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ORIGINAL CONTRIBUTION

Optimal Design of Gravity-Fed Sewer Lines Using Linear Programming

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Abstract The cost of a sewerage system is mainly governed by the size of the sewer pipe, excavation depth and manhole spacing. A linear programming model is developed to minimize the total cost comprising of the pipeline cost, excavation cost and manhole cost of the sewer line. The constraints of the optimization model are related to the distance between two consecutive manholes, and slope of the sewer line to maintain the self-cleansing velocity. The nonlinearity due to the pipe size is eliminated by considering only those available diameters that satisfy the self-cleansing velocity constraint. The model selects the combination of pipe sizes and slope of the sewer line between different manholes maintaining the self-cleansing velocity, which results in the minimum value of the total cost of the entire sewer line. The application of the developed model is illustrated with the help of an existing design problem, and the results are compared with the available solution using forward recursive dynamic programming. It is found that the linear programming model results in lesser value of the total cost of the sewer line.

Keywords Optimal design · Sewer line · Linear programming · Manholes · Self-cleansing velocity

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List of Symbols

A	Area of cross section of sewer pipe
C_e	Excavation cost per unit volume of the sewer system between two manholes
C_m	Manhole cost per unit depth of upstream manhole
C_p	Pipe cost per unit length
C_{ijk}	Cost per meter length of the ith sewer link laid between jth upstream node and kth downstream node
d	Inside diameter of pipe
E_c	Excavation cost
ΔH_{ijk}	Elevation difference between jth upstream node and kth downstream node of ith link
i	Link number
j	Upstream node number for ith link
k	Downstream node number for ith link
L	Length of pipe between two consecutive manholes
L_i	Horizontal length between two manholes connected by link
M_c	Manhole cost
M_i	Manhole no. (i = 1, 2, 3,…)
n	Manning's roughness coefficient
nd(i)	Total number of downstream nodes in the ith link
nl	Total number of links in the sewer line
nu(i)	Total number of upstream nodes of the ith link
P	Wetted perimeter
P_c	Pipe cost
Q	Peak discharge through sewer pipe

R	Hydraulic mean radius of channel
S_0	Longitudinal slope of the sewer pipe between two consecutive manholes
T	Total cost of sewer line between two consecutive manholes
V	Velocity of flow in the pipe
V_e	Volume of excavation of the sewer system between two manholes
X_{ijk}	Length of ith sewer link laid between jth upstream node and kth downstream node
Y	Depth of upstream manhole for a link
y	Depth of flow in the sewer pipe
Z	Total cost of the sewer line
α_{ijk}	Angle of ith link laid between jth u/s node and kth d/s node
θ	Half of angle subtended at the center of the pipe by water surface

Introduction

A sewerage system involves the collection, treatment and disposal of the sewage. The sewage is collected from individual homes and carried to the treatment plant through a network of sewer pipes. The network of sewer pipes may consist of house sewers, main/trunk sewers and out fall sewers. Main sewer lines are generally provided with manholes at suitable interval for facilitating their cleaning and inspection. The sewer pipes are laid at downward gradient to carry sewage under gravity up to the outfall point. The design, construction, modification, operation and maintenance of sewerage systems involve a large capital. The sewer lines being the basic unit of a sewerage system, any saving in its cost due to proper design may affect the overall cost of the sewerage system. In spite of the growing concern for the urban environment and the future expenses and effort involved, conventional design methods for storm sewers fail to explicitly account for the cost interactions of the various components of storm sewer systems [18].

The earliest effort to consider hydraulic design of sewer lines was made by Camp [3]. Since then, a large number of research workers contributed to this subject. Some of the researchers employed heuristic methodologies [4, 5, 9, 10, 15, 17, 21]. Many researchers used optimization methods for design of sewer lines. Dynamic programming approach was used by Argaman et al. [2], Walsh and Brown [31], Mays and Yen [18], Mays and Wenzel [19], Gupta et al. [12] and Kulkarni and Khanna [14], to obtain optimal design of sewer networks. Gupta et al. [13] used Powell's method of conjugate directions to optimize the cost function. Swamee [29] applied the Lagrange multiplier method for minimization of the cost function. Afshar and Rohani [1] proposed a hybrid method by considering two sub-optimization problems.

Dajani et al. [6] addressed the problem using a separable convex programming model. Dajani and Hasit [7] compared three models based on separable-convex and mixed integer programming for the optimization of drainage networks. Dajani et al. [8] minimized the cost of a wastewater collection network using separable convex, dynamic and geometric programming. Swamee and Sharma [30] addressed the optimal design of the sewer system by incorporating commercially available pipe sizes directly in the linear programming model. In the recent past, different multi-objective optimization approaches have been developed mainly to minimize the pollution load in combined sewer water and treatment costs by Rathnayake and Tanyimboh [24–26] and Rathnayhke [22, 23].

