Vibration of Stiffened Plates with Cutout Subjected to Partial Edge Loading

A. K. L. Srivastava

Abstract The buckling and vibration characteristics of stiffened plates with cutout subjected to in-plane partial edge loadings at the plate boundary are studied using finite element method. Buckling loads and vibration frequencies are determined for different plate and cutout aspect ratios, various boundary conditions, partial edge loading at different locations, cutout ratios, various parameters of stiffeners by varying the number, size and location of the stiffeners. The analysis presented determines the stresses all over the region for different kinds of loading and edge conditions. In the structural modelling, the plate and the stiffeners are treated as separate elements where the compatibility between these two types of elements is maintained. The buckling and vibration characteristics are discussed and the free vibration results available in the literature for stiffened plates with/without cutout are compared.

List of symbols

a	Plate dimension in longitudinal direction
b	Plate dimension in the transverse direction
c	Width extent of partial edge loading at the boundary
t	Plate thickness
E, G	Young's and shear moduli for the plate material

b_s, d_s	Web thickness and depth of a x-stiffener
A_s	Cross-sectional area of the stiffener
I_s	Second moment of area of the stiffener cross-section about reference axis
$[D_P]$	Rigidity matrix of plate
$[D_S]$	Rigidity matrix of stiffener
$[K_e]$, $[K_S]$	Stiffness matrix of plate, stiffness matrix of stiffener
$[M_p]$, $[M_S]$	Consistent mass matrix of plate, stiffener
$[K_G]$	Geometric stiffness matrix
$[N]_r$	Matrix of a shape function of a node r
P_{cr}	Critical buckling load
g, d	Cutout length, cutout width
g/d	Cutout width ratio
T_S, P_S	Torsional constant, polar moment of inertia of the stiffener element
ω, λ	Frequency parameter, buckling parameter
D	Plate flexural rigidity
ρ	Density of the plate material

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Introduction

Stiffened plates are structural components consisting of plates reinforced by a system of ribs to enhance their load carrying capacities. The hull of a ship, its deck and superstructure, a road bridge and a launching pedestal of rocket are examples of stiffened plate structures. Stiffened plates with cutouts in aerospace, civil, mechanical and marine structures are inevitable mainly for practical and design considerations. In many circumstances these structures are found to be exposed to in-plane loading. The applied load is seldom uniform and the boundary conditions may be completely arbitrary in practice. The buckling and vibration characteristics of stiffened plates with cutout

subjected to uniform and non-uniform in-plane edge loadings are of considerable importance. Aircraft wing skin panels, which are made of thin sheets, are usually subjected to non-uniform in-plane stresses caused by concentrated or partial edge loading at the edges, and due to panel stiffener support conditions.

The buckling and vibration characteristics of stiffened plates with cutouts pose a tremendous challenge and must be properly understood in the design of such structures. The study of buckling and vibration characteristics of stiffened plate structures with or without cutout subjected to concentrated and partial edge loadings at the plate boundary are of interest for the present work.

Large number of references in the published literature deal with the buckling, vibration behaviour of stiffened plates with/without cutout subjected to in-plane uniform loading. The problem becomes complicated when the loading is non-uniformly distributed over the edges of the plate boundary. A relatively less number of investigations deal with the buckling and vibration problems under in-plane non-uniform edge loadings.

Literature Review

Vibration and buckling calculations for rectangular plates subjected to non-uniform in-plane stress distribution have been studied by different investigators [1–3].

A number of investigation do exist on buckling under uni-axial uniform loads of stiffened plates having longitudinal equispaced stiffeners, mostly based on shear deformation theories under different classical boundary conditions. In these studies, however, a uniformly distributed load has been treated under relatively simple boundary condition. The effects of stiffener location on vibration and buckling characteristics have been discussed.

The analysis has been performed earlier on the plate subjected to locally distributed in-plane loading over a finite length of the edge at the center of two opposite edges [4]. The researchers have earlier analyzed the stability of rectangular plates subjected to a pair of patch loading at the center of two opposite edges by using Ritz method [5].

