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ORIGINAL PAPER

A remark on "Existence and uniqueness for a neutral differential problem with unbounded delay via fixed point results $\mathcal{F}$-metric spaces"

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Abstract

Very recently, Hussain and Kanwal (Trans A Razmadze Math Inst 172(3):481–490, 2018) proved some (coupled) fixed point results in this setting for $\alpha - \psi$-contractive mappings on the setting of $\mathcal{F}$-metric spaces that was initiated by Jleli and Samet (Fixed Point Theory Appl 2018:128, 2018). In this note, we underline that the proof of Hussain and Kanwal (Trans A Razmadze Math Inst 172(3):481–490, 2018) has a gap. We provide two examples to illustrate our observation. We also correct the proof and improved the result by replacing α-admissibility by orbital α-admissibility.

Keywords α-Admissible mappings · Orbital-α-admissible mappings · $\mathcal{F}$-metric space · Fixed point

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1 Introduction and preliminaries

One of the trends in fixed point theory is to replace the metric space with a more general abstract space [15,17,28–30]. Among them, we focus on the notion of $\mathcal{F}$-metric spaces that was proposed by Jleli and Samet [20]. For the sake of integrity of the note, we recollect the notion as well as the fundamental properties of $\mathcal{F}$-metric spaces.

Let $\mathcal{F}$ be the set of functions $f : (0, \infty) \to \mathbb{R}$ satisfying the following conditions:
$(\mathcal{F}_1)$ f is non-decreasing, i.e., $0 < s < t$ implies $f(s) \leq f(t)$;
$(\mathcal{F}_2)$ For every sequence $\{t_n\} \subset (0, +\infty)$, we have

$$\lim_{n\to+\infty} t_n = 0 \text{ if and only if } \lim_{n\to+\infty} f(t_n) = -\infty.$$

The notion of an $\mathcal{F}$-metric space is defined as follows.

Definition 1.1 [20] Let X be a nonempty set and $D : X \times X \to [0, +\infty)$ be a given mapping. Suppose that there exists $(f, a) \in \mathcal{F} \times [0, +\infty)$ such that
(D_1) $(x, y) \in X \times X$, $D(x, y) = 0 \Leftrightarrow x = y$;
(D_2) $D(x, y) = D(y, x)$, for all $(x, y) \in X \times X$;
(D_3) For every $(x, y) \in X \times X$, for every $N \in \mathbb{N}$, $N \geq 2$, and for every $(u_i)_{i=1}^N \subset X$ with $(u_1, u_N) = (x, y)$, we have

$$D(x, y) > 0 \text{ implies } f(D(x, y)) \leq f\left(\sum_{i=1}^{N-1} D(u_i, u_{i+1})\right) + a.$$

Then D is said to be an $\mathcal{F}$-metric on X, and the pair (X, D) is said to be a $\mathcal{F}$-metric space.

A sequence $\{x_n\}$, in a $\mathcal{F}$-metric space (X, D), is $\mathcal{F}$-convergent to $x \in X$ if $\{x_n\}$ is convergent to x with respect to the $\mathcal{F}$-metric D, that is

$$\lim_{n\to\infty} D(x_n, x) = 0.$$

A sequence $\{x_n\}$, in a $\mathcal{F}$-metric space (X, D), is called $\mathcal{F}$-Cauchy, if

$$\lim_{n,m\to+\infty} D(x_n, x_m) = 0.$$

We say that a $\mathcal{F}$-metric space (X, D) is $\mathcal{F}$-complete, if every $\mathcal{F}$-Cauchy sequence in X is $\mathcal{F}$-convergent to a certain element in X.

On what follows, let Ψ be the set of functions $\Psi : [0, \infty) \to [0, \infty)$ such that

(ψ_1) ψ is nondecreasing;
(ψ_2) $\sum_{n=1}^{\infty} \psi^n(t) < \infty$ for each $t \in \mathbb{R}^+$, where ψ^n is the nth iterate of ψ.

Remark 1.1 It is easy to see that if $\psi \in \Psi$, then $\psi(0) = 0$ and $\psi(t) < t$ for any $t > 0$.

