



Dust Acoustic Solitary Waves with Dust Charge Fluctuation in Superthermal Plasma

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Abstract

The nonlinear propagation of dust acoustic waves with dust charge fluctuation in presence of superthermal (κ) electrons and ions has been investigated. Reductive perturbation technique along with space-time stretched coordinates is used to transform the basic nonlinear partial differential equations to a modified Korteweg-de-Vries equation. The modified Korteweg-de-Vries equation governs the dynamics of the small amplitude solitary waves in a superthermal dusty plasma. It is observed that the presence of κ distributed electrons and ions significantly change the amplitude of solitons in an inhomogeneous environment. The present investigations may be useful to understand the nonlinear propagation of dust acoustic solitary waves in laboratory and space plasma.

Keywords Dust acoustic solitary waves · Superthermal plasma · Dust charge fluctuation · Reductive perturbation technique

1 Introduction

A number of research works [1–6] have been done in literature on linear and nonlinear dust acoustic (DA) waves since its first prediction by Rao et al. [7] in unmagnetized plasma. The restoring force for the propagation of DA mode comes from the pressure of inertialess electrons/ions, and inertia is provided by the charged dust. Therefore, the phase speed of these waves is much lower than the ion and electron thermal speeds [8, 9]. In nonlinear regime, these waves can give rise to the formation of both compressive and rarefactive solitons. Ergun et al. [10] argued that such type of nonlinear structures play a key role in supporting parallel electric fields in the downward current region of the auroral zone. The Viking spacecraft and Freja satellite data indicated the presence of electrostatic solitary waves in the magnetic ionosphere. Each solitary wave has a definite

velocity depending on the magnitude of the amplitude. The presence of stationary dust component gives rise to a new type of wave mode, known as the dust ion acoustic (DIA) mode. The phase velocity of this mode lies in between the ion and electron thermal velocities [11]. Dust ion acoustic solitary and shock waves also received a great deal of attention in plasma community. The static dust grains give rise to DIA waves whereas the DA waves arise as a result of mobile dust [12]. The DIA wave is the modified form of ion acoustic (IA) in the presence of static dust [13, 14]. The dust particles have different grain sizes; however, as a first approximation, the radius is assumed to be constant [15]. The interaction of electrons and ions with the dust particles makes them charged (positive or negative) by the method of either photoemission or by secondary electron emission. In many theoretical investigations, the dust charge is assumed to be constant for simplicity; however, in different realistic situations, the dust charge varies. The charge variation depends of parameters such as the electrostatic plasma potential and the number densities of electrons and ions, and in response affects the collective behavior of the plasma [6]. Thus, the effect of dust charge variation is important in understanding the nonlinear wave propagation.

Inhomogeneity is observed in most of the astrophysical as well as in laboratory plasmas having considerable effects on plasma dynamics. The inhomogeneity may arise in density, temperature, or in magnetic field [16, 17]. In dusty plasma, this inhomogeneity actively changes the DA wave

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amplitude [18]. Zakir et al. [19] studied the DA drift waves and nonlinear structures in an inhomogeneous dusty plasma with dust charge fluctuation. Recently, Googi and Deka [20] found that inhomogeneity parameters have meaningful influence on the propagation of DA solitary waves.

Generally, the linear and nonlinear dynamics of DA waves have been studied using Boltzmann distributed plasma species. But observations in space plasma environment confirmed the presence of electrons and ions which are not in thermal equilibrium. These energetic particles have revealed the fact that they are extremely nonthermal. This causes a deviation from Boltzmann distribution, the existence of such particles has been observed at high altitudes, in the solar wind and in several space plasmas [21]. An achievement in this case is the introduction and development of non-Maxwellian distribution form [22], generally modeled as the κ -distribution function. The kappa distribution is suitable for superthermal systems, e.g., high-energy particles found in solar atmosphere and Saturn's magnetosphere, etc. These observations have also been confirmed in the Vela satellite mission. This function was first used by Vasyliunas [23] which incorporates the superthermal particles, while Maxwellian in the limit $\kappa \rightarrow \infty$ is considered as a special case of the κ -distribution. The factor κ denotes the degree of superthermality having its lower and upper limits such that $2 < \kappa < 6$. In recent years, this distribution has been extensively applied by a number of authors to various types of space and laboratory plasmas in linear as well as in nonlinear limits [24–26]. Han et al. [27] have studied the superthermality effect on electron acoustic solitary/shock waves in dissipative medium and they showed that the characteristics of these waves considerably modify with these effects. Knowing the importance of kappa distribution, we have been motivated to study this effect on nonlinear DA waves in an inhomogeneous dusty plasma through charging process.

In this paper, we derive the Korteweg-de-Vries (KdV)-like equation by using the reductive perturbation technique in the presence of kappa distributed ions and electrons with dust charge fluctuation effect. We discuss that the charging processes are affected by the kappa distribution of electrons and ions. As a result, the amplitude of the DA solitary wave varies by these superthermal effects in an inhomogeneous dusty plasma. The manuscript is organized as follows: in Section 1, we introduce the dust acoustic nonlinear solitary waves, its applications, and the superthermal kappa distribution. Section 2 presents the basic model equations for these nonlinear waves, with the effect of negative dust charge fluctuations. Section 3 gives the derivation of the modified KdV equations. A solution to the modified KdV equation and discussion of the numerical results are presented in Section 4. Finally, concluding remarks are written in Section 5.

