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Gaussian Light Modulation of Translational Squeezed Quantum Light Field

Hai-Jun Yu[1,2] · Hong-Yi Fan[1]

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Abstract

It is an important topic of the coupling between quantum and classical light fields. In this paper, a classical Gaussian light field $e^{-\frac{q^2}{2\tau_1^2}-\frac{p^2}{2\tau_2^2}}$ is used to modulate the translational squeezed quantum light field. By using the integral method in the ordered operator, we find that the translational squeezed quantum light field modulated by Gaussian light is still a translational squeezed light, and its quantum properties remain unchanged, but chaotic parameters and compression parameters are changed. We also derive the formula of the Wigner function depending on the modulation parameters. It can be concluded that the integral method in the ordered operator is an effective way to deal with the mixing of classical and quantum light fields.

Keywords Translational squeezed quantum light field · Classical Gaussian-state light field · Coupling · Wignal function

1 Introduction

Einstein pointed out "what is the nature of light" in his later years. Up to now, no satisfactory conclusion has been reached on this issue. In order to understand the nature of light, physicists invented the laser in the last century. It is described by coherent state, and its properties are different from chaotic light. The density operators [1–3] describing chaotic light are

$$\rho_c \equiv \left(1-e^{\lambda}\right)e^{\lambda a^+ a}. \tag{1}$$

The photon annihilation operator and the photon generation operator satisfy the commutation relation of $\left[a, a^+\right]=1$. When the chaotic parameter $\lambda=-\frac{\omega\hbar}{kT}$, k is Boltzmann constant and T is the temperature of the chaotic light field, the average photon number of the chaotic light gives the Bose–Einstein equation

$$\overline{n}=Tr\left(\rho_c a^+ a\right)=\frac{1}{e^{-\lambda}-1}. \tag{2}$$

Theoretically, the coherent state $|\alpha\rangle$ can be obtained by applying the translation operator [4, 5] $D(\alpha)$ to the vacuum state [6–8] $|0\rangle$

$$|\alpha\rangle = D(\alpha)|0\rangle,\ D(\alpha)=exp\left(\alpha a^+ - \alpha^* a\right), \alpha=(q+ip)/\sqrt{2}. \tag{3}$$

Due to the strong coherence of Coherent States, the Heisenberg uncertainty relation is minimal, so it is closest to the classical case, so the subject of quantum optics emerges as the times require. Later, compressed light [9, 10] was prepared, and some of its non-classical properties were revealed [11, 12]

$$S(r)=exp\left[\frac{r}{2}\left(a^2-a^{+2}\right)\right]. \tag{4}$$

It acts on the vacuum state. At present, the light existing in nature (including celestial bodies) is translational compressed chaotic light [13, 14] in a broad sense. It is the result of the chaotic light field acted by the translational operator and the compression operator. The density operator of this light field is

$$\rho_s \equiv \left(1-e^{\lambda}\right)D(\alpha)S(r)e^{\lambda a^+ a}S^{-1}(r)D^{-1}(\alpha). \tag{5}$$

α is the translation parameter and $\alpha=\frac{q+ip}{\sqrt{2}}$, r is the compression parameter in formula (5).

Considering that any quantum light field may be coupled with the classical light field of its existing environment, what

✉ Hai-Jun Yu
haijun20030@163.com

1 School of Materials and Engineering, University of Science and Technology of China, Hefei 230026, China

2 School of Electronic Engineering, Huainan Normal University, Huainan 232038, China

results will be produced if a classical Gaussian light field is demodulated into a translational compressed quantum light field? How to deal with the coupling of classical light field and quantum light field (expressed by density operator) in theory?

Because of the incompatibility of operators, this problem has not been discussed effectively for a long time. Now we find that the integral theory in ordered operators [15–18] can deal with this problem well because the generation operator and annihilation operator of photons in normal product or Weyl ordering are interchangeable. The integral in ordered operators have been used to deal with the integral of Dirac ket bra operators, such as the integral of the following operators in coordinate representation, we can get the contraction operator

$$\int_{-\infty}^{\infty}\frac{dq}{\sqrt{\mu}}|q/\mu\rangle\langle q| = S(r), \mu = e^r,$$

$Q|q\rangle = q|q\rangle$, $|q\rangle$ is the coordinate eigenstate. With the help of the integral in the ordered operator, we will point out that the classical Gaussian optical modulation of the translational squeezed quantum field is still translational squeezed light, but the chaotic parameters and squeezing parameters are changed accordingly, and the density operator with new parameters is derived. The arrangement of this paper is as follows: in the second section, we derive the Weyl order [19–21] form of the translational squeezed chaotic light field, and in the third section, we bring it into the normal product form and find that it presents normal distribution, and then give its Wigner function. In the fourth section, the Gaussian convolution is studied by using the integral method in the ordered operator, and the formulas of adjusting the chaotic parameters and compression parameters of the new light field with the classical modulation parameters are given.

