



Prediction of Distribution of Dry Matter and Leaf Area of Faba Bean (*Vicia faba*) Using Nonlinear Regression Models

Najibullah Ebrahimi¹ · Ahmad Reza Salihy¹ · Sabqatullah Alipour¹ ·
Sayed Hamidullah Mozafari¹ · Jawad Aliyar² · Ibrahim Darwish³

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Abstract Growth analysis is a valuable method for quantitatively investigating the growth and development of products. To analyze plant growth during the growing season, access to accurate and regular plant information is needed, which is obtained by measuring leaf surface and dry matter accumulation. The use of nonlinear regression models is expanding due to having parameters with physiological meaning in growth analysis. Of these models, there are beta, logistic, Gomperts, Richards, linear, cut and symmetric linear models. Therefore, this study was conducted on bean plant of the variety “Barakt” under factorial experiment in the form of basic randomized complete block design with four crop densities in four replications under rainfed conditions at the research farm of Gorgan University of Agricultural Sciences and Natural Resources in 2014–2015, located in the west of Gorgan, with a latitude of 37° and 45 min north and a longitude of 54° and 30 min east and an altitude of 120 m above sea level. In this study, the nonlinear beta and logistic regression models were fitted to leaf surface data, and beta, Gompertz and logistic models were fitted to bean dry weight. The AICc criterion analysis showed that the beta model had a better fit than the logistic model for leaf area. According to this model under various crop densities, LAI_{max} was between 2.30 and 5.30 g per square meter, t_m was from 131.90 to 144.20 days after planting, and t_c was between 158.7 and 163.50 days. Also, the analysis of the AICc criterion for dry matter accumulation showed that the beta model was better in fitting the dry matter accumulation than Gomperts and logistic models. According to this model, W_{max} varied between 1.725 and 1484.3 g per square meter, t_m between 138.30 and 146.40 days after planting, and t_c between 162.60 and 179.0 days in different densities.

Keywords Analysis of growth · Faba Bean · Dry matter weight · Leaf area · Model

Introduction

Faba Bean (*Vicia faba* L.) belonging to the Fabaceae family. And it is one of the major legumes in the Middle East region, which has received a lot of attention for human

and livestock nutrition due to the presence of 23.4% protein in the dry seeds of this plant [3, 29].

Growth analysis is considered a valuable method to quantitatively investigate the growth, development and production of agricultural plants [2, 11], which can be used to justify and interpret the reaction of plants to different conditions during the growth period [25].

Analysis of plant growth is a useful quantitative method to describe plant system performance and understand biological problems. To analyze plant growth during the growing season, access to accurate and regular plant information is needed, and these data will be obtained by measuring leaf surface and dry matter accumulation. Leaf area index and dry matter are two main components of growth analysis through which parameters of growth

✉ Najibullah Ebrahimi
najebullahebrahimi@gmail.com

¹ Department of Agronomy, Faculty of Agriculture, Ghazni University, Ghazni, Afghanistan

² Department of Chemistry, Faculty of Education, Ghazni University, Ghazni, Afghanistan

³ Department of Biology, Faculty of Education, Ghazni University, Ghazni, Afghanistan

analysis such as crop growth rate (CGR), relative growth rate (RGR), net absorption rate (NAR) and leaf area duration (LAD) can be calculated [17, 31].

Leaf area is a key variable for phytological studies including plant growth, light absorption, photosynthetic efficiency, evapotranspiration and plant response to fertilizers and irrigation [6]. Therefore, leaf area strongly affects growth and production, and investigating its changes over time is one of the essential components of crop growth models [16]. The accumulation of dry matter in the aerial parts of the plant is one of the other variables through which the parameters of growth analysis can be determined, so in various investigations, it is absolutely necessary to consider the factors affecting the production and accumulation of dry matter and the relationship between them.

In this way, the identification of growth components is of special importance. On the other hand, there is a strong relationship between the increase in leaf area index with the amount of absorbed solar radiation and finally the production of dry matter [4, 22]. The slow development of the leaf surface will cause poor development of vegetation and less absorption of radiation, which will eventually reduce the growth rate of the product and then result in a decrease in yield [14].

