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ORIGINAL ARTICLE

The confluence of attitudes towards mathematics and pedagogical practice: evaluating the use of mathematical manipulatives

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Abstract
The use of manipulatives to develop conceptual understanding is a prevalent practice in many mathematical learning experiences, particularly in the early years of schooling. From *primary student perspectives*, our understanding of the impact of manipulatives in mathematics education on students' *attitudes* is limited. This study evaluates the impact of mathematical manipulatives on Young Children's Attitudes Towards Mathematics (YCATM) by examining children's drawings, as well as their written and verbal descriptions of their drawings from 106 year 2 and year 3 students. Classroom observations were conducted to investigate how attitudes towards mathematics are enacted during mathematical learning experiences. The modified three-dimensional model of attitude (MTMA) and Bruner's experiential stages were used to investigate how manipulatives influence YCATM. Data analyses used systematic, numerical coding, and thematic and comparative approaches, employing inductive, deductive, and anticipatory coding for data from both lesson and non-lesson contexts. The findings suggest that young children enjoyed using manipulatives, contributing to their vision of mathematics and perceived competence. However, the transition between enactive, iconic, and symbolic experiences can contribute to the formation of negative attitudes. The present study also emphasizes the importance of context, content, and familiarity with the use of manipulatives.

Keywords Attitudes towards mathematics · Modified three-dimensional model of attitude · Primary students · Manipulatives · Experiential learning · Children's drawings

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Children's attitudes towards mathematics are strongly related to their receptiveness to learning and understanding mathematics, their achievement, the value of the subject, self-confidence, and enjoyment (Stiles et al., 2008). Describing the nature of an individual's attitude towards mathematics has been of interest for decades. Most studies that investigate students' attitudes look at specific situations or aspects of mathematics education. For example, researchers have investigated parental influences (Köğce et al., 2009), peer influences (Quane, 2021), and inquiry-based learning in mathematics (Gómez-Chacón et al., 2023) on students' attitudes. Such studies are valuable in identifying factors that influence students' attitudes towards mathematics.

Underlining the necessity for mathematics and how children develop mathematical understanding requires a need to understand the factors that influence attitudes towards mathematics (Saritas & Akdemir, 2009). Attitude is a multi-dimensional construct with affective, cognitive, and behaviour dimensions (DeBellis & Goldin, 2006; Goldin et al., 2011; Grootenboer & Marshman, 2016; McLeod, 1992; Quane, 2021; Walker et al., 2020). Investigating attitudes towards mathematics as a multi-dimensional construct provides an erudite view between attitudes and mathematics achievement (Walker, 2020). A concerning development is the increasing number of students who are identified as having a negative attitude towards mathematics or mathematics anxiety. Mathematics anxiety is not a new phenomenon with Gough (1954) describing the condition as "Mathemaphobia", attributing a fear of mathematics to mathematics avoidance and, in some cases, underachieving in mathematics. Consequently, attitudes towards mathematics and the factors must be understood so that positive attitudes can be fostered and nurtured. While there is extensive knowledge concerning older students' attitudes towards mathematics, research about young children's attitudes towards mathematics (YCATM) is notably lacking (Ingram et al., 2020; Quane et al., 2021).

The limited research extends to investigating how the use of manipulatives by children influences their attitudes towards mathematics. Manipulatives are an established mathematics education resource that can be a "positive tool to improve student learning" (Liggett, 2017, p. 90) and a tool to develop mathematical conceptual understanding (Quane & Brown, 2022). Further, manipulatives are an established form of mathematical representation (Moyer, 2001). Goldin and Shteingold (2001) remarked: "that a mathematical representation cannot be understood in isolation" (p. 1). Rather, a representation of mathematics is part of a more comprehensive system of mathematical conventions and meaning. Mathematics representations can be a process and a product and are broadly classified as external or internal representations (Goldin & Shteingold, 2001), with manipulatives being an example of a product and an external representation. Puchner et al. (2008) suggest that manipulatives aid in formal internal representations that could provide opportunities to develop mathematical thinking and reasoning.

However, research indicates that manipulatives are more than physical, external representations (Goldin & Shteingold, 2001; Quane, 2022; Quane & Brown, 2022; West, 2018). Bruner (1966) suggests that our world can be represented and translated into experience in three stages: enactive (action), iconic (perceptual organisation), and symbolic (words and symbols). Research has focused on how teachers effectively use manipulatives to facilitate mathematical learning (Quane & Brown, 2022; West, 2018) and the challenges of using manipulatives (Moch, 2002) and warn that manipulatives do not necessarily lead to success and can even be detrimental to learning (McNeil & Jarvin, 2007). The studies discussed here have been conducted with older students. Adolescents and adults exhibit their attitudes towards mathematics differently compared to young children due to their social and psychological development (Slavin, 2014). Many of the studies focused on attitudes in a non-lesson context; however, there is a need to investigate attitudes in lesson contexts and with younger children (Aiken, 1996; Attard et al., 2016; Di Martino, 2019; Ingram et al., 2020). This research reports on a larger study that investigated YCATM from two different contexts (non-lesson and lesson), making comparisons between contexts to identify factors contributing to YCATM (Quane, 2021; Quane et al., 2023). Children's drawings, as well as their written and verbal descriptions of their drawings, are used as a research tool to document YCATM during a non-lesson context, while observations of mathematical learning experiences are used to document the enactment of attitudes during a lesson context.

This study investigated the range and nature of YCATM and, in doing so, identified a range of factors that were found to influence attitudes. Numerous factors were identified, including, but not limited to, the use of technology, game-based pedagogies, tests and assessments, and manipulatives. The focus of this paper is on the use of manipulatives and how they influence attitudes towards mathematics. The guiding research question is: "How do manipulatives and their representations used during mathematical learning experiences influence young children's attitudes towards mathematics?".

Literature review

Mathematics is often perceived as a challenging and abstract subject that elicits a diverse range of emotions and dispositions among learners, influencing their motivation, engagement, and ultimately, their success in the subject (Aiken, 1972; Ma, 1997; Kontas, 2016). Pedagogical practices encompass a wide spectrum of teaching methods, strategies, and approaches used to convey mathematical concepts to students, and these practices play a pivotal role in shaping students' experiences and understanding of the subject. In this context, the incorporation of manipulatives, tangible objects, and tools that enable students to interact with mathematical concepts concretely and visually, has become a subject of growing interest, offering the potential to enhance mathematical understanding, particularly among younger learners (Quane, 2022). This literature review aims to explore the intricate interplay between attitudes towards mathematics, and a specific pedagogical practice of using manipulatives, shedding light on the evolving landscape of mathematics education and the implications for both theory and classroom application.

Attitudes towards mathematics

Children develop their attitude towards mathematics through social interactions and educational experiences, whether through direct exposure or indirect influences (Aiken, 1972; Ajisuksmo & Saputri, 2017; Barkatsas, 2012; Kontas, 2016; Mata et al., 2012; Quane et al., 2021; Shamsuddin et al., 2018). Attitudes, particularly attitudes towards mathematics, are an essential objective of instruction affecting learning, memory, retention, and behaviour (Ma, 1997; Mata et al., 2012). Attitude can be directed towards an object, situation or behaviour and can fluctuate and vary in intensity (Chamberlin, 2010; Rokeach, 1968). In terms of attitudes being directed towards objects, there is little research that explores the confluence between children's attitudes and how these attitudes are directed towards or influence by mathematical manipulatives.