2 Model Equations

Consider a collisionless, unmagnetized, inhomogeneous dusty plasma having density gradients along the negative x -direction. To study the dust acoustic solitary waves, the negatively charged dust grains are considered dynamic while electrons and ions are assumed inertialess to follow the superthermal (κ) distribution. It is also assumed that the dust grains have charge fluctuations. The conventional isotropic, three-dimensional form of the kappa distribution function can be written as

$$F_{\kappa}(v) = A_{\kappa} \left(1 + \frac{v_j^2}{\kappa \theta_j^2} \right)^{-(\kappa+1)}, \quad (1)$$

in which κ is the spectral index.

$$A_{\kappa} = \frac{n_{j0}}{(\pi \kappa \theta_j^2)^{3/2}} \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa - 1/2)}. \quad (2)$$

The term θ_j in (1) represents the most probable (effective thermal) speed particles and is defined as

$$\theta_j = \left[\frac{(\kappa - 3/2) k_B T_j}{\kappa m_j} \right]^{1/2},$$

where T_j is the associated Maxwellian temperature of plasma species. Here, n_{j0} stands for the plasma number density and Γ is the usual gamma function, $v^2 = v_x^2 + v_y^2 + v_z^2$ denotes the three-dimensional square norm of velocity. Following the approach used by Shukla and Mamun [28], Rubab and Murtaza [29], Mishra et al. [30], Jana [31], and Taibany [32], the induced charge fluctuation relation for Z_{d1} can be written as

$$\partial_t Z_{d1} = I_{e1} + I_{i1} = I_{s1}, \quad (3)$$

where I_{i1} is the perturbed electron and ion currents. I_{s1} is the current which flows on the dust surface given by

$$I_j = \sum_j \int e_j v_j \sigma_j^d F_{\kappa} d^3 v_j, \quad (4)$$

σ_j^d is the charging cross section of dust grain surface defined by

$$\sigma_j^d = \pi a_d^2 \left(1 - \frac{2q_j \phi_d}{m_j v_j^2} \right),$$

here, ϕ_d is the dust surface potential relative to the plasma potential. For the dynamics of the DA solitary waves, we use the following set of normalized equations:

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x} (n_d u_d) = 0, \quad (5)$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} + e \frac{\partial \phi}{\partial x} = 0, \quad (6)$$

$$n_e = \left[1 - \frac{e\phi}{(\kappa_e - 3/2)T_e} \right]^{-\kappa_e + 1/2}, \tag{7}$$

$$n_i = \left[1 + \frac{e\phi}{(\kappa_i - 3/2)T_i} \right]^{-\kappa_i + 1/2}, \tag{8}$$

$$\frac{\partial^2 \phi}{\partial x^2} - (\mu n_e - Z_d n_d - \delta n_i) = 0. \tag{9}$$

In the above set of equations, n_d denotes the dust grain density normalized by the unperturbed dust number density n_{d0} , v_d is the dust fluid velocity, which is normalized by the dust acoustic speed $C_d = (Z_d T_i / m_d)^{1/2}$ and m_d represents the mass of the dust particles. The electrostatic potential is being normalized by e/T_e . At equilibrium, we can write the charge neutrality condition as $\mu = n_{i0} / Z_{d0} n_{d0} = 1 + \delta = 1 + n_{e0} / Z_{d0} n_{d0}$. Z_{d0} is the charged dust state, i.e., number of electrons(ions) residing on the dust grain surface. The kappa distributed electron and ion currents in the unperturbed environment are obtained by using (1) in (4), as

$$I_{e0}^\kappa = -2\sqrt{\pi} e a_d^2 \theta_e n_{e0} \frac{1}{\kappa^{3/2}} \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa - 1/2)} \times \frac{\kappa}{(\kappa - 1)} \left[1 + \frac{2e\phi_{d0}}{\kappa m_e \theta_e^2} \right]^{-(\kappa + 1)}, \tag{10}$$

$$I_{i0}^\kappa = 2\sqrt{\pi} e a_d^2 \theta_i n_{i0} \frac{1}{\kappa^{3/2}} \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa - 1/2)} \times \frac{\kappa}{(\kappa - 1)} \left[1 - \frac{2e(\kappa - 1)\phi_{d0}}{\kappa m_i \theta_i^2} \right]^{-(\kappa + 1)}. \tag{11}$$

Here, in these equations, a is the radius of the spherical dust grain. If $2e\phi_{d0} / \kappa m_e \theta_e^2 < 1$, the above two expressions can be simplified as

$$I_{e0}^\kappa = -2\sqrt{\pi} e a_d^2 \theta_e n_{e0} \frac{1}{\kappa^{3/2}} \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa - 1/2)} \frac{\kappa}{(\kappa - 1)} \left(1 + \alpha_e^\kappa \frac{e\phi_{d0}}{T_e} \right), \tag{12}$$

and

$$I_{i0}^\kappa = 2\sqrt{\pi} e a_d^2 \theta_i n_{i0} \frac{1}{\kappa^{3/2}} \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa - 1/2)} \frac{\kappa}{(\kappa - 1)} \left(1 - \alpha_i^\kappa \frac{e\phi_{d0}}{T_e} \right), \tag{13}$$

where $\alpha_e^\kappa = (\kappa - 1) / (\kappa - 3/2)$ and $\alpha_i^\kappa = \tau \alpha_e^\kappa$ with $\tau = T_e / T_i$. Using first-order perturbation analysis $\phi_d = Q_d / a_d$, the electron/ion currents, in a more compact form, can be expressed as

$$I_{e1}^\kappa = -|I_{e0}^\kappa| \left[\frac{n_{e1}}{n_{e0}} + \frac{\alpha_e^\kappa e\phi_d}{(T_e + \alpha_e^\kappa e\phi_{d0})} \right], \tag{14}$$

and

$$I_{i1}^\kappa = -|I_{i0}^\kappa| \left[\frac{\alpha_i^\kappa e\phi_{d1}}{(T_e - \alpha_i^\kappa e\phi_d)} - \frac{n_{i1}}{n_{i0}} \right], \tag{15}$$

where n_{i1} and n_{e1} represent the perturbed densities of electrons and ions, respectively.