There are two methods for quantifying growth analysis parameters: classical method and regression method. In the classical method of growth analysis, growth parameters are values estimated in the average time interval between two samplings. The equations that are used for calculating these parameters in the interval between two samplings were obtained through polynomial equations and integrating the formulas and then dividing by the time between two samplings [1, 8, 11, 19].

In the regression method, regression models (linear and nonlinear) are used for determining growth parameters. In this method, the regression equations of the dry matter data were fitted, and the coefficients of these equations had a physiological meaning and indicate growth parameters [11, 20, 21, 28]. But one defect that existed in the parameters of linear regression models (such as the quadratic equation) was that they did not have a special meaning from a physiological point of view [1, 2, 11, 19, 31].

Nonlinear regression models including logistic, Gompertz, Richards, linear exponential, cut linear exponential, symmetric linear exponential and beta models have been used for growth analysis [5, 8, 10–12, 15, 18, 23, 28]. In these tests, nonlinear regression models have been used for different treatments, and the treatments could be compared by estimating these parameters. This study was conducted in order for evaluating different nonlinear regression models to investigate the changes in leaf area index and dry

matter and to estimate parameters related to growth analysis.

These models are fitted to different treatments in the experiment, the parameters obtained from them, and the treatments could be compared with the help of parameter estimation. This study was conducted in order to collect and introduce nonlinear regression models (along with SAS programs to fit them) for use in growth analysis studies [8]. In the study on wheat, different non-regression models were used to estimate leaf area and dry matter accumulation [7, 13, 24]. An investigation for estimating leaf area and dry matter accumulation in saffron by the use of nonlinear regression models were done [27] and they used different regression models for this purpose.

Materials and Methods

This experiment was conducted in the research farm of Gorgan University of Agricultural Sciences and Natural Resources, located in the west of Gorgan, with latitude 37° 45 min north and longitude 54° 30 min east and an altitude of 120 m above sea level with an average rainfall of 607 mm, an average temperature of 13° Celsius and fluctuating a temperature of 10 °C was implemented in the 2014–2015 crop year. The land used for the experiment has soil with silty clay loam texture. This experiment was carried out on the bean plant “Barakt” in the form of a factorial experiment in the form of a basic design of randomized complete blocks (FRCBD) in four replications. The factors of the experiment include planting date (December 6, December 4 and February 11) and density (5, 15, 25 and 35 plants per square meters).

The seeds were planted manually with a fixed row spacing of 50 cm and a depth of 5 cm. At the time of planting, 107.14 kg/ha of potassium phosphate, 107.14 kg/ha of phosphorus and 35.71 kg/ha of urea fertilizers were added to the soil. The fight against pests, diseases and weeds was carried out when necessary.

Sampling was done from all the plots from 6 to 8 leaves stage to the end of growth stage every 7–10 days. In each sampling, the plant leaf area was measured based on 5 plants with a Deltati leaf area meter. The dry weight of the plants was measured separately from leaves and other organs at 75 °C until a constant weight was reached with an accurate scale of 0.01 g.

The following two models were used to describe the changes in the leaf area index during the plant growth period:

Beta model [31]:

$$LAI = LAI_{max} \left[\left(\frac{t_e - t}{t_e - t_m} \right) \left(\frac{t - t_b}{t_m - t_b} \right)^{\frac{t_m - t_b}{t_m - t_b}} \right]^\delta$$

In this model, t is the time after planting (days), LAI is the leaf area index value, LAI_{max} is the maximum leaf area index, t_b is the start of growth, t_m is the time when the maximum leaf area index occurs, t_e is the end time of leaf growth where the leaf area index is zero, and δ is a constant coefficient in the model.

Logistics model [8, 28]:

$$LAI = \frac{ae^{-a(t-t_m)(c)}}{(1 + e^{-a(t-t_m)})^2}$$

where a is a constant coefficient and shows the amount of rotation of the curve, t_m is the time after planting in which the maximum leaf surface index occurs and c is a constant coefficient.

The following nonlinear regression models were used to describe the changes in dry matter (w) against time after planting (t):

Beta model [8, 28, 31]:

$$W = W_{max} \left(1 + \frac{t_e - t}{t_e - t_m} \right) \left(\frac{t}{t_e} \right)^{\frac{t_e}{t_e - t_m}}$$

where W_{max} is the maximum amount of dry matter, t_m is the time when the growth rate of the product reaches its maximum value, and t_e is the time of the end of the growth period in which the amount of dry matter is equal to W_{max} .

