



Construction project planning under fuzzy time constraint

N. Ibadov¹

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Abstract

A distinctive feature of construction projects is the uniqueness of conditions for their implementation. Therefore, the durations data of the works are often formulated imprecisely. On the other hand, the time limit for the execution of the works is defined in the contract conditions by data indicating the due time and the deadline. These indicate the distribution of time values, which the contractor can allocate for the implementation of the construction project in accordance with the client preferences. For assessing the compliance of the planned project makespan with the fuzzy time constraint, it is recommended to use the fuzzy sets theory. The article presents a method for assessing the level of satisfaction of fuzzy constraint of time for the execution of construction works using the probabilistic measure, in conjunction with the concept of α -cuts of a fuzzy number. The application of the presented method is demonstrated on a numerical example, which shows the compliance of the evaluation results with those obtained with the simulation methods. The article confirms the correctness of the assumptions of the presented method, which allows for its use for the formulation and resolving schedule optimization problems in the case of imprecisely formulated schedule input data.

Keywords Construction planning · Fuzzy constraints · Fuzzy scheduling · Probabilistic measure

Introduction

In planned construction projects, there is always a necessity to foresee possible completion dates of project in relation to deadline imposed by a client. To fulfill this requirement, it is required to take under consideration various delaying factors (Ibadov 2016a, b, 2017). Calculated durations under influence of those factors are then compared with constraints deriving from the project agreement. Therefore, predicting possibility of fulfilling required completion dates is of key importance for planning construction projects. In the literature, there are various methods and techniques for calculating duration of construction projects. Chosen methods and techniques for assessing construction works duration are presented by Juszczak (2014) and Rosłon (2017). Despite many tools supporting construction management, delays keep occurring in construction projects (Głuszak and

Leśniak 2015). Therefore, there is a constant need for finding relevant methods of defining projects duration under set constraints. It is worth stressing that a distinctive feature of construction projects is the uniqueness of conditions for their implementation. Therefore, the data on durations of the works are often formulated imprecisely, for example, “about 3 weeks,” “2 to 3 weeks,” and the like. To model ill-defined durations of the works, fuzzy numbers are used (Kulejewski 2010a, 2011; Lorterapong and Moselhi 1996). On the other hand, the time limit for the execution of the works is defined in the contract conditions by data indicating the due time and the deadline. These indicate the distribution of time values, which the contractor can allocate for the implementation of the construction project in accordance with the client preferences. For assessing the compliance of the planned project makespan with the fuzzy time constraint, it is recommended to use the theory of fuzzy sets (Dubois et al. 2003). However, the result of this assessment significantly depends on the optimism or pessimism of the planner. This article demonstrates that for the neutralization of assessing the level of meeting the fuzzy time constraint, a probabilistic measure can be used, in conjunction with the concept of α -cuts of a fuzzy number. The research was done in Warsaw (Poland) in year 2017.

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✉ N. Ibadov
n.ibadov@il.pw.edu.pl

¹ The Faculty of Civil Engineering, Warsaw University of Technology, Warsaw, Poland

Materials and methods

From the methodological point of view, the article concerns uncertainty modeling in planning and scheduling of construction projects (with time constraints), with the use of mathematical tools—fuzzy set theory and probabilistic measure.

In construction industry, during project planning phase one has to take into consideration the occurrence of unfavorable events and its consequences (characterized by term “risk”). In the scientific literature, it is often underlined that risk factors are random and can be described with the use of probability theory (in contrast to “uncertainty,” which cannot be described in a quantitative way). Such approach requires the knowledge on probability distribution of these factors. However, in construction practice it is impossible to assume probability distribution for a hypothetical event. That is why, all risk factors for a construction project are subject to uncertainty. Due to this fact, the construction manager (decision maker) should know that using any probability distribution hypothesis for random variables may result in incompatibility of the results with actual conditions. In majority of cases, decision is made while goals, constraints and consequences of events (or actions) are not clear. That is why, the use of probabilistic methods for planning and scheduling is well justified when one knows the probability of possible disruptions which may lead to specific results. Due to the lack of such possibility, people responsible for creation of construction schedules use heuristic knowledge gathered thanks to identification of facts and finding dependencies between them. The rationale and conclusions of the above hypotheses are formulated imprecisely, often with the use of concepts specific to colloquial language. For example, the occurrence of a selected factor with a given severity results in extension of the given work by about 5 days or seven to eight working days.

