Sankhyā : The Indian Journal of Statistics
2019, Volume 81-B, Supplement 1, pp. S123-S145

Partial Distributional Policy Effects Under Endogeneity

Badi H. Baltagi
Syracuse University, Syracuse, USA
Pallab K. Ghosh
University of Oklahoma, Norman, USA

Abstract

Rothe (*Econometrica* 80, 2269–2301 2012) introduces a new class of parameters called 'Partial Distributional Policy Effects' (PPE) to estimate the impact on the marginal distribution of an outcome variable due to a change in the unconditional distribution of a single covariate. Since the strict exogeneity assumption of all covariates makes this approach less applicable in empirical research, we propose the identification of the PPEs for a continuous endogenous explanatory variable using the control variable approach developed by Imbens and Newey (*Econometrica* 77, 1481–1512 2009). We also apply this proposed control variable PPE approach to investigate how poverty and black-white racial wage gaps contribute to the steep increase in the incarceration rate of black men over the period 1980-2010. Our control variable PPE estimates suggest that although the fall in the racial wage gap does not explain the changes in the incarceration rate of black men, changes in the poverty rate contribute about one-third of the steep increase in the incarceration rate at the upper-tail of the distribution.

AMS (2000) subject classification. Primary 62; Secondary 60.
Keywords and phrases. Partial distributional policy effects, Distribution regression, Quantile regression, Wage inequality

1 Introduction

Rothe (2012) introduces a concept called Partial Distributional Policy Effects (PPEs) which estimates the effect of a counterfactual change in the unconditional distribution of an explanatory variable on some features of the unconditional distribution of an outcome variable. This concept has attracted lots of attention in the econometrics literature because it allows arbitrarily complex nonlinear relationships between the outcome variable

This paper is written in honor of the 125th anniversary of PC Mahalanobis, for his important contributions to statistics and econometrics. We would like to thank two anonymous referees as well as the guest editors Arnab Bhattacharjee and Taps Maiti for their helpful comments and suggestions.

and the covariates, instead of relying on a linear model. Furthermore, PPEs measure the impact on general distributional statistics, such as the mean, variance, higher moments, quantiles, quantile differences, or Gini coefficient and not only for a simple location shifts model.

This new class of PPEs contribute to the extensive literature on the analysis of counterfactual distributions, surveyed by Fortin et al. (2011). Besides their use in public policy analysis, PPEs can also be used in the popular Oaxaca-Blinder (Oaxaca, 1973) and (Blinder, 1973) decomposition method to decompose the intragroup differences in means and other distributional statistics of the outcome distribution. Firpo et al. (2009) propose a regression-based estimation method known as unconditional quantile regression (UQR) which estimates the impact of changing the distribution of an explanatory variable on the marginal distribution of the outcome variable, or other functionals of the marginal distribution. Methodologically, the Rothe (2012) PPE approach is substantially different from the unconditional quantile regression method.[1]

Rothe (2012) identifies the PPEs under the strict exogeneity assumption between the explanatory variable and the unobserved error term. One of the major goals of this study is to extend the analysis of partial distributional policy effects to allow for the presence of an endogenous regressor, which is frequently encountered in many economic applications. A possible approach to identify the PPEs for an endogenous variable is to apply the control variable techniques (see Blundell and Powell 2003, 2004; Florens et al. 2008). Imbens and Newey (2009) use such an approach under relatively general conditions.

An important feature of the Imbens and Newey (2009) control variable approach is that it can be applied to models with nonseparable multidimensional disturbances. Under the assumption of common support and strict exogeneity of the instruments, Imbens and Newey (2009) show identification of the Average Structural Function, the Quantile Structural Function, the Average Derivatives and Policy Effects in the presence of an endogenous regressor. In this paper, we show that the PPEs for a continuous endogenous explanatory variable can be identified by using the Imbens and Newey (2009) control variable approach. However, we can not identify the PPE for

[1]Rothe (2012)'s method is also different from the study by Carneiro et al. (2010) who estimated the Marginal Policy-Relevant Treatment Effect of a marginal change in the conditional probability of receiving a binary treatment variable.

a discrete regressor because the Imbens and Newey (2009) method is not applicable for a discrete or a binary endogenous explanatory variable.[2]

As a further contribution, this paper implements the PPEs of a continuous endogenous variable in an empirical application analyzing the effect of racial wage gaps and the poverty rate to explain the steep increase in the incarceration rate of black men. Carson (2013) argues that as a consequence of three decades of dynamic growth in prison population since the late 1970s, the US has the highest incarceration rate in the world. Glaze and Herberman (2013) estimate that the annual US correctional combined federal, state and local expenditures on justice-related programs exceeds $260 billion per year. Although there is a large literature investigating the role of several possible state-economic factors, surprisingly little is known empirically about the impact of the black-white wage gap and the poverty rate on the incarceration rate of black men.

To estimate the impact of the black-white wage gap and the poverty rate on black men's incarceration rate, we use the Oaxaca-Blinder (Oaxaca, 1973) and (Blinder, 1973) decomposition method and hence construct a counterfactual incarceration distribution function by following Rothe (2012)'s PPE approach. The counterfactual distribution represents a hypothetical scenario where the individual state economic variables such as the black-white wage gap or the poverty rate are distributed as in 2000-2010, and all other variables remain the same as in the 1980s. We find that there is a discrete jump in black men's incarceration rate at the upper-tail of the distribution and our estimated full policy effects can explain that sharp change in the upper-tail.

We apply the Imbens and Newey (2009) control variable techniques in order to estimate the PPE of an endogenous variable such as the poverty rate or the black-white racial wage gap. We find that a change in the poverty rate can explain approximately one-third of the change in the incarceration rate of black men in the upper-tail of the distribution. However, it does not explain much of the changes in the incarceration rate in the lower half of the distribution. Although the racial white-black wage gap decreased about 8% from 1980-1990 to 2000-2010, we find that it did not significantly affect the incarceration rate of black men. To check the validity of our results, we implement a similar distribution regression approach developed by Chernozhukov et al. (2013). As a check, we compare our estimated PPEs to those obtained using the distribution regression method.

[2] We provide a detailed explanation in Section 2.

The structure of the paper is as follows. Section 2 presents the econometric model, identification and estimation of PPE in the presence of an endogenous variable. Section 3 contains the empirical application and robustness check. We conclude in Section 4. Proofs are shown in the Appendix.

2 Econometric Model

Suppose we have an outcome variable Y which depends on a k-dimensional vector of covariates X and an unobservable η through the following general non-separable structural model,

$$Y = m(X, \eta) \tag{2.1}$$

where $m(\cdot)$ denotes the true, unknown structural function of interest and η is distributed with positive density over its support. Since we do not impose any restrictions on the dimensionality of η to identify the above nonseparable model, this model allows the structural disturbance η to enter in a fully nonseparable way in equation (1).