2 Weyl Ordering Form of Density Operators for Translational Squeezed Chaotic Light Field

In order to fully understand the properties of a quantum light field, it is necessary to investigate the different representations of its density operators under different ordering rules, so as to calculate its Wigner function distribution. In order to derive the Weyl ordering form of the translational squeezed chaotic light field, we first use the common law

$$\rho = 2\int\frac{d^2\beta}{\pi}\langle -\beta|\rho|\beta\rangle \vdots\, exp\left(2|\alpha|^2 - 2\beta a^+ + 2a\beta^*\right) \vdots\,. \tag{6}$$

$\vdots\;\vdots$ represents Weyl ordering, $|\beta\rangle = exp\left(\beta a^+ - \beta^* a\right)|0\rangle$ is coherent state and operator identity of normal product

$$e^{\lambda a^+ a} =: exp\left[\left(e^\lambda - 1\right)a^+ a\right]. \tag{7}$$

The operator $e^{\lambda a^+ a}$ is transformed into Weyl ordering, that is, from (6) and (7)

$$\begin{aligned} e^{\lambda a^+ a} &= 2\int\frac{d^2\beta}{\pi}\vdots\,\langle -\beta| : exp\left[\left(e^\lambda - 1\right)a^+ a\right] : |\beta\rangle exp\left(2a^+ a - 2\beta a^+ + 2\beta^* a\right)\vdots\,, \\ &= \frac{2}{e^\lambda + 1}\vdots\, exp\left[\frac{e^\lambda - 1}{e^\lambda + 1}\left(P^2 + Q^2\right)\right]\vdots \end{aligned} \tag{8}$$

where

$$Q = \frac{a^+ + a}{\sqrt{2}},\ P = \frac{a - a^+}{\sqrt{2}i}.$$

Note that the boson operator [22–24] is commutative in Weyl ordering $\vdots\;\vdots$. Then, we use the properties of the translation operator $\vdots\;\vdots$ and contraction operator $S(r) = exp\left[-\frac{i}{2}(QP + PQ)lnr\right]$,

$$D(\alpha)QD^{-1}(\alpha) = Q - q,\ D(\alpha)PD^{-1}(\alpha) = P - p, \tag{9}$$

$$S(r)PS^{-1}(r) = e^r P,\ S(r)QS^{-1}(r) = e^{-r}Q, \tag{10}$$

and the order invariance of Weyl ordering operator under the similar transformation (that is, unitary transformation operator can directly act on the inner operator over the "fence" $\vdots\;\vdots$)

$$\begin{aligned} \rho_s &\equiv \left(1 - e^\lambda\right)D(\alpha)(r)S(r)e^{\lambda a^+ a}S^{-1}(r)D^{-1}(\alpha) \\ &= \frac{2\left(1 - e^\lambda\right)}{e^\lambda + 1}\vdots\, exp\left\{\frac{e^\lambda - 1}{e^\lambda + 1}\left[e^{2r}(P - p)^2 + e^{-2r}(Q - q)^2\right]\right\}\vdots\,. \end{aligned} \tag{11}$$

This is the Weyl ordering form of the density operator of the translational squeezed chaotic light field. In Sect. 3, we find its normal product order, so as to find its Wigner function distribution.

3 Normal Product Ordering Form of Density Operators for Translational Squeezed Chaotic Light Field

According to Weyl quantization rule [25]

$$\rho = 2\int d^2\beta\Delta(\beta,\beta^*)h(\beta,\beta^*), \tag{12}$$

where $\Delta(\beta,\beta^*)$ is a Wigner operator and its Weyl ordering form is

$$\begin{aligned} \Delta(\beta,\beta^*) &= \frac{1}{2}\vdots\,\delta(\beta - a)\delta\left(\beta^* - a^+\right)\vdots\, = \vdots\,\delta\left(q' - Q\right)\delta\left(p' - P\right)\vdots\,, \\ \beta &= \left(q' + ip'\right)/\sqrt{2} \end{aligned} \tag{13}$$

It can be concluded that the classical function corresponding to ρ_s is

$$h(\beta,\beta^*)=\frac{2\left(1-e^{\lambda}\right)}{e^{\lambda}+1}exp\left\{\frac{e^{\lambda}-1}{e^{\lambda}+1}\left[e^{2r}\left(p-p'\right)^2+e^{-2r}\left(q-q'\right)^2\right]\right\}. \tag{14}$$

It can be seen from (2)

$$\frac{e^{\lambda}-1}{e^{\lambda}+1}=-\frac{1}{2\overline{n}+1}.$$

The normal product ordering form based on the Wigner operator [26, 27]

$$\Delta(\beta,\beta^*)=\frac{1}{\pi}:e^{-\left(q'-Q\right)^2-\left(p'-P\right)^2},$$

