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RESEARCH ARTICLE

Control of Chaos to Obtain Periodic Behaviour via Nonlinear Control

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Abstract In this paper, we propose a scheme for controlling chaotic Lorenz like system to a periodic system via nonlinear control. Our method is based on Lyapunov stability theory. We have presented the numerical simulation results to show the efficiency of our method.

Keywords Chaos · Chaos control · Chaos synchronization · Lyapunov stability

Introduction

Chaos has been observed in many nonlinear systems ([1, 2]). Experimental observations have pointed out that chaotic systems are common in nature but still unpredictable due to its sensitive dependence on initial conditions. Chaos is found in meteorology, the solar system, heart and brain of living organisms and so on [3]. Experimental realization of chaos synchronization and control have been achieved with a magnetoelastic ribbon, a heart, a thermal convection loop, a diode oscillator, an optical multimode chaotic solid-state laser, a Belousov-Zhabotinski reaction diffusion chemical system and many other experiments. Everyday examples of chaotic systems include weather and climate. Lorenz and Poincare ([1, 2]) were early pioneers of chaos theory. Chaotic system has a dense set of unstable periodic orbits. Control of chaos is the stabilization, by means of small system perturbations, of one of these unstable periodic orbits.

Chaos control and synchronization of non linear dynamics have been possible [4, 5]. Hunt [6] has studied high-period orbits in a chaotic system. Pyrages [7] has discussed continuous control of chaos by self-controlling feedback. Fuh and Tung [8] have studied controlling chaos using differential geometric method. Huang [9] has analysed controlling chaos through growth rate adjustment. Sinha [10] has introduced threshold mechanism for controlling chaos. Using backstepping design method Umut [11] has studied controlling chaos in nuclear spin generator system.

Several different regimes of chaos synchronization [12, 13], e.g. generalized synchronization [14, 15], phase synchronization [16] and lag synchronizations [17, 18], anti-synchronization [19, 20], adaptive synchronization [21] and a coupled n-dimensional time-delay system [22] of chaotic oscillators have been theoretically investigated and experimentally observed.

Control of chaotic system to a periodic system has many application in biology and engineering which motivated us to study this problem. In this paper, we have taken the modified chaotic Lorenz model proposed by Das [23] and introduce a scheme for obtaining periodic behaviour from chaotic system. Numerical simulation results are presented to show the efficiency of our method.

Synchronization via nonlinear control

Das [23] obtained the following Lorenz like model for a large Prandtl number convection problem

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$$\left.\begin{aligned} \dot{x_1} &= \sigma(-x_1+y_1)+sx_2,\\ \dot{x_2} &= \sigma(-x_2+y_2)+sx_1,\\ \dot{y_1} &= -y_1+(r-z)x_1+tx_2,\\ \dot{y_2} &= -y_2+(r-z)x_2+tx_1,\\ \dot{s} &= -\tfrac{10}{3}\sigma s+\tfrac{3}{5}\sigma t-\tfrac{3}{10}x_1x_2,\\ \dot{t} &= -\tfrac{10}{3}t+rs-\tfrac{1}{4}(x_1y_2+x_2y_1),\\ \dot{z} &= -bz+x_1y_1+x_2y_2, \end{aligned}\right\} \tag{1}$$

where x_1, x_2 represent the vertical velocity and y_1, y_2 represent the temperature field, s and t represent the coupling between two sets of mutually perpendicular rolls and z denotes the heat flux across the fluid layer. This system has chaotic dynamics for $\sigma = 7, r = 16$ and $b = \frac{8}{3}$. Our aim is to synchronize the dynamical system (1) to the following periodic system (where $u_1, u_2, u_3, u_4, u_5, u_6$ have periodic behavior with time)

$$\left.\begin{aligned} \dot{u_1} &= u_2,\\ \dot{u_2} &= -u_1,\\ \dot{u_3} &= u_4,\\ \dot{u_4} &= -u_3,\\ \dot{u_5} &= u_6,\\ \dot{u_6} &= -u_5,\\ \dot{u_7} &= -u_7. \end{aligned}\right\} \tag{2}$$