The main objective of the Rothe (2012) paper is to estimate the effect of a counterfactual (fixed or marginal) change in the unconditional distribution of a single covariate X_1 included in X on some feature of the distribution of Y, holding everything else constant. To formalize the notation of a ceteris paribus change in one of the components of a multivariate distribution, we partition the observable covariate vector as $X = (X_1, X_2)$, where X_2 is the $(k-1)$-dimensional vector of the remaining covariates. Rothe (2012) performs a counterfactual experiment in which the unconditional distribution of X_1 has been changed to a different CDF, H and everything else remains constant. In this experiment, our outcome variable can be defined as:

$$Y_H = m(X_H, \eta) \tag{2.2}$$

where $X_H = (H^{-1}(U_1), X_2)$ is the corresponding counterfactual covariate vector, and $H^{-1}(U_1) = inf\{x_1 : H(x_1) \geq U_1\}$ is the quantile function corresponding to H and $U_1 \sim U[0,1]$. Whether $H(\cdot)$ is a fixed CDF or part of a sequence of CDFs $\{H_t, t \in \mathbb{R}\}$ that approaches F_{X_1} from a different direction of X_1 as $t \to 0$ depends on the empirical application.[3]

2.1. Partial Distributional Policy Effects Once we define different distributional statistics $\nu(F_Y^H)$ of the counterfactual distributional function, we

[3]Where F_{X_1} is the CDF of X_1 and defined as $F_{X_1}(x_1) = Pr(X_1 < x_1)$.

compare them with the corresponding distributional statistics $\nu(F_Y)$ which are obtained from the distribution of Y. Rothe (2012) defines any difference between these quantities as the new class of parameters, *Partial Distributional Policy Effect*. We formally define our parameter of interest as follows:

Definition. *(i) When H is a fixed CDF, the Fixed Partial Distributional Policy Effect (FPPE) is defined as,*

$$\alpha(\nu, X_1, H) = \nu(F_Y^H) - \nu(F_Y).$$

(ii) Let F_Y^t be a counterfactual distribution function such that $H = H_t$ be an element of a continuum of CDFs indexed by $t \in \mathbb{R}$ and $H_t \to F_{X_1}$ as $t \to 0$. Then the Marginal Partial Distributional Policy Effect (MPPE) is given by

$$\beta(\nu, X_1, H_t) = \lim_{t \to 0} \left(\frac{\nu(F_Y^t) - \nu(F_Y)}{t} \right) = \left. \frac{\partial \nu(F_Y^t)}{\partial t} \right|_{t=0}$$

provided that the above limit exists.

2.2. Identification The identification of both fixed and marginal PPEs are based on the assumption that the unobserved heterogeneity η is independent of the rank variable, U_1 conditional on the set of exogenous variables X_2 . In the literature of counterfactual distributions, Firpo et al. (2009), Chernozhukov et al. (2013), & Rothe (2010) also use this strict conditional exogeneity assumption. However, in many empirical applications, X_1 can be a function of η because X_1 is an equilibrium outcome partially determined by η. To identify the PPEs for a continuous endogenous explanatory variable, we follow the control variable approach introduced by Imbens and Newey (2009).

Suppose there is a vector of instruments Z_1 and a scalar disturbance ε such that $Z = [Z_1, X_2]$ and $X_1 = h_1(Z, \varepsilon)$, where $h_1(Z, \varepsilon)$ is strictly monotonic in the error term ε. We assume that the set of instruments Z is orthogonal to both the error terms η and ε, that is $Z \perp (\eta, \varepsilon)$. Using this full independence assumption, Imbens and Newey (2009) show that there exists a control variable $V = F_{X_1|Z}(x_1, z) = F_\eta(\eta)$, where $F_{X_1|Z}(\cdot)$ is the conditional cumulative distribution (CDF) of X_1 given Z and $F_\eta(\cdot)$ is the CDF of η. Conditional independence occurs because V is a one-to-one transformation of η, and conditional on η, X_1 depends only on Z.

Under the full independence and strict monotonicity assumptions, we restate the Imbens and Newey (2009) Theorem 1 result for completeness:

$$\eta \perp X_1 | V \quad \textit{where } V = F_{X_1|Z}(x_1, z).$$

Note that the strict monotonicity assumption rules out the possibility that X_1 is a discrete variable because if X_1 takes only a finite number of discrete values, there is no guarantee that $h_1(Z, \varepsilon)$ will be strictly monotonic with respect to ε. Therefore, this approach is not applicable for a discrete or a binary endogenous regressor.

Applying the Probability Integral Transformation Theorem,[4] we rewrite $\tilde{X} = [X_1|V \ X_2]$ in terms of their quantile functions and the standard uniformly distributed latent variables $U = (U_1, U_2, ..., U_k)$, that is,

$$\begin{aligned} X &= \left(F^{-1}_{X_1|V}(U_1), F^{-1}_{X_{21}}(U_2), ..., F^{-1}_{X_{2(k-1)}}(U_k)\right) \\ &= \left(Q_{X_1|V}(U_1), Q_{X_{21}}(U_2), ..., Q_{X_{2(k-1)}}(U_k)\right) \end{aligned} \tag{2.3}$$

where $F_{X_1|V}, F_{X_{21}}, ..., F_{X_{2(k-1)}}$ are the CDFs and $Q_{X_1|V}, Q_{X_{21}}, ..., Q_{X_{2(k-1)}}$ are the quantile functions of $X_1, X_{21}, .., X_{2(k-1)}$ and $U_i \sim U[0, 1]$ for $i = 1, ..., k$. Note that there exists a one to one relationship between $X_1|V$ and U_1 when the quantile function is strictly increasing. Therefore, we perform a counterfactual experiment where we hold the copula of $X_1|V$ and X_2 fixed, while we change the marginal distribution of the variable $X_1|V$. In other words, we change $Q_{X_1|V}$ to another quantile function $\tilde{H}(\cdot)$ while keeping the joint distribution of the rank variables $U = \left(U_1, U_2, ..., U_k\right)$ fixed.

Following Imbens and Newey (2009), we also assume that the support of V conditional on X_1 is equal to the support of V for all $X_1 \in \mathcal{X}$. This common support assumption implies that the instrumental variables Z need to vary sufficiently to explain the entire support of the endogenous regressor X_1. Therefore, this restriction rules out any discrete instrumental variables because the only variation in $V|X_1$ is induced by Z and this can take only a finite number of values when the instrument is a discrete variable. This assumption also implies that in empirical applications the Imbens and Newey (2009) control function approach is not applicable for weak instruments.

We apply the Imbens and Newey (2009) control variable approach to identify the FPPE and MPPE for a continuous endogenous regressor in the above specified triangular model and present a modified version of the Rothe (2012) identification results in the following Proposition.[5]

[4]Probability Integral Transformation Theorem states that if a random variable X has a continuous CDF $F_X(x)$ and we define the random variable Z as $Z = F_X(x)$, then Z is uniformly distributed on (0, 1). Note that if F_X is strictly increasing, then F_X^{-1} is well defined by $F_X^{-1}(z) = x \Rightarrow Q_X(z) = x$ where Q_X is the quantile function of X.

[5]The proof of the following Proposition 1 is analogous to the proofs of Theorem 1 and Theorem 4 in Rothe (2012).

Proposition 1. *Suppose that in the triangular nonseparable model represented by equations (1) and (2), the regularity conditions stated in Imbens and Newey (2009) hold, then we have* $F_Y^H = E\Big(F_{Y|X,V}\big(y|\tilde{H}^{-1}(F_{X|V}(X|V)), X_2\big)\Big)$ *and the FPPE* $\tilde{\alpha}(\tilde{\nu}, X_1|V, \tilde{H})$ *is identified for any functional* $\tilde{\nu}$.

2.3. Estimation To implement the FPPE for a continuous endogenous regressor, we first estimate the control variable, $\hat{V}_i$. As discussed, we estimate the control variable, $\hat{V}_i = \hat{F}_{X|Z}(X_i|Z_i)$ where $\hat{F}_{X|Z}$ is the empirical conditional CDF of X given Z and construct $\widehat{\tilde{X}} = [X_1|\hat{V} \;\; X_2]$ when we have a valid set of instruments Z, and the common support assumption holds.