3 The mKdV Equation

For the dynamical evaluation of the small amplitude electrostatic dust acoustic perturbation, we use the reductive perturbation technique (RPT). The RPT is mostly applied to small amplitude nonlinear waves. This method rescales both space and time in the governing equations of the system in order to introduce space and time variables, which are appropriate for the description of long wavelength phenomena. In order to obtain the KdV equation for the present model, we follow the procedure used by Washimi and Taniuti (1973) [33] and choose the stretched coordinates as

$$\xi = \epsilon^{1/2} \left(\frac{x}{\lambda_0} - t \right), \quad \tau = \epsilon^{3/2} x, \tag{16}$$

where ϵ is a small (positive) parameter, which measures the weakness of the amplitude, and λ_0 is the phase velocity mode of the wave. One can then expand the variables n_d , u_d , ϕ , and Z_d about the unperturbed states in power series of ϵ as

$$\left. \begin{aligned} n_d &= n_d^{(0)} + \epsilon n_d^{(1)} + \epsilon^2 n_d^{(2)} + \dots, \\ u_d &= \epsilon u_d^{(1)} + \epsilon^2 u_d^{(2)} + \dots, \\ \phi &= \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \dots, \\ Z_d &= Z_d^{(0)} + \epsilon Z_d^{(1)} + \epsilon^2 Z_d^{(2)} + \dots \end{aligned} \right\} \tag{17}$$

These expansions develop a number of equations in various powers of ϵ . In homogenous plasma, λ_0 is considered as a function of the slow variable τ , due to which inhomogeneity arises. Using (7) and (8) together with (16) and (17), the models (5), (6), and (9) become

$$-\frac{\partial n_d}{\partial \xi} + \frac{1}{\lambda_0} \frac{\partial}{\partial \xi} (n_d u_d) + \epsilon \frac{\partial}{\partial \tau} (n_d u_d) = 0, \tag{18}$$

$$-\frac{\partial u_d}{\partial \xi} + \frac{u_d}{\lambda_0} \frac{\partial u_d}{\partial \xi} + \epsilon u_d \frac{\partial u_d}{\partial \tau} + \frac{Z_d}{\lambda_0} \frac{\partial \phi}{\partial \xi} + \epsilon Z_d \frac{\partial \phi}{\partial \tau} = 0, \tag{19}$$

$$\begin{aligned} &\frac{\epsilon}{\lambda_0^2} \frac{\partial^2 \phi}{\partial \xi^2} + \frac{2\epsilon^2}{\lambda_0} \frac{\partial^2 \phi}{\partial \xi \partial \tau} + \epsilon^3 \frac{\partial^2 \phi}{\partial \tau^2} - \frac{\epsilon^2}{\lambda_0^2} \frac{\partial \lambda_0}{\partial \tau} \frac{\partial \phi}{\partial \xi} \\ &- \mu \alpha_e^\kappa e^\phi + Z_d n_d + \delta \alpha_i^\kappa e^{-\phi} = 0. \end{aligned} \tag{20}$$

Together with the help of boundary conditions such that $n_d^{(1)}, u_d^{(1)}, \lambda_0 \rightarrow 0$ as $\xi \rightarrow \infty$, the lowest power of ϵ produces

$$u_d^{(1)} = \frac{Z_d^{(0)}}{\lambda_0} \phi^{(1)}, \quad (21)$$

$$n_d^{(1)} = \frac{\alpha_e^k (\sigma_i + 1)}{Z_d^{(0)}} \phi^{(1)}, \quad (22)$$

and

$$\lambda_0 = Z_d \sqrt{S},$$

$$S = \frac{n_d^{(0)}}{\left(\alpha_e^k \mu \sigma_i n_e^{(0)} + \alpha_i^k \delta n_i^{(0)}\right)}. \quad (23)$$

To next higher order in ϵ , we obtain a set of equations viz.

$$-\frac{\partial n_d^{(2)}}{\partial \xi} + \frac{n_d^{(0)}}{\lambda_0} \frac{\partial n_d^{(2)}}{\partial \xi} + \frac{1}{\lambda_0} \frac{\partial (n_d^{(1)} u_d^{(1)})}{\partial \xi} + \frac{\partial (n_d^{(1)} u_d^{(1)})}{\partial \tau} = 0 \quad (24)$$

$$-\frac{\partial u_d^{(2)}}{\partial \tau} + \frac{Z_d^{(0)}}{\lambda_0} \frac{\partial \phi^{(2)}}{\partial \xi} + \frac{u_d^{(1)}}{\lambda_0} \frac{\partial u_d^{(1)}}{\partial \xi} + Z_d^{(0)} \frac{\partial \phi^{(1)}}{\partial \tau} = 0 \quad (25)$$

$$\frac{1}{\lambda_0^2} \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} - \alpha_e^k (\sigma_i + 1) \phi^{(2)} + Z_d^{(0)} n_d^{(2)} = 0. \quad (26)$$

Differentiating (26) with respect to ξ , we can write

$$\frac{1}{\lambda_0^2} \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} - \alpha_e^k (\sigma_i + 1) \frac{\partial \phi^{(2)}}{\partial \xi} + Z_d^{(0)} \frac{\partial n_d^{(2)}}{\partial \xi} = 0.$$