Theoretical Considerations

Sewer pipes are designed to carry sewage such that the solid particles remain in suspension to avoid occurrence of clogging due to settlement of particles. The particles are kept in suspension by laying the sewer pipes at suitable gradients to achieve self-cleansing velocity at different possible discharges. A trunk sewer line collects sewage from branched sewer pipes at manholes causing variation of flow in its different sections. This requires use of different pipe sizes and gradients to maintain self-cleansing velocity from section to section. The cost of a trunk sewer line includes the costs of sewer pipes, excavation and manholes. The cost of sewer pipes depends on the size and material of the pipe. The cost of the excavation depends upon the gradient adopted, and the cost of the manhole depends upon the layout of sewer system, size and materials used. The aspects related to hydraulics of flow through sewer pipes and cost estimation are described in the following subsections.

Hydraulics of Sewer Pipe

The flow in a sewer pipe under gravity is considered as open-channel flow for designing the pipe size. The velocity of flow through a sewer pipe can be calculated using the Manning's formula as:

$$V = \frac{1}{n} R^{2/3} S_0^{1/2} \quad (1)$$

where V is the velocity of flow in the channel (m/s), n is the Manning's roughness coefficient, R is the hydraulic mean radius of channel (m) and S_0 is the longitudinal slope of the sewer pipe between two consecutive manholes. The hydraulic mean radius is calculated as ratio of area of cross section A (m^2) to wetted perimeter, P (m). The value of A can be calculated as [28]:

$$A = \frac{d^2}{8}(2\theta - \mathrm{Sin}2\theta) \tag{2}$$

where d is inside diameter of pipe (m) and 2θ is the angle (radian) subtended at the center of the pipe by water surface as shown in Fig. 1. The value of P can be calculated for any depth of flow y (m) as:

$$P = d\theta. \tag{3}$$

The discharge through a sewer pipe can be calculated as:

$$Q = VA \tag{4}$$

where Q is the peak discharge through the sewer pipe (m^3/s).

Using the value of θ as 2.094395 and for y as 0.75d, and arranging Eqs. (1), (2), (3) and (4), the value of d can be obtained as:

$$d = \left(3.5184\frac{n}{\sqrt{S_0}}Q\right)^{3/8} \tag{5}$$

The value of V should be such that neither the suspended particles in sewage get silted up nor the pipe material gets scoured out. Therefore, the velocity in the sewer pipe is maintained within a permissible range from 0.6 to 3 m/s [11].

Pipe Cost

The cost of a sewer pipe line can be calculated according to diameter and length as:

$$P_c = C_p L \tag{6}$$

where P_c is the pipe cost ($), C_p is the pipe cost per unit length ($/m) that depends upon the diameter and L is the length of pipe between two consecutive manholes (m).

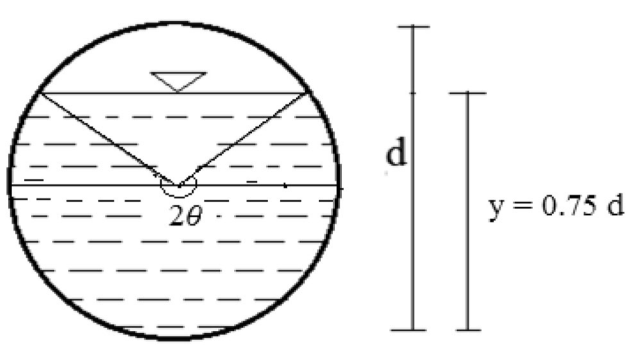

Fig. 1 Circular channel running partially full

Excavation Cost

The excavation cost based on the average depth (sum of pipe diameter and minimum safe excavation cover) and width of excavation can be calculated as:

$$E_c = C_e V_e \tag{7}$$

where E_c is the excavation cost ($), C_e is the excavation cost per unit volume of the sewer system between two manholes ($/m^3) and V_e is the volume of excavation of the sewer system between two manholes (m^3).

Manhole Cost

Manholes are generally provided at every bend, junction, change of gradient and change of sewer pipe size. The discharge between different pairs of two consecutive manholes may vary according to the addition of flow from lateral lines at the upstream manhole. Manhole cost of a sewer line between two consecutive manholes can be expressed as:

$$M_c = C_m Y \tag{8}$$

where M_c is the cost of manhole ($), C_m is the cost per unit depth of upstream manhole ($/m) and Y is the depth of upstream manhole (m).