Logistic model [8, 28, 31]:

$$w = \frac{W_{max}}{1 + e^{-k(t-t_m)}}$$

where W_{max} is the maximum amount of dry matter accumulation, k is the coefficient indicating the rapidity of dry matter increase, and t_m is the time when the growth rate of the product reaches its maximum value (at that time, the amount of dry matter has reached half of its maximum value). At time t_m , RGR is equal to $\frac{2}{k}$. The logistic equation in time t_m is symmetrical.

Gomperts model [8, 9, 31]:

$$W = w_{max} e^{-e^{-k(t-t_m)}}$$

In this model, W_{max} is the maximum amount of dry matter accumulation, k is the coefficient indicating the rapidity of dry matter increase and t_m when the growth rate of the product reaches its maximum value. According to the Gomperts model, at time t_m , the value of RGR is equal to the value of coefficient k .

The fitting of the models for the data of leaf surface and dry matter accumulation, as well as the estimation of the parameters of each model was done with the iterative

optimization method with the help of PROC NLIN procedure of SAS software [8]. In the iterative optimization method, the initial values of the parameters are entered each time, and their final values are estimated by the least squares method. The values were changed until the best estimate of the parameter was obtained. The best estimate of the model parameters was obtained based on the standard error (SE) of the model parameters. The following criteria were used to compare different models describing the leaf area index and dry matter during the growing season:

Root means square error between predicted and observed value:

$$RMSE = \sqrt{\frac{\sum(P - O)^2}{n - 1}}$$

where O is the actual value and P is the predicted value of leaf surface or dry matter production. n is the number of observations.

Simple linear regression coefficients (a and b) between predicted values and actual values: coefficients a and b , respectively, indicate the deviation of the regression line from the coordinate origin and the slope of the regression line from the 1:1 line. If the predicted points lie on the 1:1 line, it indicates that the model is ideal. The 1:1 line has a width from the origin of zero ($a = 0$) and a slope of 45° ($b = 1$).

Correlation coefficient (r) between actual and predicted values of leaf area or dry matter: higher value of r indicates the superiority of the model.

Akaike Information Criterion (AICc): The model that has a lower AICc value is more likely to be correct.

$$AICc = N.LN(SSE/n) + 2k + \frac{2k(k + 1)}{n - k - 1}$$

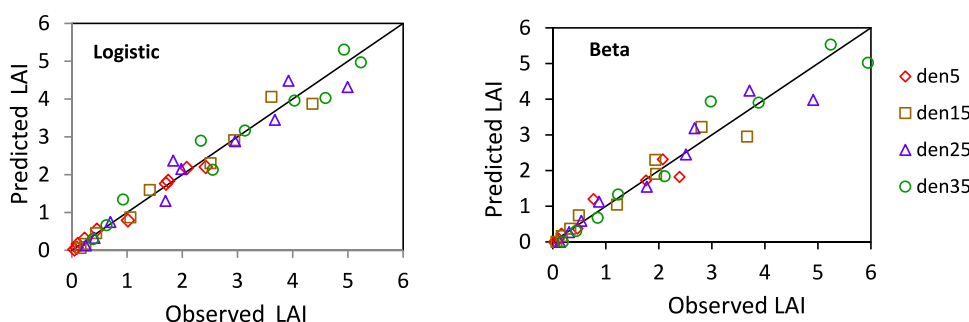
where n is the number of observations, k is the number of model parameters plus one, and SSE is the sum of squares of the model error. To calculate the probability that the model with lower AICc is more correct than any of the other models (Prob), the following relationship was used:

$$Prob = \exp(-0.5\Delta)/(1 + \exp(-0.5\Delta))$$

where Δ is the difference between the AICc values in the two investigated models.

The models were fitted on the data of leaf surface and dry matter accumulation using SAS software, and the figures were drawn using Excel software.