We use the following sample analog estimator of FPPE for a continuous variable:

$$\widehat{\alpha}(\nu, X_1|\hat{V}, \tilde{H}) = \nu(\widehat{F}_Y^{\tilde{H}}) - \nu(\widehat{F}_Y)$$

where $\nu(\widehat{F}_Y^{\tilde{H}}) = (1/n)\sum_{i=1}^{n} \hat{F}_{Y|\tilde{X}}\big(y, \tilde{H}^{-1}(\hat{F}_{X_1|\hat{V}}(\hat{X}_{1i}|\hat{V}_i)), X_{2i}\big)$ and $\hat{F}_Y, \hat{F}_{X_1|\hat{V}}$ are the empirical CDFs of Y and $X_1|\hat{V}$. Also, $\tilde{H}^{-1}(\cdot)$ is the quantile function of $X_1|\hat{V}$, while $\hat{F}_{Y|\tilde{X}}(\cdot)$ denotes the conditional CDF of Y given $\tilde{X}$. Using Foresi and Peracchi (1995) we estimate $\hat{F}_{Y|\tilde{X}}(\cdot)$ by modeling the conditional probability of the event$(Y \leq y)$ separately for each $y \in \mathbb{R}$ via a logistic regression. Under point identification, we estimate the MPPE by using the plugin estimators following the same fashion as FPPE.

3 Empirical Application

The steep increase in the incarceration rate of men since the late 1970s is one of the most alarming economic issues in the United States because incarcerating an individual imposes enormous economic costs on society (Charles and Luoh, 2010). There is a large amount of research documenting several explanations of this pattern including labor market opportunities, unemployment rates, income inequality, drug epidemic, gang violence, gun control laws and changing demographics.[6] While all these factors are important, we investigate two other factors, how changes in the poverty rate and the black-

[6]See labor market opportunities (Gould et al., 2002), unemployment (Freedman and Owen, 2016; Fougere et al., 2009; Raphael and Winter-Ebmer, 2001), income (Grogger, 1998), drug epidemic and gang violence (Grogger and Willis, 2000; Levitt and Venkatesh, 2000; Kuziemko and Levitt, 2004), Gun control laws (Lott and Mustard, 1997; Ludwig, 1998; Duggan, 2001), and changing demographics ((Sampson and Lauritsen, 1997).

white wage gap affect the incarceration rate of black men. This is important from a policy perspective.

The hypothesis of how poverty and black-white racial wage gap affect the incarceration rates is based on Merton (1938)'s strain theory and Becker (1968)'s theory of crime. Becker (1968)'s economic theory of crime suggests that low skill individuals have a higher incentive to commit crimes based on their labor market opportunities. Merton (1938)'s strain theory argues that unsuccessful individuals feel frustrated while having relatively successful people around them. Therefore, the higher the poverty rate and social inequality such as the racial wage gap, the higher is the strain and the higher are the number of crimes.

Our empirical approach differs from the existing literature in three important respects. First, instead of focusing on the crime rate, we directly estimate the impact of the poverty rate and the racial wage gap on the state-level incarceration rate. Second, the existing literature mainly estimated the mean effect of different state variables, whereas we estimate the mean effect as well as use the Rothe (2012)'s PPE approach to decompose the incarceration distribution function and estimate the FPPE of poverty and the racial wage gap at different quantiles. Third, we address the endogeneity of poverty and the racial wage gap by using the control variable approach developed by Imbens and Newey (2009).

3.1. Data The data come from three different sources. First, the household income as well as all the demographic control variables are constructed from the 5 percent sample of the Current Population Survey (CPS). Second, all crime data are collected from the National Prisoner Statistics Code book and third, all state-level control variables are obtained from the University of Kentucky Center for Poverty Research Welfare Data. Since the CPS data contain individual-level information, we use these data to construct a rich set of demographic control variables which are reported in the right panel of the summary statistics shown in Table 1. We also calculate the black-white racial wage gap from the CPS data.

The state-level variables such as the state minimum wage, the poverty rate, the unemployment rate, the state GDP, and total population come from the University of Kentucky Center for Poverty Research Welfare Data. The aggregate state-level incarceration data are obtained from the National Prisoner Statistics Codebook. We merge these three data sets by the state indicator variable. Therefore, we have 51 state-level observations for each year. To implement the Rothe (2012)'s PPE approach we split the sample into two time periods 1980-1990 and 2000-2010. Following a similar approach to Rothe (2012), we use the data from 1980-1990 as our base period and use the 2000-2010 data to estimate the direction of the counterfactual change.

The outcome variable (incarceration rate of black men) is the average number of black state prisoners per 1000 individuals of that state. The covariates X include the poverty rate, the black-white annual wage gap in 2000 dollars, the unemployment rate, the real state minimum wage, per capita state income, and a rich set of state-level demographic characteristics such as years of education, age, number of children and household members, percentage of urban population, works in unskilled occupation and ratio of racial minorities. A state's poverty rate denotes the percentage of people below the Federal Government's threshold poverty level income which depends on the number of adults and children in a household. It is also adjusted each year by the inflation factor.[7]

The summary statistics are shown in Table 1. These summary statistics show that in our data the incarceration rate of black men increased from 0.21 in 1980-1990 to .428 in 2000-2010. Meanwhile, the racial wage gap decreased from 0.20 in 1980-1990 to 0.12 in 2000-2010. The poverty rate decreased from 13.88 to 12.2 over these two periods, while the unemployment rate decreased from 6.94 to 5.52 over the same two periods. Other changes as well as the simple t-statistics showing the significance of the difference in means are reported in Table 1. Notably the increase in the real state minimum wage from 1.98 to 6.87 and the increase in log of state GDP from 10.37 to 12.03 over these two periods.

3.2. Results Using the procedure described in Section 2.3, we estimate FPPE, $\alpha(\nu, X, H)$ for various functionals of ν of the black-white wage gap, unemployment rate, poverty and the real state minimum wage. We do not report the results of other covariates for brevity. Since poverty is most likely correlated with the unobserved local labor market conditions, we first estimate the control variable V_i by using the Bartik (1991) instruments, which are based on plausible exogenous local labor demand shocks. This approach is popularised by Blanchard and Katz (1992), and since then these instruments have been widely used in many sub-fields of economics.

The Bartik instruments are constructed by estimating the local employment growth that generates from interacting regional variation in the industry employment shares with national industry employment growth rates. Suppose we have K industries, T time periods and S states, with k, t, and s denoting a particular industry, time and state. Specifically, to construct the

[7] See https://usa.ipums.org/usa/volii/poverty.shtml for the detailed definition of poverty.