Total Cost

The total cost of a sewer line between two consecutive manholes, T ($), can be expressed by combining Eqs. (6), (7) and (8) as:

$$T = P_c + E_c + M_c \tag{9}$$

Linear Programming Model

A linear programming model is formulated for minimizing the total cost of the sewer line. The sewer line is designed to carry the design discharge satisfying the requirements of the length, velocity and slope in different sections. The sewer line between two consecutive manholes is termed as a link and the manholes as nodes. The lengths of the available discrete pipe sizes satisfying the permissible values of the minimum and maximum velocities in each link are treated as decision variables. For each link, the sewer pipe can be laid at different slopes and accordingly, there will be as many possible diameters as the number of possible slopes to carry the design discharge for the selected pipe material. The preliminary value of diameter corresponding to each possible slope for each link is first calculated using Eq. (5) and then rounded to the next higher available pipe size. If the velocity for the rounded pipe size is within the permissible range, the diameter is selected as candidate diameter for the link. This process eliminates direct use of nonlinear cost functions in the optimization model.

Objective Function

The objective function of the linear programming model to minimize the total cost of the sewer line can be written as:

$$\text{Minimize } Z = \sum_{i=1}^{\text{nl}} \sum_{j=1}^{\text{nu}\,(i)} \sum_{k=1}^{\text{nd}\,(i)} C_{ijk} X_{ijk} \tag{10}$$

where Z is the optimal total cost of the sewer line ($), i is link number, nl is the total number of links in the sewer line, j is upstream node number of the ith link, nu(i) is the total number of upstream nodes of the ith link, k is the downstream node number of the ith link, nd(i) is the total number of downstream nodes of the ith link, C_{ijk} is the total cost per meter length of the ith sewer link laid between the jth and the kth nodes ($/m) and X_{ijk} is the length of the ith sewer link laid between the jth and the kth nodes (m). For each link, the combination of upstream and downstream nodes is chosen in such a way that downstream node is at a lower elevation. For each set of i, j, k, there will be a single diameter for which C_{ijk} is calculated from Eq. (9).

Constraints

The length constraint for each link is imposed by making the horizontal length between the two consecutive manholes equal to the sum of the horizontal equivalent of lengths of different candidate pipe sizes as:

$$\sum_{j=1}^{\text{nu}\,(i)} \sum_{k=1}^{\text{nd}(i)} X_{ijk} \text{Cos}\alpha_{ijk} = L_i \quad \forall,\ i = 1, nl \tag{11}$$

where L_i is the horizontal length between two manholes connected by link i and α_{ijk} is the angle in degree for X_{ijk} such that:

$$\tan \alpha_{ijk} = \frac{\Delta H_{ijk}}{L_i} \tag{12}$$

where ΔH_{ijk} is the elevation difference between jth upstream node and kth downstream node of the ith link. The non-negativity constraints of the decision variables are included as:

$$X_{ijk} \geq 0 \quad \forall\ i, j, k \tag{13}$$

Design Example

To illustrate the applicability of the proposed linear programming model, a design problem is taken for Goodwin Avenue sewer from Mays and Tung [20]. The data related to the length and peak design inflows for the sewer links named 1, 2 and 3 are given in Table 1. The sewer line along with the ground elevation of manholes and possible elevations of the u/s and d/s ends of different links considered between two consecutive manholes are shown in Fig. 2. The manholes are named M_1, M_2, M_3 and M_4, respectively, from upstream (u/s) to downstream (d/s). The minimum cover depth over the sewer pipe is 1.0668 m (3.5 ft). At each manhole, the crown elevation of the downstream pipe draining out the manhole is equal to or less than the crown elevation of the upstream pipe draining into the manhole. Three possible elevations are considered for both upstream and downstream ends of each sewer link between two consecutive manholes. Thus, total number of possible paths for each sewer link is nine. The difference between two consecutive possible elevations of the sewer link at upstream (or downstream) end is taken as 0.3048 m. The unit costs of excavation and manholes are $ 6.00/yd^3 and $ 100.00/ft depth, respectively. The average excavation width for all pipes is taken as 1.524 m (5 ft). The unit cost of available commercial pipe sizes is given in Table 2. The value of Manning's coefficient is assumed as 0.014 for all pipe sizes.

