#### GENERAL AND APPLIED PHYSICS





# Analysis of Co-Tunneling Current in Fullerene Single-Electron Transistor

Vahideh KhademHosseini<sup>1</sup> · Daryoosh Dideban<sup>1,2</sup> · MohammadTaghi Ahmadi<sup>3,4,5</sup> · Razali Ismail<sup>5</sup>

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#### Abstract

Single-electron transistors (SETs) are nano devices which can be used in low-power electronic systems. They operate based on coulomb blockade effect. This phenomenon controls single-electron tunneling and it switches the current in SET. On the other hand, co-tunneling process increases leakage current, so it reduces main current and reliability of SET. Due to co-tunneling phenomenon, main characteristics of fullerene SET with multiple islands are modelled in this research. Its performance is compared with silicon SET and consequently, research result reports that fullerene SET has lower leakage current and higher reliability than silicon counterpart. Based on the presented model, lower co-tunneling current is achieved by selection of fullerene as SET island material which leads to smaller value of the leakage current. Moreover, island length and the number of islands can affect on co-tunneling and then they tune the current flow in SET.

Keywords Co-tunneling . Coulomb blockade . Fullerene . Leakage current . Multiple Islands . Single-electron transistor

# 1 Introduction

The single-electron transistor (SET) as a new electronic device operates based on the coulomb blockade (CB) which prevents electron tunneling through tunnel junctions, so current flow stops. Coulomb blockade occurs when thermal energy is lower than charging energy. The other conditions are when tunneling resistance is lower than quantum resistance or island charge is higher than bias voltage  $[1-4]$  $[1-4]$  $[1-4]$  $[1-4]$ . Moreover, the leak-

 $\boxtimes$  Vahideh KhademHosseini [vahideh\\_khademhosseini@yahoo.com](mailto:vahideh_khademhosseini@yahoo.com)

- <sup>2</sup> Department of Electrical and Computer Engineering, University of Kashan, Kashan, Iran
- <sup>3</sup> Department of Electrical Engineering, Pardis of Urmia University, Urmia, Iran
- Nanotechnology Research Center, Nano Electronic Research Group, Physics Department, Urmia University, Urmia, Iran
- <sup>5</sup> Faculty of Electrical Engineering, Universiti Teknologi Malaysia, 81310 Johor Bahru, Johor, Malaysia

age current rises due to co-tunneling process; therefore, SET reliability decreases. Co-tunneling happens when an electron tunnels from source to island while another electron leaves island to drain at the same time. This phenomenon is shown in Fig. [1](#page-1-0).

This process has two types based on the number of electrons of island: elastic co-tunneling and inelastic co-tunneling. The elastic co-tunneling occurs when an electron tunnels from source to island and then another electron moves from island to source at the same time. Therefore, this process depends on island structure. It occurs for an even number of electrons on the island. The second process is inelastic co-tunneling that an electron tunnels to island but another electron at a lower energy level moves to drain. This electron transfer produces an electron-hole excitation in the island. These processes are shown in Fig. [2](#page-1-0).

Co-tunneling depends on bias voltage and temperature. The inelastic co-tunneling is dominant in small bias voltage and low temperature  $[5-10]$  $[5-10]$  $[5-10]$  $[5-10]$ . The multiple islands increase probability of co-tunneling in SET [\[11](#page-4-0)–[13](#page-4-0)]. The decreasing of co-tunneling is required for reliable operation of SET. Therefore, selection of fullerene as an island material with special properties such as high electron mobility can suppress this operation limitation. Thus, using this material can rise SET reliability [[14](#page-4-0), [15](#page-4-0)].

<sup>1</sup> Institute of Nanoscience and Nanotechnology, University of Kashan, Kashan, Iran

<span id="page-1-0"></span>

Fig. 1 The co-tunneling process in SET

## 2 Modelling of Co-Tunneling Current on Fullerene Single-Electron Transistor

Single-electron transistor is nanoscale device. Therefore, it needs material with special properties such as nanometre size, high carrier mobility and high speed of electron transfer. Fullerene as a graphene-based material has these unique properties. It has high stability in nanoscale and also its size is controllable in nanometre range. Thus, fullerene is chosen for the island part of SET.