The notion of α-admissible mappings [32] and triangular α-admissible mappings [25] are refined by Popescu [31] as follows:

Definition 1.2 [31] Let $\alpha : X \times X \to [0, \infty)$ be a mapping and $X \neq \emptyset$. A self-mapping $T : X \to X$ is said to be an $\alpha-$orbital admissible if for all $s \in X$, we have

$$\alpha(s, Ts) \geq 1 \Rightarrow \alpha(Ts, T^2s) \geq 1. \tag{1.1}$$

Furthermore, an $\alpha-$orbital admissible mapping T is called triangular α-orbital admissible if the following condition holds:

(TO) $\alpha(s,t) \geq 1$ and $\alpha(t,Tt) \geq 1$ implies that $\alpha(s,Tt) \geq 1$, for all $s,t \in X$.

Each $\alpha-$admissible mapping is an α-orbital admissible mapping. For more details and interesting examples, see e.g. [1–9,11–14,16,19,21–23,26,27].

Very recently, Hussain and Kanwal [24] generalized the main result of Jleli and Samet [20] in the class of $\mathcal{F}$-metric spaces by considering $\alpha - \psi$-contractive mappings.

Definition 1.3 [24] Let (X,D) be an $\mathcal{F}$-metric space and $T : X \to X$ be a given mapping. We say that T is a generalized $\alpha - \psi$-contraction if there exist $\alpha : X \times X \to [0,\infty)$ and $\psi \in \Psi$ such that, for all $x,y \in X$, with $\alpha(x,y) \geq 1$, we have

$$D(Tx,Ty) \leq \psi(M(x,y)) \tag{1.2}$$

where

$$M(x,y) = \max\{D(x,y), D(x,Tx), D(y,Ty)\}. \tag{1.3}$$

Unfortunately, there is a gap in the statement of Theorem 2.1 in [24] and its proof. In this paper, we provide counter-examples showing their gap and we give the corrected proofs.

2 Main results

The essential main result in [24] is the following theorem.

Theorem 2.1 [24] *Let (X,D) be an $\mathcal{F}$-complete $\mathcal{F}$-metric space and $T : X \to X$ be an α-admissible generalized $\alpha - \psi$-contraction mapping. If there exists $x_0 \in X$ such that $\alpha(x_0,Tx_0) \geq 1$, then T has a fixed point in X.*

Theorem 2.1 contains a gap. The following examples explain this fact. In fact, we will consider self-mappings satisfying all hypotheses of Theorem 2.1, but having no fixed point.

Example 2.1 Consider $X = \mathbb{R}$. Define $D : X \times X \to [0,\infty)$ as

$$D(x,y) = |x-y|, \quad x,y \in X.$$

Note that D is an $\mathcal{F}$-metric with $f(t) = \ln t$ and $a = 0$. Consider

$$Tx = \begin{cases} \frac{x-1}{2} & \text{if } x > -1 \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\alpha(x,y) = \begin{cases} 1 & \text{if } x,y \in (-1,\infty) \\ 0 & \text{otherwise.} \end{cases}$$

Choose $\psi(t) = \frac{t}{2}$ for $t \geq 0$. How is it $T((-1,\infty)) = (-1,\infty)$, we obtain that T is α-admissible. For $x_0 = 1$, we have $\alpha(x_0,Tx_0) \geq 1$. We shall show that T is a generalized $\alpha - \psi$-contraction. Let $x,y \in X$ such that $\alpha(x,y) \geq 1$, so $x,y \in (-1,\infty)$. Here,

$$D(Tx,Ty) = \frac{1}{2}D(x,y) \leq \frac{1}{2}M(x,y) = \psi(M(x,y)).$$

So, all hypotheses of Theorem 2.1 hold, but T has no fixed point.