Fig. 1 Observed vs. predicted leaf area index in Barkat bean plant in four densities in the 1:1 line, where triangle, square, triangle and circle marks, respectively, indicate the density of 5, 15, 25 and 35 plants per square meter



Results and Discussion

Figure 1 shows the observed versus predicted leaf area index around the 1:1 line with two beta and logistic models. As shown in the figure, the data are well placed around the 1:1 line. The non-significance of the coefficients *a* and *b* of the linear regression line between the observed and predicted leaf area index data is zero and one, respectively. It shows the appropriate efficiency of these two models to describe the process of leaf surface changes over time (Table 1). Akaike’s Information Criterion (AICc) was also used for comparing the beta and logistic models.

The results showed that in all densities, the beta model has a lower AICc than the logistic model, which indicates that the beta model is more accurate than the logistic model in describing the changes in leaf area over time (Table 1). Also, further investigation showed that the beta model at densities of 5, 25 and 35 plants per square meter described the change in leaf area over time with a probability of more than 94% describe more correctly the change in the leave surface than the logistic model, while at a density of 15 plants per square meter with a probability of 55%, the logistic model was more correct (Table 1), but due to the small difference between the beta model and the logistic model in the density of 15 plants per square meter and also because the beta model was correct in other densities, the beta model was selected for describing the trend of surface index changes over the time.

Also, the results showed that the beta model has a lower RMSE and CV and a larger *r* value than the logistic model,

which indicates a relatively better accuracy of the beta model (Table 2).

The description of the changes in the leaf area index of the bean plant using two nonlinear beta and logistic regression models is shown in Fig. 2. Based on the beta model, it was observed that with the increase in density, the maximum leaf area index has an upward trend, so that the maximum leaf area index at a density of 5 number of plants per square meter increased to 2.3, and at a density of 35 plants per square meter, it increased to 5.3. This indicates that the plant has probably been able to produce the maximum index of leaf area through an increase in the number of leaves or the area of single leaves [13].

The time of the start of leaf growth (*t_b*) between the densities studied, due to the lack of fit of the data model, *t_b* was considered as a constant 50 days (Table 3). The occurrence time of the maximum leaf area index (*t_m*) decreased from the density of 5–35 plants per square meter, so that the plant community at the density of 35 plants per square meter reached the maximum leaf area index about 131 days after planting. The amount of leaf area index in the early stages of plant growth is low due to the few and small leaves and incomplete vegetation cover, but gradually with the growth and increase of the leaves of the plant, the leaf area index increases until it reaches its limit. That this time of occurrence of the maximum index of leaf area at higher densities usually happens earlier due to the interference and competition inside and outside the plants, and it remains in this state for a shorter period of time [7].

The end time of leaf growth (*t_e*) varied between 158.7 and 163.5 days in different densities and no significant

Table 1 The correctness of beta ratio model in logistic and Gompertz models

Growth indicators	Density	Density			
		5	15	25	35
Leaf surface index	Beta/Logistic	0.94	0.55	0.96	0.96
Dry matter accumulation	Beta/Gompertz	0.99	0.62	0.94	0.98
Dry matter accumulation	Beta/Logistic	0.94	0.93	0.85	0.50

Table 2 Root mean square error (RMSE), correlation coefficient (*r*) and regression coefficient (*a,b*) for nonlinear beta and logistic regression models in describing the change trend of leaf area index (LAI) and day after planting in Barkat bean plant

Model	Density	<i>a</i> ± SE	<i>b</i> ± SE	RMSE	CV	<i>r</i>	AICc
Beta	5	− 0.01 ± 0.047	1.00 ± 0.035	0.1	10.1	0.99	− 24.4
	15	0.01 ± 0.150	0.98 ± 0.065	0.3	15.9	0.98	− 2.9
	25	− 0.09 ± 0.165	0.01 ± 0.061	0.3	3.13	0.99	− 2.4
	35	− 0.11 ± 0.159	1.03 ± 0.047	0.3	10.0	0.99	− 4.6
Logistic	5	0.02 ± 0.061	0.98 ± 0.046	0.1	13.0	0.99	− 19.0
	15	0.01 ± 0.148	0.98 ± 0.064	0.3	15.6	0.98	− 3.3
	25	0.05 ± 0.225	0.97 ± 0.083	0.4	18.1	0.97	3.7
	35	0.13 ± 0.233	0.95 ± 0.070	0.4	13.1	0.98	2.9