Table 1 Data related to Goodwin Avenue sewer line [20]

U/S–D/S manholes	Sewer link	Sewer length (m)	Peak inflow (m^3/s)
M_1–M_2	1	70.104	1.0423
M_2–M_3	2	48.768	1.1896
M_3–M_4	3	76.505	1.3312

Solution

The cost coefficients of Eq. (10) for different possible paths of each link of the sewer line are computed in Table 3. The slope for all possible paths of each link is taken using the elevation of the u/s and d/s ends. The values of slope and peak flow rate are used in Eq. (5) to calculate the value of diameter for all possible paths of each link. The pipe diameters are then rounded to just larger available pipe size, and their corresponding costs are taken from Table 2 to obtain the pipe cost from Eq. (6). To compute the excavation cost using Eq. (7), V_e is obtained by multiplying the average width of excavation with the depth of excavation (equal to sum of pipe diameter and minimum safe excavation cover). The computation of total cost of the sewer line using Eq. (9) is given in Table 3. The diameters given in Table 3 for each link are satisfying the permissible velocity limits. Further, the cases of adverse slope (i.e., d/s crown elevation higher than u/s) are considered infeasible as represented by dashed lines in

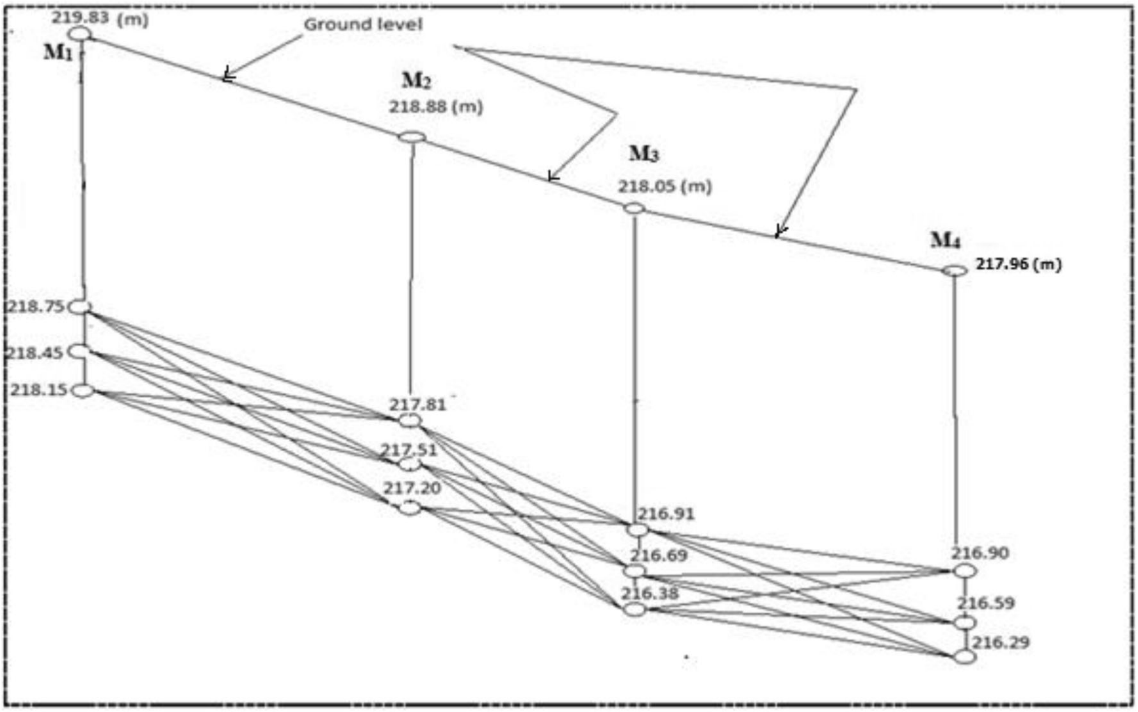

Fig. 2 Schematic diagram showing elevations and possible paths of sewer links

Table 2 Cost of different pipe sizes of sewer line

Pipe size (cm)	30.48	38.1	45.72	53.34	60.96	68.58	76.2	91.44	106.68	121.92	137.16
Cost ($/m)	11.155	14.6	19.357	24.279	30.184	36.254	46.588	62.5	82.021	127.461	129.43

Table 3. Therefore, the cost per unit length of the links corresponding to adverse slope is taken as a large number B in Table 3 using the concept of big-M method [27]. The values of total cost per unit length and slope from Table 3 with B as 2×10^{20} and the lengths from Table 1 are substituted in the linear programming model (Eqs. 10–13) for all the three links resulting in Eqs. (14–20).