Example 2.2 Consider $X = \mathbb{R}$. Define $D : X \times X \to [0, \infty)$ as

$$D(x, y) = \begin{cases} e^{|x-y|} & \text{if } x \neq y \\ 0 & \text{if } x = y. \end{cases}$$

Note that D is an $\mathcal{F}$-complete $\mathcal{F}$-metric with $f(t) = -\frac{1}{t}$ and $a = 1$. Consider

$$Tx = \begin{cases} \frac{2x-1}{3} & \text{if } x \in (-1, \infty) \\ 0, & \text{otherwise.} \end{cases}$$

and

$$\alpha(x, y) = \begin{cases} 1, & \text{if } x, y \in (-1, \infty) \quad \text{with } x \neq y \\ 0, & \text{otherwise.} \end{cases}$$

Choose

$$\psi(t) = \begin{cases} \frac{2}{3}t, & \text{if } t < 1 \\ t^{\frac{2}{3}}, & \text{if } t \geq 1. \end{cases}$$

Now using the fact that $T((-1, \infty) = (-1, \infty)$, we obtain that T is α-admissible. For $x_0 = 1$, we have $\alpha(x_0, Tx_0) \geq 1$. We shall show that T is a generalized $\alpha - \psi$-contraction. Let $x, y \in X$ such that $\alpha(x, y) \geq 1$, so $x, y \in (-1, \infty)$ and $x \neq y$. Here, $M(x, y) \geq D(x, y) \geq 1$. Thus,

$$D(Tx, Ty) = (D(x, y))^{\frac{2}{3}} \leq (M(x, y))^{\frac{2}{3}} = \psi(M(x, y)).$$

All hypotheses of Theorem 2.1 are verified, but T has no fixed point.

Remark 2.1 Note that Example 2.2 in [24] is incorrect. Indeed, the authors considered $\psi(t) = \sqrt{t}$. For $t \in (0, 1)$, we have $\psi(t) > t$, and so $\psi \notin \Psi$. There is also gap in [24]. In fact, the authors used the hypothesis (H) (stated below) without mentioning it in their theorem. Moreover, to show that the map T has a fixed point, there is a gap because the authors [24] passed to the limit as $n \to \infty$ in the three given cases, which are only true for some n.

Now, we give the corrected and improved version of Theorem 2.1 together with its appropriate proof. For the corrected version, we need the following condition:

(H) Let (X, D) be an $\mathcal{F}$-metric space. The function $f \in \mathcal{F}$ verifying (D_3) is assumed to be continuous. Also, ψ is chosen to be continuous and to satisfy that $f(u) > f(\psi(u))+a$ for all $u \in (0, \infty)$, where a is also given in (D_3).

Theorem 2.2 *Let (X, D) be a an $\mathcal{F}$-complete $\mathcal{F}$-metric space and $T : X \to X$ be a generalized $\alpha - \psi$-contraction. Suppose that*

(i) *T is orbital α-admissible;*
(ii) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$;*
(iiia) *either, T is continuous,*
(iiib) *or (H) holds.*

Then T has a fixed point.

Proof By (ii), there is x_0 such that $\alpha(x_0, Tx_0) \geq 1$. Define $\{x_n\}$ by $x_{n+1} = Tx_n = T^{n+1}x_0$ for $n = 0, 1, 2, \ldots$. Regarding that T is orbital α-admissible, we derive that

$$\alpha(x_0, Tx_0) \geq 1 \text{ yields } \alpha(Tx_0, T^2x_0) = \alpha(x_1, x_2) \geq 1.$$

Recursively, we have

$$\alpha(x_n, x_{n+1}) \geq 1 \quad \text{for each } n \in \mathbb{N}.$$

After then, we can follow the proof of Theorem 2.1 in [24], and we conclude that

$$\lim_{n\to\infty} D(x_n, x_{n+1}) = 0. \tag{2.1}$$

Also, $\{x_n\}$ is an $\mathcal{F}$-Cauchy sequence. Since (X, D) is $\mathcal{F}$-complete, there exists some point $z \in X$ such that

$$\lim_{n\to\infty} D(x_n, z) = 0. \tag{2.2}$$

We have two cases.

Case $(iiia)$. Suppose that T continuous.

We have that

$$z = \lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} Tx_n = T(\lim_{n\to\infty} x_n) = Tz.$$

That is, z is a fixed point of T.