Table 1: Summary statistics of state and demographic variables (Mean and Standard Deviation)

	State Variables					Demographic Variables			
	All	2000-2010	1980-1990	Diff (t-stat)		All	2000-2010	1980-1990	Diff (t-stat)
Incarceration Rate of Black Men	0.324	0.428	0.213	0.215	Education	13.150	13.474	12.806	0.668
	(0.200)	(0.161)	(0.176)	(21.022)		(0.497)	(0.344)	(0.391)	(29.880)
Racial Wage Gap	0.160	0.120	0.202	-0.082	Family Size	3.099	3.058	3.144	0.086
	(0.233)	(0.179)	(0.273)	(-5.851)		(0.203)	(0.201)	(0.195)	(7.133)
Poverty	13.010	12.195	13.875	-1.680	No of Children	0.989	1.007	0.971	0.036
	(3.966)	(3.279)	(4.425)	(-7.127)		(0.113)	(0.110)	(0.114)	(5.268)
Unemployment	6.215	5.527	6.944	-1.416	Urban Population	0.656	0.734	0.574	0.160
	(2.281)	(1.987)	(2.348)	(-10.743)		(0.249)	(0.189)	(0.276)	(11.211)
State Migration	0.032	0.028	0.035	-0.007	Full Time Workers	0.878	0.897	0.858	0.040
	(0.020)	(0.014)	(0.025)	(-5.745)		(0.031)	(0.029)	(0.029)	(26.883)
State Minimum Wage	4.498	6.868	1.984	4.883	Labor Force Participation	0.917	0.932	0.901	0.031
	(2.752)	(1.702)	(0.500)	(63.273)		(0.024)	(0.014)	(0.022)	(28.182)
Log State GDP	11.222	12.028	10.366	1.662	Unskilled Occupation	0.831	0.869	0.790	0.079
	(1.334)	(1.052)	(1.035)	(26.201)		(0.075)	(0.081)	(0.039)	(20.209)
Log Population	15.016	15.080	14.948	0.132	Black/White Ratio	0.148	0.162	0.134	0.027
	(1.015)	(1.031)	(0.994)	(2.145)		(0.245)	(0.227)	(0.262)	(1.850)
No of Observation	1,084	558	526		No of Observation	1,084	558	526	

The summary statistics table reports mean and standard deviations (in parentheses). The data are obtained for the period 1980 to 2010 from the Current Population Survey, National Prisoner Statistics Code book and University of Kentucky Center for Poverty Research. Each year we have 51 observations. Annual income and state minimum wages are expressed in 2000 US dollars using the Personal Consumption Expenditures price index

Bartik instruments we estimate the predicted change in log employment $\widehat{B}_{st}$ of a state s between years t_0 and t_1 as

$$\widehat{B}_{st} = \sum_{k=1}^{K} \theta_{skt_0} \delta_{-skt_1} \tag{3.1}$$

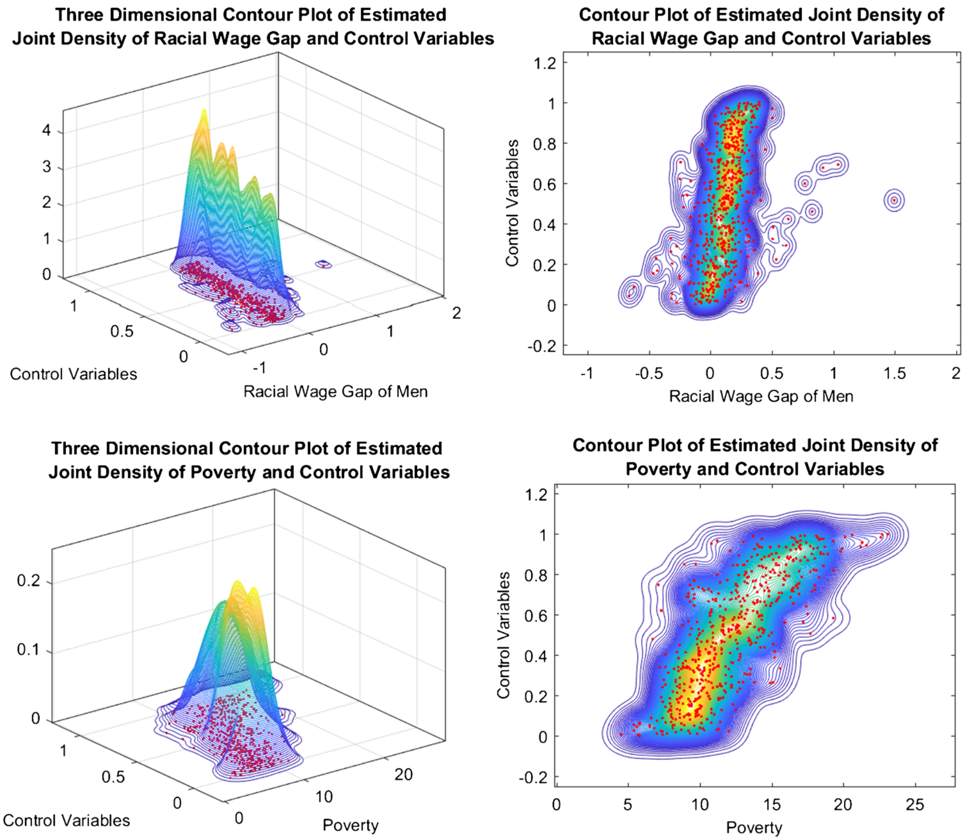
where θ_{skt_0} is the employment share in industry k in state s at period t_0 and δ_{-skt_1} is the national employment growth rate.[8] Equation 3.1 suggests that the difference in predicted employment growth across states stems almost entirely from variations in the regional industry composition. Therefore, the strength of the Bartik instruments relies on the association of the location-specific industry shares, not the national employment growth rate (Blanchard and Katz, 1992) and (Autor and Duggan, 2003).[9] In Appendix Fig. 1, we show that the predicted black-white racial wage gap and poverty rates obtained from using the Bartik instruments are highly correlated with the observed values.[10]

We estimate the conditional CDF of Y given X by using a flexible parametric approach developed by Foresi and Peracchi (1995). We also estimate the full distributional policy effect of a change in the joint covariate distribution from the period 1980-1990 to 2000-2010. The standard errors are calculated using 500 bootstrap replications. To verify the common support assumption, we follow the approach proposed by Imbens and Newey (2009). In Figure 1, we show the contour plot of the joint density function of $(X_1, \hat{V})$ where $\hat{V} = \hat{F}_{X_1|Z}(x_1|z)$ and X_1 denotes the black-white wage gap in the upper panel and poverty rate in the lower panel. In both the panels, the left figure shows a three-dimensional plot of the joint density function of $(X_1, \hat{V})$ and the right figure shows the same plot in two dimensions where the height

[8]We use the Current Population Survey (CPS) two-digit industry classification to determine the total number of industries. In this case, $K = 14$. Also, the notation δ_{-skt_1} implies that when we calculate the national employment growth of industry k we consider all the states except the particular state s.

[9]Currie and Gruber (1996a), Currie and Gruber (1996b), Bound and Holzer (2000), Gould et al. (2002), Autor and Duggan (2003), Acemoglu et al. (2010), & Autor et al. (2013), among others use cross-state differences in industrial composition and national changes in employment to predict each state's employment growth.

[10]As shown in the Appendix Fig. 1, the slope of the best linear fitted line is 0.21 for the black-white wage gap and 0.61 for poverty.