Researchers acknowledge that attitudes towards mathematics is a multi-dimensional construct (Quane et al., 2021; Quane, 2021, 2022). The naming of these dimensions often differs between models or frameworks, but the consensus is that attitudes towards mathematics contain the core elements of affective, behaviour, and cognitive dimensions (DeBelis & Goldin, 2006; Grootenboer & Marshman, 2016; McLeod, 1992; Quane et al., 2023; Walker et al., 2020;). One model that incorporates affective, behavioural and cognitive factors is the Affective-Behavioural-Cognitive or ABC model of attitudes towards mathematics. A second model is the Three-dimensional Model of Attitude (TMA) developed by Zan and Di Martino (2007) who developed TMA out of necessity to address "the lack of theoretical clarity that characterizes research on attitude and the inadequacy of most instruments" (p. 157). According to Walker et al (2020), a multi-dimensional model has the potential to provide greater insight into individuals' attitudes towards mathematics as "opposed to a single-factor (unidimensional model)" (p. 12).

A definition that is been widely adopted is a multi-dimensional definition comprising of emotions, beliefs, and the value individuals place on mathematics (Hannula, 2012; Zan & Di Martino, 2007). Di Martino and Zan (2010) broaden the definition of attitude by describing three key dimensions: emotional dimension (ED), vision of mathematics (VM) and perceived competence (PC) contributing to students' attitudes towards mathematics. The three dimensions together are known as the three-dimensional model of attitude (TMA). By broadening the definition of attitude, relationships between the dimensions can be described (Di Martino & Zan, 2010). Quane et al. (2023) further modified TMA for use with young children and primary-aged students by dividing each original dimension of TMA into two sub-dimensions. Quane (2021) describes the modification as a four-prong, integrated approach whereby "an extensive literature review, deductive, anticipatory and inductive processes" were conducted to theorize each sub-dimension (p. 246). Quane (2021) provides the following description the two sub-dimensions of the emotional dimension as Emotional Tendency, "children's feelings and emotional responses towards mathematics" and Overall Sentiment: "children's reactions to mathematics, including posture, gestures and body language" (p. 246). For the other two dimensions, the naming provides an indication of what each dimension encapsulates. That is the vision of mathematics dimension is separated into topic, tasks and processes, and value and appreciation of mathematics, whereas the perceived competence dimensions is divided into mathematical mindset and self-concept. The MTMA (Quane, 2021; Quane et al., 2023) has been used as the theoretical framework for the research reported in this paper.

In ascertaining an individual's attitude towards mathematics, numerous research techniques have been employed. The use of participant drawings as a research technique to ascertain affective factors such as attitudes towards mathematics has gained increasing attention since the research conducted by Picker and Berry in 2000 who used drawings to investigate pupil's images of mathematicians. Subsequently, numerous researchers have adopted the use of participant drawings as a research technique to investigate attitudes towards mathematics. Kuzle (2021) investigated students' attitudes towards geometry while Di Martino (2019) and Pehkonen et al. (2016) used participants' drawings to explore attitudes towards mathematical problem-solving. Doğan and Sönmez (2019) investigated attitudes towards mathematical games, while Bachman and colleagues (2016) explored attitudes in two different learning areas, mathematics and dance. Howell (2012, 2016, 2017) investigated attitudes towards standardised testing in Australia. Foley (2016) explored female, primary-aged students' attitudes towards mathematics, and Quane (2021) investigated the influence of peers on students' attitudes towards mathematics. These researchers and others advocate for the use of participants' drawings as a research technique claiming that drawings are rich sources of data that can convey nuanced, subtle, abstract expressions, feelings, and ideas and recreate aspects of everyday and mathematical experiences (Cherney et al., 2006; Jolley et al., 2004; Martin & Murtagh, 2017; Thom, 2018; Thom & McGarvey, 2015). Furthermore, social and strong emotions can be shown in children's drawings, and drawings can be used across cultures (Bonoti & Misalidi, 2015; Laine et al., 2015; Stiles et al., 2008; Sullivan et al., 2017).

Pedagogical practice and attitudes towards mathematics

In defining and outlining how attitudes towards mathematics develop over time, we see that there are external factors at play in the formation and development of children's attitudes. Several researchers posit that pedagogical practice influences students' attitudes towards mathematics (Kontas, 2016; Pepin, 2011; Philippou & Christou, 1998; Quane, 2021; Reyes, 1984). The term pedagogical practice describes the teaching and learning strategies, resources, connections to other mathematical content, authentic situations or other learning areas that a teacher uses and makes during mathematical learning experiences (Farquhar, 2003).

A premise of quality teaching is good pedagogical practice (Farquhar, 2003). Research has examined the effect of a wide range of pedagogical practices and their influence on students' attitudes. For example, Pepin (2011) found group work, playing games and problem solving positively influenced attitudes, whereas repetition in mathematics negatively influenced attitudes. Kontas (2016) found the use of concrete learning materials (manipulatives) to positively influence secondary students' attitudes towards algebra. In contrast, mathematical learning experiences that students viewed as boring or irrelevant negatively impacted on attitudes towards mathematics (Philippou & Christou, 1998; Reyes, 1984). Additionally, Howell (2016) reported students in the primary years to have negative attitudes towards mathematical assessment.

The aforementioned studies have been conducted with predominately with older students, exploring specific pedagogical practices and strands of mathematics. However, adolescents and adults exhibit their attitudes towards mathematics differently compared to young children due to their social and psychological development (Slavin, 2014). Many of the studies focused on attitudes in a non-lesson context and as previously discussed, there is a need to investigate attitudes in lesson contexts and with younger children (Aiken, 1996; Attard et al., 2016; Di Martino, 2019; Ingram et al., 2020; Quane et al., 2023). The aim of this research is to explore how a prolific pedagogical practice of using manipulatives influences young children's attitudes towards mathematics.

Mathematical manipulatives

A manipulative is "an object that can be handled by an individual in a sensory manner during which conscious and unconscious mathematical thinking will be fostered" (Swan & Marshall, 2010, p. 14). Manipulatives can be physical or virtual objects (Durmus & Karakirik, 2006) that act as an enabling tool for students and teachers to integrate their knowledge and associated it with mathematical concepts (Kontas, 2016). The use of manipulatives can include concrete materials that provide tactile experiences to allow students to model, describe, engage, explore, reflect, and justify mathematics and mathematical thinking (Cranston, 2020; Quane & Buhren, 2024). Manipulatives could be in the form of mathematical resources or everyday items or toys such as a Pop-it fidget toy (Quane & Brown, 2022). Some researchers classify the use of fingers as a "physical manipulative" to develop mathematical understanding as a form of embodied and constructivist approach to learning (Soylu et al., 2018, p. 127). Manipulatives can also be virtual which are "interactive, Web-based visual representation of a dynamic object" (Moyer et al., 2002, p. 373).

The benefits of using manipulatives in mathematics have been well documented. For example, Kontas (2016) examined the use of manipulatives with middle school students and their achievement and attitudes finding an increase in student achievement and attitudes towards mathematics as measured quantitatively using the Attitude Towards Mathematics Lesson Scale. The value of using manipulatives as concrete representations has been known for decades, with Bruner and Kenney (1965) advocating the use of such materials. Less is known about how manipulatives influence the enactment of young children's attitudes and, in particular, the nature of these influences. Further, little is known about how children's attitudes develop or change or if any change occurs when transitioning from the physical use of a manipulative to the symbolic representation and abstraction that may follow.