$$\begin{aligned} \text{Minimize } Z = {} & 77.0227X_{111} + 67.2390X_{112} + 69.017X_{113} \\ & + 98.77205X_{121} + 82.0906X_{122} \\ & + 72.31086X_{123} + 101.9619X_{131} \\ & + 103.7878X_{132} + 87.1625X_{133} + 80.6871X_{211} \\ & + 70.7543X_{212} + 72.5769X_{213} + 103.3010X_{221} \\ & + 86.3701X_{222} + 76.4373X_{223} \\ & + 129.5241X_{231} + 108.9844X_{232} \\ & + 92.05303X_{233} + 169.0491X_{311} \\ & + 118.5110X_{312} + 98.3365X_{313} \\ & + 2 \times 10^{20}X_{321} + 174.0015X_{322} \\ & + 123.4636X_{323} + 2 \times 10^{20}X_{331} \\ & + 2 \times 10^{20}X_{332} + 178.9475X_{333} \end{aligned} \tag{14}$$

subject to:

$$\begin{aligned} & 0.9999X_{111} + 0.9998X_{112} + 0.99989X_{113} + 0.9999X_{121} \\ & \quad + 0.9998X_{122} + 0.9997X_{123} + 0.9999X_{131} + 0.9998X_{132} \\ & \quad + 0.9998X_{133} \\ & \quad = 70.104 \end{aligned} \tag{15}$$

$$\begin{aligned} & 0.9998X_{211} + 0.9997X_{212} + 0.9996X_{213} + 0.9999X_{221} \\ & \quad + 0.9998X_{222} + 0.9997X_{223} + 0.9999X_{231} + 0.9999X_{232} \\ & \quad + 0.9998X_{233} \\ & \quad = 49.0728 \end{aligned} \tag{16}$$

$$\begin{aligned} & 0.9999X_{311} + 0.9999X_{312} + 0.9999X_{313} + 0.0X_{321} \\ & \quad + 0.9999X_{322} + 0.9999X_{323} + 0.0X_{331} + 0.0X_{332} \\ & \quad + 0.9999X_{333} \\ & \quad = 76.5048 \end{aligned} \tag{17}$$

$$X_{111}, X_{112}, X_{113}, X_{121}, X_{122}, X_{123}, X_{131}, X_{132}, X_{133} \geq 0 \tag{18}$$

$$X_{211}, X_{212}, X_{213}, X_{221}, X_{222}, X_{223}, X_{231}, X_{232}, X_{233} \geq 0 \tag{19}$$

$$X_{311}, X_{312}, X_{313}, X_{321}, X_{322}, X_{323}, X_{331}, X_{332}, X_{333} \geq 0 \tag{20}$$

Table 3 Computation of cost of sewer system

Links no.	U/s manhole	D/s manhole	U/s elevation	Crown elevation u/s (m)	D/s elevation	Crown elevation d/s (m)	Slope m/m	Diameter (cm)	Pipe cost (*P*) ($)	Excavation cost (*E*) at u/s manhole ($)	U/s manhole cost (*M*) ($)	Total cost (*T*) ($)	Cost/ length
I	M_1	M_2	1	218.75	1	217.81	0.01341	76.20	3266.00	1533.60	600.00	5399.60	77.0227
			1	218.75	2	217.51	0.01768	68.58	2541.50	1597.22	575.00	4713.72	67.2390
			1	218.75	3	217.20	0.02211	68.58	2541.50	1735.00	575.00	4841.50	69.0617
			2	218.45	1	217.81	0.00913	91.44	4381.50	1788.89	750.00	6920.39	98.7205
			2	218.45	2	217.51	0.01341	76.20	3266.00	1788.89	700.00	5754.89	82.0906
			2	218.45	3	217.20	0.01768	68.58	2541.50	1852.78	675.00	5069.28	72.3109
			3	218.15	1	217.81	0.00485	91.44	4881.50	1916.66	850.00	7148.16	101.9619
			3	218.15	2	217.51	0.00913	91.44	4381.50	2044.44	850.00	7275.94	103.7878
			3	218.15	3	217.20	0.01355	76.20	3266.00	2044.44	800.00	6110.40	87.1620
II	M_2	M_3	1	217.81	1	216.99	0.01671	76.20	2286.20	1073.33	600.00	3959.53	80.6871
			1	217.81	2	216.69	0.02282	68.58	1779.05	1118.06	575.00	3472.11	70.7543
			1	217.81	3	216.38	0.02914	68.58	1779.05	1207.50	575.00	3561.55	72.5769
			2	217.51	1	216.99	0.01060	91.44	3067.05	1252.22	750.00	5069.27	103.3010
			2	217.51	2	216.69	0.01671	76.20	2286.20	1252.22	700.00	4238.42	86.3701
			2	217.51	3	216.38	0.02303	68.58	1779.05	1296.94	675.00	3750.99	76.4373
			3	217.20	1	216.99	0.00428	106.68	4025.00	1431.11	900.00	6356.11	129.5241
			3	217.20	2	216.69	0.01040	91.44	3067.05	1431.11	850.00	5348.16	108.9844
			3	217.20	3	216.38	0.01671	76.20	2286.20	1431.11	800.00	4517.31	92.05303
III	M_3	M_4	1	216.99	1	216.90	0.00118	137.16	9901.95	2231.12	800.00	12,933.07	169.0491
			1	216.99	2	216.59	0.00523	106.68	6275.00	2091.66	700.00	9066.66	118.5110
			1	216.99	3	216.29	0.00915	91.44	4781.55	2091.66	650.00	7523.21	98.3365
			2	216.69	1	216.90	Adverse	–	–	–	–	–	*B*
			2	216.69	2	216.59	0.00131	137.16	9901.95	2510.00	900.00	13,311.95	174.0015
			2	216.69	3	216.29	0.00523	106.68	6275.00	2370.56	800.00	9445.56	123.4636
			3	216.38	1	216.90	Adverse	–	–	–	–	–	*B*
			3	216.38	2	216.59	Adverse	–	–	–	–	–	*B*
			3	216.38	3	216.29	0.00118	137.16	9901.95	2788.88	1000.00	13,690.33	178.9475