Case $(iiib)$. We argue by contradiction. Suppose that $D(Tz, z) > 0$. Using (1.2), we have

$$\begin{aligned} f(D(z, Tz)) &\leq f(D(z, Tx_n) + D(Tx_n, Tz)) + a \\ &\leq f(\alpha(x_n, z)D(Tx_n, Tz) + D(z, Tx_n)) + a \\ &\leq f(\psi(\max\{D(x_n, z), D(z, Tz), D(x_n, Tx_n)\}) \\ &\quad +D(z, x_{n+1})) + a \\ &= f(\psi(\max\{D(x_n, z), D(z, Tz), D(x_n, x_{n+1})\}) \\ &\quad +D(z, x_{n+1})) + a \end{aligned}$$

Letting $k \to \infty$ and using (2.1) and (2.2) together with continuity of f and ψ, we get

$$f(D(z, Tz)) \leq f(\psi(D(z, Tz))) + a.$$

which is a contradiction with respect to $f(u) > f(\psi(u)) + a$ for all $u \in (0, \infty)$. Hence, we obtain $D(Tz, z) = 0$, so $z = Tz$, that is, z is a fixed point of T. □

Example 2.3 Consider $X = \mathbb{R}$. Define $D : X \times X \to [0, \infty)$ as

$$D(x, y) = |x - y|, \quad x, y \in X.$$

Note that D is an $\mathcal{F}$-metric with $f(t) = \ln t$ and $a = 0$. Consider

$$T(x) = \frac{x+2}{2}$$

and

$$\alpha(x, y) = \begin{cases} \frac{3}{2}, & \text{if } x, y \in [1, \infty) \\ 0, & \text{otherwise.} \end{cases}$$

Choose $\psi(t) = \frac{3t}{4}$ for $t \geq 0$. Using the fact that $T([1, \infty)) = [\frac{3}{2}, \infty) \subset [1, \infty)$, we can conclude that T is triangular α-admissible. For $x_0 = 2$, we have $\alpha(x_0, Tx_0) \geq 1$. We shall show that T is a generalized $\alpha - \psi$-contraction. Let $x, y \in X$ such that $\alpha(x, y) \geq 1$. Then $x, y \in [1, \infty)$, so we have

$$D(Tx, Ty) \leq \frac{3}{4}D(x, y) \leq \frac{3}{4}M(x, y) = \psi(M(x, y)).$$

So, all hypotheses of Theorem 2.2 hold. It can be seen that $x = 2$ is a fixed point of T.

Remark 2.2 Note that the second part of condition $(iiib)$) in Theorem 2.2 is not superfluous. To be clear, we have the following.

1. Following Example 2.1 in [20], consider $D : X \times X \to [0, \infty)$ defined as

$$D(x, y) = \begin{cases} (x - y)^2 & \text{if } (x, y) \in [0, 3] \times [0, 3] \\ |x - y| & \text{if } (x, y) \notin [0, 3] \times [0, 3]. \end{cases}$$

where $X = \{0, 1, 2, \ldots\}$. Such D is a $\mathcal{F}$-metric with $f(t) = \ln(t)$, $t > 0$, and $a = \ln(3)$. Note that f is continuous on $(0, \infty)$ and the condition on ψ, which is, $f(u) > f(\psi(u)) + a$ for all $u > 0$, becomes $\ln(u) - \ln(\psi(u)) > \ln(3)$, that is, ψ is chosen to be continuous such that

$$\psi(u) < \frac{1}{3}u.$$

2. Following Example 2.4 in [20], consider the $\mathcal{F}$-metric $D : X \times X \to [0, \infty)$ given as

$$D(x, y) = \begin{cases} e^{|x-y|} & \text{if } x \neq y \\ 0 & \text{if } x = y. \end{cases}$$

where $X = \{0, 1, 2, \ldots\}$, $f(t) = -\frac{1}{t}$ for $t > 0$, and $a = 1$. Note that f is continuous on $(0, \infty)$ and the condition on ψ, which is, $f(u) > f(\psi(u)) + a$ for all $u > 0$, becomes $-\frac{1}{u} > \frac{1}{\psi(u)} > 1$, that is, ψ is chosen to be continuous such that

$$\psi(u) < \frac{u}{u + 1}.$$

A simple consequence of Theorem 2.2 is stated as follows.