The investigators have solved the problem of buckling of simply supported rectangular and circular plates subjected to a pair of oppositely directed concentrated compressive and tensile loads anywhere on two opposite edges [6]. The vibration of flat thin plates subjected to complex middle force system under arbitrary stress system based on Kirchoff's theory is studied by earlier researchers [7].

The procedure has been followed to obtain the buckling loads for simply supported rectangular plates subjected to a partially distributed normal edge loading at the center or near the two ends of the opposite edge, or by shear loading

along the two longitudinal (with respect to applied load) edges [8]. The researchers have applied yet another approach, called the conjugate load displacement method to the problem of buckling of plates under concentrated edge loading [9].

The vibration characteristics of rectangular plates subjected to non-uniform loading has been studied using power series method by earlier investigators [10]. Vibration and dynamic stability of stiffened plates subjected to in-plane uniform harmonic edge loading has been studied using the finite element analysis by previous researchers [11].

A finite element analysis of a clamped thin plate with different cutout sizes, along with experiments has been carried out by the earlier investigators [12] using holographic interferometry.

The researchers have developed the finite difference method for obtaining the natural frequencies and mode shapes for rectangular plates, of varying stiffnesses causing re-entrant corners, by assuming average curvature at the corners [13].

The scientists have studied the natural frequencies of simply supported rectangular plates with rectangular cutouts using the Rayleigh–Ritz method [14].

The researchers have also studied the vibration of square plates with square cutouts by using boundary characteristics orthogonal polynomials satisfying the boundary conditions [15].

It has also been studied the vibration of a rectangular plate by dividing the total domain into smaller areas and using a modified form of Rayleigh–Ritz method. The same method has been employed by the researchers [16] to study the vibrations of plates with stiffened openings using orthogonal polynomials and partitioning method.

Prediction of natural frequencies of rectangular plates with rectangular cutouts has been done by earlier investigators [17]. In this the assumed deflection functions are no longer continuous throughout the whole plate. The deflections are, however, made to satisfy all or part of the internal free edge condition along the four edges of the cutout. The researchers have investigated dynamic stability behaviour of curved panels with cutouts [18].

However, little attention has been given on buckling, vibration behaviour of stiffened plates with/without cutout subjected to non-uniform loading of practical interest such as concentrated and patch loading. The vibration and static stability analysis of stiffened plates with/without cutout subjected to partial loading has been sparsely treated and can be extended to practically important patch loading under different boundary conditions. The authors could not find any work in the literature on the buckling and vibration characteristics of stiffened plate with cutout subjected to non-uniform in-plane edge loading at the plate boundary. Thus the study of buckling and vibration analysis of stiffened plate with cutout subjected to partial edge loading is

new. In the earlier researches, the authors have analyzed the buckling and vibration characteristics of stiffened plates with cutout subjected to uniform edge loading at the plate boundary.

The present work deals with the buckling and vibration analysis of eccentrically stiffened plates with cutout subjected to central partial edge loading. 'The finite element technique is adopted to study the effects of different boundary conditions, aspect ratios of plate and cutout, various parameters of stiffened plates, cutout size, position of partial edge loadings and their extent. In the present analysis, the plate is modelled with the nine noded isoparametric quadratic element with five degrees of freedom, where the contributions of bending and membrane actions are taken into account. The formulation of the stiffener is done in such a manner so that it may lie anywhere within a plate element [19].

In order to maintain the compatibility between the plate and the stiffener, the interpolation functions used for the plate are used for the stiffener as well.

Proposed Finite Element

The plate skin and the stiffeners are modelled as separate elements but the compatibility between them is maintained. Vibration and buckling of rectangular stiffened plates subjected to uniform edge loading have been studied by many investigators [20–25] in details. The nine noded isoparametric quadratic elements with five degrees of freedom (u, v, w, θ_X and θ_y) per node have been employed. A study of the stiffness and mass matrices of the stiffener element reveals that the contribution of the beam element is reflected in all nine nodes of the plate element, which contains the stiffener. The contribution of the stiffener to a particular node depends on the proximity of the stiffener to that node. The element matrices for the plate and stiffener are generated separately and then added up to form overall matrices.