Combining (24), (25), and (27), and ignoring the second-order perturbed quantities, one can readily obtain the nonlinear partial differential equation as

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + B \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} + C \frac{\partial n_d^{(1)}}{\partial \tau} \phi^{(1)} = 0. \quad (27)$$

This is a type of KdV equation with the coefficients

$$A = \frac{3}{2\lambda_0^2 S^{1/2}},$$

$$B = \frac{Z_d^{(0)} S^{3/2}}{2n_d^{(0)} \lambda_0^4},$$

and

$$C = \frac{Z_d^{(0)}}{2(n_{e0} - n_{i0})}.$$

Equation (27) is a modified Korteweg-de-Vries equation having the effect of superthermal plasma particles. The nonlinear coefficients A and B depend on the inhomogeneity

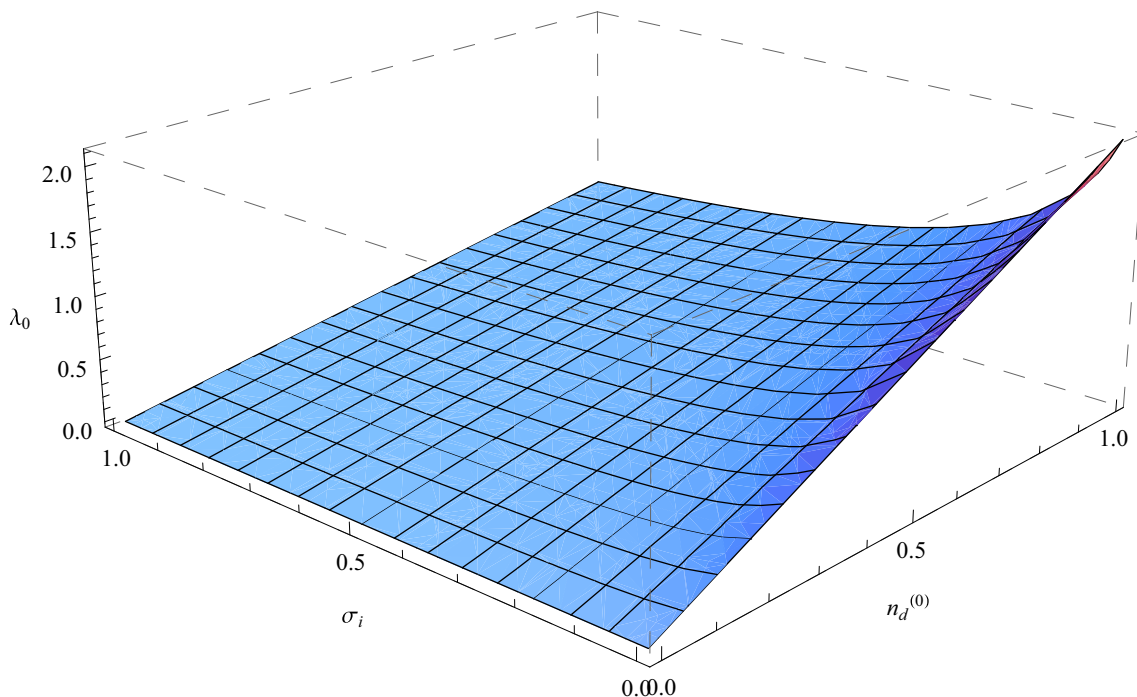


Fig. 1 (Color online) Dependence of phase velocity λ_0 on dust density $n_d^{(0)}$ and σ_i for $n_e^{(0)} = 1$, $n_i^{(0)} = 0.1$, $Z_d^{(0)} = 0.9$, and $\kappa = 3$