Mathematical basis of the fuzzy sets theory allows using imprecisely determined relationships between the frequency and intensity of a given disturbance, and its effects, to determine fuzzy numbers showing the distribution of possible times of performing individual works.

The existence of such a possibility (in light of the imperfection of the probabilistic methods of scheduling) has become the reason for the development of network model analysis methods with fuzzy durations of activities, and fuzzy scheduling and project planning methods. Numerous articles are devoted to this topic, and various authors try to model the uncertainty using fuzzy set theory separately or together with other mathematical tools. Worth mentioning is the work of Afsordegan et al. (2016) which takes into account linguistic data provided by the

decision makers without any previous aggregation. Li et al. (2015) suggest an interval fuzzy-robust two-stage stochastic-robust programming model for management in conditions of uncertainty. The model can enhance the robustness for the optimization process under fuzzy constraints. Some adjustments for decision making in conditions of uncertainty were made by Xu et al. (2012) by creation of a compromise optimization model, in which total duration of the project is considered to be a fuzzy variable. In this model a fuzzy-based adaptive-hybrid genetic algorithm is developed to find feasible solutions for time–cost–environment trade-off problem for large-scale construction projects. For various levels of project planning, Masmoudi and Haït (2013) developed a solving procedure by considering a fuzzy modeling of the workload inspired from the fuzzy/possibilistic approach. Plebankiewicz and Karcińska (2016) suggested scheduling methods basing on fuzzy values of working time norms and employee numbers. The proposed procedure allows for determining a real duration of a project taking into account various factors affecting durations of single activities. Castro-Lacouture et al. (2009) analyzed the implementation of fuzzy sets in schedule planning when time, costs and resources were limited. Each of the above-mentioned models supports decision making in the applied field.

Analysis of current scheduling methods that include uncertainty (probabilistic methods, fuzzy sets theory methods) can lead to the following assumption. Due to the uniqueness of construction contract conditions, even for the most advanced predictive–reactive scheduling methods with use of probability (Herroelen and Leus 2005), subjective data should be used based on activity duration probability distribution. However, expert knowledge is rough and hypothesis related to entered data for scheduling is not explicit, very often with use of verbal definitions. Theory of fuzzy sets allows for modeling and processing of data which is difficult to be quantified by use of probabilistic or statistical methods. According to Kulejewski (2010b), existing methods of schedule creation with fuzzy sets which are presented in articles by Chanas and Zieliński (2001, 2002), Hapke and Słowiński (1996, 2000), Lorterapong and Moselhi (1996), Wang (1999, 2002, 2004), Slyeptsov and Tyshchuk (2003) do not provide satisfying results related to:

- calculations of latest completion times and identification of critical activities and paths in fuzzy model of construction networks,
- assessment of time-constrained compliance level in case when activities completion time is not precisely set,
- transformation of fuzzy schedule to regular schedule with assurance of a level for imprecisely set constraints of completion time.



Taking the above under the consideration, there is a need to develop such methods of scheduling which will include fuzzy modeling of imprecise planning data and which assure the requested level of completion time constraint which is set in an imprecise way.

