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ORIGINAL CONTRIBUTION

Optimization of Power Coefficient of Wind Turbine Using Genetic Algorithm

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Abstract In the design of a wind turbine, the goal is to attain the highest possible power output under specified atmospheric conditions. The optimization of power coefficient of horizontal axis wind turbine has been carried out by integration of blade element momentum method and genetic algorithm (GA). The design variables considered are wind velocity, angle of attack and tip speed ratio. The objective function is power coefficient of wind turbine. The different combination of design variables are optimized using GA and then the Power coefficient is optimized. The optimized design variables are validated with the experimental results available in the literature. By this optimization work the optimum design variables of wind turbine can be found economically than experimental work. NACA44XX series airfoils are considered for this optimization work.

Keywords Wind turbine · Angle of attack · Genetic algorithm · Power coefficient

List of symbols

C_p	Power coefficient
a'	Angular induction factor
a	Axial induction factor
N_b	Number of blade
C	Chord
C_L	Coefficient of lift
C_D	Coefficient of drag
ρ	Density of air
θ	Inflow angle
TSR	Tip speed ratio
AOA	Angle of attack
v	Wind velocity
$p_{\_size}$	Population size
N_{ran}	Random numbers
$p_{\_mut}$	Probability of mutation
r_{cp}	Cutting point random number
$p_{\_cro}$	Probability of crossover

Introduction

The importance of clean energy sources was realized rapidly after the negative effects of the pollution caused by generators on the environment became clear. Wind energy is a clean and renewable energy source whose applications exist worldwide. Some countries changed their national energy forward planning for renewable clean energy projects. In developed countries some researchers have worked on developing new kinds of wind turbines in order to produce a part of their power from renewable wind energy [1]. The research activities on wind energy seem to increase throughout the world as the wind energy is clean and cheap.

The globally installed capacity of wind energy has been increasing steadily over the past decade, and the price of electricity derived from it has been continuously declining.

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While viable site selection with high quality wind resources still exist, improved turbine technology and cost reductions can expand the number of locations that are economically viable [2].

The maximum output of the wind turbine at two different operating conditions (i.e. at two different values of the ratio of tip speed to wind speed) is investigated using the vortex blade element theory. The theory of optimum loading is developed, including the effect of profile drag. For the range of parameters considered, it is shown that the effect of profile drag on blade loading is quite small, although the effect on efficiency is significant [3].

When designing a wind turbine, the goal is to attain the highest possible power output under specified atmospheric conditions. From the technical point of view, this depends on the shape of the blade. The change of the shape of blade is one of the methods to modify stiffness and stability, but it may influence aerodynamic efficiency of wind turbine [4]. Practical aerodynamic shape design problems must balance performance optimization over a range of on-design operating conditions with constraint satisfaction at off-design operating conditions. A multipoint optimization formulation can be used to represent on-design and off-design conditions with corresponding objective or constraint functions. Two methods are presented for obtaining optimal airfoil designs that satisfy all design objectives and constraints. The first method uses an unconstrained optimization algorithm where optimal design is achieved by minimizing a weighted sum of objective functions for each of the conditions. To address competing design objectives between on-design and off-design conditions, an automated procedure is used to weight off-design objective functions to limit their influence on the overall optimization. The second method used the constrained optimization algorithm SNOPT (Sparse Nonlinear OPTimizer) allowing aerodynamic constraints imposed at off-design conditions to be treated explicitly. Both methods are applied to the design of an airfoil for a hypothetical aircraft, which is formulated as an 18-point multipoint optimization [4]. The numerical simulation of Horizontal Axis Wind Turbines (HAWTs) with untwisted blade was performed to determine the optimal angle of attack that produces the highest power output. The numerical solution was carried out by solving conservation equations in a rotating reference frame wherein the blades and grids were fixed in relation to the rotating frame [5].