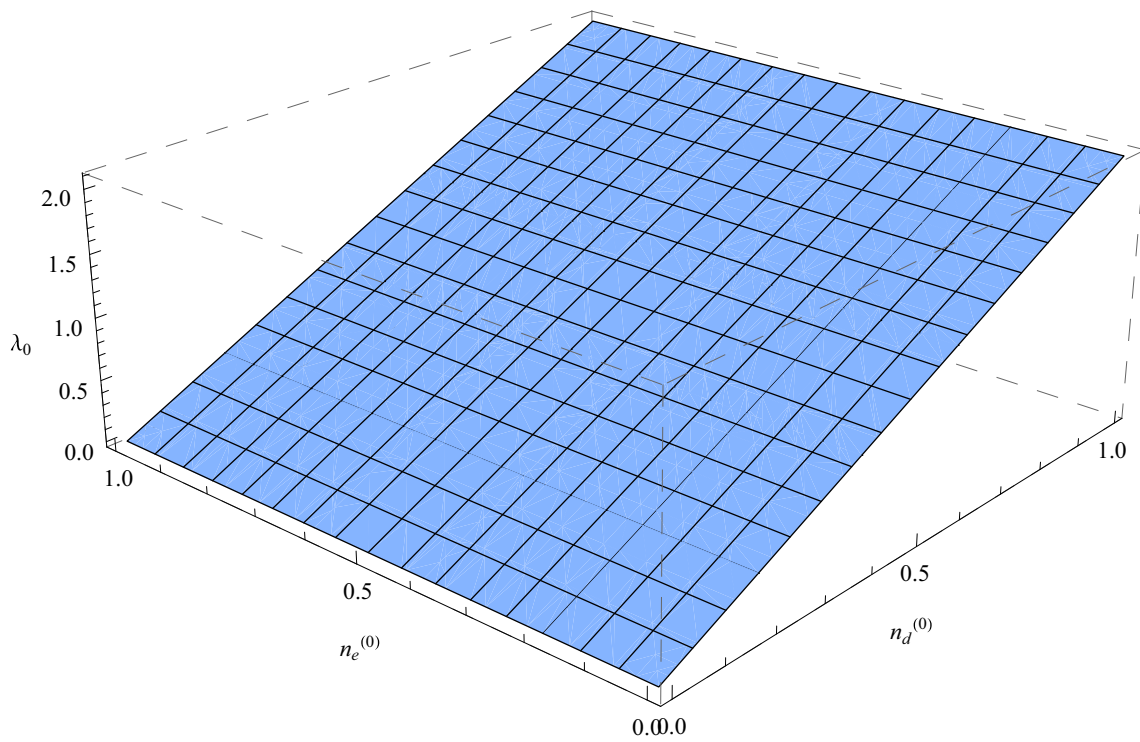


Fig. 2 (Color online) Dependence of phase velocity λ_0 on dust density $n_d^{(0)}$ and $n_e^{(0)}$ for $\sigma_i = 0.01$, $n_i^{(0)} = 0.1$, $Z_d^{(0)} = 0.9$, and $\kappa = 3$

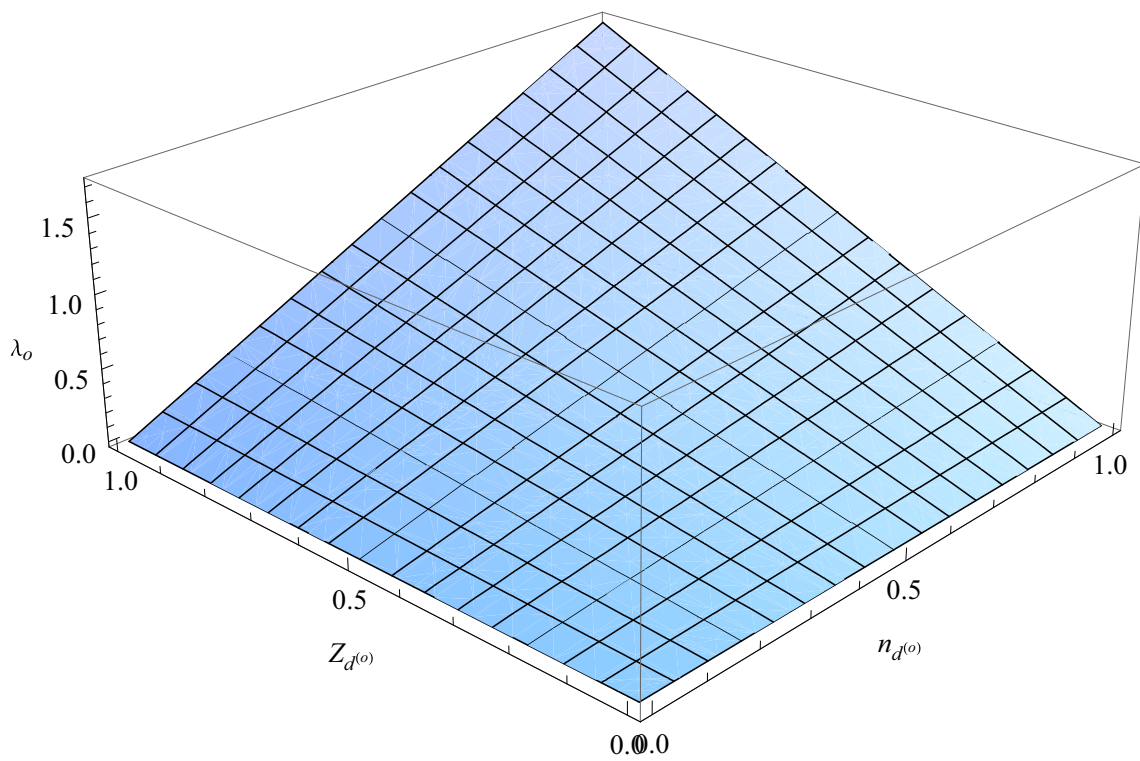
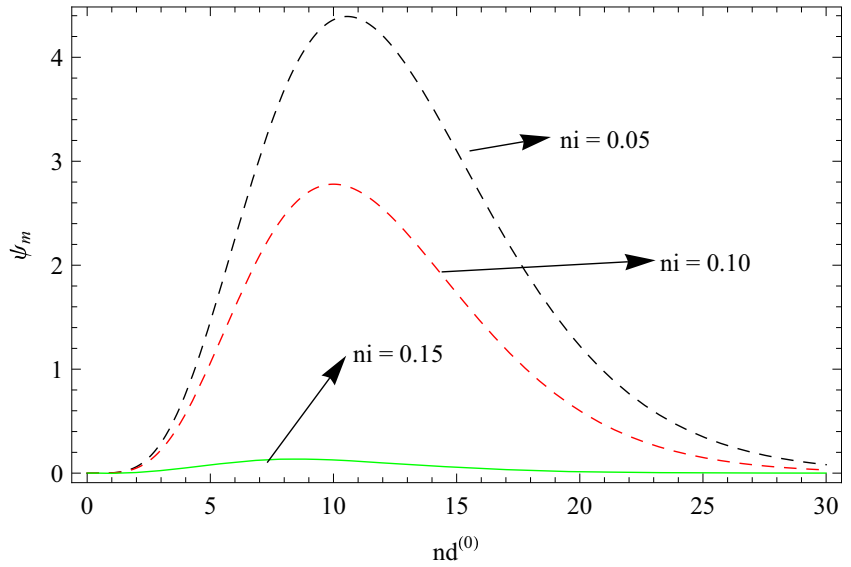


Fig. 3 (Color online) Dependence of phase velocity λ_0 on dust density $n_d^{(0)}$ and $Z_d^{(0)}$ for $n_e^{(0)} = 1$, $n_i^{(0)} = 0.1$, $\sigma_i = 0.01$, and $\kappa = 3$

Fig. 4 (color online) Variation of soliton amplitude ψ_m against $n_d^{(0)}$ for different values of $n_i^{(0)}$ ($= 0.05, 0.10, 0.15$) with $\sigma_i = 0.06, \kappa = 2, n_e^{(0)} = 1$ and $Z_d^{(0)} = 0.9$



and dust charge fluctuation. In the following section, we present the solution and numerical discussion of the modified KdV equation.