Children need the time to develop confidence in using physical representations before introducing "conventional notation" (Perry & Atkins, 2002, p. 201). Bruner and Kenney (1965) recommend that children begin a mathematical representation "by constructing an embodiment of some concept" that is "building a concrete form of operational definition" (p. 56). This can be achieved through a range of manipulatives that provide the conduit for the "construction of images" that can be used to represent the concrete representation. Bruner (1966) referred to this stage as "enactive" (p. 11). Only once the concept has been represented concretely, that is enacted, then it can be represented through pictorial representations or "iconic" (Bruner, 1966, p. 11). Bruner (1966) suggests that "iconic" representations can be created using "words or language" that is in the form of "symbolic" (p. 10) representations. In this way, children are creating insight and developing conceptual understanding. Bruner and Kenney (1965) claim that "the growth of such abstraction is important" as children store a series of mental images of their abstractions which further aids their conceptual understanding. Further, the use of manipulatives provides children with the opportunity to explore mathematical ideas through the process of "construction, unconstruction and reconstruction" (Bruner & Kenney, 1965, p 52). That is through creating concrete representations there is a "reversibility" that aids in the internalisation of mathematical operations. This entire process of "constructing", "unconstructing", and "reconstructing" can lead to "new manipulations" to make further discoveries which can provide the foundation for future symbolic notations. In addition, by representing mathematical concepts through concrete materials, mathematical language is developed by starting rather informally about the concrete materials and then becoming more sophisticated as the mathematical concepts develop (Bruner & Kenney, 1965).

Tall (1994) represents Bruner's (2007) experiential stages as outlined above starting with a physical representative object, representations using imaging, and then using symbolic notation as shown in Fig. 1. In doing so, Tall (1994) suggests that this process is not linear rather it can be cyclic.

Larkin (2016) further refined Bruner's three stages of experiential learning to map types of manipulatives to each stage. According to Larkin (2016), the enactive stage includes three types of manipulatives: familiar objects such as toys, sport equipment and household items; substituted objects mainly in the form of commercial mathematical manipulatives; and digital objects which compose of virtual manipulatives and digital applications. Larkin (2016) suggests that it is at the enactive stage that students spend a considerable amount of time to foster deep conceptual mathematical understanding. The iconic stage is also categorised into three subgroups of photographs, graphics, and diagrams. Tall's (1994) depiction of Bruner's (2007) three stages of experiential learning recognises that the use of manipulatives is a recurring, dynamic process whereby interaction between internal and external representations is pivotal to developing mathematical understanding. In terms of manipulative use, Donovan and Alibali (2021) found that the way that manipulatives are used produces an affective response in children. However, the nature of this affective response is not documented. The focus of this research explored the confluence between children's attitudes towards mathematics and the influence of *physical* manipulatives. Bruner's (2007) three stages of experiential learning are used to examine the enactment of YCATM.

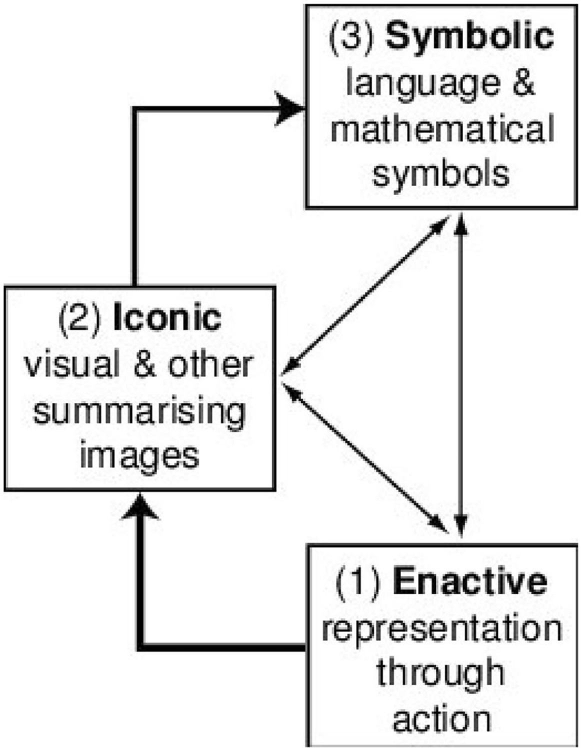


Fig. 1 Bruner's three stages of experiential learning as depicted by Tall (1994)

Theoretical framework

To explore the confluence of YCATM and the use of manipulatives, the Modified Three-dimensional Model of Attitude (MTMA) was used to define the construct of attitudes towards mathematics (Quane et al., 2023). Bruner's (1966's) experiential stages of learning were used to analyse how children used manipulatives and their attitudinal response to using manipulatives. To further categorise and develop a more nuanced understanding, the mapping mathematical materials framework by Larkin (2016) was applied to describe how children used manipulatives.

The MTMA moves beyond the dichotomies of "liking" versus "disliking" (Capps & Cox, 1969) and "positive" versus "negative" (Lipnevich et al., 2013) to capture the complexity of attitudes in three broad dimensions. These three broad and interconnected dimensions were conceptualised by Di Martino and Zan (2010) and encompassed the emotional disposition (ED), vision of mathematics (VM) and perceived competence (PC) dimensions. The MTMA provides six explicit sub-dimensions that can be used to classify and describe YCATM, placing a premium on the *developmental* aspects of children (Quane et al., 2023). Further, moving the definition of attitude away from a dual classification system to include a more extensive and nuanced description of attitude affords the opportunity to also identify factors that influence their attitudes. Each original dimension of the original TMA was modified to include two sub-dimensions, as shown in Table 1. The six sub-dimensions of MTMA were used to identify how the use of manipulatives influences YCATM.

Method

A mixed-methods approach was employed to explore the impact of manipulative usage on YCATM. Qualitative methodologies encompassed children's drawings, written descriptions, interviews, and observations, drawing upon established research techniques (Quane et al., 2019, 2023). These child-centric methods enabled children to express their attitudes towards mathematics and discuss influencing factors such as using manipulatives. Quantitative analyses provided insights into the frequency and distribution of attitudinal scores or the *range* of attitudes. Qualitative analyses developed into narratives constructed from children's accounts, actions, artifacts, actors, and significant events depicted in their drawings and discussed during the semi-structured interviews in a non-lesson context. Together, data from the non-lesson context and observations contributed to understanding the *nature* of children's attitudes. Data collection and analyses occurred over three phases: an exploratory study involving 25 children, a main study with 81 children, and an overarching analysing involving all 106 children.

Participants

The 106, year 2 and 3 children attended three South Australian schools, representing ten classes. Participants were aged between 7 and 9 years old, with 73 children (69%) enrolled in year 2 and 33 children (31%) in year 3. A mix of male ($n = 56$, 53%) and female ($n = 50$, 47%) children, including children from diverse language, cultural, and geographical backgrounds participated in the study. In Australia, years 2–3 are key transitional years where children move from junior primary to middle primary. Year 3 also sees the introduction of national standardised testing in the form of the National Assessment Program: Literacy and Numeracy (NAPLAN). There is limited research into children's attitudes towards mathematics prior to and after year 3 NAPLAN testing. The year groups have been strategically and purposively selected to develop a detailed understanding of YCATM. Table 2 provides details of the demographic data for the participating children.