Studies for optimum airfoil characteristics showed that the airfoil sections should have a relative high maximum lift at the entire span including the tip region. An increase in the swept area should therefore involve a complete redesign of the rotor blades, and avoid the use of low maximum lift airfoils at the tip, which so far has been widely used to control peak power [6]. An optimum design has been defined for horizontal axis wind turbines by treating the lifting line theory based on a variational problem. Confrontation of the results obtained with those of Maekawa, Sharpe, a Glauert's blade element momentum (BEM) theory calculations and a developed simplified model, have revealed the potential of the optimum design in predicting improved and higher rotor performances even at relatively high wind speeds [7].

The power optimization objective is accomplished by computing optimal control settings of wind turbines using data mining and genetic algorithms (GA). Data mining algorithms identify a functional mapping between the power output and controllable and non-controllable variables of a wind turbine. An evolutionary strategy algorithm is applied to determine control settings maximizing the power output of a turbine based on the identified model [8]. The performance of horizontal axis wind turbines can be accurately and efficiently modeled using the vortex lattice method. It has also been shown that this prediction scheme is viable as a prediction tool used in a GA based optimization process. This effort clearly represents the first step toward generating a multidisciplinary optimization scheme which would allow the entire wind turbine to be optimized simultaneously including the aerodynamic shapes, the structure and the power plant [9].

The factors affecting the power coefficient of wind turbine with specified airfoil geometry are wind velocity, angle of attack and tip speed ratio. The process of optimization of power coefficient includes consideration of all the factors simultaneously with its wide range of variations. Hence, the solution of optimum power coefficient has to be arrived by considering the different values of the above factors leading to hectic calculations. The optimization of power coefficient uses GA with optimum angle of attack and tip speed ratio for a range of wind velocities for a specified airfoil section of wind turbine.

The variables affecting the power coefficient of wind turbine such as wind velocity, angle of attack and tip speed ratio have minimum and maximum range of its values. The different combinations of these variables have to be analyzed for optimization along with other parameters such as inflow angle, twist angle, coefficient of lift and drag, axial and tangential flow factor, tip loss correction factor. In such situations, the exhaustive search algorithms can be used effectively in optimization process. These search algorithms can search and yield the optimum results from the larger solution area.

The proposed GA performs optimization based on BEM method. The Chord and Twist angle are optimized by using GA as these two variables determine the blade shape. For a specified geometry, the optimization method provides optimum power coefficient with the optimum angle of attack, tip speed ratio at a particular wind velocity. The various NACA 4 series airfoils are considered blade geometry in root and tip. The thicker airfoil is considered for the root portion and thin airfoil is considered for tip portion. The next part describes the Methodology of implementing BEM theory, optimization of chord and twist angle and the implementation of GA. In the chapter 3, the results of the optimization are discussed.

Implementation of Genetic Algorithm

A horizontal axis wind turbine of one MW capacity is considered for optimization and the optimum power coefficient is obtained. The mathematical model implemented in this work for the fluid dynamics design of a wind turbine is based on the BEM theory. By applying the momentum and angular momentum conservation equations, it is possible to obtain the forces acting on the blades, and so the torque and power at the rotor shaft. Figure 1 shows a cross-section of the rotor blade and the velocities relating to the airfoil. It also shows the axial and tangential induction factors (a and a′) that significantly affect the real value of the velocities.

The mathematical model for the fluid dynamics wind turbine design was developed in a previous work [10], is based on BEM Theory. By applying momentum and angular momentum conservation equations, the axial force and torque acting on the blade sector is obtained. By applying the BEM theory, it is possible to evaluate the force dN and the torque dM for each blade element, as given in Eqs. (1) and (2):

$$\mathrm{dN} = \frac{\rho}{2}\frac{V_o^2(1-a)^2}{\sin\theta}N_b(C_L\cos\theta + C_D\sin\theta)\,cdr_1 \tag{1}$$

$$\mathrm{dM} = \frac{\rho}{2}\frac{V_o(1-a)}{\sin\theta}\frac{\omega r_1(1+a')}{\cos\theta}N_b(C_L\sin\theta - C_D\cos\theta)\,cr_1dr_1 \tag{2}$$