Basic concepts of fuzzy sets theory and uncertainty modeling

The following information includes only a part of fuzzy sets theory, which is directly connected to this article. The area considered in the fuzzy sets theory is a certain non-empty space X , which is a non-fuzzy set. In this space, a certain set A is fuzzy if elements $x \in X$ belong that set with some grade of membership μ (Rutkowski 2006). A fuzzy number is a fuzzy set $A \subseteq R$, whose membership function $\mu_A(x)$ is at least segmentally continuous, and fuzzy set A is convex and normalized. The sum and maximum of two fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ can be calculated using the following equations (Dubois and Prade 1978):

$$\tilde{A} \oplus \tilde{B} = (a_1 + b_1, \dots, a_4 + b_4) \tag{1}$$

$$\max\{\tilde{A}, \tilde{B}\} = (\max\{a_1, b_1\}, \dots, \max\{a_4, b_4\}) \tag{2}$$

In turn, α -cuts of a fuzzy set $A \subseteq X$ are such non-fuzzy set A^α (Rutkowski 2006):

$$A^\alpha = \{x \in X : \mu_A(x) \geq \alpha\}, \quad \forall \alpha \in [0,1] \tag{3}$$

Each non-fuzzy set A^α can be presented in the interval notation $[a_L^\alpha, a_U^\alpha]$ in the following manner:

$$a_L^\alpha = \inf_{x \in R}(A^\alpha), \quad a_U^\alpha = \sup_{x \in R}(A^\alpha) \tag{4}$$

To compare fuzzy numbers, one can use terms like degree of necessity $N(Z)$ and degree of possibility $\Pi(Z)$ for occurrence of a specified event Z (Dubois and Prade 1992). Relations between degrees of necessity and possibility are as follows:

$$N(Z) = \inf_{x \notin Z}(1 - \mu_Z(x)) = 1 - \Pi(\neg Z) \tag{5}$$

$$\Pi(Z) = \sup_{x \in Z}(\mu_Z(x)) = 1 - N(\neg Z) \tag{6}$$

where $(\neg Z)$ is an event opposite to event Z , $\mu_Z(x)$ is a membership function Z .

Basic concepts of fuzzy network modeling analysis

In the fuzzy network, modeling analysis durations of single activities are fuzzy numbers. Figure 1 shows an example of modeling activity duration as trapezoidal number.

Fuzzy critical path method (FCPM) is used for most projects. The principles of this method are like the conventional

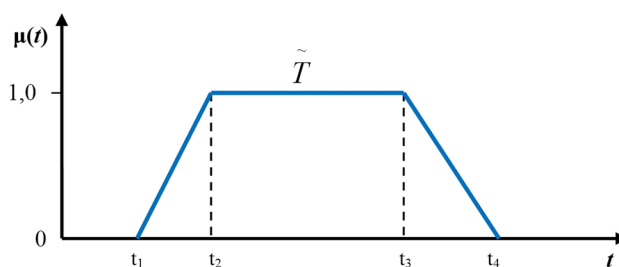


Fig. 1 An example of using a trapezoidal fuzzy number for activity duration modeling

critical path method (CPM) (Soltani and Haji 2007). The length of the longest network path (in terms of duration) is a so-called critical path of the project. It is important to note that there are many flaws of determining a critical path. Many articles were devoted to this subject. In order to determine critical path with fuzzy activities' durations, authors use various methods of ranking fuzzy numbers (Elizabet and Sujatha 2013; Chandra and Kumar 2014; Shankar et al. 2010a, b). The described methods have some flaws. First of all, in a case that there are multiple critical paths there is a problem in selecting one of the alternative critical durations. Furthermore, in previously proposed algorithms an activity can have different durations if it is located on several paths, which is a major flow of solution. Moreover, there are some problems in determining slack.

It seems that the key aspect of fuzzy project planning is calculating finish time of the whole project. In this regard, Huang, Oh and Pedrycz (2013) propose three fuzzy programming models for estimating the overall completion time of project. The proposed models are handled through techniques that combine mechanisms of fuzzy simulation and genetic optimization. In this setting, fuzzy simulation is exploited to estimate the value of uncertain functions. Khalilzadeh et al. (2017) develop an algorithm for project scheduling with fuzzy time and resources. This algorithm first calculates the latest start times of activities under fuzzy environment and then constructs a feasible schedule by using the parallel scheduling method. Taking into consideration above-mentioned flaws, the author of this article is sticking only to the basic formulas and concepts useful for solving the network models presented later in this article.