Data collection

This study examined YCATM and the factors that influence attitudes towards mathematics in both lesson and non-lesson contexts, addressing the gap in understanding the enactment of attitudes towards mathematics. Children's drawings, written descriptions, and interview responses ($N = 106$) were collected in a non-lesson context. That is, data collection occurred in a common area outside of the class. Each child was assigned an alpha-numerical code to ensure anonymity with the letter denoting the school the child attended and the number indicating the order of drawing

Table 1 The Modified Three-dimensional Model of Attitude (MTMA) with reference to Bruner's experiential stages of learning adapted by Quane et al. (2023)

MTMA dimension	Bruner's experiential stages of learning	**Attitude classifications**
ED: emotional tendency	Children's initial emotional response and reaction during enactive learning experiences using manipulatives and emotions towards iconic and symbolic representations of manipulatives	Cannot be classified Extremely negative Negative Neutral Positive Extremely positive
ED: overall sentiment	Children's general reactions and emotional beliefs regarding mathematics, including non-verbal cues (posture, gestures, and body language) and verbal cues to the use of manipulatives (enactive, iconic, and symbolic representations of experience)	Cannot be classified Extremely negative Negative Neutral Positive Extremely positive
VM: topics, tasks and processes	Types of mathematical learning experiences and processes identified by children; the number of mathematical topics and how children communicate their mathematical understanding and learning. For example, children's use of manipulatives during mathematical learning experiences (enactive); children's drawings of manipulatives to represent mathematical concepts, ideas, and thinking (iconic); children using or discussing the use of manipulatives to then represent mathematics in written form using words and symbols (symbolic)	Cannot be classified Minimal vision of mathematics Low vision of mathematics Developing vision of mathematics High vision of mathematics Exemplary vision of mathematics
VM: value and appreciation	How and what children view as important and acknowledge as worthwhile about mathematics. For example, the worth or importance children place on using manipulatives in mathematics as a direct sensory experience (enactive), using pictorial representations of manipulatives (iconic), using manipulatives to aid in the representation of mathematics abstractly (symbolic)	Cannot be classified No value of mathematics Low value of mathematics Some value of mathematics High value of mathematics Very high value of mathematics
PC: mathematical mindset	Children's mathematical mindset and perceptions of themselves related to their ability to do mathematics. Children's mathematical mindset when using manipulatives, drawing, or making iconic representations of their use of manipulatives or recording their use in symbolic form	Cannot be classified Fixed mindset Low growth mindset Mixed mindset Growth mindset High growth mindset

Table 1 (continued)

MTMA dimension	Bruner's experiential stages of learning	**Attitude classifications**
PC: self-concept	Children's beliefs in their mathematical ability and their expectancy for success when using and representing manipulatives	Cannot be classified Extremely low perceived Competence Low perceived competence Neutral perceived competence High perceived competence Very high perceived competence

Table 2 Participant demographics and numbers

School	School sector	Class	Year level(s)	Male	Female	% of class population[b]
A	Government, inner regional with total enrolment of 285[a] students $n=25$	1	2/3	1 (yr 2) 4 (yr 3)	2 (yr 2) 4 (yr 3)	11/24 (46%)
		2	3/4	3	4	Yr 3: 7/11 (64%)
		3	1/2	3	4	Yr 2: 7/18 (39%)
B	Government, major city with total enrolment of 861[a] students $n=58$	4	1/2	8	5	13/24 (54%)
		5	2	13	10	23/25 (92%)
		6	2	2	3	5/25 (20%)
		7	1/2	5	3	8/25 (32%)
		8	3	3	6	9/26 (35%)
C	Government, inner regional with total enrolment of 163[a] students $n=23$	9	2/3	7 (yr 2) 1 (yr 3)	4 (yr 2) 2 (yr 3)	14/24 (58%)
		10	2/3	4 (yr 2) 2 (yr 3)	0 (yr 2) 3 (yr 3)	9/25 (36%)
Total	**3 schools**	**10**		**56**	**50**	**106/227 (47%)**

[a]Total enrolment is taken from myschool website

[b]Percentage rounded to the nearest whole number

and interview was conducted. A total of 27 observations of mathematical learning experiences in nine classes were conducted subsequently to the collection of non-lesson data. Three children with different attitude classifications from each class were purposefully selected and observed during mathematical learning experiences.

Children's drawings were obtained through the use of a drawing prompt "draw yourself doing mathematics" (Quane et al., 2019, 2023; Quane, 2022). This prompt has been found to elicit children's "personal stories about their complex relationship with mathematics, revealing their attitude towards mathematics" (Quane et al., 2021, p. 121). Children were given an A3 piece of paper and 24 coloured Textas, and the prompt was read to children with the written version placed on the table. Upon completion of the drawing, all 106 children were asked a series of questions via a semi-structured interview. The interview provided opportunities to ask clarifying questions regarding elements drawn and their use of manipulatives as well as additional questions to further ascertain their attitude towards mathematics. Excerpts from the children's interviews are used to further illustrate the confluence between children's attitudes and how they use and represent manipulatives.

The observations targeted overt behaviours discernible to the observer. Concealed or unobserved actions, reactions or behaviours are not dismissed as inconsequential; rather, they are challenging to discern in a classroom environment. An observational procedure was employed to ensure consistency between observations:

1. Children's drawings, written descriptions, and interview responses were analysed to determine the nature of each child's attitude towards mathematics.
2. Three children were selected from each class who had completed research phases 1 and 2 and were classified as having three different attitudes towards mathematics.
3. Descriptive observations of the classrooms were conducted.
4. Focused systematic observations were conducted:
    a. Five-minute intervals observing the three children.
    b. Observations of children conducted on a rotational basis.
5. Selective observations were conducted when a significant event occurred, or overt behaviour exhibited by one of the three children.
6. Return to focused observations when the child returns to the act before exhibiting overt behaviour or when the significant event ends.

An initial broad and *descriptive observation* of the classroom environments was conducted documenting the space, planned activities, actors, objects, goals, and times (Spradley, 2016). Descriptive observations occurred before and after the focused and selective observations and without the presence of children. Consultation between the researcher and classroom teachers was an essential part of the descriptive observations and provided the opportunity to document the variables accurately and to clarify any information. An observational protocol and field notes were used to record observations. *Focused observations* were conducted systematically at 5-min intervals observing the three children. During focused observations, children's emotions, facial expressions, gestures, interactions, and talk were documented. The observer switched to *selective observations* whenever a significant event occurred with one of the identified children or what Cohen et al. (2018) refer to as critical incidents. A critical incident is one whereby a child may behave unexpectedly, offer valuable insights or may reveal an emotional response (Cohen et al., 2018).

Data analysis

Children's attitudes were classified for both the lesson and non-lesson contexts. In reporting the findings, the context where the data was generated is indicated providing transparency regarding the source of data. Data analyses employed a combination of systematic, numerical coding, thematic, and comparative approaches. Systematic analysis involved analysing data generated from the non-lesson and lesson contexts using inductive, deductive, and anticipatory coding methods.

Each drawing, written description, and transcribed interview ($N=106$) was viewed initially as three separate pieces of data and then as a collection of work. Two main coding systems were then employed. First, a systematic analysis using the principle of atomism was used to examine each drawing (Bachman et al., 2016) using the non-lesson context rubric. Children's drawings were analysed to identify indicators for each attitude classification within each modified MTMA sub-dimension, starting with the child's face and moving to the whole child to ascertain their emotional tendency and overall sentiment. Several iterations of analysis of the children's drawings were completed for the remaining five sub-dimensions. Once the drawing was examined at the atomic level, drawings were viewed holistically (Bachman et al., 2016). The process of analysing at the atomic and holistic levels was repeated with the child's written description and interview responses. Once the data generated from the three individual data collection techniques were analysed, they were combined to form a more comprehensive picture of YCATM (Quane, 2021). The drawings were annotated with clarifying and supportive statements that the child made during the interview. A rubric was developed to ensure consistent coding.