The dynamical systems (1) and (2) are uncoupled, therefore to synchronize the system (1) to the system (2) we have to couple the systems by suitable coupling functions w_i. Here w_i may be functions of $x_1, x_2, y_1, y_2, s, t, z$ and $u_1, u_2, u_3, u_4, u_5, u_6, u_7$. We coupled the systems (1) and (2) in the following manner

$$\left.\begin{aligned} \dot{x_1} &= \sigma(-x_1+y_1)+sx_2+w_1,\\ \dot{x_2} &= \sigma(-x_2+y_2)+sx_1+w_2,\\ \dot{y_1} &= -y_1+(r-z)x_1+tx_2+w_3,\\ \dot{y_2} &= -y_2+(r-z)x_2+tx_1+w_4,\\ \dot{s} &= -\tfrac{10}{3}\sigma s+\tfrac{3}{5}\sigma t-\tfrac{3}{10}x_1x_2+w_5,\\ \dot{t} &= -\tfrac{10}{3}t+rs-\tfrac{1}{4}(x_1y_2+x_2y_1)+w_6,\\ \dot{z} &= -bz+x_1y_1+x_2y_2+w_7, \end{aligned}\right\} \tag{3}$$

and

$$\left.\begin{aligned} \dot{u_1} &= u_2,\\ \dot{u_2} &= -u_1,\\ \dot{u_3} &= u_4,\\ \dot{u_4} &= -u_3,\\ \dot{u_5} &= u_6,\\ \dot{u_6} &= -u_5,\\ \dot{u_7} &= -u_7. \end{aligned}\right\} \tag{4}$$

Let us define the synchronization error between the system (3) and (4) as $e_1 = x_1 - u_1, e_2 = x_2 - u_2, e_3 = y_1 - u_3, e_4 = y_2 - u_4, e_5 = s - u_5, e_6 = t - u_6, e_7 = z - u_7$, then dynamical system of the synchronization error will follow the following differential equations

$$\left.\begin{aligned} \dot{e_1} &= -\sigma e_1+\sigma(y_1-u_1)+sx_2-u_2+w_1,\\ \dot{e_2} &= -\sigma e_2+\sigma(y_2-u_2)+sx_1+u_1+w_2,\\ \dot{e_3} &= -e_3-u_3+(r-z)x_1+tx_2-u_4+w_3,\\ \dot{e_4} &= -e_4-u_4+(r-z)x_2+tx_1+u_3+w_4,\\ \dot{e_5} &= -\tfrac{10}{3}\sigma(e_5+u_5)+\tfrac{3}{5}\sigma t-\tfrac{3}{10}x_1x_2-u_6+w_5,\\ \dot{e_6} &= -\tfrac{10}{3}(e_6+u_6)+rs-\tfrac{1}{4}(x_1y_2+x_2y_1)+u_5+w_6,\\ \dot{e_7} &= -b(e_7+u_7)+x_1y_1+x_2y_2+w_7. \end{aligned}\right\} \tag{5}$$

Now our aim is to find suitable controllers $W = (w_1, w_2, w_3, w_4, w_5, w_6, w_7)^T$ such that the fixed point $(0,0,0,0,0,0,0)$ of the error system become globally asymptotically stable. If $lim_{t\to\infty} e_i = 0, i = 1,2,3,4,5,6,7,$ then the system (3) and (4) will synchronize identically. Now we choose controllers as

$$\left.\begin{aligned} w_1 &= -\sigma(y_1-u_1)-sx_2+u_2,\\ w_2 &= -\sigma(y_2-u_2)-sx_1-u_1,\\ w_3 &= u_3-(r-z)x_1-tx_2+u_4,\\ w_4 &= u_4-(r-z)x_2-tx_1-u_3,\\ w_5 &= \tfrac{10}{3}\sigma u_5-\tfrac{3}{5}\sigma t+\tfrac{3}{10}x_1x_2+u_6,\\ w_6 &= \tfrac{10}{3}u_6-rs+\tfrac{1}{4}(x_1y_2+x_2y_1)-u_5,\\ w_7 &= bu_7-x_1y_1-x_2y_2. \end{aligned}\right\} \tag{6}$$

Now we choose Lyapunov function for the error system as $L = \frac{1}{2}(e_1^2+e_2^2+e_3^2+e_4^2+e_5^2+e_6^2+e_7^2)$ then $\frac{dL}{dt} = (e_1\dot{e_1} + e_2\dot{e_2} + e_3\dot{e_3} + e_4\dot{e_4} + e_5\dot{e_5} + e_6\dot{e_6} + e_7\dot{e_7}) = -\sigma e_1^2 - \sigma e_2^2 - e_3^2 - e_4^2 - \frac{10}{3}\sigma e_5^2 - \frac{10}{3}\sigma e_6^2 - be_7^2 < 0$ since $\sigma > 0, b > 0$. Therefore, by Lyapunov stability theory error system is asymptotically stable at the origin.