The strain components of the plate can be written as:

$$\{\varepsilon\} = \sum [B_p]_r\{q\}_r = [B_p]\{q\} \tag{1}$$

where

$$[B_P] = \begin{bmatrix} [B_P]_1 & [B_P]_2 & \dots & [B_P]_r & \dots & [B_P]_9 \end{bmatrix} \tag{2}$$

Using the isoparametric coordinates, the element stiffness matrix is expressed as

$$[K_b]_p = \int\limits_{-1}^{+1}\int\limits_{-1}^{+1} [B_p]^T [D_p][B_p]|J_p|d\xi\, d\eta \tag{3}$$

The element mass matrix can be expressed in isoparametric coordinate as

$$[M_p]_e = \int\limits_{-1}^{1}\int\limits_{-1}^{1} [N]^T [m_p][N]|J_p|d\xi\, d\eta \tag{4}$$

where

$$[m_p] = \begin{bmatrix} \rho t & 0 & 0 & 0 & 0 \\ 0 & \rho t & 0 & 0 & 0 \\ 0 & 0 & \rho t & 0 & 0 \\ 0 & 0 & 0 & \frac{\rho t^3}{12} & 0 \\ 0 & 0 & 0 & 0 & \frac{\rho t^3}{12} \end{bmatrix} \tag{5}$$

For buckling analysis the bending strains developed due to the action of in-plane loads are considered.

The strain vector can be expressed as:

$$\{\varepsilon_P\} = \sum_{r=1}^{9} [B_{GP}]_r\{q\}_r \tag{6}$$

where

$$[B_{GP}] = \begin{bmatrix} [B_{GP}]_1 & [B_{GP}]_2 & \dots & [B_{GP}]_r & \dots & [B_{GP}]_9 \end{bmatrix} \tag{7}$$

When expressed in iso-parametric coordinates the geometric stiffness matrix can be expressed as:

$$[K_G]_P = \int\limits_{-1}^{+1}\int\limits_{-1}^{+1} [B_{Gp}]^T[\sigma_P][B_{Gp}]|J_p|d\xi\, d\eta \tag{8}$$

The element matrix for the stiffener element can be found out following the steps similar to those of plate. Similar to the plate element, the elastic stiffness matrix $[K_S]$, geometric stiffness matrix $[K_{GS}]$ and mass matrix $[M_S]$ of a stiffener element placed anywhere within a plate element and oriented in the direction of x may be derived as follows:

The strain is given by

$$\{\overline{\varepsilon_S}\} = \sum_{r=1}^{9} [B_S]_r\{\delta\}_r = [B_S]\{\delta\} \tag{9}$$

where

$$[B_S] = \begin{bmatrix} [B_S]_1 & [B_S]_2 & \dots & [B_S]_r & \dots & [B_S]_9 \end{bmatrix} \tag{10}$$

Element stiffness matrix for stiffener

$$[K_S] = \int\limits_{-1}^{+1} [B_S]^T[D_S][B_S]|J_S|d\xi, \tag{11}$$

$$[D_S] = \begin{bmatrix} EA_S & EF_S & 0 & 0 \\ EF_S & ES_S & 0 & 0 \\ 0 & 0 & GT_S & 0 \\ 0 & 0 & 0 & GA_S/1.2 \end{bmatrix} \tag{12}$$

The element mass matrix can be expressed in iso-parametric co-ordinate as

$$[M_S] = \int\limits_{-1}^{1} [N]^T [m_S][N]|J|d\xi \qquad (13)$$

The geometric stiffness matrix of the stiffener element can be determined similar to those of the plate. The geometric stiffness of the stiffener element can be expressed in isoparametric co-ordinate as:

$$[K_G]_S = \int\limits_{-1}^{+1} [B_{GS}]^T [\sigma_S][B_{GS}]|J_S|d\xi \qquad (14)$$