While modeling a course of the project using a single-point fuzzy network with finish-to-start dependencies between activities, earliest starting times and finish times of individual activities can be determined based on the dependencies similar to those used in the classic CPM method (Chanas and Kamburowski (1981):

$$\tilde{ES}_j = \max_{i \in \text{Prec}\{j\}} \{ \tilde{ES}_i \oplus \tilde{D}_i \} \tag{7}$$

$$\widetilde{EF}_j = \widetilde{ES}_j \oplus D_j, \quad j = 1, \dots, J \tag{8}$$

where \widetilde{ES}_j -fuzzy early start time for activity j, $\text{Prec}\{(j)\}$ -set of j activity predecessors, \widetilde{EF}_j -fuzzy early finish time for activity j, D_j -fuzzy duration of activity j.

Adding fuzzy numbers can be done according to Eq. (1). The maximum of fuzzy numbers is determined using Eq. (2).

Fuzzy duration of a project can be calculated in the following manner (Lorterapong and Moselhi 1996):

$$\tilde{T} = \widetilde{EF}_j \tag{9}$$

Assessing the compliance with the fuzzy time constraint

According to Dubois et al. (2003) and Zadeh (1999), to assess the compliance with the fuzzy time constraint one should estimate the degree of necessity and the degree of possibility of the relationship $\tilde{T} \leq \tilde{T}_D$, where fuzzy number \tilde{T} models the estimated construction project makespan and fuzzy number \tilde{T}_D models the fuzzy time constraint. The shapes of these fuzzy numbers are shown in Fig. 2.

It should be noticed that the assessment of the possibility degree of the relation $\tilde{T} \leq \tilde{T}_D$ is not complementary to the assessment of the degree of occurrence of the opposite relation. This means that $\Pi(\tilde{T} \leq \tilde{T}_D)$ need not be equal to $1 - \Pi(\tilde{T} \geq \tilde{T}_D)$. For this reason, there is a search in the literature for a synthetic indicator, having (in line with the intuition of the planner) the property of complementarity. Such a property characterizes a schedule performance measure, introduced by Wang (2002) as the schedule risk:

$$SR = \beta \Pi(\tilde{T} > \tilde{T}_D) + (1 - \beta)N(\tilde{T} > \tilde{T}_D) \tag{10}$$

where β is a coefficient, characterizing the level of optimism. It should be noticed that using Eq. (10), the planners at different levels of optimism are not unanimous in their assessment of the level to meet the fuzzy constraint on the construction project makespan.

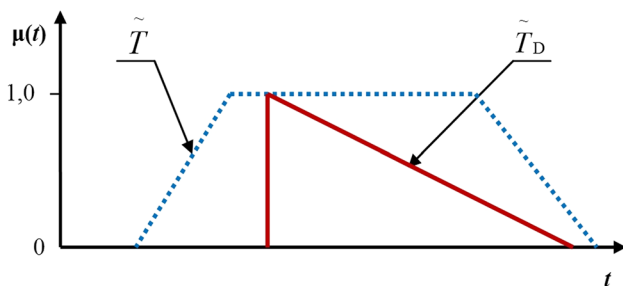


Fig. 2 Fuzzy numbers \tilde{T} and \tilde{T}_D

Probabilistic assessment of the compliance with the fuzzy time constraint

The main idea of the described below method for compliance assessment with the fuzzy time constraint is to use the concept of α -cuts of fuzzy numbers \tilde{T} and \tilde{T}_D to designate interval numbers T^α and T_D^α . Those interval numbers are further compared with the use of the probabilistic measure and probability $P(T^\alpha > T_D^\alpha)$ is calculated. By the aggregation of probability $P(T^\alpha > T_D^\alpha)$ calculated for the finite number of α -cuts of fuzzy numbers \tilde{T} and \tilde{T}_D , probability $P(\tilde{T} > \tilde{T}_D)$ is obtained. The probability that the planned construction project makespan is no longer than the project makespan preferred by the client is:

$$P(\tilde{T} \leq \tilde{T}_D) = 1 - P(\tilde{T} > \tilde{T}_D) \tag{11}$$

Figure 3 depicts an example of interval numbers T^α and T_D^α designation for some α -cut of fuzzy numbers \tilde{T} and \tilde{T}_D . Choosing a real number t in the interval T^α and a real number t_d in the interval T_D^α , one will receive a pair of real numbers (t, t_d) . Figure 3 shows that a real number t may take a value from one of the subintervals $T_{(1)}^\alpha = [t_L^\alpha, t_{d_L}^\alpha]$, $T_{(2)}^\alpha = [t_{d_L}^\alpha, t_{d_U}^\alpha]$ or $T_{(3)}^\alpha = [t_{d_U}^\alpha, t_U^\alpha]$, whereas, a real number t_d will always take value from the subinterval $[t_{d_L}^\alpha, t_{d_U}^\alpha]$. As a result, there may be one of the events Z_q such that $Z_q = (t \in T_{(q)}^\alpha, t_d \in T_D^\alpha)$, $q = 1, 2, 3$. The events $t \in T_{(q)}^\alpha$ and $t_d \in T_D^\alpha$ are independent, because:

$$P(Z_q) = P(t \in T_{(q)}^\alpha)P(t_d \in T_D^\alpha) \tag{12}$$

The probability $P(t_d \in T_D^\alpha) = 1$. The probability $P(t \in T_{(q)}^\alpha)$ can be assessed geometrically, comparing the length of the subinterval $T_{(q)}^\alpha$ and the length of the subinterval T^α . On this basis, one obtains:

$$P(Z_1) = \frac{t_{d_L}^\alpha - t_L^\alpha}{t_U^\alpha - t_L^\alpha}; \quad P(Z_2) = \frac{t_{d_U}^\alpha - t_{d_L}^\alpha}{t_U^\alpha - t_L^\alpha}; \quad P(Z_3) = \frac{t_U^\alpha - t_{d_U}^\alpha}{t_U^\alpha - t_L^\alpha} \tag{13}$$

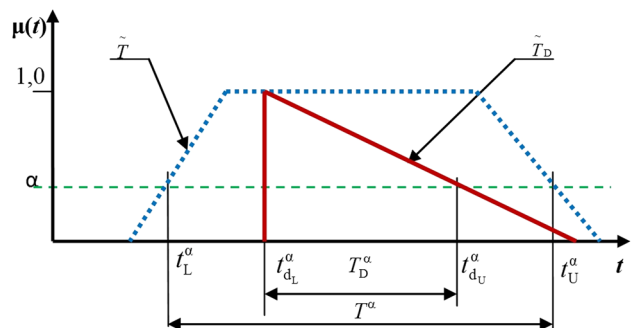


Fig. 3 Example of designation of interval numbers T^α and T_D^α

The probability that a real number t chosen from the interval $[t_L^\alpha, t_U^\alpha]$ proves to be greater than the real number t_d chosen from the interval $[t_{d_L}^\alpha, t_{d_U}^\alpha]$ is the conditional probability $P(T^\alpha > T_D^\alpha | Z_q)$. An event Z_2 is selected as a real number t and a real number t_d from the same subinterval $T_{(2)}^\alpha = [t_{d_L}^\alpha, t_{d_U}^\alpha]$. It can therefore be assumed that when the event Z_2 will occur, the probability that the chosen real number t will be greater than the chosen real number t_d , is $P(T^\alpha > T_D^\alpha | Z_2) = 0,5$. In the case of occurrence of the event Z_1 , a real number t will be always lower than a real number t_d . Therefore, $P(T^\alpha > T_D^\alpha | Z_1) = 0$. Finally, if the event Z_3 will occur, a real number t will be always greater than a real number t_d . Therefore, $P(T^\alpha > T_D^\alpha | Z_3) = 1$.