The proposed structure of single-electron transistor has linear island arrays as shown in Fig. 3. They are located



Fig. 2 The elastic co-tunneling and the inelastic co-tunneling

Fig. 3 The structure of singleelectron transistor with multiple

fullerene islands

**Tunnel Junction**  $a)$ Island Drain Source Ą 岔  $\overline{\mathbf{u}}$  $V(x)$  $_{\rm b}$  $V<sub>0</sub>$  $\overline{L}$  $\Omega$ 

Fig. 4 a The regions of fullerene SET. b Potential across the island length

between source and drain electrodes. The linear island arrays cause higher critical temperature in SET. It contains a gate electrode that islands are placed above it. Island length is equal to fullerene diameter and the size of SET electrode is 10 to 20  $A<sup>0</sup>$ . When an electron tunnels from source to first island, it will cross dominant conducting path between islands. If it is transferred to the drain electrode, a cycle of single-electron tunneling will be completed and the current will flow in SET.

Quantum mechanics expresses when wave crosses from different regions, the wave functions will be changed. Based on this point, SET is divided in three regions: two tunnel junctions and one fullerene island. The different parts of SET are shown in Fig. 4.

Therefore, Schrodinger equations could be written for its regions as

$$
\frac{\partial^2}{\partial x^2} \psi_I(x) - \alpha^2 \psi_I(x) = 0 \quad x \le 0 \tag{1}
$$

$$
\frac{\partial^2}{\partial x^2} \psi(x) - k^2 \psi(x) = 0 \quad 0 \lt x \lt L \tag{2}
$$

$$
\frac{\partial^2}{\partial x^2} \psi(x) - \alpha^2 \psi_I(x) = 0 \quad x \ge L \tag{3}
$$

where 
$$
\alpha = \frac{(2mE)^{\frac{1}{2}}}{h}
$$
 and  $k = \frac{(2m(V-E))^{\frac{1}{2}}}{h}$ .



<span id="page-2-0"></span>

Fig. 5 The comparison between current versus voltage characteristics of proposed model and experimental data of fullerene SET

Transmission coefficient of fullerene SET can be given by:



Fig. 7 I-V curves of fullerene SET with different island lengths

$$
T = \frac{\cos\sqrt{\frac{2mE}{h^2}} - \sqrt{\frac{2(E-E_g)}{ta}} - \cos\sqrt{\frac{2mE}{h^2}} + \sqrt{\frac{2(E-E_g)}{ta}}}{\omega}
$$
  
\n
$$
e^{i}\sqrt{\frac{2mE}{h^2}} \cdot \sin(NL-L)\sqrt{\frac{2(E-E_g)}{ta}} + e^{-i\sqrt{\frac{2mE}{h^2}}}\sin(NL+L)\sqrt{\frac{2(E-E_g)}{ta}} - \sin NL\sqrt{\frac{2(E-E_g)}{ta}}
$$
\n(4)

Where " $m$ " is effective mass, " $h$ " is plank's constant, "E" is energy of fullerene and " $E_g$ " is energy band gap of fullerene, "L" is the length of island,  $a = 1.42A<sup>0</sup>$  is carboncarbon bond length,  $t = 2.7ev$  is the nearest neighbour carbon-carbon tight binding overlap energy and " $N$ " is the number of islands.