The equation of equilibrium for vibration of a structure subjected to in-plane loads can be written in matrix form as

$$[M]\{\ddot{q}\} + [[K_b] - P(t)[K_G]]\{q\} = 0 \qquad (15)$$

Results and Discussion

The problem considered here consists of a rectangular plate (a × b) with longitudinal stiffeners having a central rectangular cutout of size (g × d) subjected to harmonic in-plane partial edge loading at the plate boundary as shown in Fig. 1. The stiffener cross-section is shown in Fig. 2. To study the vibration and buckling behaviour of stiffened plates with cutout under general non-uniformly distributed in-plane edge loading, rectangular stiffened plates with different types of edge loading are considered as shown in Fig. 3. In Fig. 3a, b, the plates are subjected to a pair of partial loads on the two opposite edges, which are distributed over a part of the loaded edges of length c. The partial edge loadings have different locations relative to the ends or center of the loaded edges in different cases.

The non-dimensionalisation of different parameters like vibration, buckling is taken as: Non-dimensional load, $\lambda = Pb/D$ and Buckling load parameter ($\lambda = N_X b^2/\pi^2 D$).

Non-dimensional frequency, $\varpi = \omega b^2 \sqrt{\rho t/D}$, where D is the plate flexural rigidity, $D = Et^3/12(1 - v^2)$, P is the applied load, P_{cr} is the buckling load, ρ is the density of the plate material and t is the plate thickness. Assuming a general case of several longitudinal stiffener at a distance (D_x) from the edge y = 0, the stiffener parameter terms δ and γ are defined as: $\delta = A_S/bt$ = Ratio of cross-sectional area of the stiffener to the plate, where A_S is the area of the stiffener. $\gamma = EI_S/bD$ = Ratio of bending stiffness rigidity of stiffener to the plate, where I_S is the second moment of area of the stiffener cross-section about reference axis.

Convergence and Comparison with Previous Results

The accuracy and efficiency of the present finite element formulation are established through comparison of frequency parameters of simply supported rectangular plates
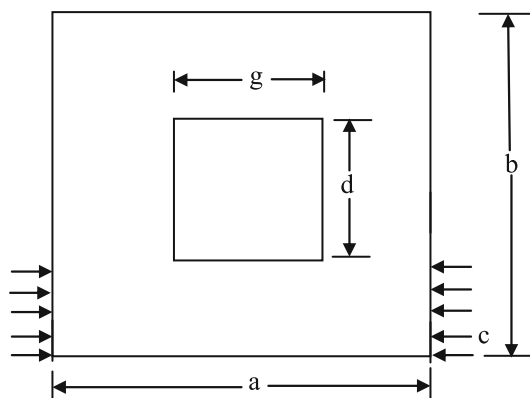

Fig. 1 Stiffened plate with cutout under in-plane partial edge loading at one end at plate boundary
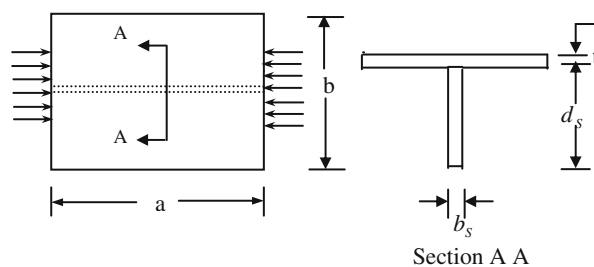

Fig. 2 Stiffened plate cross-section

with different cutout sizes with the numerical scheme using orthogonal polynomial functions by Lam and Hung [16], the numerical method based on Rayleigh quotient by the prior investigations [17] and are presented in Table 1. Good agreement exists between the present FEM results with those in the literature. In Table 2, the natural frequency results obtained by the present method for different cutout sizes are compared with earlier researches as studied the vibration of square plates with square cutouts by using boundary characteristics orthogonal polynomials satisfying the boundary conditions [15].

The non-dimensional fundamental frequencies of simply supported isotropic square plates with concentric square cutouts as obtained are presented in Table 3 and compared with some investigators and found in good agreement.