The total probability that in the case shown in Fig. 3, a real number t chosen from the interval T^α will be greater than the real number t_d chosen from the interval T_D^α is:

$$P(T^\alpha > T_D^\alpha) = \sum_q P(Z_q)P(T^\alpha > T_D^\alpha | Z_q) = 0,5 \frac{t_{d_U}^\alpha - t_{d_L}^\alpha}{t_U^\alpha - t_L^\alpha} + \frac{t_U^\alpha - t_{d_U}^\alpha}{t_U^\alpha - t_L^\alpha} \tag{14}$$

In a similar way, one can calculate the probability $P(T^\alpha > T_D^\alpha)$ for other cases than the one shown in Fig. 3. By aggregating the probability $P(T^\alpha > T_D^\alpha)$, calculated for the finite number of α -cuts of fuzzy numbers \tilde{T} and \tilde{T}_D , one can obtain:

$$P(\tilde{T} > \tilde{T}_D) = \frac{\sum_i \alpha_i P(T^{\alpha_i} > T_D^{\alpha_i})}{\sum_i \alpha_i} \tag{15}$$

where i is the index of the given α -cut.

Results and discussion

Numerical example

The scope of an exemplary construction project covers the finishing works in 6 buildings. Duration of activities is modeled in the form of trapezoidal fuzzy numbers. The network model is shown in Fig. 4. Fuzzy durations of works (in working days) and the planned earliest dates of the execution of

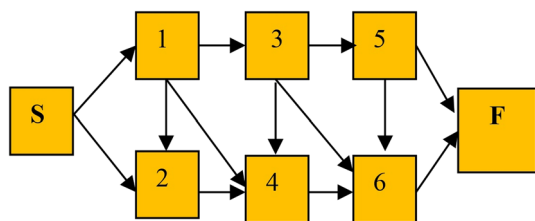


Fig. 4 Network model of exemplary construction project

works calculated on the base of Eqs. (7) and (8) are given in Table 1.

Suppose that the project due time is 25 working days from the date of commencement of works. The delay results in payment of liquidated damages by the contractor for the customer, but if the construction makespan will be greater than 30 working days, the client will withdraw from the contract due to the fault of the contractor. This constraint for the construction project makespan is modeled by the fuzzy number $\tilde{T}_D = (25, 25, 25, 30)$. The fuzzy schedule with early time for each activity and time constraints is presented in Fig. 5.

To determine the probability $P(\tilde{T} \leq \tilde{T}_D)$, one can introduce α -cuts of numbers \tilde{T} and \tilde{T}_D on the levels varying from $\alpha = 0,1$ to $\alpha = 1,0$, with the grading for example at 0,1. For individual α -cuts, one can determine the probability $P(T^{\alpha_i} > T_D^{\alpha_i})$ using Eq. (14). Then, aggregate the results using Eq. (15). Finally, using the Eq. (11), one can determine the probability of compliance of the planned project makespan with the fuzzy time constraint. In this example, $P(\tilde{T} \leq \tilde{T}_D) = 0,75$.

The correctness of the result can be checked by simulation. For this purpose, one can generate random α -cuts of fuzzy numbers \tilde{T}_j , modeling durations of individual works. Then for every generated α -cut, determine the lower limit $t_{j_L}^\alpha$ and the upper limit $t_{j_U}^\alpha$ of an interval T_j^α . Also, one can generate random values of coefficients β_j , characterizing the risk attitude of a planner. On this basis, it is possible to determine the duration of each activity t_j :

$$t_j = \beta_j t_{j_U}^\alpha + (1 - \beta_j) t_{j_L}^\alpha \tag{16}$$

Then, for each simulation, one should determine the earliest start and finish dates for the individual activities and for the whole project (project start date was set to zero). In a similar manner, one can determine the value of the time constraint in each simulation. After the prescribed number of simulations, one should determine the relative frequency of cases in which the duration of the project does not exceed the project time limit, in