Optimal Design

The optimization model formulated as per Eqs. 14–20 is solved using LINGO 8.0 software [16] in demo mode. Table 4 shows the optimal lengths for the first, second and third links as 70.115 m, 49.086 m and 76.508 m having diameters as 68.52 cm (27 inch), 68.58 cm (27 inch) and 91.44 cm (36 inch) with their corresponding paths as 1–2, 1–2 and 1–3, respectively. The optimal cost of sewer line is obtained as $ 15,712.25. The optimal solution of the same problem obtained by Mays and Tung [20] using the forward recursive dynamic programming optimization technique is also given in Table 4. The comparison of the optimal results shows that the linear programming model resulted in lesser cost of the sewer line with the same values of optimal diameters due to different optimal path obtained for link II.

Effect of Slope on Optimal Results

The developed linear programming model is solved for two additional sets of slopes, set I and set II, as given in Table 5. The set I and set II of slopes are taken by lowering the downstream elevation of each link by 0.1524 m (half a foot) and 0.3048 m (one foot), respectively, from the values for the case shown in Fig. 2. The computations of total cost for set I and set II of slopes of the sewer line are given in Tables 6 and 7, respectively. The optimal solutions for these two sets of slopes using the developed linear programming model are given in Table 8. On comparing the results of Tables 4 and 8, it is observed that the increase in optimal costs for set I and set II of slopes is $ 253.53 and $ 491.64, respectively, showing nonlinear nature of increase. Further, the results show there is no change in optimal diameters and optimal lengths for both the sets of slope whereas there is change in optimal path of link III for the

Table 4 Comparison of results by linear programming and dynamic programming models

S.no.	Optimal results from linear programming model				Optimal results from dynamic programming model			
Link no.	Optimal length (m)	Optimal path	Optimal diameter (cm)	Optimal cost ($)	Optimal length (m)	Optimal path	Optimal diameter (cm)	Optimal cost ($)
I	70.115	1–2	68.58	15,712.25	70.104	1–2	68.58	16,882.35
II	49.086	1–2	68.58		49.073	2–3	68.58	
III	76.508	1–3	91.44		76.104	1–3	91.44	

Table 5 Set of slopes for sensitivity analysis of sewer line design

Set of slopes	Path	Link I		Link II		Link III	
		U/S elevation	D/S elevation	U/S elevation	D/S elevation	U/S elevation	D/S elevation
I	1–1	218.75	217.66	217.66	216.84	216.84	216.90
	1–2	218.75	217.35	217.66	216.53	216.84	216.59
	1–3	218.75	217.05	217.66	216.23	216.84	216.29
	2–1	218.45	217.66	217.35	216.84	216.53	216.90
	2–2	218.45	217.35	217.35	216.53	216.53	216.59
	2–3	218.45	217.05	217.35	216.23	216.53	216.29
	3–1	218.15	217.66	217.05	216.84	216.23	216.90
	3–2	218.15	217.35	217.05	216.53	216.23	216.59
	3–3	218.15	217.05	217.05	216.23	216.23	216.29
II	1–1	218.75	217.51	217.51	216.68	216.68	216.59
	1–2	218.75	217.20	217.51	216.38	216.68	216.29
	1–3	218.75	216.90	217.51	216.07	216.68	215.98
	2–1	218.45	217.51	217.20	216.68	216.38	216.59
	2–2	218.45	217.20	217.20	216.38	216.38	216.29
	2–3	218.45	216.90	217.20	216.07	216.38	215.98
	3–1	218.15	217.51	216.90	216.68	216.07	216.59
	3–2	218.15	217.20	216.90	216.38	216.07	216.29
	3–3	218.15	216.90	216.90	216.07	216.07	215.98