Qualitative analysis generated themes to describe YCATM in-depth in the form of narrative analysis. Geertz's (1993) notion of "thick descriptions" was used as children's attitudes are "not reducible to simplistic interpretation" (Cohen et al., 2018, p. 17). "Thick descriptions were used to describe the complexities of the two contexts and the nature of YCATM describing the complexity of a situation, as classroom events and interactions" (Cohen et al., 2018, p. 17). After the observations, a rubric was used to classify children's attitudes for each MTMA dimension.

Two analytical rubrics were used to analyse the generated data from the non-lesson and lesson contexts. The rubrics featured a main idea and indicative criteria placed along a continuum of attitude from cannot be classified, extremely negative, negative, neutral, positive to extremely positive. Brookhart (2018) describes analytical rubrics as taking into consideration "criteria one at a time" (p. 1). The two rubrics were then used to numerically code children's attitudes, to analyse children's manipulative use and for thematic and comparative analyses for both contexts. The overall attitude classification was given a numerical value, with 0 representing cannot be classified and 5 representing an extremely positive attitude towards mathematics. The attitude classifications used for each MTMA sub-dimension are more reflective of the dimension as shown in Table 1.

The data generated has been re-analysed using open coding and MTMA and Bruner's experiential stages to explicate how manipulatives influence YCATM. After the generated data was coded, indicators were developed aligning with the framework outlined in Table 1. This approach address the broad research question: "How do manipulatives and their representations used during mathematical learning experiences influence young children's attitudes towards mathematics?".

Findings

A total of 13 different types of mathematical tasks or activities were identified by children as "doing mathematics". The types of tasks and activities are part of a child's vision of mathematics and, in particular, how they view the type of topics, tasks, and processes. Mathematics representations, in particular the use of manipulatives to engage and complete a task, emerged from the non-lesson context as a theme of interest. Forty-one (39%) children discussed the use of manipulatives either communicating their use in their drawing, written description or during the interview (Fig. 2).

In presenting these findings, it is important to acknowledge that the use of manipulatives varied between classes with the type of manipulatives used during mathematics governed by teachers. Teachers implement the Australian Curriculum: Mathematics (AC:M) which provides content descriptors and elaborations of content. While the AC:M does not stipulate pedagogical practices, the curriculum provides guidance and suggestions in the form of elaborations which are not mandated content. At the time of the study, teachers were using Version 8.4 of the Australian Curriculum. Examining the elaborations of the content descriptors identified that the AC:M suggested a range of manipulatives, mostly physical but also iconic. In year 2, the suggested manipulatives included an abacus, linking blocks,

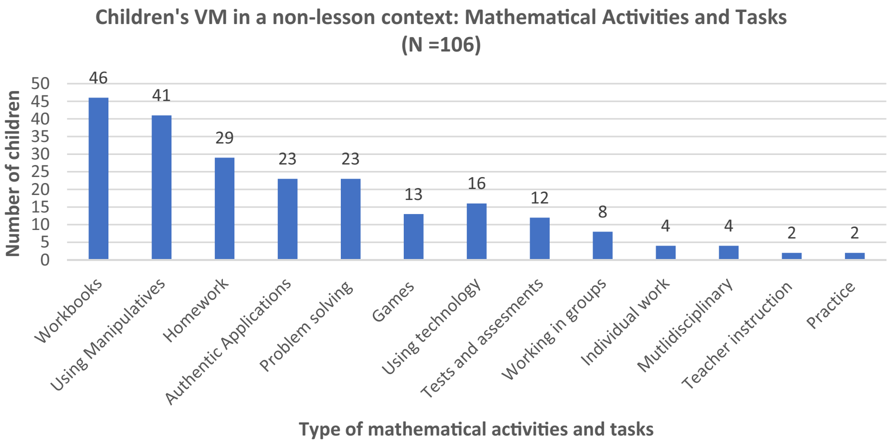
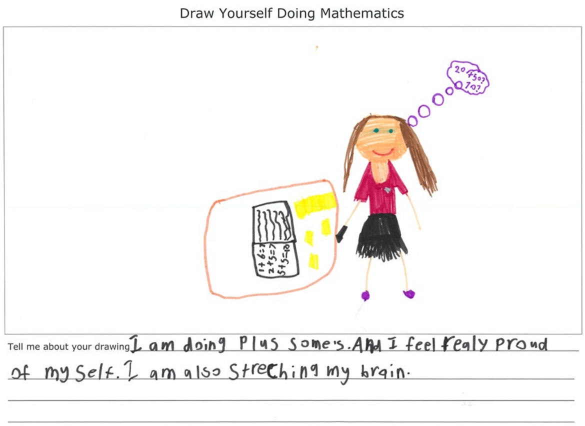


Fig. 2 Children's vision of mathematics in a non-lesson context

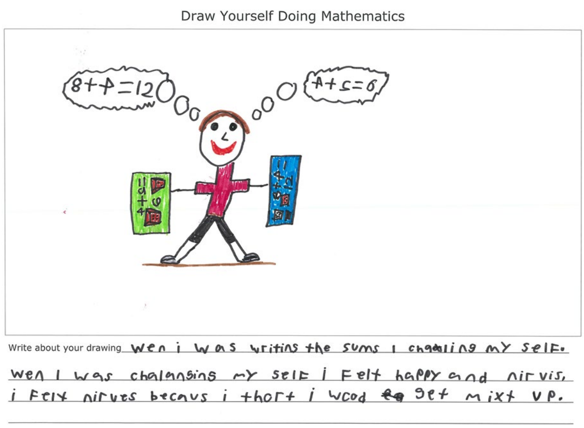
Fig. 3 A22, female, year 2



coins, stones, balance scales, hands, fingers, pieces of string, tens frames, and number lines. Objects were suggested to explore content from the number and algebra, and measurement and geometry strands, but the type of objects was not stipulated. In year 3, fewer manipulatives were suggested and included the use of calculators, maps, doors, coins, and number lines. Again, objects were also suggested.

Two main generalised findings are notable differences between the non-lesson and lesson contexts. First, in a non-lesson context, children discussed using manipulatives such as unifix cubes, dice, counters, and geoboards, to represent and solve mathematical questions and problems. Figures 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12 are *selected* children's drawings that depict the use of these manipulatives. These 10 drawings show the range and nature of attitudes towards mathematics and the influence of physical manipulatives. In the non-lesson context, children went beyond identifying different manipulatives to describe how they used manipulatives in relation to specific mathematical topics, their emotions towards using particular manipulatives and their perceived competence in using the manipulatives. Second, it was noted that during the 27 observations of mathematical learning experiences, a more extensive range of manipulatives were utilised in the lesson context, including

Fig. 4 A15, male, year 3



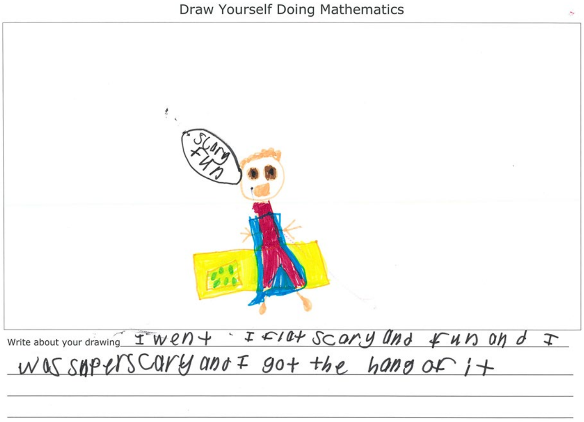


Fig. 5 A17, male, year 3

unifix cubes, dice, counters, geoboards, Polydrons, measuring instruments, attribute blocks, paddle pop sticks, dice, clocks, number charts, and tens frames. From these observations, the confluence between children's enacted attitude and the use of manipulatives was noted. These generalised results are further examined using the MTMA and Bruner's experiential learning stages to describe children's attitudes as outlined in Table 1.