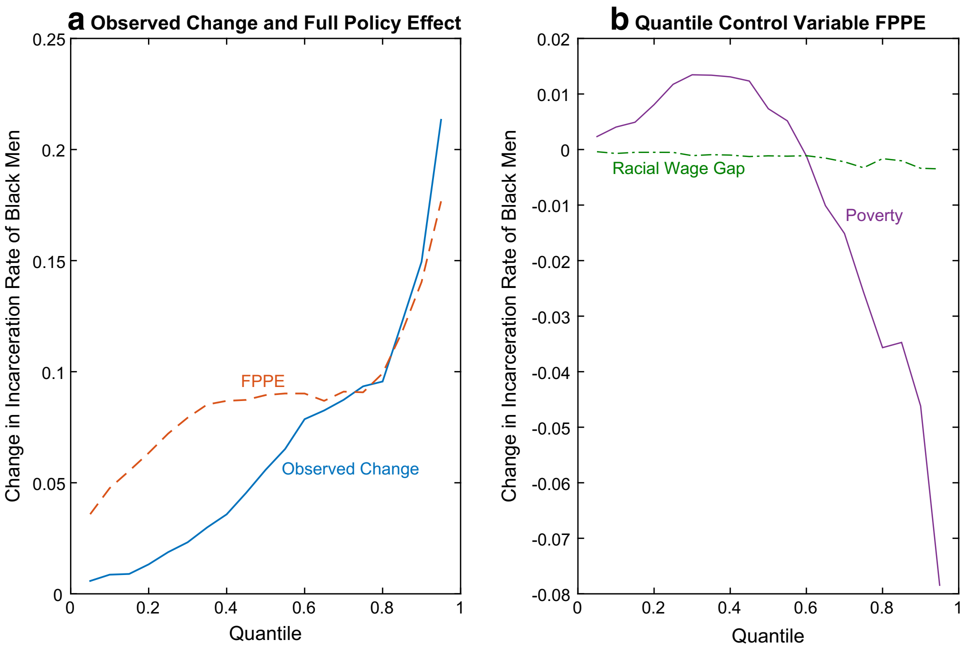
The plots show the Kernel Joint Density Estimator of $(X_1, \hat{V})$ based on X_{1i} and the control variable estimates $\hat{V}_i = \hat{F}_{X_1|Z}(x_{1i}|z_i)$. The red dots indicate the observations of $(X_{1i}, \hat{V}_i)$. In the right panel the color intensity denotes the height of the joint density estimator which is shown in the left panel

Figure 1: The contour plot of the estimated joint density function of control variables and black-white racial wage gap in the upper panel and control variables and poverty in the lower panel

of the joint density curve is shown by the color intensity of the shaded area. In both the upper and lower panels the left-hand side plot shows that the estimated density is close to zero for a limited number of pair of observations and the right-hand side figures show that those points don't have any blue shaded area. Thus, in our empirical analysis the common support assumption is satisfied for a wide range of values of X_1.

The left panel of Fig. 2 compares estimates of the observed change in the incarceration rate and the full distributional policy effects from the 5th to the 95th quantiles of the black men's incarceration rate distribution function. We see that the observed change in the incarceration rate of black men monotonically increases and there is a discrete jump of the change in the incarceration rate at the upper-tail of the distribution. The full policy effect also has a similar pattern, but it marginally overestimates the impact in the lower half of the distribution. However, as shown in the left panel of the FPPE curve, one can trace out the discrete jump of the change in the incarceration rate in the upper-tail of the distribution.

Next, we consider the FPPEs of the poverty rate and the black-white racial wage gap. Table 1 shows that racial wage gap decreased about 8% from 1980-1990 to 2000-2010. However, as shown in the right panel B of Fig. 2, across all quantiles it does not explain the changes in the incarceration rate of black men. Note that the FPPE estimate of poverty at the 90th quantile - 0.046 is obtained from the difference between the counterfactual distribution



Relative change in τ-quantile of US black men's incarceration rate from 1980 to 2010 for $\tau \in (0.05, 0.95)$. Left panel: Observed change (bold line) and estimated full distributional policy effect (dashed line). Right panel: FPPE of changes in poverty (bold line); FPPE of changes in racial wage gap (dashed line)

Figure 2: Observed and estimated change in incarceration rate of black men and impact of poverty and black-white racial wage gap from the control variable FPPE method

and black men's observed incarceration distribution of period 1980-1990. The counterfactual distribution is defined as a hypothetical scenario where the distribution of the poverty rate is that of period 2000-2010 and all other covariates remain the same as in 1980-1990. Table 1 shows that the poverty rate fell from 13.87 in 1980-1990 to 12.19 in 2000-2010. Thus, the FPPE estimate -0.046 implies that the incarceration rate of black men would have fallen by 0.046 if everything else remain the same as in 1980-1990 and only the poverty rate is distributed as in 2000-2010. Since at the 90th quantile black men's incarceration rate increases by 0.149 from 1980-1990 to 2000-2010, the FPPE estimates of poverty shows that changes in the poverty rate account for about one-third of the increase in black men's incarceration rate in the 90th quantile of the distribution. This is also shown in the right panel B of Fig. 2. In the lower panel of Table 2, we report the Rothe (2012) FPPE results when we do not take into account the endogeneity of the poverty rate. We note that if we do not address the endogeneity issue, the FPPE estimates of the poverty rate are biased.

3.3. Comparison to Chernozhukov et al. (2013) Using the distribution regression (DR) approach developed by Chernozhukov et al. (2013), we can also construct counterfactual distribution functions for changes in the distribution of the covariates, or changes in the conditional distribution of the outcome given the covariates or both. Therefore, we also implement the Chernozhukov et al. (2013) distribution regression method and compare the results with those obtained using the FPPE method. We estimate the impact of an individual covariate X by using the following expression:

$$Effect\ of\ X = \nu(\widehat{F}_Y^C) - \nu(\widehat{F}_{Y,t})$$

where $\widehat{F}_Y^C$ is the estimated counterfactual distribution function of black men's incarceration rate and $\widehat{F}_{Y,t}$ is the estimated unconditional incarceration rate distribution function of black men for period t. The counterfactual distribution function of black men's incarceration rate, F_Y^C is defined as,

$$F_Y^C = \int F_{Y|X_{t=0}}(y|x)\, dF_{X_{t=1}}(x)$$

where $t = 0$ and 1 denote respectively the base period and the last period of the data and $F_{Y|X_{t=0}}(\cdot)$ is the conditional distribution of Y given $X_{t=0}$.

Wenote that both Rothe (2012) & Chernozhukov et al. (2013) approaches can incorporate a counterfactual experiment to construct the unconditional

Table 2: Estimated fixed partial policy effects of changes in the unconditional distribution of state economic variables from that in 1980-1990 to that in 2000-2010 on the distribution of incarceration rate of black men

	Mean	Q10	Q25	Q50	Q75	Q90
I. Total change	0.063	0.009	0.019	0.056	0.093	0.149
	(0.006)	(0.001)	(0.001)	(0.006)	(0.005)	(0.008)
II. Control variable FPPE						
Full policy effect	0.097	0.048	0.072	0.089	0.091	0.140
	(0.021)	(0.001)	(0.002)	(0.003)	(0.015)	(0.044)
Partial policy effect						
Poverty	−0.012	0.004	0.012	0.007	−0.026	−0.046
	(0.003)	(0.002)	(0.003)	(0.003)	(0.004)	(0.005)
Racial wage gap	0.001	−0.001	−0.001	−0.001	−0.003	−0.003
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Unemployment	0.003	0.000	−0.001	−0.007	0.001	−0.003
	(0.001)	(0.000)	(0.000)	(0.001)	(0.001)	(0.002)
Per capita state income	0.008	−0.003	0.000	0.005	0.004	0.018
	(0.002	(0.000)	(0.000)	(0.001)	(0.003)	(0.006)
III. Rothe FPPE						
Full policy effect	0.105	0.048	0.073	0.092	0.111	0.174
	(0.025)	(0.002)	(0.002)	(0.006)	(0.021)	(0.053)
Partial policy effect						
Poverty	0.016	0.001	0.015	0.026	0.024	0.011
	(0.003)	(0.001)	(0.002)	(0.003)	(0.002)	(0.004)
Racial wage gap	0.001	−0.001	−0.001	−0.001	−0.003	−0.003
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)
Unemployment	0.002	0.000	−0.001	−0.006	−0.001	−0.006
	(0.001)	(0.000)	(0.000)	(0.001)	(0.001)	(0.004)
Per capita state income	−0.004	−0.003	−0.003	0.007	−0.016	−0.022
	(0.001	(0.000)	(0.000)	(0.001)	(0.001)	(0.012)