Table 6 Computation of cost of sewer system for set I of slopes

Links no.	U/s manhole	D/s manhole	U/p elevation	Crown elevation u/s (m)	D/s elevation	Crown elevation d/s (m)	Slope m/m	Diameter (cm)	Pipe cost (*P*) ($)	Excavation cost (*E*) at u/s manhole ($)	U/s manhole cost (*M*) ($)	Total cost (*T*) ($)	Cost/ length
I	M_1	M_2	1	218.75	1	217.66	0.01555	76.20	3266.00	1597.22	600.00	5463.22	77.930
			1	218.75	2	217.35	0.01998	68.58	2541.50	1661.11	575.00	4777.61	68.151
			1	218.75	3	217.05	0.02425	68.58	2541.50	1788.89	575.00	4905.39	69.973
			2	218.45	1	217.66	0.01127	76.20	3266.00	1725.00	700.00	5691.00	81.179
			2	218.45	2	217.35	0.01569	76.20	3266.00	1852.78	700.00	5818.78	83.002
			2	218.45	3	217.05	0.01997	68.58	2541.50	1916.67	675.00	5133.17	73.222
			3	218.15	1	217.66	0.00699	91.44	4881.50	1980.56	850.00	7712.06	110.009
			3	218.15	2	217.35	0.01141	76.20	3266.00	1980.56	850.00	6096.56	86.965
			3	218.15	3	217.05	0.01569	76.20	3266.00	2108.33	800.00	6174.33	88.074
II	M_2	M_3	1	217.66	1	216.84	0.01671	76.20	2286.20	1162.78	650.00	4098.98	83.529
			1	217.66	2	216.53	0.02303	68.58	1779.05	1207.50	625.00	3611.55	73.596
			1	217.66	3	216.23	0.02914	68.58	1779.05	1296.94	625.00	3640.99	74.196
			2	217.35	1	216.84	0.01039	76.20	3067.05	1252.22	750.00	5069.27	103.301
			2	217.35	2	216.53	0.01671	76.20	2286.20	1341.67	750.00	4377.87	89.117
			2	217.35	3	216.23	0.02283	68.58	1779.05	1386.39	725.00	3890.44	79.279
			3	217.05	1	216.84	0.00428	91.44	3067.05	1431.11	900.00	5398.16	110.003
			3	217.05	2	216.53	0.01060	76.20	2286.20	1431.11	850.00	4567.31	93.072
			3	217.05	3	216.23	0.01671	76.20	2286.20	1520.56	850.00	4656.76	94.895
III	M_3	M_4	1	216.84	1	216.90	0.00131	121.92	7743.35	2231.11	800.00	10,774.46	140.834
			1	216.84	2	216.59	0.00523	91.44	4781.55	2091.67	700.00	7573.22	98.991
			1	216.84	3	216.29	0.00928	91.44	4781.55	2231.11	700.00	7712.66	100.813
			2	216.53	1	216.90	adverse	–	–	–	–	–	*B*
			2	216.53	2	216.59	0.00118	121.92	7743.35	2510.00	900.00	11,153.35	145.786
			2	216.53	3	216.29	0.00523	91.44	4781.55	2370.56	750.00	7902.11	103.289
			3	216.23	1	216.90	Adverse	–	–	–	–	–	*B*
			3	216.23	2	216.59	Adverse	–	–	–	–	–	*B*
			3	216.23	3	216.29	0.00131	121.92	7743.35	2788.89	1000.00	11,532.24	150.739