Attitudes towards mathematics and manipulative use in the enactive stage

In the non-lesson context, children connected the use of manipulatives to a variety of mathematical topics, predominately number (operations and place value; see Figs. 3, 4, 5, 6, and 11), followed by 2D shapes (Figs. 7 and 8), time and reading analogue clocks (Figs. 9 and 12), and money. Children's emotional tendency towards using manipulatives varied greatly depending on the topic and

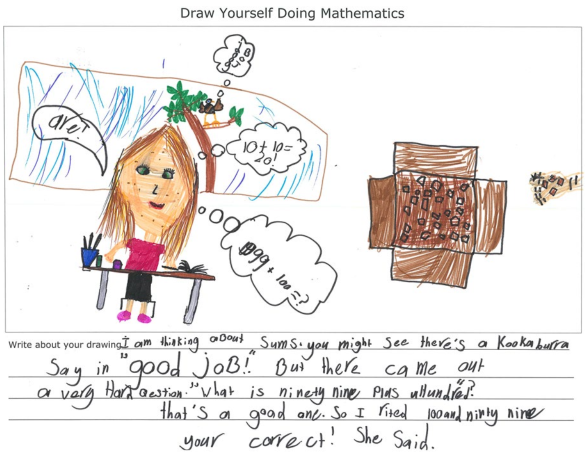
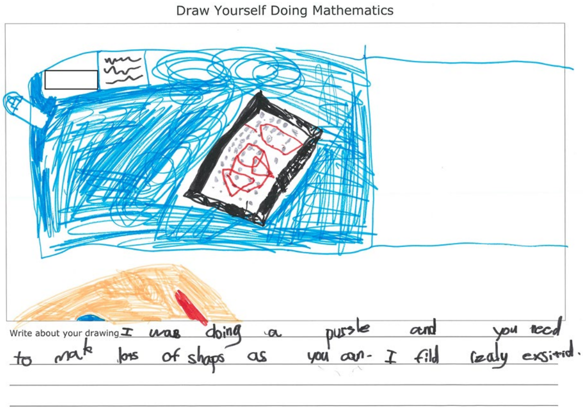


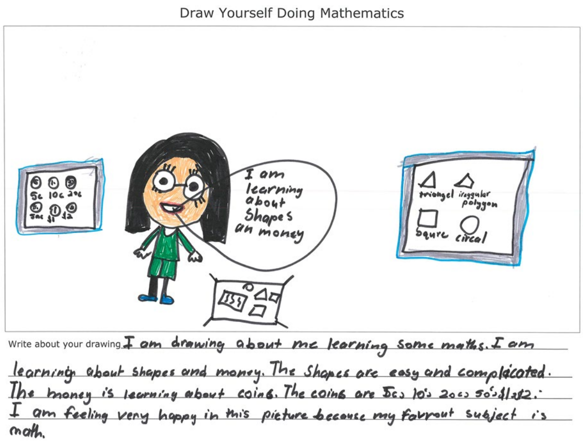
Fig. 6 A18, female, year 2

Fig. 7 B2, male, year 2



manipulative used, which contributed to children's overall emotional sentiment towards mathematics. The majority of children, even children who were classified as having a negative attitude towards mathematics, appeared to exhibit positive attitudes towards manipulatives during enactive learning experiences. The following examples are taken from field notes written of an observation conducted in a year 2 and 3 class 1 at school A during the third observation. The learning experience began with the teacher reading part of Uno's Garden by Graeme Base. This book includes representing the number of items in numerical form as well as addition and multiplication number sentences. The teacher poses a question similar to the questions presented in the book, providing children with tens frames, counters, beads, mini white boards, and markers. A10 used counters and her fingers as manipulatives to solve the question in three different ways and appeared to enjoy the sharing of the book and using manipulatives, using the mathematical representations within the book to develop connections to help her solve the question. The child's language during this task was positive, "look,

Fig. 8 B10, female, year 2



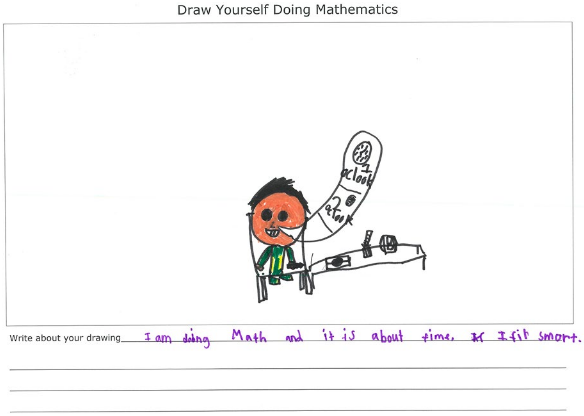


Fig. 9 B17, male, year 2

I've found another way" which contrasts with her previous evaluations of her work which were negative. As such, A10 showed perseverance and interest in what she was doing, and this was a stark contrast to the other observed learning experiences where A10 exuded disdain and negativity.

In contrast, not all children appear to appreciate using manipulatives in the same way as A10, leading to negative emotions and views of manipulatives. For example, C9 found the enactive phase frustrating and prohibitive, and this was seen in both the non-lesson and lesson contexts. During the non-lesson context, C9 drew no iconic representations of manipulatives. However, he did speak extensively about using manipulatives. C9 described his "hate-love-hate" relationship with mathematics, attributing number concepts, particularly multiplication, as the cause of his disdain. C9 provided several examples in a non-lesson context of what he thought was annoying "cause [sic] you need to make like one hundred groups of sixty-five", referring to using counters to represent multiplication as an array.