Fig. 2 The trend of changes in the leaf surface index of the faba bean plant and its description with two beta and logistic models, where the quadrilateral, square, triangle and circle marks, respectively, indicate the density of 5, 15, 25 and 35 plants per square

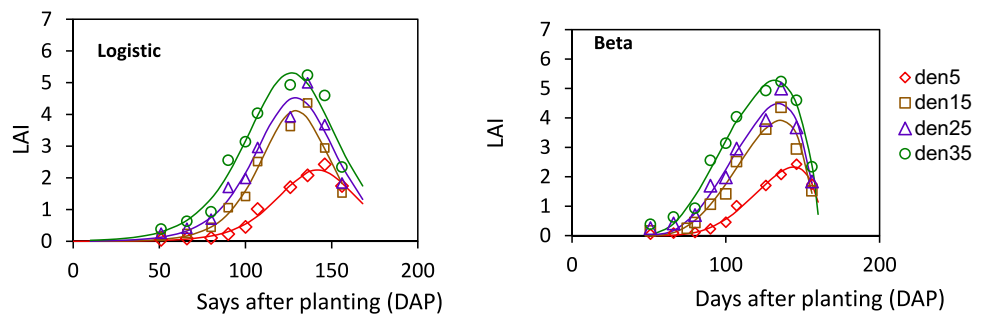


Table 3 Parameter values of beta and logistic models in describing the change trend of leaf area index in Barkat bean plant

Model	Parameter	Density			
		5	15	25	35
Beta	LAI _{max} (SE)	2.3 (0.07)	3.9 (0.19)	4.5 (0.20)	5.3 (0.17)
	<i>t</i> _b (SE)*	50	50	50	50
	<i>T</i> _m (SE)	144.2 (0.76)	135.7 (1.22)	134.6 (1.14)	131.9 (0.96)
	<i>T</i> _e (SE)	163.5 (1.33)	158.7 (0.99)	159.4(1.07)	160.9 (1.09)
	Delta	0.75	0.75	0.75	0.75
Logistic	<i>a</i> (SE)	0.1 (0.004)	0.1 (0.01)	0.1 (0.01)	0.1 (0.004)
	<i>c</i> (SE)	140.5 (9.21)	228.5 (12.75)	228.5 (12.75)	379.0 (21.71)
	<i>t</i> _m (SE)	141.7 (1.70)	129.2 (1.39)	129.2 (1.39)	126/8 (1.68)

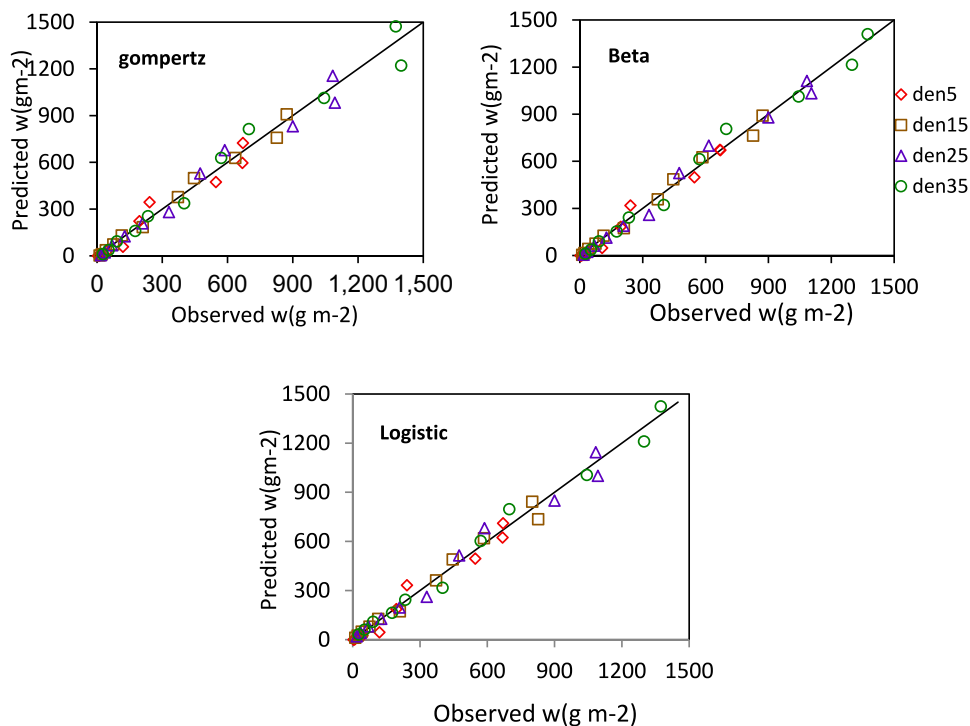
difference was observed between the densities in this respect (Table 3). The increase in density reduces the light passing to the lower part of the canopy, and the competition of more plants to receive light causes the leaves to get old and fall faster [7, 13].