Table 7 Computation of cost of sewer line for set II of slopes

Link no.	U/s manhole	D/s manhole	U/p elevation	Crown elevation u/s (m)	D/s elevation	Crown elevation d/s (m)	Slope m/m	Diameter (cm)	Pipe cost (*P*) ($)	Excavation cost (*E*) at u/s manhole ($)	U/s manhole cost (*M*) ($)	Total cost (*T*) ($)	Cost/ length
I	M_1	M_2	1	218.75	1	217.51	0.01769	68.58	2541.50	1597.26	575.00	4713.52	67.2256
			1	218.75	2	217.20	0.02211	68.58	2541.50	1981.04	575.00	5097.57	72.6968
			1	218.75	3	216.90	0.02625	68.58	2541.50	1853.72	575.00	4970.22	70.8735
			2	218.45	1	217.51	0.01341	76.20	3266.00	1788.05	700.00	5754.05	82.0717
			2	218.45	2	217.20	0.01783	68.58	2541.50	1853.07	675.00	50,769.57	72.3036
			2	218.45	3	216.90	0.02211	68.58	2541.50	1981.036	675.00	5197.54	74.1224
			3	218.15	1	217.51	0.00913	91.44	4381.68	2044.54	850.00	7276.036	103.7849
			3	218.15	2	217.20	0.01355	76.20	3266.00	2044.62	800.00	6110.62	87.1576
			3	218.15	3	216.90	0.01783	68.58	2541.5	2108.66	775.00	5425.16	77.3752
2	M_2	M_3	1	217.51	1	216.69	0.01692	76.20	2286.20	1252.40	700.00	4238.60	86.3611
			1	217.51	2	216.38	0.02303	68.58	1779.05	1297.29	675.00	3751.16	76.4202
			1	217.51	3	216.07	0.029344	68.58	1779.05	1386.98	675.00	3841.03	78.2383
			2	217.20	1	216.68	0.01060	91.44	3067.05	1431.19	850.00	5348.24	108.9788
			2	217.20	2	216.38	0.01671	76.20	2286.20	1431.32	800.00	4517.52	92.0441
			2	217.20	3	216.07	0.02303	68.58	1779.05	1476.23	775.00	4030.28	82.1283
			3	216.90	1	216.68	0.00448	106.68	4025.00	1610.02	1000.00	6635.016	135.1988
			3	216.90	2	216.38	0.01060	91.44	3067.05	1610.09	950.00	5633.14	114.7841
			3	216.90	3	216.07	0.01692	76.20	2286.20	1610.23	900.00	4796.43	97.7268
3	M_3	M_4	1	216.68	1	216.59	0.00118	121.92	7743.35	2370.556	850.00	10,963.906	143.3100
			1	216.68	2	216.29	0.005098	106.68	6275.08	2370.588	900.00	11,013.94	143.9543
			1	216.68	3	215.98	0.00915	91.44	4781.55	2370.66	850.00	8002.21	104.45904
			2	216.38	1	216.59	Adverse	–	–	–	–	–	*B*
			2	216.38	2	216.29	0.00118	121.92	7743.35	2649.44	950.00	11,342.79	148.2625
			2	216.38	3	215.98	0.00523	106.68	6275.08	2649.48	1000.00	11,392.83	148.9064
			3	216.07	1	216.59	Adverse	–	–	–	–	–	*B*
			3	216.07	2	216.29	Adverse	–	–	–	–	–	*B*
			3	216.07	3	215.98	0.00118	121.92	7743.35	2928.33	1050.00	11,721.68	153.2019

Table 8 Optimal solutions for two sets of slopes

Set of slopes	Link	Diameter	Paths	U/S elevation	D/S elevation	Length (m)	Total cost ($)
I	I	68.58	1–2	218.75	217.35	70.12504	15,965.78
	II	68.58	1–2	217.66	216.53	49.08753	
	III	91.44	1–2	216.84	216.59	76.51245	
II	I	68.58	1–2	218.75	217.20	70.118	16,457.42
	II	68.58	1–2	217.51	216.38	49.088	
	III	91.44	1–3	216.68	215.98	76.528	

slopes of set II. However, the optimal cost of the sewer line is increased due to higher excavation cost at larger slope. This indicates that the optimal solution in terms of length and diameter of the sewer line may or may not be affected due to change in slope.

Conclusions

The developed linear programming model can be used to minimize the total cost comprising of pipe cost, excavation cost and manhole cost of a sewer line connecting a series of manholes. The developed model results in optimal values of pipe diameter and slope from different alternatives for carrying the desired discharge and satisfying the constraints of self-cleansing velocity and lengths of sewer line between each pair of successive manholes. The optimal solution of the developed linear programming model is found economical over the optimal solution obtained by forward recursive dynamic programming model. The optimal cost of the sewer line increases nonlinearly with an increase in slope. The developed model can also be used to obtain the optimal range of discharge for an existing sewer line. The developed linear programming model will be useful for wastewater management.

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