Kecojevic, Injuries in U.S. mining operations—a preliminary risk analysis. Saf. Sci. 46, 792–801 (2008). <https://doi.org/10.1016/j.ssci.2007.01.012>
- 6. S. Shariati, Underground mine risk assessment by using FMEA in the presence of uncertainty. Decis. Sci. 3, 295–304 (2014)
- 7. P. Kumar, Risk analysis by using failure mode and effects analysis for safe mining. Int. J. Sci. Res. 3(11), 2512–2515 (2014)
- 8. D.P. Tripathy, C.K. Ala, Investigation of equipment related fatal accidents in Indian coal mines. Saf. Equip. Rev. 16, 14–19 (2015)
- 9. H. Kinilakodi, R.L. Grayson, A methodology for assessing underground coal mines for high safety-related risk. Saf. Sci. 49(6), 906–911 (2001). <https://doi.org/10.1016/j.ssci.2011.02.007>
- 10. R. Kumar, A.K. Ghosh, Mines systems safety improvement using an integrated event tree and fault tree analysis. J. Inst. Eng. India Ser. D. 8(1), 101–108 (2017). [https://doi.org/10.1007/](https://doi.org/10.1007/s40033-016-0121-0) [s40033-016-0121-0](https://doi.org/10.1007/s40033-016-0121-0)
- 11. S. Thompson, Risk Assessment in Mines. Paper presented at Queensland Mining Industry Health and Safety Conference (1999)
- 12. A. Iannacchione, F. Varley, T. Brady, The Application of Major Hazard Risk Assessment (MHRA) to Eliminate Multiple Fatality Occurrences in the US Minerals Industry (Department of Health and Human Services, 2008). [https://www.cdc.gov/niosh/mining/](https://www.cdc.gov/niosh/mining/userfiles/works/pdfs/2009-104.pdf) [userfiles/works/pdfs/2009-104.pdf](https://www.cdc.gov/niosh/mining/userfiles/works/pdfs/2009-104.pdf). Accessed 28 Dec 2017
- 13. J. Joy, Occupational safety risk management in Australian mining. Occup. Med. 54(5), 311–315 (2004). [https://doi.org/10.1093/](https://doi.org/10.1093/occmed/kqh074) [occmed/kqh074](https://doi.org/10.1093/occmed/kqh074)
- 14. DGMS, Safety Management System—A Guideline for Implementation (2002). [https://elibrarywcl.files.wordpress.com/2015/](https://elibrarywcl.files.wordpress.com/2015/02/dgmstechst-circular-no-13-of-2002-safety-management-system.pdf) [02/dgmstechst-circular-no-13-of-2002-safety-management-system.](https://elibrarywcl.files.wordpress.com/2015/02/dgmstechst-circular-no-13-of-2002-safety-management-system.pdf) [pdf.](https://elibrarywcl.files.wordpress.com/2015/02/dgmstechst-circular-no-13-of-2002-safety-management-system.pdf) Accessed 3 Oct 2017
- 15. M. An, Y. Chen, C.J. Baker, A fuzzy reasoning and fuzzy-analytical hierarchy process based approach to the process of railway risk information: a railway risk management system. Inf. Sci. 181(18), 3946–3966 (2011). [https://doi.org/10.1016/j.ins.2011.](https://doi.org/10.1016/j.ins.2011.04.051) [04.051](https://doi.org/10.1016/j.ins.2011.04.051)
- 16. M. Hadjimichael, A fuzzy expert system for aviation risk assessment. Expert Syst. Appl. 36(3), 6512–6519 (2009). <https://doi.org/10.1016/j.eswa.2008.07.081>
- 17. E. Ghasemi, M. Ataei, Application of fuzzy logic for predicting roof fall rate in coal mines. Neural Comput. Appl. 22, 311–321 (2013)
- 18. M. Monjezi, M. Rezaei, A.Y. Varjani, Prediction of rock fragmentation due to blasting in Gol-E-Gohar iron mine using fuzzy logic. Int. J. Rock Mech. Min. Sci. 46, 1273–1280 (2009)
- 19. J. Aroba, J.A. Grande, J.M. Andu´jar, M.L. Torre, J.C. Riquelme, Application of fuzzy logic and data mining techniques as tools for qualitative interpretation of acid mine drainage processes. Environ. Geol. 53, 135–145 (2007)
- 20. E.H. Mamdani, S. Assilian, An experiment in linguistic synthesis with a fuzzy logic controller. Int. J. Man Mach. Stud. 7(1), 1-13 (1975). [https://doi.org/10.1016/S0020-7373\(75\)80002-2](https://doi.org/10.1016/S0020-7373(75)80002-2)
- 21. L.A. Zadeh, Fuzzy sets. Inf. Control 8(3), 338–353 (1965). [https://doi.org/10.1016/S0019-9958\(65\)90241-X](https://doi.org/10.1016/S0019-9958(65)90241-X)
- 22. T.J. Ross, Fuzzy Logic with Engineering Applications (Wiley, New York, 2004)
- 23. S. Mandal, K. Singh, R.K. Behera, S.K. Sahu, N. Raj, J. Maiti, Human error identification and risk prioritization in overhead crane operations using HTA, SHERPA and fuzzy VIKOR method. Expert Syst. Appl. 42(20), 7195–7206 (2015). <https://doi.org/10.1016/j.eswa.2015.05.033>
- 24. M. Fasanghari, F.H. Roudsari, The fuzzy evaluation of e-commerce customer satisfaction. World Appl. Sci. J. 4(2), 164–168 (2008)
- 25. Y.M. Wang, Centroid defuzzification and the maximizing set and minimizing set ranking based on alpha level sets. Comput. Ind. Eng. 57(1), 228–236 (2009). [https://doi.org/10.1016/j.cie.2008.](https://doi.org/10.1016/j.cie.2008.11.014) [11.014](https://doi.org/10.1016/j.cie.2008.11.014)
- 26. J. Ma, D. Ruan, Y. Xu, G. Zhang, A fuzzy-set approach to treat determinacy and consistency of linguistic terms in multi-criteria decision making. Int. J. Approx. Reason. 44(2), 165–181 (2007). <https://doi.org/10.1016/j.ijar.2006.07.007>
- 27. J.S.R. Jang, C.T. Sun, E. Mizutani, Neuro-fuzzy and soft computing: a computational approach to learning and machine intelligence, in Fuzzy Inference Systems, ed. By T. Robbins, P. Daly (Prentice Hall, Englewood Cliffs, 1997), pp. 73–91
- 28. R. Saneifard, R. Saneifard, A method for defuzzification based on centroid point. Turk. J Fuzzy Syst. 2(1), 36–44 (2011)