Partial policy effects are reported for selected covariates only. Other covariates included on the model are listed in the descriptive statistics table. Bootstrapped standard errors in parentheses (B = 500 bootstrap replications)

distribution of the outcome variable by integrating the conditional cumulative distribution function (CDF) of Y given X with respect to the counterfactual covariate distribution and then we can directly calculate the distributional feature of interest. Another similarity is that both approaches use the conditional exogeneity condition of a covariate to perform a counterfactual exercise. However, there are also some dissimilarities between these two approaches. First, Rothe (2012) introduces a rank invariance condition which implies that in the counterfactual experiment the joint distribution of individuals' respective covariate ranks remain constant. Thus, the PPE approach holds the copula of the covariate distribution constant by preserving the dependence structure of individuals' observable characteristics. While performing the counterfactual experiment, unlike the Rothe (2012) method, the Chernozhukov et al. (2013) approach does not necessarily satisfy the rank invariance condition.

Second, because of the rank invariance condition it is more difficult to construct a counterfactual distribution of Y for an exogenous change of a discrete variable in Rothe (2012)'s PPE approach than the Chernozhukov et al. (2013)'s distribution regression method. For a discrete variable, PPE parameter can be only partially identified because the rank of an individual in the respective unconditional distribution is not uniquely determined by the data. Thus, we can only obtain upper and lower bounds. In contrast, we can fully identify the distribution regression parameter for a discrete variable.

In our empirical setup, the base period is 1980-1990 and the last period is 2000-2010. Thus, the counterfactual distribution function F_Y^C corresponds to the distribution of black men's incarceration rate that would have been observed in 1980-1990 if an individual covariate X was distributed as in 2000-2010 and all other variables remain the same as in 1980-1990. To estimate $F_{Y|X_{t=0}}(\cdot)$ by using the distribution regression approach, we use the logistic link function $\Lambda(\cdot)$ and then estimate $F_{X_{t=1}}$ by the empirical distribution. In the upper and middle panels of Table 3, we show the observed and estimated change in the incarceration rate of black men at different quantiles. The numbers in parentheses show the standard errors obtained from 500 bootstrap replications. Looking at the last column of Table 3 for Q90, we note that the estimated change in the incarceration rate of black men from the Chernozhukov et al. (2013) distribution regression method accounts for only one-third of the observed change in incarceration rate in the upper-tail of the distribution (0.045 estimated change compared to 0.130 total change). In contrast, the last column of Table 2 for Q90 shows that the control variable FPPE approach can explain almost the entire change in the incarceration rate in the upper-tail of the distribution (0.140 for the full policy effect compared to 0.149 total change).

In the lower panels of Table 3, we report the estimates from the distribution regression decomposition method to show to what extent the racial wage gap, poverty, the unemployment rate and the real state minimum wage can explain changes in the unconditional incarceration rate distribution of

Table 3: Distribution regression based estimated change in incarceration rate of black men and impact of individual state economic variables on the changes in the incarceration distribution function from 1980-1990 to 2000-2010

	Mean	Q10	Q25	Q50	Q75	Q90
I. Total change	0.066	0.007	0.013	0.051	0.102	0.130
	(0.001)	(0.003)	(0.001)	(0.009)	(0.002)	(0.004)
II. DR decomposition						
estimated change	0.009	0.029	0.036	0.039	0.042	0.045
	(0.002)	(0.007)	(0.005)	(0.004)	(0.002)	(0.004)
Impact of individual covariates						
poverty	−0.023	0.004	−0.012	−0.021	−0.016	−0.002
	(0.001)	(0.001)	(0.004)	(0.003)	(0.003)	(0.002)
Racial wage gap	0.021	−0.004	0.042	0.049	0.021	−0.004
	(0.001)	(0.002)	(0.004)	(0.002)	(0.003)	(0.003)
Unemployment	−0.011	−0.043	−0.011	0.002	−0.010	0.009
	(0.001)	(0.001)	(0.004)	(0.002)	(0.003)	(0.003)
Per Capita state income	0.001	0.002	−0.004	0.029	0.012	−0.001
	(0.001)	(0.002)	(0.003)	(0.003)	(0.002)	(0.003)

We report the estimated change in incarceration rate at different quantiles of the distribution and impact of four state economic variables to explain the change in incarceration distribution function from 1980-1990 to 2000-2010 by using distribution regression methods. Bootstrapped standard errors in parentheses (B = 500 bootstrap replications).

black men. Note that the impact of poverty is much smaller than what we obtain from the control variable FPPE approach in the upper-tail of the distribution. This finding is not surprising because Chernozhukov et al. (2013) based decomposition method cannot explain the discrete jump in the incarceration rate of black men in the upper-tail of the distribution.

4 Conclusion

Using the control variable approach introduced by Imbens and Newey (2009), we extend the identification and estimation of PPEs to allow for endogeneity, i.e., the case when the strict exogeneity assumption between an explanatory variable and the unobserved error term does not hold. We also implement the control variable PPE approach to estimate the impact of the poverty rate and the black-white racial wage gap on the incarceration rate of black men. We find that changes in the regional variations in the poverty rate can explain about one-third of the changes in the incarceration rate of black men in the upper-tail of the distribution. In contrast, changes in black-white racial wage gap from 1980-1990 to 2000-2010 do not have much impact on the steep increase in black men's incarceration rate. Comparing our results with a similar method developed by Chernozhukov et al. (2013), we find that the estimated change in the incarceration rate of black men from the Chernozhukov et al. (2013) distribution regression method accounts for only one-third of the observed change in incarceration rate in the upper-tail of the distribution. In contrast, the control variable FPPE approach can explain almost the entire change in incarceration rate in the upper-tail of the distribution.

Publisher's Note. Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

References

ACEMOGLU, D., FINKELSTEIN, A. and NOTOWIDIGDO, M. (2010). Income and health soending: Evidence from oil price shocks. *Rev. Econ. Stat.* **95**, 4, 1079–1095.

AUTOR, D., DORN, D. and HANSON, G. (2013). The China syndrome: Local labor market effects of import competition in the united states. *Am. Econ. Rev.* **103**, 6, 2121–2168.

AUTOR, D. and DUGGAN, M. (2003). The rise in the disability rolls and the decline in unemployment. *Q. J. Econ.* **118**, 1, 157–205.

BARTIK, T. (1991). Who benefits from state and local economic development policies?, W.E. Upjohn Institute.

BECKER, G. (1968). Crime and punishment: an economic approach. *J. Polit. Econ.* **76**, 2, 169–217.

BLANCHARD, O. and KATZ, L. (1992). Regional evolutions, Brookings Papers on Economic Activity.