Figure 3 shows the trend of observed and predicted changes in dry matter accumulation around the 1:1 line with three beta, logistic and Gompertz models. As shown in the figure, the data placed well around the 1:1 line. The non-significance of the coefficients *a* and *b* of the linear regression line between the observed and predicted dry

matter accumulation data with zero and one, respectively, indicates the appropriate efficiency of these two models to describe the change process of dry matter accumulation over time.

The results of the comparison of the models on the fitting of the dry matter data over time showed that in all densities, the beta model has a lower AICc than the Gompertz and logistic models, which indicates that the beta model is more correct than the other two models in describing the adjustment process of dry matter over time (Table 1). Also, further investigation showed that the beta

Fig. 3 The trend of changes in dry matter of Barkat bean plant and its description with three beta, Gompertz and logistic models, where the quadrilateral, square, triangle and circle symbols, respectively, indicate the density of 5, 15, 25 and 35 plants per square



model at densities of 5, 25 and 35 plants per square meter with a probability of more than 94% and at a density of 15 plants per square meter with a probability of 62% showed the change trend of dry matter accumulation over time compared to the Gompertz model. As well as the beta model in the densities of 5, 15 and 25 plants per square meter with a probability of more than 85%, but with a very small difference between the beta model and logistics in the density of 35 plants per square meter, and also because the beta model is more accurate than other densities, the beta model changes the trend dry matter accumulation was selected over time (Table 1).

Also, the results showed that the beta model has a lower RMSE and CV and a larger r value than the Gompertz and logistic model, which indicates a relatively better accuracy of the beta model (Table 4). The description of changes in the accumulation of dry matter of the bean plant using three nonlinear regression models, beta, logistic and Gompertz is shown in Fig. 4.

The change process of dry matter accumulation in all different densities was similar (Fig. 4), because the plant has a slow growth in the early stages due to its rosette stage and the increase in dry weight in this period is insignificant compared to time. In this period, the plant was limited to producing leaves and increasing the dry weight of the leaves. After the rosette stage, the plant enters the linear growth stage, in this stage the plant grows rapidly and the dry weight of the whole plant increases rapidly.

This is due to the increase in the accumulation of dry matter in the leaves and the plant entering the stem growth stage and the rapid increase in the dry weight of the stems. The third stage of growth begins after linear growth. In this stage, due to the aging and reduction of the leaf surface, the accumulation of dry matter slows down. The results showed that there is a significant difference between the densities in terms of the maximum accumulation of dry matter (Table 5).

Hence, the density of 35 plants per square meter was the highest (1484.3 g per square meter) and the density of 5 plants per square meter had the lowest dry matter accumulation (725.1 g per square meter). The increase of dry matter in the density of 35 plants per square meter can be considered due to receiving more solar radiation and more growth and development of leaves and finally increasing the speed of growth and accumulation of photosynthetic materials [19].