4 Solution of mKdV-Like Equation and Discussion

Using the transformation $\phi^{(1)} = \psi \exp(-Cn_d^{(0)})$ in (27), one can get the following well-known KdV equation:

$$\frac{\partial \psi}{\partial \tau} + A^* \frac{\partial \psi}{\partial \xi} + B \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0, \tag{28}$$

with $A^* = A \exp(-Cn_d^{(0)})$. To solve (28), we assume the new variable η which depends on ξ and τ such that $\eta = \xi - \lambda\tau$, here, λ is the frame velocity. Finally, we obtain a solution of the form

$$\psi = \phi_m^{(1)} \operatorname{sech}^2 \left[\frac{\eta}{\Delta_s} \right].$$

The above equation reveals small amplitude waves either in $\phi > 0$ for $A > 0$ or for $\phi < 0$ if $A < 0$. The amplitude $\phi_m^{(1)}$ and the width Δ_s are given by $3\lambda/A$ and $\Delta_s = \sqrt{4B/\lambda}$.

In this study, we have discussed the nonlinear dust acoustic waves in an inhomogeneous dusty plasma having

Fig. 5 (Color online) Variation of soliton amplitude ψ_m against $n_d^{(0)}$ for different values of σ_i ($= 0.01, 0.05, 0.10$) with $n_i^{(0)} = 0.10, \kappa = 2, n_e^{(0)} = 1$, and $Z_d^{(0)} = 0.9$

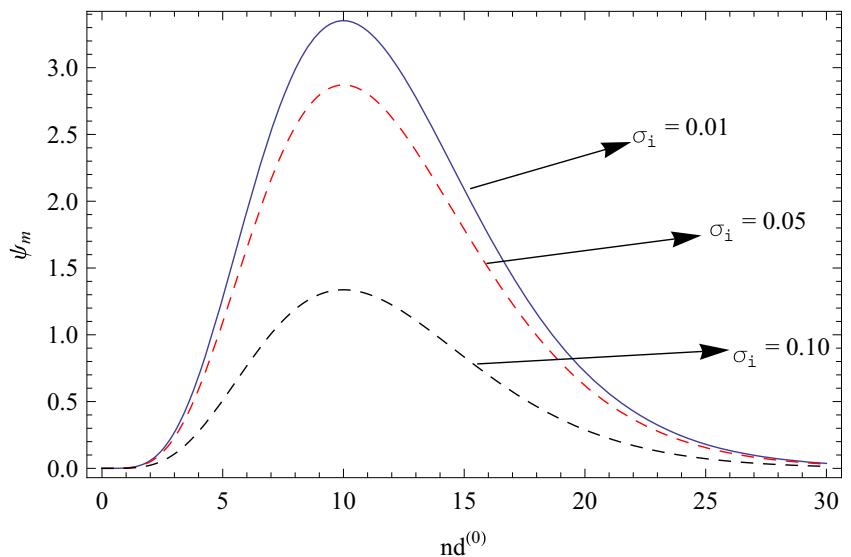
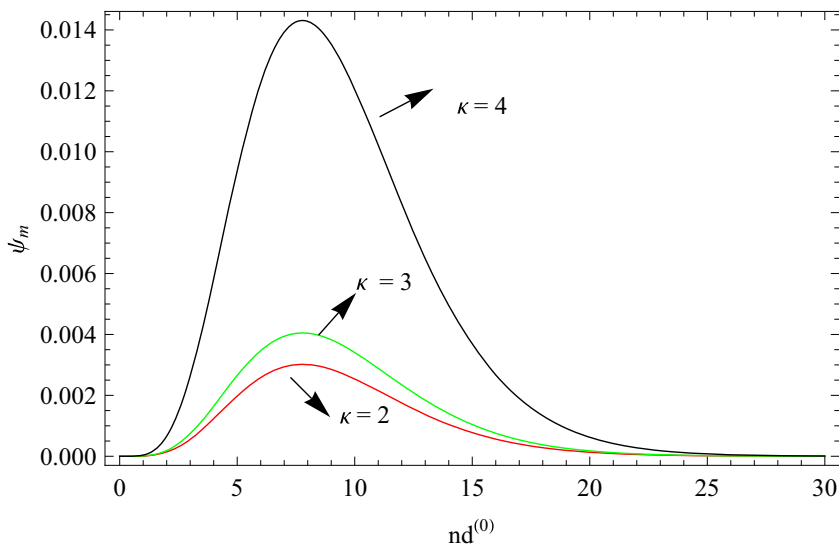


Fig. 6 (Color online) Variation of soliton amplitude ψ_m against $n_d^{(0)}$ for different values of κ ($= 2, 3, 4$) with $n_i^{(0)} = 0.3, \sigma_i = 0.6, n_e^{(0)} = 1$, and $Z_d^{(0)} = 0.9$



the dust charge fluctuation effect. A modified version of KdV equation by using the reductive perturbation technique together with stretched variables for space and time is obtained for kappa distributed ions and electrons. The numerical results for the modified KdV equation show that only refractive solitons can form. From Figs. 1, 2, and 3, we show the dependency of the phase velocity on $n_d^{(0)}, n_i^{(0)}$, and σ_i . Figure 1 indicates the variation of phase velocity λ_0 on $n_d^{(0)}$ and σ_i . It is clear that with increase in $n_d^{(0)}$, the phase velocity increases. For the same case, a gradual decrease is observed for higher value of σ_i . Figure 2 reflects the dependency of phase velocity on electron density but in this

case, the variation is less compared to the effect of the dust density.