BLINDER, A. (1973). Wage discrimination: reduced form and structural estimates. *J. Hum. Resour.* **8**, 436–455.

BLUNDELL, R. and POWELL, J. (2003). Endogeneity in nonparametric and semiparametric regression models. Advances in Economics and Econometrics: Theory and Applications (Eighth World Congress) 2.

BLUNDELL, R. and POWELL, J. (2004). Endogeneity in semiparametric binary response models. *Rev. Econ. Stud.* **71**, 3, 655–679.

BOUND, J. and HOLZER, H. (2000). Demand shifts, population adjustments, and labor market outcomes during the 1980s. *J. Labor Econ.* **129**, 1, 1–59.

CARNEIRO, P., HECKMAN, J. and VYTACIL, E. (2010). Evaluating marginal policy changes and the average effect of treatment for individuals at the margin. *Econometrica* **78**, 377–394.

CARSON, E. (2013). Impresonment Rate of Sentenced Prisoners under the Jurisdiction of State Or Correctional Authorirites per 100,000 U.S Residents., National prisoner statistics program, Bureau of Justice Statistics.

CHARLES, K. and LUOH, M. (2010). Male incarceration, the marriage market, and female outcomes. *Rev. Econ. Stat.* **92**, 3, 614–627.

CHERNOZHUKOV, V., FERNANDEZ-VAL, I. and MELLY, B. (2013). Inference on counterfactual distribution. *Econometrica* **81**, 6, 2205–2268.

CURRIE, J. and GRUBER, J. (1996a). Health insurance eligibility, utilization, medical care and child health. *Q. J. Econ.* **111**, 2, 431–466.

CURRIE, J. and GRUBER, J. (1996b). Saving babies: The efficacy and cost of recent changes in the medicaid eligibilty of pregnant women. *J. Polit. Econ.* **104**, 6, 1263–1296.

DUGGAN, M. (2001). More guns, more crime. *J. Polit. Econ.* **109**, 5, 1086–1114.

FIRPO, S., FORTIN, N. and LEMIEUX, T. (2009). Unconditional quantile regressions. *Econometrica* **77**, 953–973.

FLORENS, J., HECKMAN, J., MEGHIR, C. and VYTLACIL, E. (2008). Identification of treatment effects using control functions in models with continuous, endogenous treatment and heterogeneous effects. *Econometrica* **76**, 5, 1191–1206.

FORESI, S. and PERACCHI, F. (1995). The conditional distribution of excess returns: An empirical analysis. *J. Am. Stat. Assoc.* **90**, 451–466.

FORTIN, N., LEMIEUX, T. and FIRPO, S. (2011). Decomposition methods in economics. *Handbook of Labor Economics* **4**, 1–102.

FOUGERE, D., KRAMARZ, F. and POUGET, J. (2009). Youth unemployment and crime. *J. Eur. Econ. Assoc.* **7**, 909–938.

FREEDMAN, M. and OWEN, E. (2016). Your friends and neighbors: Localized economic development and criminal activity. *Rev. Econ. Stat.* **98**, 2, 233–253.

GLAZE, L. and HERBERMAN, E. (2013). Correctional Population in the United States, 2012., Teachnical Report U.S. Department of Justice, Bureau of Justice Statistics.

GOULD, E., WEINBERG, B. and MUSTARD, D. (2002). Crime rates and local labor market opportunities in the United States 1977-1997. *Rev. Econ. Stat.* **84**, 1, 45–61.

GROGGER, J. (1998). Market wages and youth crime. *J. Labor Econ.* **16**, 4, 756–791.

GROGGER, J. and WILLIS, M. (2000). The emergence of crack cocaine and the rise in urban crime rates. *Rev. Econ. Stat.* **82**, 11, 519–529.

IMBENS, G. and NEWEY, W. (2009). Identification and estimation of triangular simultaneous equations models without additivity. *Econometrica* **77**, 1481–1512.

KUZIEMKO, I. and LEVITT, S. (2004). An empirical analysis of imprisoning drug offenders. *J. Public Econ.* **88**, 2043–2066.

LEVITT, S. and VENKATESH, S. (2000). An economic analysis of a drug-selling gang's finances. *Q. J. Econ.* **115**, 8, 755–789.

LOTT, J. and MUSTARD, D. (1997). Right-to-carry concealed guns and the importance of deterrence. *J. Leg. Stud.* **26**, 1, 1–68.

LUDWIG, J. (1998). Concealed gun carrying laws and violent crime: Evidence from state panel data. *Int. Rev. Law Econ.* **18**, 9, 239–254.

MERTON, R. (1938). Social structure and anomie. *Am. Sociol. Rev.* **3**, 672–682.

OAXACA, R. (1973). Male-female wage differentials in urban labor markets. *Int. Econ. Rev.* **14**, 693–709.

RAPHAEL, S. and WINTER-EBMER, R. (2001). Identifying the effect of unemployment on crime. *J. Law Econ.* **44**, 1, 259–283.

ROTHE, C. (2010). Nonparametric estimation of distributional policy effects. *J. Econ.* **156**, 56–70.

ROTHE, C. (2012). Partial distributional policy effects. *Econometrica* **80**, 2269–2301.

SAMPSON, R. and LAURITSEN, J. (1997). Racial and ethnic disparities in crime and criminal justice in the united states. *M. Tonry, ed. Chicago: University of Chicago Press* **21**, 9, 311–374.

Appendix

Proposition 2. *Suppose that in the triangular nonseparable model represented by* Eqs. 2.1 *and* 2.2*, the regularity conditions stated in Imbens and Newey* (2009) *hold, then we have* $F_Y^H = E\left(F_{Y|X,V}\left(y|H^{-1}(F_{X|V}(X|V)), X_2\right)\right)$ *and the FPPE* $\tilde{\alpha}(\tilde{\nu}, X_1|V, \tilde{H})$ *is identified for any functional* $\tilde{\nu}$*.*

Proof. For convenience, we restate all the necessary regularity conditions we need from Imbens and Newey (2009) & Rothe (2012).

- A1: In the triangular model of Eqs. 2.3 and 3.1 introduced in Section 2, suppose (i) (independence) of the error terms of both equations, η and ε are independent of the instrumental variables Z and (ii) (monotonicity) ε is a continuously distributed scalar with CDF that is strictly increasing in the support of ε.
- A2: Let the endogenous variable X_1 have a continuous CDF, $H(x_1)$ which is monotonically increasing in the entire support of X_1 .
- A3: Common Support: For all $X_1 \in \mathcal{X}_1$, the support of V conditional on X_1 equals the support of V.
- A4: The unknown structural function $m(\cdot)$ is continuously differentiable of order d and the support of the derivatives are uniformly bounded in x and z where $d \geq 2$.
- A5: Let $W = [X, V]$ then Var(Y | W) is bounded.

Using the nonseparable structural model introduced in Section 2, and the definition of cumulative distribution function, we get

$$F_Y^{\tilde{H}}(y) = Pr\left(m\left(\tilde{X}_H, \eta\right) \leq y\right) \tag{4.1}$$

where $\tilde{X}_H = \left[\tilde{X}_1 \ X_2\right]$, $\tilde{X}_1 = X_1|V$, V is the control variable defined as $V = F_{X_1|Z}(X_1, Z) = F_\varepsilon(\varepsilon)$ and Z is the vector of instruments. Using the full independence and monotonicity assumptions from A1, Imbens and Newey (2009) show that there exist a control variable V such that $\tilde{X}_1$ and η are independent.