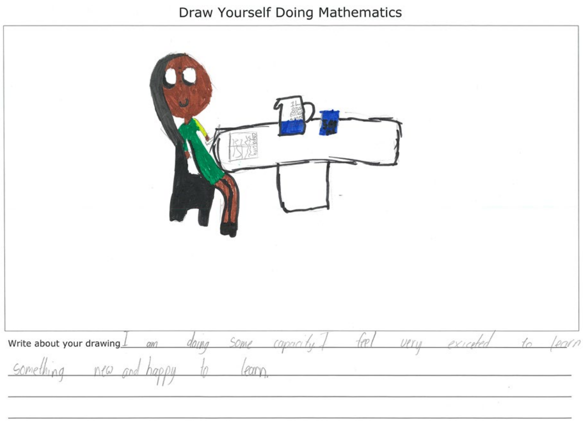
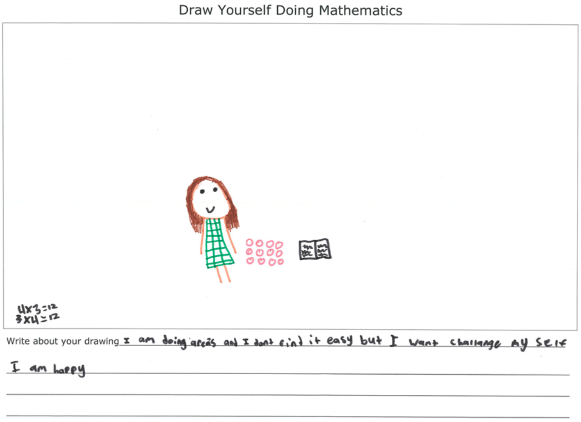


Fig. 10 B57, female, year 3

Fig. 11 C3, female, year 3

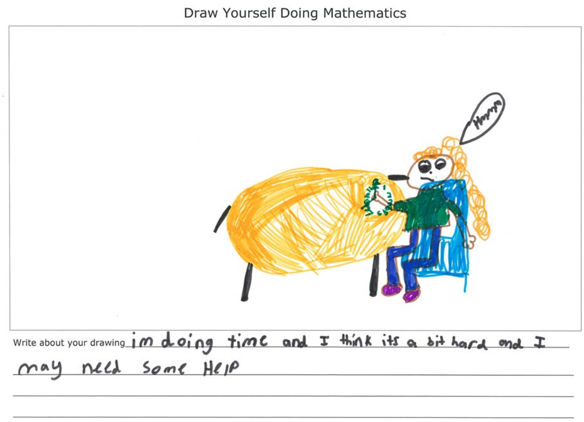


C17 (Fig. 12) indicated a combination of emotions in the non-lesson context, stating that she “sometimes feel a little bit anxious, anxious where I just want to give up” and related these feelings to using “a fake clock and making time on it” to tell the time. C17 elaborates “I’m making times on it and um for me sometimes I think time is a bit hard, so um sometimes I need some help that’s why I wrote this down here, so um sometimes I think time is a bit complicated in maths”. C17 found using the “fake clock… a bit tricky when there’s some in between here” indicating the minute increments and using these increments to tell time to the nearest minute. C17 indicated that using the “fake clock” did not help her but also showed some optimism, stating, “I feel like I need some help, and I need to get my brain thinking more”.

Attitudes towards mathematics and manipulative representation in the iconic stage

Children’s drawings were noteworthy sources to examine the confluence of manipulative use, representation, and attitude formation. The use of iconic imagery in the non-lesson context was documented by children in their written descriptions of

Fig. 12 C17, female, year 3



their drawings. Figures 3–12 show a range of iconic representations of manipulatives representing topics from addition and skip counting, subitising, 2D shapes, capacity, and clocks. A range of emotions was depicted in the drawings, ranging from happiness and enjoyment (A22, B2, and B57) to feeling nervous and scared (A15, A17, and C17). These sources provided opportunities for further inquiry. For example, A22 (Fig. 3) liked to use manipulatives, such as the blocks that she has drawn, to help her find a solution, "well, why I chose to do plus sums is because I really like umm solving them with different things and I especially like using the blocks". A22 was able to represent the manipulatives iconically, seeing value in representing her mathematical thinking in multiple ways, contributing to her vision of mathematics and her perceived competence. This is evidenced in A22's written statement "I feel really proud of myself. I am also stretching my brain".

However, during the lesson context, many children appeared to struggle to create iconic representations of the representations used in the enactive stage. Further, they were yet to understand the notion of a productive struggle (Warshauer, 2014). For example, B8 (year 2, male) was observed during mathematical rotations where he was given the task to find possible ways to make a cube given six squares. When using Polydrons to model a cube, B8 quickly realised that it was not possible to draw multiple 1:1 scale iconic representations of the physical representations that he was creating on an A4 piece of paper. This impediment led to outward signs of frustration, anger, and ultimately not pursuing more than two possible solutions. Similarly, B52 struggled to create iconic representations of familiar objects during a fraction learning experience. While other children in B52's group acted as enablers and shared their strategies on how they substituted the familiar object with a diagram, B52 was adamant that it was not achievable. It appears that transitioning from enactive to iconic caused some children to outwardly exhibit negative emotions resulting in disengaging, especially in tasks with higher levels of cognitive demand.

The transition between enactive and iconic experiences was exacerbated by introducing iconic experiences before children were ready or had developed the necessary conceptual understanding, as seen in the following vignette. C9, C11, and C21 could identify 3D solids when engaging in a learning experience with 3D wooden attribute blocks and use these blocks to identify and describe related objects in their environment. The children used the attribute blocks to determine the number of faces, corners, or edges by rotating the block to assist in identifying features. However, all struggled with visualising iconic representations of 3D solids as a 2D representation in the form of a net. This struggle is contextualised in the discussion.

Attitudes towards mathematics and manipulative representation in the symbolic stage

Children created symbolic representations of manipulative use in both the lesson and non-lesson contexts. As seen in Figs. 3–12, children wrote a range of number sentences to accompany the representation on manipulatives. For example, A15 writing the number sentence 4 + 2 = 6 to match the numbers shown on the dice that formed part of the enactive stage. In doing so, we see the number formation, including number reversal and how he feels about the symbolic stage, where he admits to feeling nervous about getting “mixed up”.

In a non-lesson context, children with positive attitudes depicted more complex number sentences and were able to describe mathematical processes to perform the operations depicted. Children with positive attitudes were more likely to draw iconic representations of manipulatives and showed how they enacted their use in their drawings. For example, B1 (year 2) wrote 636 + 636 and depicted the process of partitioning (“chunking”) to work out the answer. Other children used number lines to show repeated addition of two-digit numbers, while others drew MAB (longs and units) to show the processes they used to add numbers. These children were confident in using multiple representations (an indicator of the TTP) and in their answers to the questions that they depicted (an indicator of their SC). Further, children with a positive or extremely positive attitude classification embraced the notion of creating and developing their own symbolic representations, confidently showing their mathematical thinking and working. Children with neutral, negative, and extremely negative attitude classifications are yet to develop this confidence. Children with a negative and neutral attitude classification struggled in transitioning from physical representations (manipulatives) to visual and symbolic representations.

The lesson context confirmed the results from the non-lesson context, providing further examples of how children used symbolic representations of manipulatives and how creating symbolic representations influenced their attitude towards mathematics. The following vignette is from two children in the same class attempting the same task that requires multiple ways of adding numbers to 12. A22 worked independently on the task, regularly making statements about what she was doing. Her self-talk is audible to others but appears to be directed at no one in particular. The child works on the task, continuing to self-talk when the teacher clarifies the instruction about the task, stating that a number can only be used once. A22 stands up to get an eraser from a different desk and returns to her seat, uttering a mild expletive before erasing some of her work. A22 resumes the task independently and soon resumes the self-talk uttering “9 + 2 + …” and “I’ve got two done”, quickly followed by “I’ve got four questions done”. The child continues to work on the task. In contrast, A18 was reluctant to make a start and appeared to be avoiding and delaying work. A18 pushes her chair backwards, away from the table, physically distancing herself from her work, finding other unrelated reasons for not completing the task. Even with teacher prompting, A18 is hesitant and exhibits signs of discomfort and distress. She stands in the doorway, arms folded on her chest, frowning, and huffing. She moves further into the corridor so that she cannot be seen, occasionally glancing back into the room, remaining there for approximately two minutes. A18 returns to her desk, stating that she has only got one number sentence (3 + 4 + 5) with two children suggesting two solutions. A18 ignores their assistance and begins counting out some pop sticks and proceeds to write a second number sentence in her workbook, opting not to share her solutions with the other children.

