

## Cohort Analysis' Unholy Quest: A Discussion

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### Some Personal History With APC Analysis

I am pleased to be able to join in the discussion of the article by Luo (in this issue) on age, period, and cohort (APC) modeling in demographic research. Demographers have had a long fascination with various forms of accounting frameworks, and thus APC modeling was a natural topic for the demographic literature.

My interest in these models arose sometime in the mid-1970s, when I was part of a team teaching a short course on categorical data analysis. One of the “students” was especially persistent, both wanting to know precisely how to implement the computer programs but also the many details not in the lectures or the notes I distributed. His questions were always penetrating, and my answers tended to stretch out into the breaks and dinners. We spent a couple of evenings talking about research problems. That was my introduction to William (Bill) Mason.

One of Bill’s late-night questions was about how to address what was coming to be known as the “age-period-cohort problem” in a categorical data context using log-linear models. Thus began a collaboration that stretched over close to a decade, produced a couple of major papers (Fienberg and Mason 1978, 1985), a Social Science Research Council conference, and an edited volume based on papers presented at the conference (Mason and Fienberg 1985).

I tend to write relatively terse descriptions of methodology, and although my two main papers with Mason appear to be lengthy, I always thought of them as summarizing the problem in a rather pithy way. What we essentially said was

1. There is no technical way to solve the APC problem.  $A = P - C$  is a linear constraint, and the only way to make it go away is by fiat—that is, by specifying some conditional linear contrast whose appropriateness could never be tested. Luo appears to be in full agreement with this view.

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2. The APC problem is a linear effects problem, and nonlinear effects and possibly some interactions are estimable, depending on the nature of the data, and so on. Adding covariates to the linear APC model changes the model but not the identification problem.

We went on to explain what all this meant and illustrated the enterprise through careful calculations and analyses. We chose to illustrate the untangling of linear components using substantively motivated equality constraints, but we also checked on sensitivity via sequences of over-constraints, thus capturing aspects of the nonlinear components at the same time.

Much to my surprise, few people truly understood, neither old hands who believed that they “owned” cohort analyses nor others who came later and still tried to solve the unsolvable problem. In response to one of these latter efforts, Mason and I collaborated with Herb Smith on a commentary (Smith et al. 1982). In effect, we said that waiting for a solution to the so-called APC problem is like “Waiting for Godot.” Perhaps we needed Samuel Beckett as a coauthor because the response to our commentary by Rodgers (1982) totally missed the point.

In many senses, Bill Mason already knew the answer to his question about cohort analysis back in the mid-1970s (e.g., see Mason et al. 1973), but our papers and collaboration fleshed out his ideas, hopefully in new and productive ways, along with a few of my own.

Over the years, I have seen many papers on the APC problem by a wide variety of authors in a remarkably diverse collection of fields. These tend to be of two sorts: either they redo Bill Mason’s and my work with slightly different notation or a different computer program, or they try anew to solve the unsolvable problem. Luo references a number of these attempts, although she does not reference any of my work with Mason nor the paper by Hobcraft et al. (1982/1985) and the sensible discussions of APC models by Mason and Smith (1985) and Smith (2004, 2008). All these papers argue for resolving the APC dilemma using substantive judgment and knowledge. Thus I am in full agreement with the Luo’s penultimate comment, in which she “encourage[s] the development of APC models that are informed by social theories.”

## The New APC Literature

After running an online search for recent contributions to the APC literature, I was surprised at the extent of publications and, of course, their content. I recognized several papers that I had refereed. Various new solutions have been proposed involving partial least square regression, generalized inverses, eigenvector analysis, random effects, approximating splines and related smoothing methods, latent variables, Bayesian priors and hierarchical models (both Bayesian and frequentist), and so on. I classified the proposed solutions into three categories: constrained estimation based on restricting the model, making the model nonlinear, and sleight of hand. Of course, some solutions have elements of two or more categories. The literature is now filled with algebra, matrix manipulations, and geometrical pictures arguing for and against specific technical solutions. None of these, to my reading, solves the APC problem in a meaningful way.

I am an unrepentant subjective Bayesian, and I was trained in these matters by Howard Raiffa and Robert Schlaifer (1961/2000) as a graduate student at Harvard in the 1960s, although my work with Bill Mason may have not offered a hint of my statistical religious fervor. Nonetheless, I looked at the hierarchical modeling approaches to the APC problem with interest because it is closely related to much of my current work, and I know that Bill has been enamored of them as well. Thus, I simply note that when one changes a model via random effects, introduction of latent variables, exchangeability, or some other modeling device without strong prior information, one is still changing the model by fiat. (See also Mason and Wolfinger (2001) on this point.) More specifically, it is especially important to understand that for Bayesian hierarchical APC models, the exchangeability on any one or more of A, P, or C turns a linear model into a nonlinear one. The same is true of introducing random effects instead of fixed effects in a frequentist framework: the only difference is that Bayesians essentially treat everything as random variables.

Rather than quote from the collection of papers I assembled, let me quote from my referee's report on one relatively recent paper (not by Luo):

This paper builds on a series of manuscripts by X, Y, and Z which to my mind are fundamentally flawed in conception and ignore the principal point from the earlier literature on APC models. Namely, it is the linear components of A, P, and C that are completely confounded and they can only be resolved by fiat. On the other hand, the non-linear components are estimable and should be the focus of statistical and substantive investigation. Although early authors emphasized this point, it seems to have been lost on the present authors. The proposed method for resolving the identification problem solves it by fiat, gussied up in the language of Moore-Penrose inverses and null-spaces. Nothing more or nothing less. The authors seem to build a mountain out of a molehill. The literature is littered with variations on this misinterpretation.

I view the intrinsic estimator of Yang and her collaborators (e.g., see Yang et al. 2004) as yet another of these efforts to solve the APC problem by fiat, and thus find myself in sympathy with Luo in her effort to explain why this is the case. I differ from Luo mainly because I see the failure of almost all these efforts as being rooted in the APC accounting model itself, and not with the method of estimation. We also appear to agree that the search for methodological solutions to the APC identity is an endless and fruitless quest. It is surely time to move onto substantively focused considerations of the meaning of the three components in settings of interest.

## Moving Forward

The APC problem appears in many different guises in the demographic and sociological literatures, as Fienberg and Mason (1985) pointed out. Thus, it behooves us to understand what formal accounting models can and cannot do, and when we need to bring to bear substantive judgment, especially prior to data collection and analysis.

I have encountered variations of APC issues in the context of the study of victimization using the National Crime Survey (Fienberg 1980) where a slight generalization

involves the simultaneous modeling of effects of month of collection, time lag to reference month, panel-rotation group number, and time-in-panel (five variables with two linear restrictions!). It has also arisen in more recent work in the context of the National Long-Term Care Survey, involving extensions of the work of Manrique-Valier and Fienberg (2009) that apply longitudinal mixed-membership models to capture individual-level mixtures of disability profiles as a function of age, generation, and period of data collection. In both contexts, the special interest is on the nonlinear components, which are identifiable.

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