Corollary 2.1 *Let (X, D) be a an $\mathcal{F}$-complete $\mathcal{F}$-metric space and $T : X \to X$ be a given mapping such that*

$$\alpha(x, y)D(Tx, Ty) \leq \psi(M(x, y)),$$

for all $x, y \in X$, where $\psi \in \Psi$ and $M(x, y)$ was defined by (1.3). Suppose that

(i) *T is orbital α-admissible;*
(ii) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$;*
(iiia) *either, T is continuous,*
(iiib) *or (H) holds.*

Then T has a fixed point.

Following [31], we consider the following condition in order to get a uniqueness fixed point result.

(K) For all $x, y \in X$, there exists $v \in X$ such that $\alpha(x, v) \geq 1$, $\alpha(y, v) \geq 1$ and $\alpha(v, Tv) \geq 1$.

Theorem 2.3 *Adding to the hypotheses of* Theorem 2.2*:*

(A) *condition* (K)*;*

(B) *for all* x *and* $\{x_n\}$ *in* X *such that* $\{x_n\}$ *converging to some* $h \in X$*, we have* $f(D(x, h) > f(\psi(\limsup_{n\to\infty} D(x, x_n)) + a$ *with* $\limsup_{n\to\infty} D(x, x_n) > 0$*,*

then the fixed point of T *is unique.*

Proof From Theorem 2.2, T has a fixed point. Suppose on the contrary that there are two fixed points of T, say ρ and ζ with $\rho \neq \zeta$. Then by condition (K), there exists $v \in X$ such that $\alpha(\rho, v) \geq 1$, $\alpha(\zeta, v) \geq 1$ and $\alpha(v, Tv) \geq 1$. The triangular α-orbital admissibility of T implies that

$$\alpha(\rho, T^n v) \geq 1 \quad \text{and} \quad \alpha(\zeta, T^n v) \geq 1 \quad \text{for all} \quad n \geq 1.$$

By Theorem 2.2, the sequence $\{T^n v\}$ converges to a fixed point of T, say σ. Assume that $\rho \neq \sigma$. Then using (D_3), (1.2) and the fact that $\alpha(\rho, T^n v) \geq 1$, one writes

$$\begin{aligned} f(D(\rho, \sigma)) &\leq f(D(\rho, T^{n+1}v) + D(T^{n+1}v, \sigma)) + a \\ &= f(D(T\rho, T(T^n v)) + D(T^{n+1}v, \sigma)) + a \\ &\leq f(\psi(\max\{D(\rho, T^n v), D(\rho, T\rho), D(T^n v, T^{n+1}v)\}) \\ &\quad + D(T^{n+1}v, \sigma)) + a \\ &= f(\psi(\max\{D(\rho, T^n v), D(T^n v, T^{n+1}v)\}) + D(T^{n+1}v, \sigma)) + a. \end{aligned}$$

Letting $n \to \infty$ and using (2.1), (2.2) and continuity of f and ψ, we find that

$$f(D(\rho, \sigma)) \leq f(\psi(\limsup_{n\to\infty} D(\rho, T^n v)) + a. \tag{2.3}$$

Necessarily, $\limsup_{n\to\infty} D(\rho, T^n v) > 0$. The inequality (2.3) is a contradiction with respect to condition (B). Thus, $\rho = \sigma$. Similarly, we obtain that $\zeta = \sigma$, so $\rho = \zeta$, which is again a contradiction. Therefore, there is a unique fixed point of T. □

3 Conclusion

In this note, we correct and improve the recently reported results in [24]. We also want to underline that the given results have a number of consequences in different structures, see e.g. [27]. For example, if we take $\alpha(x, y) = 1$ for all $x, y \in X$, we observe an analog of Corollary 2.1 in the setting of standard $\mathcal{F}$-metric spaces. On the other hand, it is easy to get the analog of these results in the setting of a partially ordered $\mathcal{F}$-metric space, by choosing $\alpha(x, y)$ properly as,

$$\alpha(x, y) = \begin{cases} 1 & \text{if } x \preceq y \text{ or } x \succeq y, \\ 0 & \text{otherwise.} \end{cases}$$

Thus, the following theorem is an apparent consequence of Corollary 2.1.