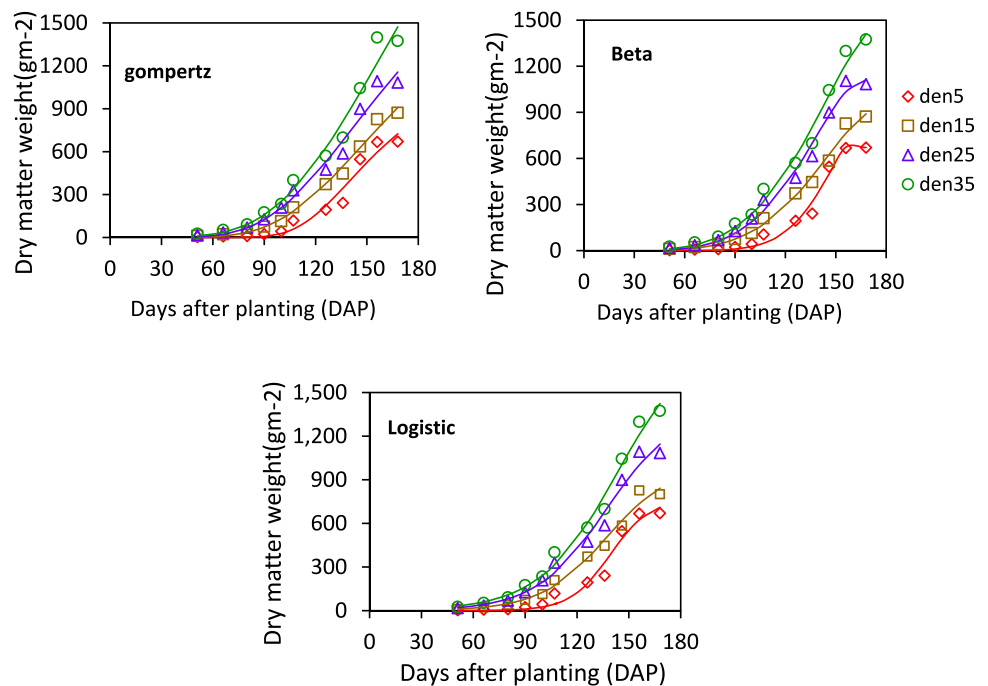
As previously stated, the density of 5 plants per square meter produced the lowest leaf area index compared to other densities, which caused a decrease in the photosynthetic area, a decrease in net photosynthesis, and as a result, a decrease in dry matter accumulated in this density [2]. In terms of the time to reach the half maximum dry matter (t_m) among different densities, it did not show any particular trend and varied between 138.3 and 146.4 days (Table 5).

In comparison of beta 1 model with six other models (logistic, Richards, Gompertz, Weibull and two

Table 4 Root mean square error (RMSE), correlation coefficient (*r*) and regression coefficient (*a* and *b*) for nonlinear beta, Gompertz and logistic regression models in describing the trend of changes in dry matter and days after planting in Barkat bean plant

Nonlinear models	Density	<i>a</i> ± SE	<i>b</i> ± SE	RMSE	CV	<i>R</i>	AICc
Beta	5	4.45 ± 14.061	0.99 ± 0.032	32.1	9.9	1.00	99.9
	15	4.45 ± 14.061	0.99 ± 0.032	32.1	9.9	1.00	96.9
	25	13.63 ± 21.783	0.98 ± 0.036	48.4	10.7	0.99	106.3
	35	12.23 ± 25.886	0.99 ± 0.036	57.2	10.5	0.99	109.7
Gompertz	5	11.31 ± 22.379	0.98 ± 0.066	57.1	24.4	0.98	109.3
	15	4.96 ± 14.677	0.99 ± 0.033	33.5	10.1	1.00	97.9
	25	6.1 ± 28.668	0.99 ± 0.048	63.0	14.1	0.99	111.7
	35	7.69 ± 38.066	0.99 ± 0.052	84.1	15.2	0.99	118.0
Logistic	5	14.50 ± 18.259	0.97 ± 0.053	46.1	20.1	0.99	105.5
	15	0.1 ± 18.169	1.00 ± 1.004	12.8	40.7	0.99	102.0
	25	1.50 ± 26.505	1.00 ± 0.045	58.0	12.9	0.99	109.8
	35	0.66 ± 26.502	1.00 ± 0.037	57.8	10.7	0.99	109.7

Fig. 4 Observed vs. predicted dry matter in Barkat bean plant in four densities in the 1:1 line, where triangle, square, triangle and circle marks, respectively, indicate the density of 5, 15, 25 and 35 plants per square meter



symmetrical truncated exponential equations), each of the model fitted to the data of seed dry weight accumulation (in six wheat genotypes), dry weight accumulation of single plant of (corn) and also accumulation of total dry weight per unit of surface area (peas and wheat) [31]. And it is stated that all equations correctly described the sigmoidal changes of seed filling, plant growth and plant total dry matter. In his study, truncated exponential model and beta 1 model had a better fit than other models. However, the logistic model was compared with 7 other models (beta 1, beta 2, Weibull, Richards, symmetrical, cut and Gompertz

models) [8] and their results showed that all the equations accurately described the sigmoidal changes of plant growth, plant dry matter production and wheat leaf area index. In their study, the logistic model had a better fit than other models. Another study compared the logistic model with five other models (Gompertz, beta, Richards, truncated exponential and symmetric exponential). These models were fitted to the data of accumulation of total dry weight per unit area (safflower), and the results showed that all the equations described the sigmoidal changes of the