Based on the integral technique in the ordered operator, we can get

$$\begin{aligned}\rho_s&=\iint\frac{2}{2n+1}dp'dq'exp\left\{-\frac{1}{2\overline{n}+1}\left[e^{2r}\left(p-p'\right)^2+e^{-2r}\left(q-q'\right)^2\right]\right\}\Delta\left(q',p'\right)\\&=\iint\frac{2}{2n+1}dp'dq'exp\left\{-\frac{1}{2\overline{n}+1}\left[e^{2r}\left(p-p'\right)^2+e^{-2r}\left(q-q'\right)^2\right]\right\}:\frac{1}{\pi}e^{-\left(q'-Q\right)^2-\left(p'-P\right)^2},\\&=:\frac{1}{\sigma_1\sigma_2}exp\left\{-\frac{(q-Q)^2}{2\sigma_1^2}-\frac{(p-P)^2}{2\sigma_2^2}\right\}:\end{aligned} \tag{15}$$

where σ_1,σ_2 meets

$$2\sigma_1^2-1\equiv\left(2\overline{n}+1\right)e^{2r},\ 2\sigma_2^2-1\equiv\left(2\overline{n}+1\right)e^{-2r}. \tag{16}$$

This is the normal product ordering form of the density operator of the translational squeezed chaotic light field, which is a two-dimensional normal distribution.

It is easy to verify that

$$tr\rho_s=\frac{1}{\sigma_1\sigma_2}\int\frac{d^2z}{\pi}\langle z|:exp\left[-\frac{(q-Q)^2}{2\sigma_1^2}-\frac{(p-P)^2}{2\sigma_2^2}\right]:|z\rangle=1. \tag{17}$$

From formula (16), we obtain

$$\overline{n}=\frac{1}{2}\left[\sqrt{\left(2\sigma_1^2-1\right)\left(2\sigma_2^2-1\right)}-1\right] \tag{18}$$

and

$$e^{4r}=\frac{2\sigma_1^2-1}{2\sigma_2^2-1},r=\frac{1}{4}ln\frac{2\sigma_1^2-1}{2\sigma_2^2-1}. \tag{19}$$

The edge distribution of two-dimensional normal distribution can be derived

$$\int\frac{dq}{\sqrt{2\pi}}\rho_s=\int\frac{dq}{\sqrt{2\pi}}\frac{1}{\sigma_1\sigma_2}:exp\left[-\frac{(q-Q)^2}{2\sigma_1^2}-\frac{(p-P)^2}{2\sigma_2^2}\right]:=\frac{1}{\sigma_2}:exp\left[-\frac{(p-P)^2}{2\sigma_2^2}\right]:, \tag{20}$$

$$\int\frac{dp}{\sqrt{2\pi}}\rho_s=\int\frac{dp}{\sqrt{2\pi}}\frac{1}{\sigma_1\sigma_2}:exp\left[-\frac{(q-Q)^2}{2\sigma_1^2}-\frac{(p-P)^2}{2\sigma_2^2}\right]:=\frac{1}{\sigma_1}:exp\left[-\frac{(q-Q)^2}{2\sigma_1^2}\right]:. \tag{21}$$

4 Wigner Function of Translational Squeezed Quantum Light Field

In order to calculate the expected photon number of the translational squeezed light field, we first obtain the Wigner function of the light field

$$W(\alpha'^*,\alpha')=2\,\pi\,tr\left[\Delta(\alpha'^*,\alpha')\rho_s\right], \tag{22}$$

where $\alpha'=\frac{q'+ip'}{\sqrt{2}}$, $\Delta(\alpha'^*,\alpha')$ is the Wigner operator, and its representation in the coherent state representation $|z\rangle$ is

$$\Delta(\alpha'^*,\alpha')=\int\frac{d^2z}{\pi}\left|\alpha'+z\right\rangle\left\langle\alpha'-z\right|e^{\alpha'z^*-\alpha'^*z}. \tag{23}$$

From the inner product of two coherent states

$$\langle z'|z\rangle=exp\left[-\frac{1}{2}(|z|^2+\left|z'\right|^2)+z'^*z\right], \tag{24}$$

we calculate the Wigner function of the translational squeezed light field

$$
\begin{aligned}
&2\pi tr\left[\Delta(\alpha'^*,\alpha')\rho_s\right] \\
&= \int \frac{2d^2z}{\sigma_1\sigma_2\pi}\langle\alpha'-z\right| : exp\left[-\frac{(q-Q)^2}{2\sigma_1^2}-\frac{(p-P)^2}{2\sigma_2^2}\right] : \left|\alpha'+z\rangle e^{\alpha' z^*-\alpha'^* z} \\
&= \frac{2\int dz_1\int dz_2}{\sigma_1\sigma_2\pi} exp\left[-2\left(z_1^2+z_2^2\right)-\frac{\left(q-q'-\sqrt{2}iz_2\right)^2}{2\sigma_1^2}-\frac{\left(p-p'-\sqrt{2}iz_1\right)^2}{2\sigma_2^2}\right] \\
&= \frac{2}{\sqrt{(2\sigma_1^2-1)(2\sigma_2^2-1)}} exp\left[-\frac{\left(q-q'\right)^2}{2\sigma_1^2-1}-\frac{\left(p-p'\right)^2}{2\sigma_2^2-1}\right].
\end{aligned}
\tag{25}
$$