For each aerodynamic airfoil, C_L and C_D depend on the Reynolds number and the angle of attack. The torque and normal forces depend on the tangential and axial induction factors. To evaluate them it is necessary to implement the momentum and angular momentum conservation equations. From the conservation of the momentum in the axial direction, between the far upstream section and the far downstream section, it is possible to obtain two further expressions for the force dN and the torque dM. Equalizing two equations, Eqs. (3) and (4) can be arrived as

$$a = \frac{1}{\frac{4F\sin^2\theta}{\sigma(C_L\cos\theta + C_D\sin\theta)} + 1} \tag{3}$$

$$a' = \frac{1}{\frac{4F\sin\theta\cos\theta}{\sigma(C_L\sin\theta - C_D\cos\theta)} + 1} \tag{4}$$

where F is the Prandtl tip loss factor, as defined in earlier citations [11, 12]

$$F = \frac{2}{\pi}\sigma rcos\left[exp\left(\frac{N_b(r_1 - R_1)}{2r_1 sin\theta}\right)\right] \tag{5}$$

and σ is the rotor solidity, defined as

$$\sigma = \frac{cN_b}{2\pi r_1} \tag{6}$$

Equation (3) only gives reliable results for axial induction factor values between 0 and 0.4. For axial induction factors greater than 0.4 the BEM theory does not yield reliable results. Glauert [13] gave a correction for determination of the axial induction factor when $a > 0.4$, valid only for $F = 1$. If the losses at the tip of the blade are taken into account ($F < 1$), it is necessary to consider the correction

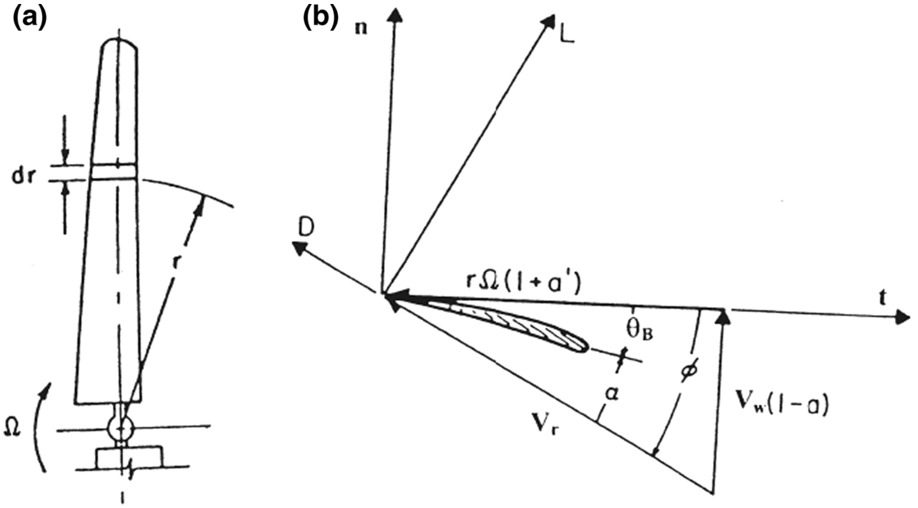


Fig. 1 Velocities relating to the airfoil

$$a = \frac{18F - 20 - 3\sqrt{C_N(50 - 36F) + 12F(3F - 4)}}{36F - 50} \tag{7}$$

This correction is necessary to eliminate the numerical instability that occurs when the Glauert correction is implemented in conjunction with the presence of tip losses. Glauert [13] initiated the calculation of the optimum wind turbine by making the power integral equation stationary. He considered an ideal actuator disk model and obtained the optimum variations of the axial and rotational induction factors (a and a'), as well as the inflow angle θ as follows

$$a' = \frac{(1 - 3a)}{(4a - 1)} \tag{8}$$

$$a'(1 + a')\lambda_r^2 = a(1 - a) \tag{9}$$

$$tan\phi = \frac{(1 - a)}{\lambda_r(1 + a')} \tag{10}$$

where $\lambda_r = V_r/V_w$ is the local speed ratio at the rth station along the blade, V_r is the rotational speed, and V_w is the net wind velocity. Figure 1 shows a front view of a rotating blade and the velocity triangle of an arbitrary airfoil section. In deriving Eqs. (8), (9) and (10) secondary effects of drag and tip losses were neglected. If one further neglects rotation behind the rotor (i.e. $a = 1/3$ and $a' = 0$), the inflow angle may be determined from the relation:

$$\phi = tan^{-1}\left(\frac{2}{3}\lambda_r\right) \tag{11}$$

which may be used only for $\lambda_r > 1$.