Table 1 Fuzzy durations of works and the planned earliest dates of the execution of works

Activity	Fuzzy duration	Earliest start	Earliest finish
Start	(0, 0, 0, 0)	(0, 0, 0, 0)	(0, 0, 0, 0)
1	(2, 4, 6, 8)	(0, 0, 0, 0)	(2, 4, 6, 8)
2	(3, 4, 5, 6)	(2, 4, 6, 8)	(5, 8, 11, 14)
3	(4, 7, 8, 11)	(2, 4, 6, 8)	(6,11,14,19)
4	(6, 7, 8, 9)	(6, 11, 14, 19)	(12, 18, 22, 28)
5	(4, 5, 7, 8)	(6, 11, 14, 19)	(10, 16, 21, 27)
6	(2, 3, 4, 5)	(12, 18, 22, 28)	(14, 21, 26, 33)
Finish	(0, 0, 0, 0)	(14, 21, 26, 33)	(14, 21, 26, 33)

Fig. 5 The fuzzy project schedule with time constraints

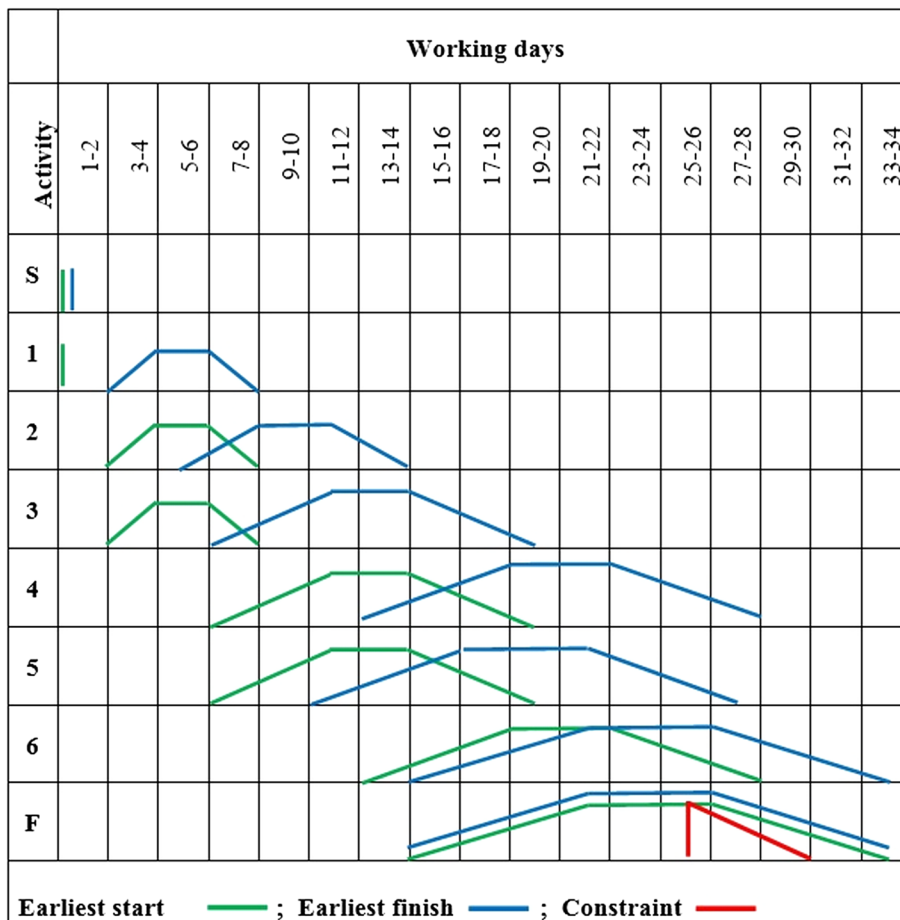


Table 2 The probability $P(T^{\alpha_i} > T_D^{\alpha_i})$

α_i	$P(T^{\alpha_i} > T_D^{\alpha_i})$
0,1	0,286889
0,2	0,283904
0,3	0,280355
0,4	0,276063
0,5	0,270771
0,6	0,26408
0,7	0,255353
0,8	0,243494
0,9	0,226445
1,0	0,19985