Table 5 Parameter values of beta, Gompertz and logistic models in describing the process of dry matter changes in Barkat bean plant

Model	Parameter	Density			
		5	15	25	35
Beta	W_{\max}	725.1 (33.992)	921.0 (81.063)	1112.0 (46.842)	1484.3 (179.1)
	T_m	146.4 (1.379)	145.2 (4.370)	138.3 (2.350)	144.3 (6.259)
	T_e	162.6 (1.361)	175.9 (8.243)	167.2 (4.547)	179.0 (11.845)
Gompertz	W_{\max}	1098.6 (443.5)	1762.5 (495.9)	2111.4 (791.3)	3500.6 (2048.0)
	t_m	140.6 (14.544)	147.6 (14.786)	142.4 (20.477)	159.2 (35.563)
	k	0.03 (0.015)	0.02 (0.005)	0.02 (0.006)	0.02 (0.007)
Logistic	W_{\max}	770.2 (91.730)	1010.9 (124.7)	1386.0 (191.4)	1906.0 (285.0)
	T_m	139.2 (4.070)	137.0 (7.1010)	137.2 (6.080)	143.5 (7.903)
	k	0.09 (0.021)	0.05 (0.008)	0.05(0.008)	0.04 (0.006)

Numbers in parentheses are standard error (SE)

production of plant dry matter. Also, in this study, the logistic model had a better fit than other models [27].

Conclusions

In general, the results showed that the beta model can describe the growth pattern well for densities of 5, 15, 25 and 35 plants per square meter. In the four densities, estimation of parameters at a density of 5 plants per square meter, for LAI_{\max} , T_e and T_m , was, respectively, 2.3, 163.5 and 144.2, for a density of 15 plants per square meter 3.9, 158.7 and 135.7 square meters, for a density of 25 plants per square meter, 4.5, 159.4 and 134.6, and for a density of 35 plants per square meter, 5.3 and 160.9 and 131.9. The maximum LAI_{\max} and T_e were at the density of 35 plants per square meter, and the maximum LAI_{\max} and T_e were obtained at a density of 35 plants per square meter; and the maximum T_m was obtained at a density of 5 plants per square meter. The achievement of the maximum leaf area index and the time of the end of leaf growth in this density are probably due to the distance between the plants or the lower density; it seems that the occurrence of the maximum leaf area index in the density of 5 plants is due to the absence of intraspecific competition and eventually spread foliage was obtained in a single plant.

Also, the beta model was able to show the trend of dry matter accumulation for densities of 5, 15, 25 and 35 plants per square meter. In the four densities, estimation of parameters in 5 plants per meter square, respectively, for W_{\max} , T_e and T_m was 725.1, 162.6 and 146.4; for 15 plants per square meter was 921.0, 175.9 and 145.2; for 25 plants per meter square was 1112.0, 167.2 and 138.3, and for the density of 35 plants per square meter was 1448.3, 179.0 and 144.3. The maximum W_{\max} and T_e were observed at

the density of 35 plants per meter square, but the maximum T_m was observed at a density of 5 plants per square meter.

Therefore, the beta model can be used to estimate the leaf area and dry matter accumulation. Also, these relations can be used in simulation models of growth and dry matter accumulation of beans. Based on the results of this experiment, farmers can be advised to use a density of 35 plants per square meter to achieve maximum leaf area index and more accumulation of dry matter and finally to achieve maximum yield.

Author Contributions NE performed data collection, data analysis, and preparing manuscript draft. ARS performed data analysis and preparing result and discussion. SA reviewed and edited the manuscript draft. SHM reviewed the draft and arranged the references. JA reviewed the manuscript draft. ID reviewed the manuscript draft.

Declarations

Conflict of interest The authors declare that they have no competing interests.

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