Then the controlled Lorenz like system and periodic system obey the following dynamical system

$$\left.\begin{aligned} \dot{x_1} &= \sigma(u_1-x_1)+u_2,\\ \dot{x_2} &= \sigma(u_2-x_2)-u_1,\\ \dot{y_1} &= -y_1+u_3+u_4,\\ \dot{y_2} &= -y_2+u_4-u_3,\\ \dot{s} &= -\tfrac{10}{3}\sigma s-\tfrac{10}{3}\sigma u_5+u_6,\\ \dot{t} &= -\tfrac{10}{3}t+\tfrac{10}{3}u_6-u_5,\\ \dot{z} &= -bz+bu_7,\\ \dot{u_1} &= u_2,\\ \dot{u_2} &= -u_1,\\ \dot{u_3} &= u_4,\\ \dot{u_4} &= -u_3,\\ \dot{u_5} &= u_6,\\ \dot{u_6} &= -u_5,\\ \dot{u_7} &= -u_7. \end{aligned}\right\} \tag{7}$$

It is observed after numerical simulation with the above choice of controller the Lorenz like dynamical system synchronized to a periodic system.

Results and Discussions

To show that feasibility and effectiveness of this method, numerical simulations are carried out by fourth order Runge-Kutta method. Time step is taken as 0.005. Values of the parameters are taken as $\sigma = 7.0, r = 16.0, b = 8/3$. We have chosen the initial conditions as $x_1(0) = 0.01, x_2(0) = 0.01, \quad y_1(0) = 0.01, y_2(0) = 0.20, s(0) = 0.30, \quad t(0) = 0.25, z(0) = 0.20$ and $u_1(0) = u_2(0) = u_3(0) = u_4(0) = u_5(0) = u_6(0) = u_7(0) = 0.01$ for coupled system and $x_1(0) = 0.01, \quad x_2(0) = 0.01, y_1(0) = 0.01, y_2(0) = 0.20, s(0) = 0.30, t(0) = 0.25, z(0) = 0.20$ for uncoupled system. From our scheme it is clear that except the variable z of the coupled Lorenz system all the variables synchronized to a periodic system under control. Here we draw figures for some of the variables. In Figs. 1 and 2 the time evolution of x_1 is plotted for the uncoupled system and the coupled system with control respectively. Figs. 3 and 4 show that the time evaluation of s in the uncoupled system and the coupled system with control respectively. It is clear from the figures that the controlled trajectories are periodic. The phase diagrams of x_2 vs. s are shown in the uncoupled system and coupled system with control respectively in Figs. 5 and 6. Figures 7 and 8 show the phase diagram of y_2 vs. s in the uncoupled system and coupled system with control respectively. Figures of phase diagram prove that the controlled system is periodic. Therefore we have successfully controlled a chaotic system to a periodic system via nonlinear control.

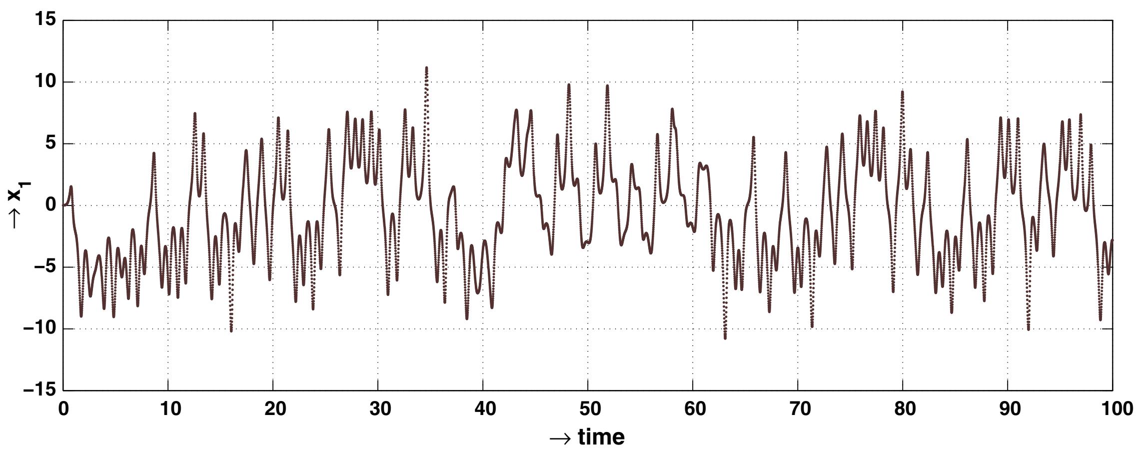

Fig. 1 Time evolution of x_1 in the uncoupled system for the parameters $\sigma = 7.0, r = 16.0$ and $b = 8/3$ with same initial conditions