accordance with customer preferences. This leads to the determination of the probability $P(\tilde{T} \leq \tilde{T}_D)$. In this example, after 100,000 simulations, $P(\tilde{T} \leq \tilde{T}_D) = 0,75$. The probability $P(T^{\alpha_i} > T_D^{\alpha_i})$, determined analytically for the chosen α -cuts of fuzzy numbers $\tilde{T} = (14, 21, 26, 33)$ and $\tilde{T}_D = (25, 25, 25, 30)$, is given in Table 2.

The results of simulations for the determination of the probability $P(\tilde{T} \leq \tilde{T}_D)$ are shown in Fig. 6.

The results show the compliance of the assessment using the presented method and the simulation method.

Conclusion

The sound literature sources recommend to use the elements of fuzzy sets theory for assessing the compliance of the planned project makespan with the fuzzy time constraint. In some cases, such an evaluation may not lead to indicating the timing option, which provides a higher level to meet the time constraints of construction in accordance with client preferences. As a tool to assist decision making in such cases, this article presents a method combining elements of fuzzy set theory and probability theory. In contrast to the method of assessing the risk of schedule with the measures of necessity and possibility, the presented method enables the direct determination of the probability of meeting the fuzzy time constraint. The numerical example shows the compliance of the assessment using the presented method and the simulation method. This confirms the correctness of the assumptions of the presented method, which allows for its use for the formulation and resolving schedule

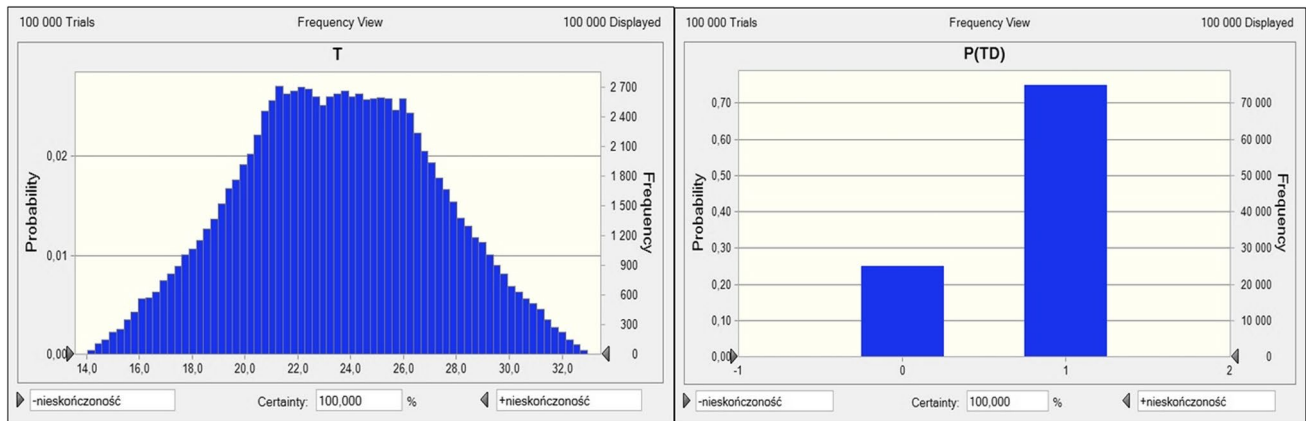


Fig. 6 Results of simulations for the probability $P(\tilde{T} \leq \tilde{T}_D)$ determination

optimization problems in the case of imprecisely formulated schedule input data.

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