Figure 3 shows that the phase velocity increases smoothly with $n_d^{(0)}$ and $Z_d^{(0)}$ in superthermal environment. The variation in amplitude ψ_m against $n_d^{(0)}$ for different values of $n_i^{(0)}, n_e^{(0)}, \sigma_i$, and spectral index κ are shown in Figs. 4, 5, 6, and 7. It is clear from Fig. 4 that the soliton amplitude decreases by increasing $n_i^{(0)}$. This change in amplitude of the soliton is maximum for larger values of $n_i^{(0)}$, where the dispersion of the soliton occurs. Figure 5 describes the effect of temperature ratio σ_i on the soliton amplitude. We observe in our investigations that the change

Fig. 7 (Color online) Variation of soliton amplitude ψ_m against $n_d^{(0)}$ for different values of $n_e^{(0)}$ ($= 1, 1.2, 1.3$) with $n_i^{(0)} = 0.2, \sigma_i = 0.6, \kappa = 2$, and $Z_d^{(0)} = 0.9$

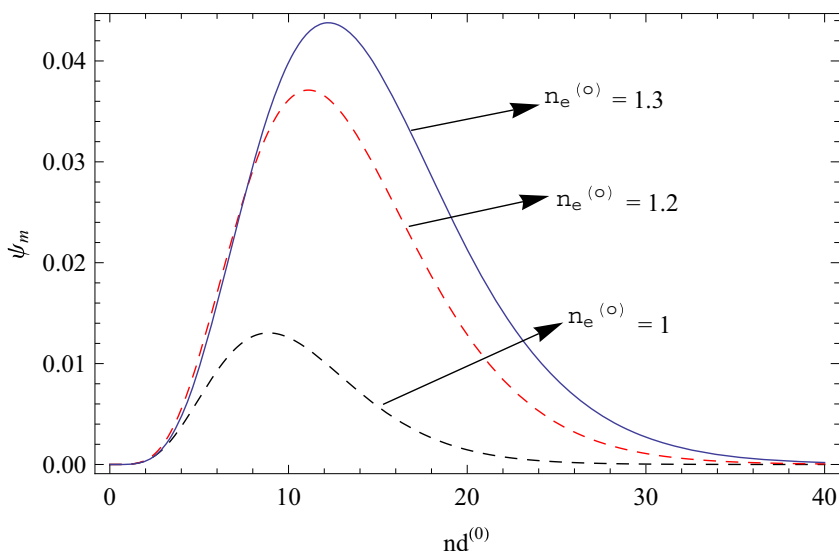
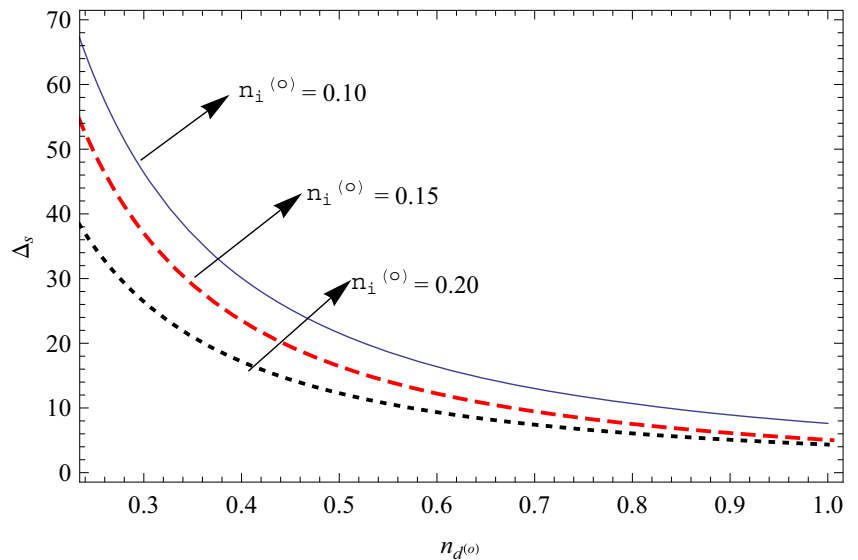