Note that $\left(q',p'\right)$ here is the translation parameter of the original light field and $\left(q',p'\right)$ is the variable in phase space.

5 Translational Squeezed Light Field Modulated by Classical Gaussian Light Field

According to the convolution definition of two arbitrary functions $u(x)$ and $v(x)$ [28]

$$
(u * v) = \int u(x-y)v(y)dy = \int v(x-y)u(y)dy. \tag{26}
$$

The Fourier transform of convolution function $(u * v)$ is denoted as F which has the property

$$
F(u * v) = F(u)F(v), \tag{27}
$$

such the integral formula is

$$
\frac{1}{2\pi\sigma\tau}\int e^{-\frac{(X-x)^2}{2\sigma^2}} e^{-\frac{x^2}{2\tau^2}} dx = \frac{1}{\sqrt{2\pi\left(\sigma^2+\tau^2\right)}} e^{-\frac{X^2}{2\left(\sigma^2+\tau^2\right)}}. \tag{28}
$$

A classical Gaussian field $e^{-\frac{q^2}{2\tau_1^2}-\frac{p^2}{2\tau_2^2}}$ is used to modulate the translation compression quantum field. By using the integral method in the ordered operator, we do the convolution

$$
\begin{aligned}
\iint \rho_s e^{-\frac{q^2}{2\tau_1^2}-\frac{p^2}{2\tau_2^2}} dqdp &= \iint \frac{1}{\sigma_1\sigma_2\tau_1\tau_2} : exp\left[-\frac{(q-Q)^2}{2\sigma_1^2}-\frac{(p-P)^2}{2\sigma_2^2}\right] : e^{-\frac{q^2}{2\tau_1^2}-\frac{p^2}{2\tau_2^2}} dqdp \\
&= \frac{1}{2\pi\sqrt{(\sigma_1^2+\tau_1^2)(\sigma_2^2+\tau_2^2)}} : exp\left[-\frac{(q-Q)^2}{2(\sigma_1^2+\tau_1^2)}-\frac{(p-P)^2}{2(\sigma_2^2+\tau_2^2)}\right] : \\
&\equiv \rho'_s.
\end{aligned}
$$

By comparing $\rho_s = \frac{1}{\sigma_1\sigma_2} exp\left[-\frac{(q-Q)^2}{2\sigma_1^2}-\frac{(p-P)^2}{2\sigma_2^2}\right]$ in (15), we can see that

$$
\rho'_s = \left(1-e^{\lambda'}\right)D(\alpha)S\left(r'\right)e^{\lambda' a^+ a}S^{-1}\left(r'\right)D^{-1}(\alpha). \tag{30}
$$

This is the convolution invariant property of the translational squeezed light field, but the squeezing parameter becomes:

$$
e^{4r} \to e^{4r'} = \frac{2\left(\sigma_1^2+\tau_1^2\right)-1}{2\left(\sigma_2^2+\tau_2^2\right)-1}, r' = \frac{1}{4}ln\frac{2\left(\sigma_1^2+\tau_1^2\right)-1}{2\left(\sigma_2^2+\tau_2^2\right)-1}.
$$

The chaotic parameter changes to.

$$
\lambda' = ln\frac{n'}{n'+1}, \overline{n}' = \frac{1}{2}\left\{\sqrt{\left[2\left(\sigma_1^2+\tau_1^2\right)-1\right]\left[2\left(\sigma_2^2+\tau_2^2\right)-1\right]}-1\right\}.
$$

This shows that a classical Gaussian field $e^{-\frac{q^2}{2\tau_1^2}-\frac{p^2}{2\tau_2^2}}$ is used to modulate the translation compression quantum field. The results show that a new translation compression quantum field is obtained, and its nature is not changed, but the chaos parameters and the compression parameters are changed accordingly.

6 Conclusion

In this paper, a classical Gaussian field is used to modulate the translation compressed quantum field by integrating technology in the ordered operator. It is found that the modulation result is still translation compressed light, its quantum properties have not changed, but the chaos parameters and compression parameters are changed accordingly. The Wigner function formula of translation compression quantum field is calculated. This provides a theoretical basis for the subsequent study of other quantum light fields of classical light modulation.

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