Wilson et al. performed a local optimization analysis by maximizing the power output at each radial station along the blade. The axial induction factor was varied until the power contribution became stationary [14]. Rohrbach and Worobel investigated the effect of blade number and section lift-to-drag ratio on the maximum turbine performance [15]. Their results have been found to yield slightly lower maximum performance than that found in Ref. [14]. Optimum power coefficient was also investigated in earlier studies [16] who derived an approximate relationship between the inflow angle (θ) and the local speed ratio (λ_r). The relation was given by the following 5th order polynomial:

$$\phi = 57.51 - 35.56\lambda_r + 10.61\lambda_r^2 - 1.586\lambda_r^3 + 0.114\lambda_r^4 - 0.00313\lambda_r^5 \tag{12}$$

where θ is measured in degrees. Nathan's equation was obtained for a lift-to-drag ratio ranging from 28.6 to 66.6 and the effects of secondary flows in the tip and hub regions were not included in the analysis.

The maximum power coefficient is derived as

$$C_p = \int_0^1 8(1 - a)a'\left[\frac{a(1 - a)}{\lambda^2\mu^2}\right]\lambda^2\mu^3 d\mu = 4a(1 - a)^2 = \frac{16}{27} \tag{13}$$

The various steps involved in optimization of power coefficient (C_p) using BEM method in GA are illustrated as a block diagram in Fig. 2. In the Input module, the values of wind velocity, angle of attack and tip speed ratio are given as an input. The wind velocity varies from 3 to 25 m/s, angle of attack varies from 0° to 15° and tip speed ratio varies from 3 to 10 as the tip speed ratio varies within this range for three blade system. The Optimization of power coefficient is performed at operating wind speed of 10 m/s and is carried out in the initial calculation module. The coding for this optimization is developed in C++ programming language.

There are two important parameters in wind turbine blade design namely chord and twist. Choosing the airfoil throughout the blade together with those parameters, the blade shape would be determined. These parameters were considered through the optimization [17]. The chord length (C) and Twist angle (θ) of wind turbine is calculated by using the following polynomial equation. The Chord and Twist angles are optimized using genetic algorithm. The comparison of optimized Chord and Twist angles are presented in Table 1.