Fig. 8 (Color online) Variation of soliton width Δ_s against $n_d^{(0)}$ for different values of $n_i^{(0)}$ ($= 0.10, 0.15, 0.20$) with $n_e^{(0)} = 1, \kappa = 2$ and $\sigma_i = 0.01$, and $Z_d^{(0)} = 1$



in amplitude is the same as in Fig. 4 but this time the shift is less compared to the dependency on $n_i^{(0)}$. In Figs. 6 and 7, the change in soliton amplitude is shown with respect to spectral index κ and $n_e^{(0)}$. Here, the soliton amplitude gets larger by increasing the superthermal spectral index value κ and the electron density $n_e^{(0)}$. This may be due to the presence of large population of superthermal particles. Similar effect is observed for inhomogeneity in electron density. Further, the variation in soliton width Δ_s against $n_d^{(0)}$ for different values of $n_i^{(0)}$, σ_i , and spectral index κ are shown in Figs. 8, 9, and 10. In Figs. 8 and 9, we see that the soliton width decreases by increasing the values of ions density $n_i^{(0)}$ and temperature ratio σ_i . This change

is reversed in the case of the spectral index κ . Our results show that the change in the soliton width is drastic for low dust density, while it becomes less for large dust density. As the expressions for the coefficients A and B given after (27) have the dust charging effect, so, we can say that the charging effect is also strongly credited for the modification of solitary wave profile in superthermal environment. Furthermore, it is observed that the influence of superthermality on the soliton width and amplitude can play a vital role in forming and destabilizing the DASWs in different space environments, namely, Earth's magnetosphere, interstellar and circumstellar clouds, etc. [28, 34].

Fig. 9 (Color online) Variation of soliton width Δ_s against $n_d^{(0)}$ for different values of σ_i ($= 0.01, 0.05, 0.10$) with $n_e^{(0)} = 0.1, \kappa = 2, n_i^{(0)} = 0.1$, and $Z_d^{(0)} = 1$

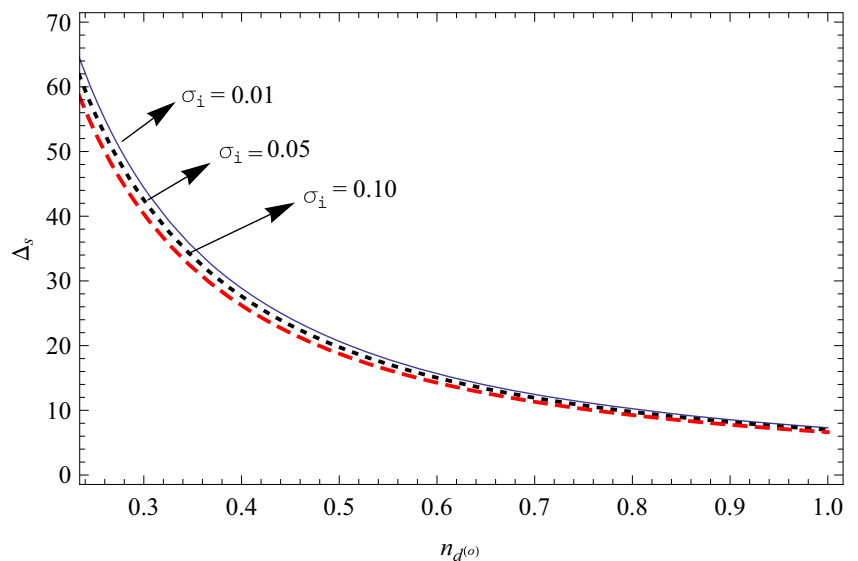
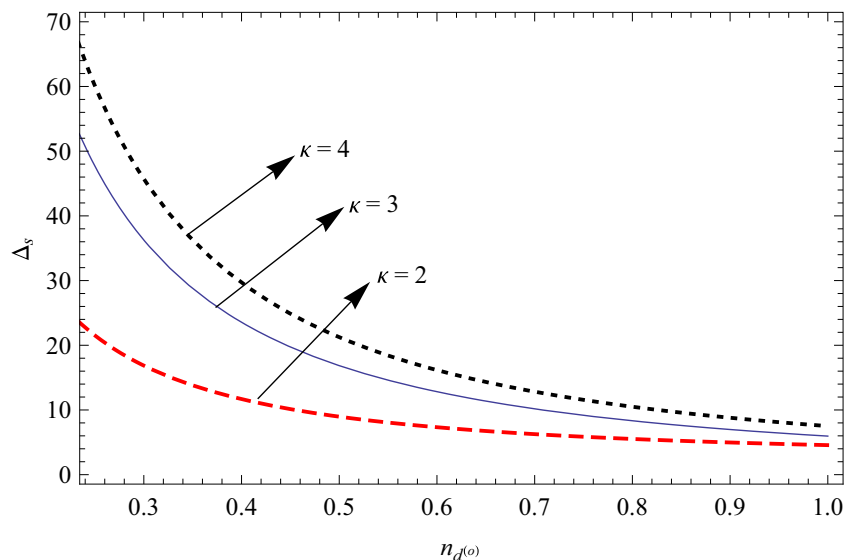


Fig. 10 (Color online) Variation of soliton width Δ_s against $n_d^{(0)}$ for different values of κ ($= 2, 3, \text{ and } 4$) with $n_e^{(0)} = 0.1, \sigma_i = 0.01, n_i^{(0)} = 0.1, \text{ and } Z_d^{(0)} = 1$



5 Conclusion

In the present study, we addressed the propagation of nonlinear dust acoustic waves in unmagnetized inhomogeneous dusty plasma with kappa distributed ions, electrons, and negatively charged dust fluctuation. The standard reductive perturbation method is employed to derive the mKdV equation from the basic governing equations which are used to investigate the propagation of solitary wave structures. In this case, the mKdV equation contains an additional term showing the effect of non-Maxwellian plasma particle distribution. We also concluded that the presence of this additional term changes the soliton profile, and these changes are numerically investigated using different parameters such as density, temperature ratio, and superthermality factor. Our investigations show that plasma inhomogeneity adversely modifies the solitary wave structures which may be interesting in understanding the nonlinear solitary structures. The present investigation can be helpful in understanding the salient features of small amplitude electrostatic wave structures [35–41] in space, e.g., in certain heliospheric environments and laboratory plasma systems.

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