The Landauer formalism is used for modelling of cotunneling current in fullerene SET as [\[16\]](#page-4-0):

$$
I_d = \int_0^n F(E) \cdot T(E) dE \tag{5}
$$

where " $F(E)$ " is the Fermi probability function and "T (E)" is the transition coefficient. Based on the Landauer formalism, co-tunneling current of multiple fullerene SET is modelled as:

$$
I_d = \int_0^n \frac{2\left[6\sqrt{c_2x} - \sqrt{c_2x^3}\right] \cdot \left[6\sqrt{c_1x} - \sqrt{c_1x^3}\right]}{e^{-i\sqrt{c_2x}} \cdot \left[6(N+1)L\sqrt{c_1x} - (N+1)^3L^3\sqrt{c_1x^3}\right] - 2\left[6NL\sqrt{c_1x} - N^3L^3\sqrt{c_1x^3}\right] + e^{\sqrt{c_2x}} \cdot \left[6(N-1)L\sqrt{c_2x} - (N-1)^3L^3\sqrt{c_2x^3}\right]} \cdot \frac{k_BTdx}{e^{x-\eta} + 1}
$$
(6)

where " $m$ " is the effective mass, " $h$ " is plank's constant, "T" is temperature, " $K_B$ " is Boltezmann's constant, "L" is length of

Fig. 6 I-V curves for fullerene SET with different number of islands

island, Eta is  $\eta = \frac{E_F - E_g}{K_B T}$ ,  $x = \frac{E - E_g}{K_B T}$ ,  $C_1 = \left(\frac{2m}{\eta}\right)^2$  $\mathbb{R}^n \times \mathbb{R}^n$ ,  $C_2 = \left(\frac{32.mK_B.T}{9\eta(t.a_{c-c})^2}\right)$ , Eta i ,  $a_{c-c} = 1.42A^0$  is carbon-carbon bond



<span id="page-3-0"></span>Fig. 8 Current versus drain voltage characteristics of two SETs. SET with multiple islands which is marked in red points and the blue points show the current of silicon SET with three islands



length,  $t = 2.7ev$  is the nearest neighbour carbon-carbon tight binding overlap energy and " $N$ " is the number of islands.

To measure the accuracy of proposed model, the currentvoltage characteristics of the model and experimental data for fullerene SET extracted from reference [[14](#page-4-0)] are compared together in Fig. [5.](#page-2-0)

The comparison between two current-voltage characteristics indicates good agreement between them in Fig. [5,](#page-2-0) so it confirms suitable accuracy of suggested model.

Proposed model depends on some parameters. Two effective factors are investigated in this research. The impact of the number of islands on the proposed model is reported and plotted in Fig. [6.](#page-2-0)

The second factor is island length. Its effect on SET current is shown in Fig. [7](#page-2-0)

The physical phenomena in Figs. [6](#page-2-0) and [7](#page-2-0) are investigated. Increasing number of islands rises free electrons and probability of co-tunneling in SET. Therefore, it increases the coulomb blockade and zero current range in SET. Moreover, island with bigger length has more carbon atoms and electrons. Therefore, their probability of co-tunneling and coulomb blockade range are higher than small islands. On the other hand, decreasing co-tunneling and leakage current depends on island material. The comparison study in Fig. 8 shows that fullerene island has lower leakage current than its silicon counterpart.

In addition, co-tunneling current versus drain voltage characteristic of suggested model for multiple fullerene islands and experimental data extracted from reference [\[17](#page-4-0)] are compared in Fig. 8.

The analysis of I–V characteristic in Fig. 8 shows that cotunneling process is observed in silicon SET in the drain bias between 0.03 and 0.09 V and its leakage current causes to reduce main current in this range, but leakage current of fullerene SET is lower than silicon SET. The comparison study indicates that the fullerene island has better suppression of cotunneling, and thus, the performance is better than silicon one. Hence, development of nanotechnology predicts the replacement of silicon SET with fullerene SET and the number of electronic devices comprising new materials such as fullerene can increase.

## 3 Conclusion

The single-electron transistor (SET) as a fast nano device works by coulomb blockade effect, so it controls the current. In this device, the number of islands as an important factor of SET plays effective role on its operation. Island arrays cause co-tunneling phenomenon which leads to the leakage current in SET. In this paper, fullerene-based SET with multiple islands is modelled and it is compared to experimental data of fullerene SET, so its good accuracy is confirmed. Moreover, comparison study between the proposed model and silicon-based SET is carried out which indicates cotunneling current of fullerene SET is lower than that of silicon SET, and hence, it shows higher reliability than silicon one. In other words, fullerene islands can reduce undesirable effect of co-tunneling in contrast to the silicon SET. Furthermore, island array SET has higher critical temperature than single island SET. The presented results recommend the replacement of the silicon SET with the fullerene one.