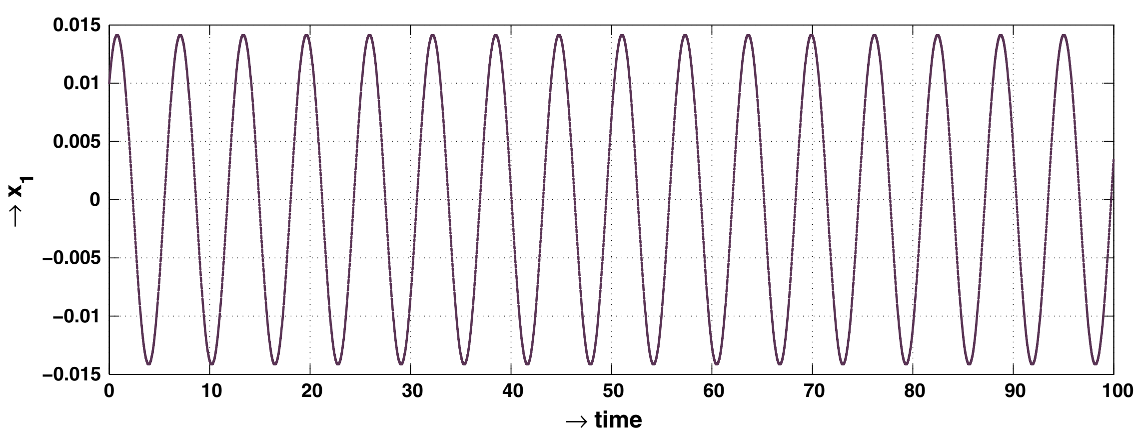

Fig. 2 Time evolution of x_1 in the coupled system with control for the parameters $\sigma = 7.0, r = 16.0$ and $b = 8/3$ with same initial conditions

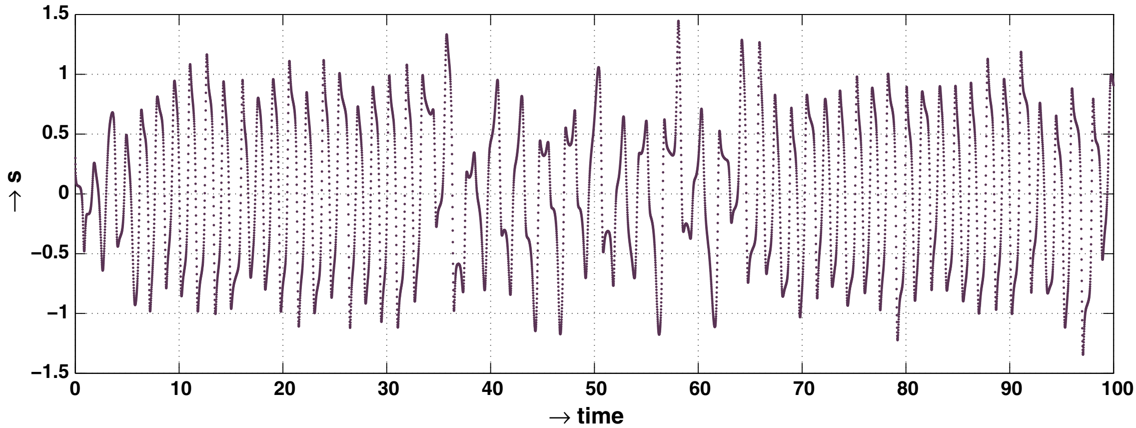

Fig. 3 Time evolution of s in the uncoupled system for the parameters $\sigma = 7.0, r = 16.0$ and $b = 8/3$ with same initial conditions