$$X_1r^2 + Y_1r + Z_1 = C_{Root} \tag{14}$$

$$X_2r^2 + Y_2r + Z_2 = C_{Mid} \tag{15}$$

$$X_3r^2 + Y_3r + Z_3 = C_{Tip} \tag{16}$$

$$C_{Mid} = \frac{C_{Root} + C_{Tip}}{2} \tag{17}$$

$$X_1r^2 + Y_1r + Z_1 = \theta_{Root} \tag{18}$$

$$X_2r^2 + Y_2r + Z_2 = \theta_{Mid} \tag{19}$$

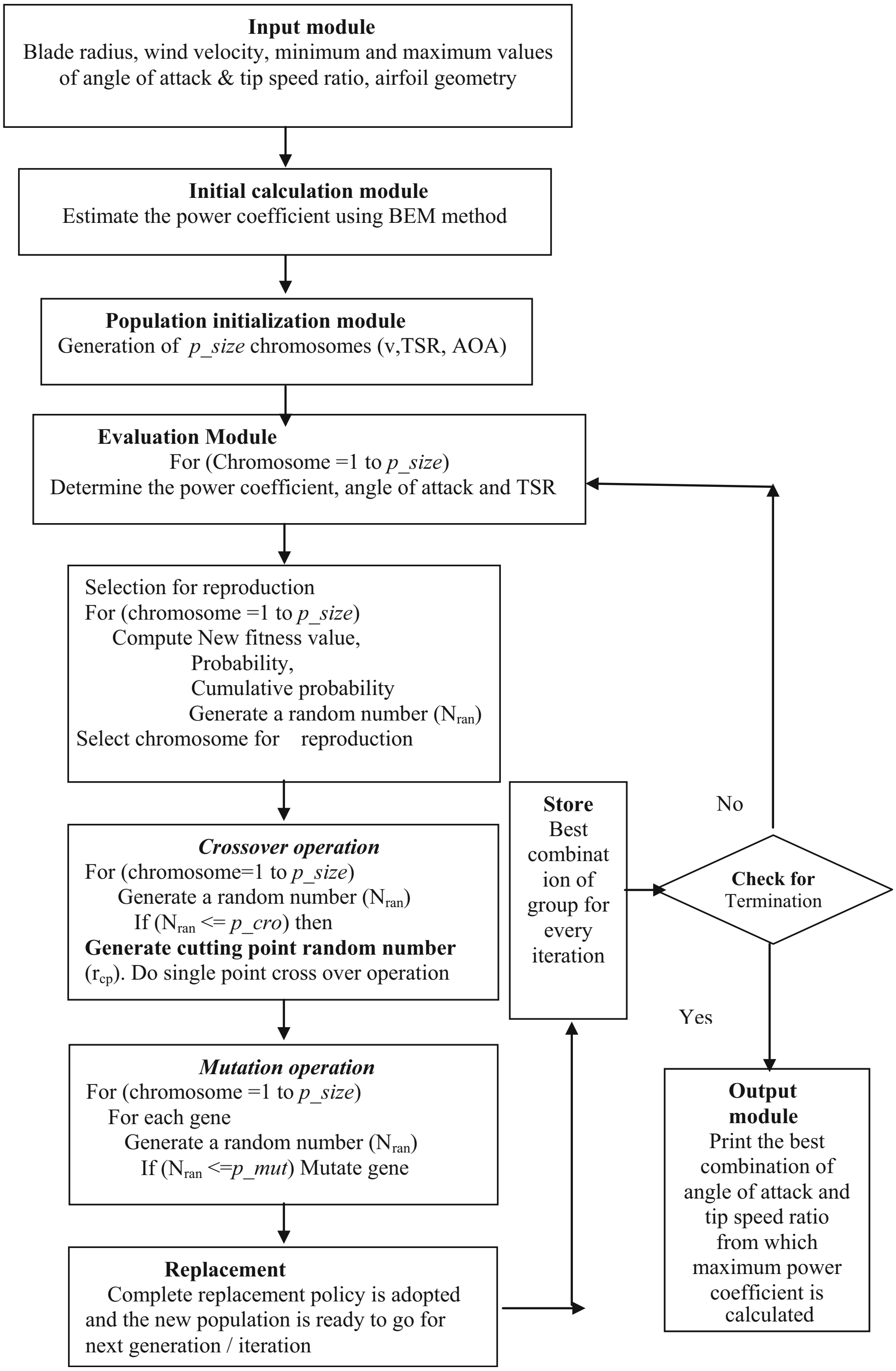
$$X_3r^2 + Y_3r + Z_3 = \theta_{Tip} \tag{20}$$

$$\theta_{Mid} = \frac{\theta_{Root} + \theta_{Tip}}{2} \tag{21}$$

The variation of original and optimized chord length is shown in Fig. 3 and the variation of original and optimized twist angle is shown in Fig. 4.

The optimized chord and twist angle of wind turbine blade are considered for further optimization of power coefficient with respect to angle of attack and tip speed ratio. In the evaluation module, optimum angle of attack and tip speed ratio are obtained to yield optimum power coefficient. The selection of reproduction of the parent string is adopted using the roulette wheel selection (as random). The single point cross over technique was adopted for developing new strings in the population. The crossover and mutation probability were set as 0.6 and 0.05 respectively since the cross over

Fig. 2 Optimization process



about 60 % is best and the mutation rate should be low (0.5–1 %) for better results [18].