# **References**

- 1. W.A. Schoonveld, J. Wildeman, D. Fichou, P.A. Bobbert, B.J. van Wees, T.M. Klapwijk, Coulomb-blockade transport in singlecrystal organic thin-film transistors. Nature 404, 977–998 (2000)
- 2. J. Park, A.N. Pasupathy, J.I. Goldsmith, C. Chang, Y. Yaish, J.R. Petta, M. Rinkoski, J.P. Sethna, H.D. Abruña, P.L. McEuen, D.C. Ralph, Coulomb blockade and the Kondo effect in single-atom transistors. Nature 417, 722–725 (2002)
- 3. F. Wang, J. Fang, S. Chang, S. Qin, X. Zhang, H. Xu, Room temperature Coulomb blockade mediated field emission via selfassembled gold nanoparticles. Phys. Lett. A 381, 476–480 (2017)
- 4. V. Khadem Hosseini, M.T. Ahmadi, S. Afrang, R. Ismail, Current analysis and modelling on fullerene single electron transistor at room temperature. J. Electron. Mater. 46(7), 4294 (2017)
- 5. D.V. Averin, Y.V. Nazarov, Virtual electron diffusion during quantum tunneling of the electric charge. Phys. Rev. Lett. 65, 2446– 2449 (1990)
- 6. D. V. Averin, Yu V. Nazarov, Single charge tunneling, Coulomb blockade phenomena in nanostructures. NATO ASI Series B, Vol 294, edited by Grabert H and Devoret Plenum M H, New York Chapter 6 pp 217–247 (1992)
- <span id="page-4-0"></span>7. H. Matsuoka, S. Kimura, Thermally enhanced co-tunneling of single electrons in a Si quantum dot at 4.2 K.Jpn. J. Appl. Phys. 34(Part 1), 2B (1995)
- 8. Y. Takahashi, S. Horiguchi, A. Fujiwara, Co-tunneling current in very small Si single-electron transistors. Physica B: ,Elsevier 227, 105–108 (1996)
- 9. S. De Franceschi, S. Sasaki, J. M. Elzerman, W. G. van der Wiel, S. Tarucha, and L. P. Kouwenhoven. Electron cotunneling in a semiconductor quantum dot.Phys. Rev. Lett. , Vol.86, p.878 (2001), 881
- 10. T.B. Tran, I.S. Beloborodov, X.M. Lin, T.P. Bigioni, V.M. Vinokur, H.M. Jaeger, Multiple cotunneling in large quantum dot arrays. Phys. Rev. Lett. 95, 076806 (2005)
- 11. A.A. Elabd, E.-R. ESM, A.T. Shalaby, Analysis of rare events effect on single-electronics simulation based on orthodox theory. J. Comput. Elect,Springer 14, 604–610 (2016)
- 12. M. Jo, T. Uchida, A. T-Fukuchi, M. Arita, A. Fujiwara, Fabrication and single-electron-transfer operation of a triple-dot single-electron transistor. J. Appl. Phys. 118, 214305 (2015)
- 13. M. Manheller, S. Karthäuser, R. Waser, K. Blech, Electrical transport through single nanoparticles and nanoparticle arrays. J. Phys. Chem. C 116(39), 20657–20665 (2012)
- 14. H. Park, J. Park, A.K.L. Lim, E.H. Anderson, A.P. Alivisatos, P.L. McEuen, Nanomechanical oscillations in a single-C60 transistor. Nature 407, 57 (2000)
- 15. E. Ulloa. Introduction to nanotechnology. spring 4138296 (2013)
- 16. S. Datta. Electronic transport in mesoscopic systems. Cambridge University Press (2002)
- 17. T. Kitade, K. Ohkura, A. Nakajima, Room-temperature operation of an exclusive-OR circuit using a highly doped Si single-electron transistor. Appl. Phys. Lett. 86, 123118 (2005)