Partial Edge Loading at One End

The influence of various parameters on buckling, vibration and dynamic stability characteristics of stiffened plates with cutout subjected to partial edge load at one end are studied. The data used for its geometry are as follows: b = 600 mm, t = 1 mm, b_S = 3.31 mm, d_S = 20.25 mm, v = 0.34, E = 6.87 × 10⁴ N/mm², ρ = 2.78 × 10⁻⁶ kg/mm³.

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Fig. 3 **a** Partial edge loading at both end. **b** Partial edge load at the centre
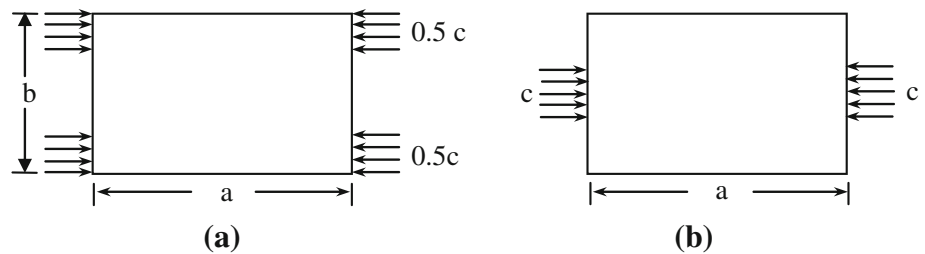
Table 1 Non-dimensional frequencies for simply supported square plate with central rectangular cutout

Non-dimensional frequency parameter (ω)

Cutout size g × d	Lam and Hung [16]	Lee et al. [17]	Present
g/d = 1			
0.0a × 0.0a	19.752	19.739	19.739
0.2a × 0.2a	19.120	18.901	19.125
0.4a × 0.4a	20.732	20.556	20.742
0.6a × 0.6a	28.241	28.491	28.249
0.8a × 0.8a	57.452	58.847	57.829
g/d = 2			
0.4a × 0.2a	19.008	18.980	19.07
0.8a × 0.4a	23.583	23.807	23.58
g/d = 3			
0.6a × 0.2a	18.981	19.117	18.984

a/b = 1, b/t = 100, ν = 0.3, $\omega = \varpi b^2 \sqrt{\rho t/D}$

Table 2 Comparison of natural frequency parameter of simply supported and clamped plates

Frequency parameter (ω)

g/a	SSSS		CCCC	
	Ref. [15]	Present	Ref. [15]	Present
0.0	19.739	19.732	35.985	35.981
0.167	20.070	19.87	37.425	36.06
0.33	20.9633	20.12	43.867	43.02
0.5	24.2434	24.24	65.715	65.27

Table 3 Non-dimensional fundamental frequencies for simply supported isotropic square plate with square cutout

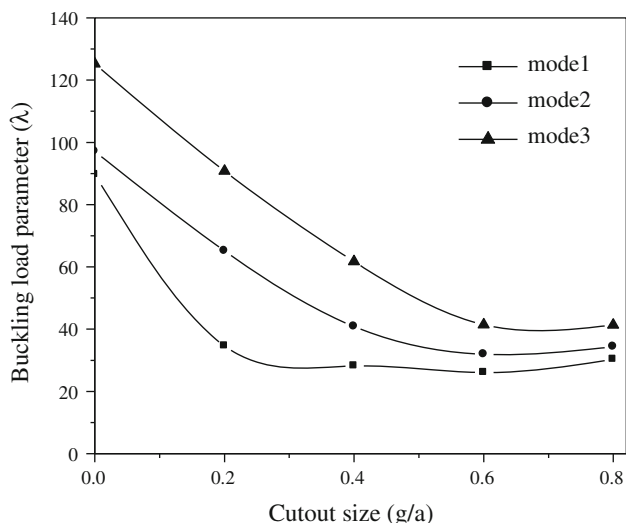| Cutout size (g/a)	Frequency parameter (ω)						
	Ali and Atwal [14]		Reddy [27]	Lee et al. [17]	Laura et al. [26]	Nayak [28]	Present
	FEM	Rayleigh–Ritz method	FEM	Rayleigh-method	Rayleigh–Ritz method	FEM	FEM
0.0	19.816	19.737	19.752	19.739	19.739	19.740	19.737
0.2	18.446	19.274	19.220	18.901	19.634	19.127	19.278
0.4	20.650	20.705	20.807	20.556	21.152	20.752	20.712
0.5	23.241	23.364	23.515	23.329	23.842	23.452	23.368
0.6	28.445	27.857	28.453	28.491		28.373	27.864
0.8	57.706	58.885	57.512	58.846		57.414	58.128

the edge restraint effect decreases and consequently the buckling load decreases. However as the load width (c/b) increases from 0.4 to uniform full width (c/b = 1), no appreciable variation is noticed.