Results and Discussion

A horizontal axis wind turbine of one MW capacity with airfoils NACA 4410, 4420, 4430 and 4440 have been considered for optimization using GA procedure explained in the previous section. The coding for the GA process is performed in C++ programming language. The analysis is performed in the following three categories.

The wind velocity is kept constant as 10 m/s as the turbine is designed for this particular velocity. Further, the angle of attack and tip speed ratio are made to vary from 0° to 15° and 3 to 10 respectively in the GA programming code and the power coefficient is optimized. The other parameters of crossover and mutation probabilities are kept

Table 1 Comparison of Optimized Chord and Twist angles

Radius	Chord (m)	Chord_{opt} (m)	Twist (°)	Twist_{opt} (°)
4	4.10	3.42	37.90	41.59
6	3.40	2.79	26.78	30.06
8	2.70	2.22	19.92	22.37
10	2.00	1.72	15.39	17.06
12	1.50	1.28	12.21	13.23
14	1.10	0.91	9.87	10.36
16	0.80	0.61	8.08	8.14
18	0.50	0.37	6.10	6.38
20	0.30	0.20	4.40	4.95
22	0.12	0.10	3.33	3.77
24	0.10	0.05	2.50	2.77
26	0.05	0.03	1.30	1.93
28	0.03	0.02	0.90	1.20
30	0.02	0.02	0.30	0.56