Discussion

The integration of manipulatives in mathematics education has long been recognised as a valuable tool for fostering conceptual understanding. However, the impact of manipulatives on young children's attitudes towards mathematics remains an unexplored area. This study addressed this gap by evaluating the influence of manipulatives on the attitudes of years 2 and 3 children's towards mathematics through an analysis of children's drawings, along with their accompanying written and verbal descriptions and observations of mathematical learning experiences. Of the 106 children participating in this study, 41 children (39%) explicitly discussed the use of manipulatives. While this is a significant proportion of children associating using manipulatives with "doing mathematics", it is not until we evaluate the use of manipulatives that we understand the nuance and complex association between using manipulatives and children's attitudes towards mathematics. Drawing upon the modified three-dimensional model of attitude (MTMA) and Bruner's experiential stages, the study examined the confluence between manipulatives and students' attitudes, highlighting both positive and potentially negative impacts. In acknowledging these impacts, it is important to recognise that the use of manipulatives was not done in isolation; as previously discussed, enactive manipulative use was accompanied by symbolic and iconic representations.

Evaluating the positive impacts, the enactive phase of manipulatives made the intended learning accessible and enjoyable, thereby fostering positive attitudes towards mathematics. Moch (2002) found similar results, reporting that children who were previously reluctant were more eager and enthusiastic. The eagerness of the children in this study manifested in many ways, where we see children such as A10 and A22 wanting to complete tasks that involve the use of manipulatives whereas previously, they were reluctant to engage.

However, the use of manipulatives by children did not always yield a positive impact. Some children appeared reluctant to use manipulatives to work through cognitively demanding tasks, even after the teacher prompted the use of specific manipulatives. It seemed that, for these children, the use of manipulatives was viewed as a last resort and not a useful mathematical tool. For a minority of children who described discomfort with mathematics, manipulatives were a tool that was used to actively or passively disengage from mathematical learning experiences. McNeil and Jarvin (2007) found similar results, reporting that manipulatives, in some cases, can be detrimental to learning.

The findings regarding the impact of manipulatives on students' attitudes towards mathematics hold significant implications for both teachers and students. The positive effects observed during the enactive phase underscore the importance of incorporating manipulatives into mathematical instruction, not only from a conceptual stance but also from an affective perspective. This suggests that educators can leverage manipulatives effectively to cultivate positive attitudes towards mathematics, as evidenced by the increased eagerness and enthusiasm displayed by previously reluctant students. However, the study also highlights a crucial challenge: Not all students respond positively to manipulatives, with some demonstrating reluctance or discomfort, especially when faced with cognitively demanding tasks. For teachers, this underscores the importance of understanding individual student preferences and readiness levels when integrating manipulatives into instruction. Additionally, educators should be mindful of the potential pitfalls associated with the overreliance on manipulatives, as some students may perceive them as a crutch rather than a valuable mathematical tool. Thus, a balanced approach that considers both the benefits and limitations of manipulatives is essential for promoting positive attitudes towards mathematics among all students.

While the findings were reported using the three distinct phases of Bruner's experiential stages of learning, an interesting finding emerged regarding the transitioning between the iconic and symbolic stages with several factors identified that contributed to this variation. The transition from physical to internal representations (Goldin & Shteingold, 2001) via the enactive, iconic, and symbolic stages (Bruner, 1966) influenced children's attitudes. The transition between representations is a vital development in the learning and acquisition of mathematics, especially as one of the goals of mathematics education is for "children to create and think critically about mathematics" (Perry & Atkins, 2002, p. 201). Children need time to develop confidence in using physical representations before introducing "conventional notation" (Perry & Atkins, 2002, p. 201). Further, children need the connection between the enactive, iconic, and symbolic representations or informal and formal representations to be made explicit (McNeil & Jarvin, 2007). Conversely, spending too long on a particular representation or method can result in frustration and boredom in children, causing negative or neutral attitudes towards mathematics. The transition between enactive and iconic was further inhibited by introducing the iconic representation of 3D solids before it was formally introduced, which is currently located in the Year 5 Australian Curriculum: Mathematics. Consequently, introducing iconic representations too early was shown to cause confusion and frustration. The children's emotional tendency was not the only dimension of attitude impacted. Children's mathematical mindset and self-concept were also negatively impacted.

Limitations

It is important to acknowledge the methodological limitations of this research. Limitations were noted around the type of observations and the analysis of children's emotional tendencies when observing mathematical learning experiences. The observations focused on overt behaviours that were observable and did not attend to small actions or reactions. It is possible that even the smallest of actions or reactions can be an indicator of how manipulative use influences children's attitudes. Further, the observations did not provide opportunities for the researcher to question children regarding the enactment of their attitudes during mathematical learning experiences.

This research was delimited to Australian primary students attending South Australian State schools. This delimitation identifies a potential threat to the external validity regarding the "lack of representativeness of available and target populations" (Cohen et al., 2018, p. 186). Logistically, a delimitation was imposed on the number of children observed per class. The number of participants varied between classes and were purposively selected based on their non-lesson attitude classification. Purposive sampling was used to "enable comparisons" between the two contexts and to "achieve representativeness" (Cohen et al., 2018, p. 156). As a result of observing three children per class, several children went unobserved, and their attitude towards mathematics in a lesson context was not analysed. Observing a larger range of children in different contexts is recommended. Further research is recommended to identify the relationship between disengagement and manipulatives, as it is possible that students are unfamiliar with the manipulative and do not know how to use it to support their conceptual development.

Conclusion

The Modified Three-dimensional of Attitude was a flexible framework that moved beyond the identification and classification of children's attitudes to analyse factors that influenced attitudes. The children in this study used many types of representations, with manipulatives emerging as a predominant representation. Investigating how students use and create with manipulatives went beyond gaining insights into their mathematical thinking. Rather, insights were gained about how children viewed manipulatives, how children felt about them and their confidence in using these materials. Several themes emerged when investigating the confluence of attitudes towards mathematics and the use of manipulatives. When used effectively and timely, manipulatives provide a solid basis for developing flexible external and internal representations of mathematics and contribute to the formation of positive attitudes towards mathematics. However, other variables influenced how children viewed and used manipulatives. Disengagement was documented in several cases, where children used manipulatives as a façade for doing mathematics and were a means to actively and passively disengage. A third theme related to the transition between enactive, iconic, and symbolic representations was noted, as well as how this can contribute to negative attitudes towards mathematics. These contextual variables and the cues and signs children provide need to be considered when planning mathematical learning experiences.

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Data availability The data that support the findings of this study are available on request from the corresponding author, KQ. The data are not publicly available due to restrictions e.g. their containing information that could compromise the privacy of research participants.

Declarations

Ethical approval The research has been approved by the Government of South Australia: Department of Education and Child Development (reference number 2008–0004) and the University of South Australia Human Ethics Research Committee (protocol number 200051).

Informed consent Informed consent was obtained from all participants and was required as part of the ethical approval process. Informed consent involved school, teacher, parental, and participant consent to participate and to have their data published. The authors affirm that human research participants provided informed consent for publication of the images in Table 2.

Human and animal rights The research has been approved by the Government of South Australia: Department of Education and Child Development (reference number 2008–0004) and the University of South Australia Human Ethics Research Committee (protocol number 200051).

Conflict of interest The author declares no competing interests.

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