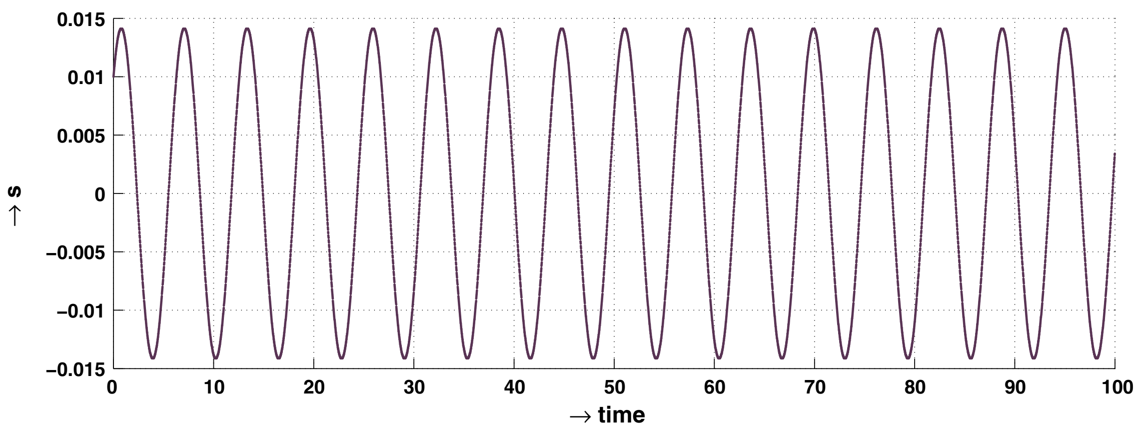

Fig. 4 Time evolution of s in the coupled system with control for the parameters $\sigma = 7.0, r = 16.0$ and $b = 8/3$ with same initial conditions

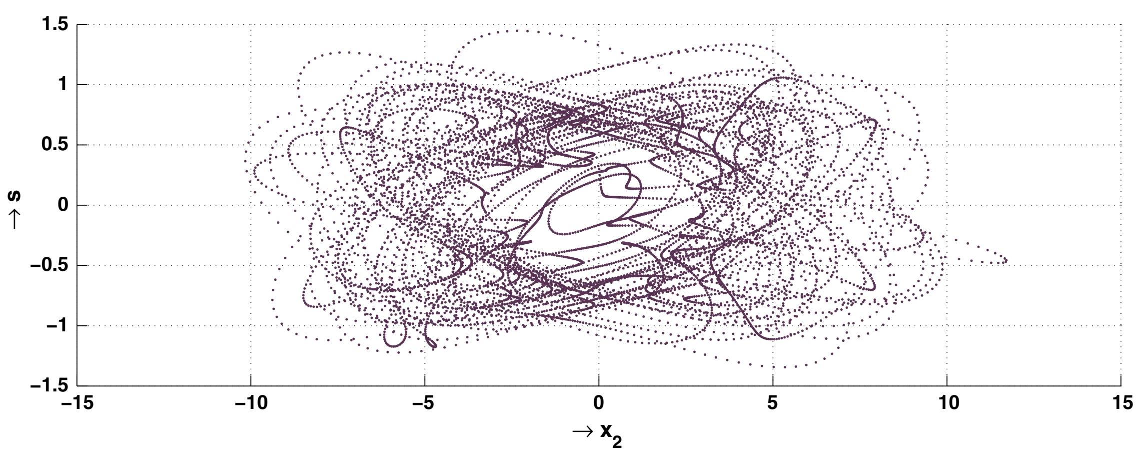

Fig. 5 Phase diagram x_2 versus s without control for the parameters $\sigma = 7.0, r = 16.0$ and $b = 8/3$ with same initial conditions