Theorem 3.1 *Let* $\preceq$ *be a partial order on* X *that is equipped with an* $\mathcal{F}$*-metric* D*. Assume that* (X, D) *is* $\mathcal{F}$*-complete. Suppose that* $T : X \to X$ *is a nondecreasing mapping with respect to* $\preceq$ *such that*

$$D(Tx, Ty) \leq \psi(M(x, y)),$$

for all $x, y \in X$ *with* $x \preceq y$*, where* $\psi \in \Psi$ *and* $M(x, y)$ *was defined by* (1.3)*. Moreover, we suppose that*

(I) *there exists* $x_0 \in X$ *such that* $x_0 \preceq Tx_0$*;*
(IIa) *either,* T *is continuous,*
(IIb) *or* (H) *holds. Moreover, the function* $f \in \mathcal{F}$ *verifying* (D_3) *is assumed to be continuous. Also,* ψ *is chosen to be continuous and to satisfy that* $f(u) > f(\psi(u)) + a$ *for all* $u \in (0, \infty)$*, where* a *is also given in* (D_3)*.*

Then T *has a fixed point.*

In a similar way, we can transfer the main results of this paper to setting of cyclic mappings by letting $\alpha(x, y)$ as

$$\alpha(x, y) = \begin{cases} 1 & \text{if } (x, y) \in (A_1 \times A_2) \cup (A_2 \times A_1), \\ 0 & \text{otherwise.} \end{cases}$$

Theorem 3.2 *Let* $\{A_i\}_{i=1}^2$ *be nonempty closed subsets of a* $\mathcal{F}$*-complete* $\mathcal{F}$*-metric space and* $T : Y \to Y$ *be a given mapping, where* $Y = A_1 \cup A_2$*. Suppose that the following conditions hold:*

(I) $T(A_1) \subseteq A_2$ *and* $T(A_2) \subseteq A_1$*;*
(II) *there exists* $\psi \in \Psi$ *such that*

$$D(Tx, Ty) \leq \psi(M(x, y)), \textit{ for all } (x, y) \in A_1 \times A_2,$$

where $M(x, y)$ *was defined by* (1.3)*;*
(III) *either,* T *is continuous,*
(IIb) *or the function* $f \in \mathcal{F}$ *verifying* (D_3) *is assumed to be continuous. Also,* ψ *is chosen to be continuous and to satisfy that* $f(u) > f(\psi(u)) + a$ *for all* $u \in (0, \infty)$*, where* a *is also given in* (D_3)*.*

Then T *has a fixed point that belongs to* $A_1 \cap A_2$*.*

We refer to [27] for more explicit details on proofs of Theorems 3.1 and 3.2.

Finally, it is clear that various corollaries can be added when replacing the generalized $\alpha - \psi$-contraction by

(a) $D(Tx, Ty) \leq \psi(D(x, y))$
(b) $D(Tx, Ty) \leq \psi(\max\{D(x, Tx), D(y, Ty)\})$
(c) $D(Tx, Ty) \leq \psi(\frac{D(x,Tx)+D(y,Ty)}{2})$

and more by letting $\psi(t) = kt$, where $k \in [0, 1)$,

(a) $D(Tx, Ty) \leq kD(x, y)$
(b) $D(Tx, Ty) \leq k \max\{D(x, Tx), D(y, Ty)\}$
(c) $D(Tx, Ty) \leq k\frac{D(x,Tx)+D(y,Ty)}{2}$,

in the setting of Corollary 2.1, Theorems 3.1 and 3.2.

Author contributions All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

Compliance with ethical standards

Conflict of interest The authors declare that they have no competing interests.

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