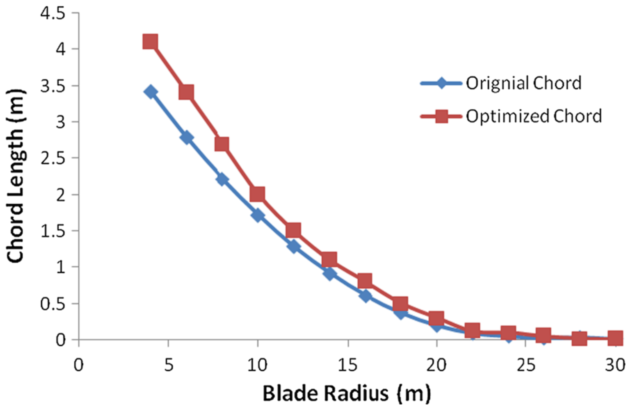


Fig. 3 Variation of original and optimized Chord Length

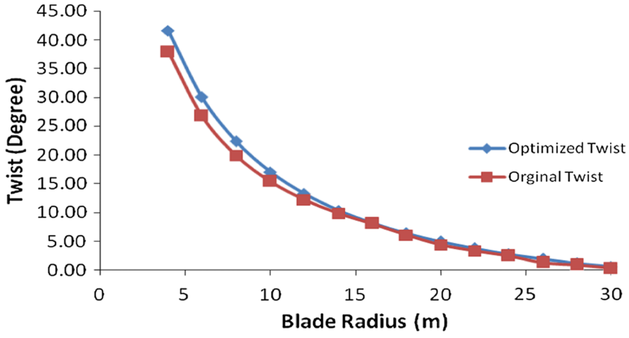


Fig. 4 Variation of original and optimized Twist angle

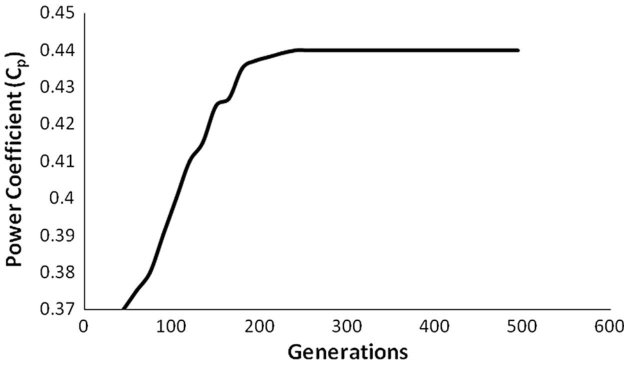


Fig. 5 Maximum fitness with generation

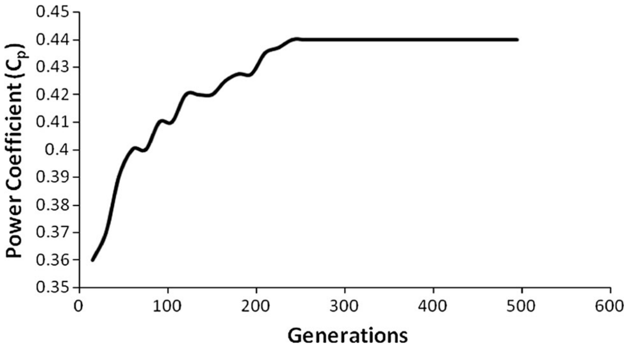


Fig. 6 Average fitness with generation

Table 2 Optimum power coefficient for NACA 4410 at 10 m/s

Angle of attack (°)	Tip speed ratio	Power coefficient
1.92	6.983	0.329
4.17	6.899	0.394
6.60	8.404	0.444
4.48	6.199	0.380
5.32	6.794	0.416
6.62	**8.173**	**0.450**
5.52	7.109	0.415
6.55	8.530	0.435
7.89	4.932	0.428
4.82	7.456	0.398

as 0.60 and 0.05 respectively. The number of iterations is considerably reduced as one of the major variables of wind velocity is kept constant. However, the increase in the number of iterations will tend to yield better results, the convergence of the variables take place in less number of iterations. Hence, in this category 500 generations have been allowed to perform to achieve optimum power coefficient. The maximum fitness with generation and average fitness with generation is shown in Figs. 5, 6. The optimum power coefficient is computed with wind a velocity of 10 m/s for NACA 4410, NACA 4420, NACA 4430 and NACA 4440 airfoils and the optimum power coefficient at various trails are shown in the Tables 2, 3, 4 and 5 respectively. The optimum power coefficient for various wind turbine profiles are shown in bold letters in the tables mentioned above.

Table 3 Optimum power coefficient for NACA 4420 at 10 m/s

Angle of attack (°)	Tip speed ratio	Power coefficient
1.34	7.123	0.252
7.15	6.584	0.334
8.87	4.617	0.373
6.74	4.309	0.310
6.58	8.138	0.433
7.36	5.471	0.361
6.13	**8.803**	**0.443**
7.32	7.053	0.372
9.28	4.876	0.373
4.43	6.934	0.356

Table 4 Optimum power coefficient for NACA 4430 at 10 m/s

Angle of attack (°)	Tip speed ratio	Power coefficient
7.31	4.267	0.266
8.11	4.813	0.299
3.95	6.227	0.237
6.74	**7.872**	**0.436**
7.91	6.255	0.289
9.10	4.736	0.310
7.29	6.857	0.297
6.50	7.830	0.333
8.50	4.234	0.302
5.56	7.234	0.332

Table 5 Optimum power coefficient for NACA 4440 at 10 m/s

Angle of attack (°)	Tip speed ratio	Power coefficient
2.08	6.787	0.135
5.04	7.823	0.180
9.03	3.504	0.221
7.88	**7.130**	**0.444**
6.39	4.610	0.197
9.30	4.547	0.247
8.64	5.457	0.227
8.94	5.492	0.240
5.86	8.747	0.417
4.34	6.346	0.397

Conclusions

A horizontal axis wind turbine of one MW capacity with NACA 4 series airfoils has been considered for optimization under three different categories. The airfoils are considered for evaluating the coefficients of lift and drag using proposed correlation and optimization of power coefficient. The proposed optimization method determined the optimum chord, twist angle, angle of attack, tip speed ratio and power coefficient at a particular wind velocity using GA. The coding for the optimization based on numerical methods have been developed in C++ programming language and tested. Various parameters and constraints related to optimization have been considered during testing. The maximum power coefficient is achieved as 0.45 for NACA 4410 at wind velocity of 10 m/s.

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