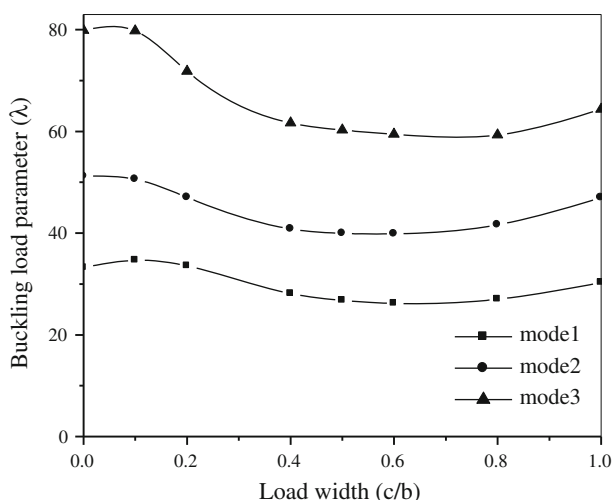### Partial Edge Load at Both Ends

The influence of various parameters on buckling, vibration and dynamic stability characteristics of stiffened plates with cutout subjected to partial edge load at both ends are studied. The data used for its geometry are as follows: b = 600 mm, d_S = 20.25 mm, t = 1 mm, b_S = 3.31 mm, ν = 0.34, E = 6.87 × 10⁴ N/mm², ρ = 2.78 × 10⁻⁶ Kg/mm³.

Vibration Studies of Stiffened Plates with Cutout

Numerical results for vibration frequencies are presented for stiffened square plate having one central stiffener subjected to in-plane partial edge load at both ends for simply supported and clamped edge conditions, for the cases (i) with central square cutout of different sizes (g/a = 0.2, 0.4, 0.6) for load width (c/b = 0.4) in Fig. 6 and (ii) for various values of load width (c/b = 0.2, 0.4, 0.6, 0.8, 1) with cutout size ratio (g/a = 0.4) in Fig. 7. It is observed from Fig. 6 that the vibration frequencies increase with the increase of cutout size (g/a) for simply supported case. On the other hand, reverse trend of

Buckling Studies of Stiffened Plates with Cutout

Numerical results for buckling load parameter are presented for simply supported stiffened square plate having one central stiffener in three higher modes for the cases:

(i) Various central square cutout sizes (g/a = 0.2, 0.4, 0.6, 0.8) and partial edge load width (c/b = 0.4) at one end in Fig. 4 and (ii) For cutout of size (g/a = 0.4) at various load width (c/b = 0.2, 0.4, 0.5, 0.6, 0.8) in Fig. 5.