The counterfactual distribution function of Y, $\tilde{F}_Y^H(y)$ is defined as a scenario where ceteris paribus holding the copula of the $\tilde{X}_1$ and X_2 fixed, while changing the marginal distribution of $\tilde{X}_1$. This can be formalized using the probability integral transformation theorem which implies that

$\tilde{X}_1 = Q_{\tilde{X}_1}(U_1)$, where $Q_{\tilde{X}_1}$ is the quantile function of $\tilde{X}_1$ and $U_1 \sim U(0,1)$. Therefore, in this setup, Rothe (2012)'s counterfactual experiment amounts to changing $Q_{\tilde{X}_1}$ to another quantile function $\tilde{H}(\cdot)$ while keeping the joint distribution of the rank variables $U = (U_1, U_2, ..., U_k)$ fixed.

Under assumptions A2 and A3, $\tilde{H}(x_1|v)$ is also monotonically increasing. Therefore, $\tilde{H}^{-1}(\cdot)$ also exists and is identified in the support of (X_1, V). Again, by using the probability integral transformation theorem, we rewrite $\tilde{X}_1$ in terms of their unconditional quantile function:

$$\tilde{X}_1 = \tilde{H}^{-1}(U_1). \tag{4.2}$$

Thus, the covariate vector $\tilde{X}_H$ can be rewritten as $\tilde{X}_H = \left[\tilde{H}^{-1}(U_1) \quad X_2\right]$ and hence we get,

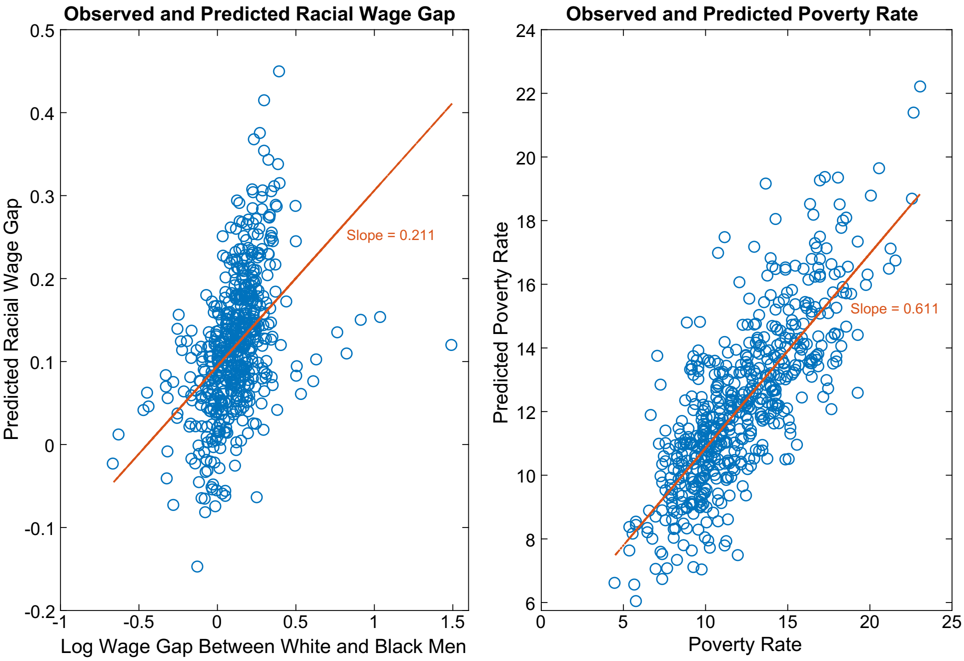
$$\begin{aligned} F_Y^{\tilde{H}}(y) &= Pr\left(m\left(\tilde{X}_H, \eta\right) \leq y \mid \tilde{X}_1 = x_1|v, X_2 = x_2\right) \\ &= \int Pr\left(m\left(\tilde{H}^{-1}(U_1), x_2, \eta\right)\right. \\ &\leq \left. y \mid U_1 = u_1, X_2 = x_2\right) dF_{U_1X_2}(u_1, x_2) \end{aligned} \tag{4.3}$$

Under the assumption A2 and A3, Eq. 4.2 shows that there exists a one to one correspondence between $\tilde{X}_1$ and U_1 over the range of $\tilde{H}^{-1}(\cdot)$. Hence, following Rothe (2012) we get,

$$\begin{aligned} F_Y^{\tilde{H}}(y) &= \int Pr\left(m(\tilde{x}_1, x_2, \eta) \leq y \mid Q_{\tilde{X}_1}(U_1)\right. \\ &= \left.\tilde{x}_1, X_2 = x_2\right) dF_{U_1X_2}\left(\tilde{H}_1(\tilde{x}_1), x_2\right) \\ &= \int Pr\left(m(\tilde{x}_1, x_2, \eta) \leq y \mid \tilde{X}_1\right. \\ &= \left.\tilde{x}_1, X_2 = x_2\right) dF_{U_1X_2}\left(\tilde{H}_1(\tilde{x}_1), x_2\right) \\ &= \int F_{Y|\tilde{X}}\left(y, \tilde{X}_1, X_2\right) dF_{U_1X_2}\left(\tilde{H}_1(\tilde{x}_1), x_2\right) \end{aligned} \tag{4.4}$$

The regularity conditions A4 and A5 imply that $m\left(\tilde{X}_1, X_2, \eta\right) = m(X_1|V, X_2, \eta)$ is contained in the support of (X, V). Then by Imbens and Newey (2009) Theorem 9, $F_{Y|X,V}(\cdot)$ is identified. Therefore, from Eq. 3.1 we get,

$$F_Y^{\tilde{H}}(y) = \int F_{Y|X,V}\left(y, \tilde{H}_1^{-1}(U_1), X_2\right) dF_{U_1X_2}(u_1, x_2)$$



In both the panels the dashed line represents the best linear fits which are obtained by regressing the outcome variables racial wage gap and poverty rates on the Bartik instruments and a set of control variables listed in Table 1

Appendix Fig. 1 scatter plot of observed and predicted black-white racial wage gap and observed and predicted poverty rate

$$= E\left(F_{Y|X,V}\left(y, \tilde{H}^{-1}(U_1), X_2\right)\right) \tag{4.5}$$

Again, under the assumption A2 and A3, Eq. 4.2 shows that there exists a one to one correspondence between $\tilde{X}_1$ and U_1 over the range of $\tilde{H}^{-1}(\cdot)$. Hence we get,

$$F_Y^{\tilde{H}}(y) = E\left(F_{Y|X,V}\left(y, \tilde{H}^{-1}\left(F_{X_1|V}(X_1|V)\right), X_2\right)\right) \tag{4.6}$$

The proof of the second part is similar to part 1. Using the above steps it can be easily shown that $F_{Y|X,V}$ is identified over the area of integration as shown in the right hand side of Eq. 4.5. Hence, both $F_Y^{\tilde{H}}(y)$ and $\tilde{\nu}_H = \nu\left(F_Y^{\tilde{H}}(y)\right)$ are identified.

Badi H. Baltagi
Department of Economics and Center
for Policy Research, 426 Eggers Hall,
Syracuse University,
Syracuse, NY 13244, USA

Pallab K. Ghosh
Department of Economics,
University of Oklahoma,
308 Cate Center Drive,
Room 158 Cate 1 (CCD1),
Norman, OK 73072, USA
E-mail: pallab.ghosh@ou.edu

Paper received: 29 April 2018.