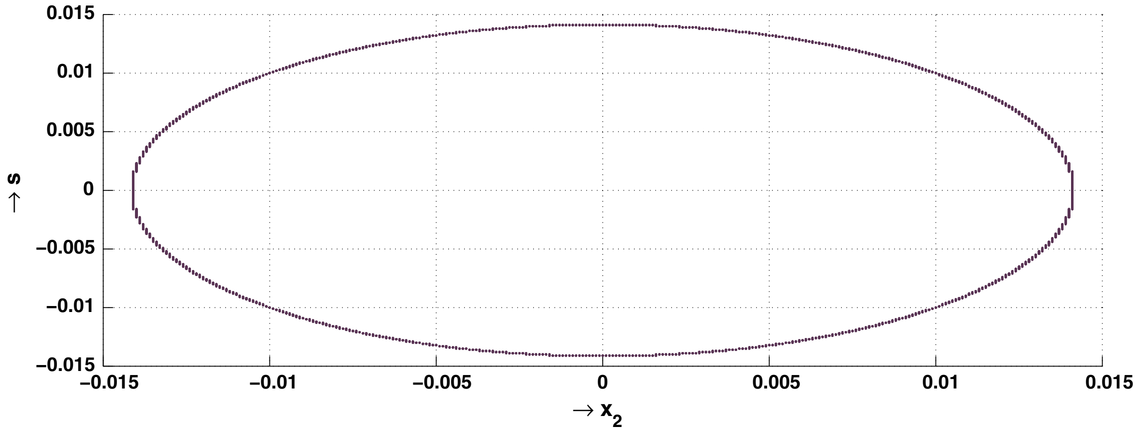

Fig. 6 Phase diagram x_2 versus s with control for the parameters $\sigma = 7.0, r = 16.0$ and $b = 8/3$ with same initial conditions

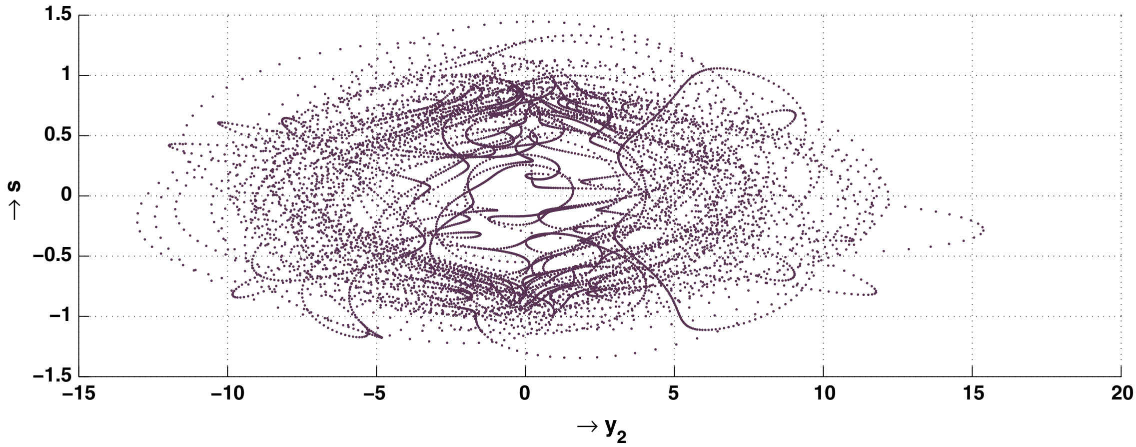

Fig. 7 Phase diagram y_2 versus s without control for the parameters $\sigma = 7.0, r = 16.0$ and $b = 8/3$ with same initial conditions

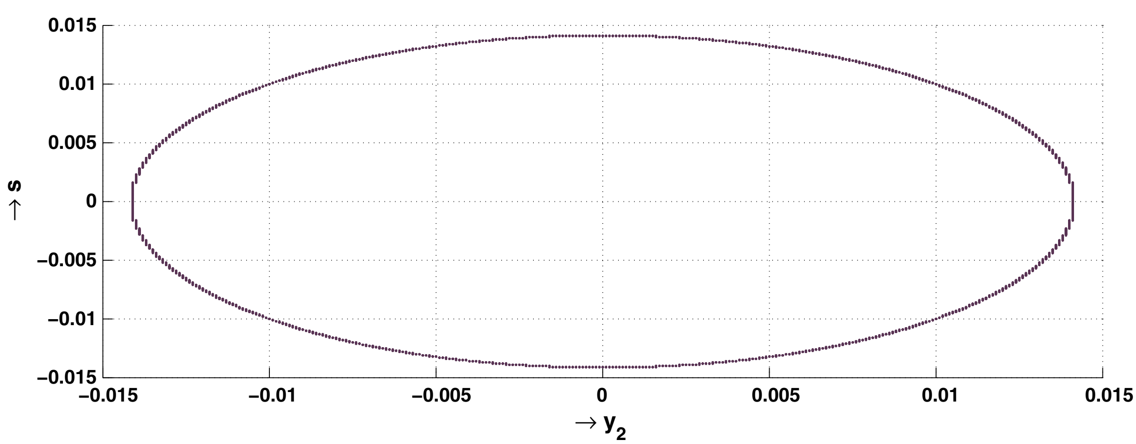

Fig. 8 Phase diagram y_2 versus s with control for the parameters $\sigma = 7.0, r = 16.0$ and $b = 8/3$ with same initial conditions

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