It is observed from Fig. 4 that buckling resistance decreases as cutout size increases in all the modes. It is observed from Fig. 5 that buckling resistance decreases as the load bandwidth (c/b) increases from 0 to 0.4. This can be explained by the fact that as the load width increases,
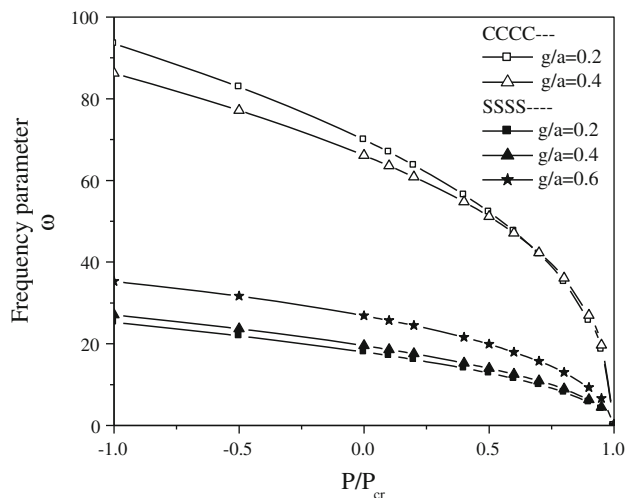
Fig. 4 Effect of cutout (g/a) on buckling load parameter for simply supported stiffened plate with cutout having one central stiffener subjected to partial edge load at one end. a/b = 1, c/b = 0.4
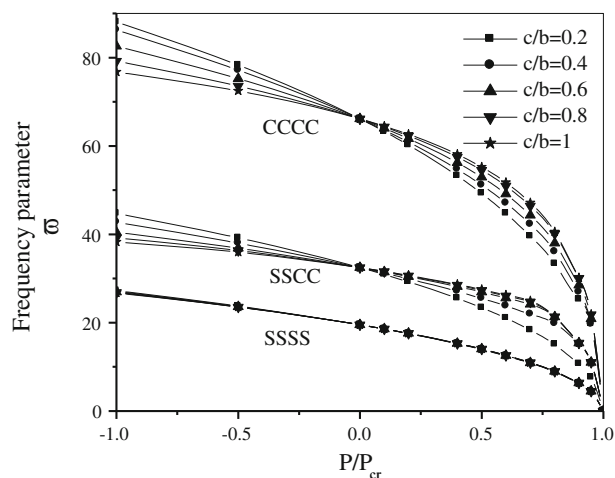

Fig. 6 Variation of frequency parameter versus P/P$_{cr}$ of stiffened plate with cutout sizes (g/a) having one central stiffener subjected to partial edge load at both ends. The edges are SSSS and CCCC. c/b = 0.4

Fig. 5 Effect of load width (c/b) on buckling load parameter with load width (c/b) for simply supported stiffened plate with cutout having one central stiffener subjected to partial edge load at one end. g/a = 0.4, a/b = 1

Fig. 7 Variation of frequency parameter versus P/P$_{cr}$ for load width (c/b) of stiffened plate with cutout having one central stiffener subjected to partial edge load at both ends. g/a = 0.4

variation of vibration frequency with cutout size is observed for clamped case.

It is observed from Fig. 7 that frequency decreases with the increase of P/P$_{cr}$ and it becomes zero at critical buckling. At P/P$_{cr}$ = 0, it becomes fundamental frequency. The vibration frequencies for clamped edge condition are found to be more than the edge conditions SSSS and SSCC. It is observed that the vibration frequencies increase as the load width increase for boundary conditions (SSSS, SSCC, CCCC). It is also observed here that for simply supported edge condition, this increase is not very appreciable.

Conclusion

The investigations of the buckling, vibration behaviour of stiffened plates with cutout, subjected to partial edge loadings have been carried out using the finite element formulation. Conclusions based on the results and discussions are summarized below.

The stability resistance increases with the increase of restraint at the edges for all types of loading, stiffener parameters and plate aspect ratios. The stability resistance increases with the increase of number of stiffeners. Natural frequencies of stiffened plates always decrease with the increase of the in-plane compressive load. The fundamental

frequency becomes zero at the respective values of the buckling load.

For partial edge load at one end, the vibration frequencies decrease with the increase of load width. The buckling load parameter of stiffened plate with cutout reduces with increase in the size of the hole. The frequency parameter values increase with the increase of cutout size. The rate of increase of frequencies also increases with increase in the size of the cutout. The effect of cutout is to increase the vibration frequency of stiffened plate in almost all the cases. The trend is different for clamped case subjected to partial edge load at both ends. For partial edge load at both ends, the vibration frequencies increase as the load width increases. For simply supported edge condition, this increase is not very appreciable. For partial edge load at the center, the buckling resistance increases as the load width increases. The buckling strength for clamped edges increases faster than simply supported edges. For partial edge load at one end and both ends, the vibration frequencies increase as the load width increases for the different boundary conditions.

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