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ORIGINAL ARTICLE

Reframing anxiety and uncertainty in the mathematics classroom

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Abstract
In this article, educational and psychological perspectives are used to examine how anxiety and uncertainty in the mathematics classroom can be reframed to benefit mathematical teaching and learning. Links between anxiety and uncertainty are discussed and from this discussion, two methods are proposed for reducing the negative impact of mathematics anxiety on learning. Firstly, it is argued that initiatives designed to teach students to use emotion regulation skills can help to reframe anxiety and improve students' ability to regulate negative emotion. In particular, the skill of reappraisal is discussed. Secondly, it is proposed that pedagogical approaches that encourage students to embrace challenge will help to normalise uncertainty in classrooms and promote more positive affect and engagement with mathematics. These approaches include choosing suitable tasks, differentiating those tasks, orchestrating classroom discussions that include all students and consolidating the learning with further tasks suitably varied. Rather than teachers seeking to remove anxiety, students can be encouraged to manage their anxiety in order to promote more positive mathematical learning and to develop mathematical resilience.

Keywords Mathematics anxiety · Mathematics instruction · Mathematics pedagogy · Psychological factors · Emotion regulation · Challenging tasks

Introduction

Across the educational and psychological research literature, anxiety experienced during mathematical learning has been studied and reported more than in other subject areas. Many researchers have demonstrated relationships between mathematics

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anxiety and mathematics avoidance, disengagement, negative attitudes and poorer mathematical learning (Dowker et al., 2016; Eden et al., 2013; Ma, 1999). Further to this, mathematics anxiety is well-researched because of the propensity for the subject area to evoke negative emotion. In this article, educational and psychological perspectives are combined to emphasise the link between anxiety and uncertainty, and how this link relates to the experience of anxiety and to the culture of mathematics in the classroom. Combining educational and psychological theory and using a multidisciplinary perspective to consider mathematics anxiety allows for a more in-depth study of this significant phenomena in mathematics classrooms. Furthermore, networking of theories and concepts has become a constructive and valuable way to integrate different theoretical perspectives in mathematics education research (Scheiner, 2020). Thus, drawing on educational and psychological theories and conceptualisations, this article proposes two methods for reducing the negative impact of mathematics anxiety. Firstly, it is argued that initiatives designed to teach students to use emotion regulation skills can help to reframe anxiety and improve students' ability to regulate negative emotion. Secondly, it is argued that pedagogical approaches that encourage students to embrace challenge can help to normalise uncertainty in classrooms and promote more positive affect and engagement with mathematics. Both the teaching of emotional regulation skills and the use of these specific pedagogical approaches are useful for all levels of mathematics teaching.

Theoretical considerations: anxiety, uncertainty and emotion regulation

Mathematics anxiety and the importance of mathematical beliefs

Mathematics anxiety has been conceptualised in different ways. In the research literature, it is described as a negative affective reaction or negative attitudinal factor that is elicited in some individuals when they are required to complete mathematics tasks (Ashcraft & Moore, 2009; Baloglu & Kocak, 2006). More specifically, mathematics anxiety is considered to be a negative emotional response characterised by worry and nervousness that is associated with participating in mathematics or thinking about mathematics (Richardson & Suinn, 1972). Buckley et al. (2016) differentiate between state and trait mathematics anxiety. State mathematics anxiety is transient and is the type of anxiety felt on-task such as when an individual is participating in mathematics, whereas trait mathematics anxiety is a more persistent characteristic connected to the level of fear and threat that an individual associates with mathematics. Buckley et al. (2016, 2020) suggest that one way of conceptualising differences between the two types is to think of state mathematics anxiety as anxiety's symptoms and trait mathematics anxiety as linked to causal factors and more attitudinal in nature.

The benefit of differentiating between state and trait mathematics anxiety is that each has a different impact on mathematical learning. For state mathematics anxiety, the impact occurs on-task during mathematical learning and performance. Various researchers have reported that the physiological symptoms of state mathematics anxiety (e.g. increased heart rate and breathing rate) as well as cognitive symptoms (e.g. intrusive thoughts or worries) can negatively impair performance and learning (Ashcraft & Kirk, 2001; Hembree, 1990; Ho et al., 2000; Wigfield & Meece, 1988). Thoughts such as 'I am hopeless at maths' consume working memory capacity which would otherwise be used to perform mathematical tasks or think mathematically (Ashcraft & Kirk, 2001; Ramirez et al., 2018).

Trait mathematics anxiety is more stable and enduring and has an impact through its effects on learning-related choices (Buckley et al., 2020). Students with higher levels of trait mathematics anxiety are less likely to choose to study mathematics or pursue careers that involve mathematics (e.g. see Daker et al., 2021). Trait mathematics anxiety is also tied more closely to the causes of mathematics anxiety. At a psychological level, these are persistent negative beliefs and patterns of thinking that revolve around the idea that mathematics ability is predetermined and unchanging, for example, 'I inherited my maths ability from my parents'; 'I'm not a maths person' (Buckley et al., 2016). These beliefs have various labels in educational and psychological research literature. For instance, some call this a fixed mindset in mathematics, which refers to students' thinking mathematics ability is unable to be changed, while others say they represent beliefs of an external locus of control for mathematics or thinking that mathematics ability is related to factors external to the individual/factors beyond their control (Claro et al., 2016; Wentzel & Wigfield, 1998). Whatever the label, these ideas feed directly into the sense of mathematics anxiety and are shaped by experiences of struggling with mathematics, through interactions with family and peers, and via teachers' attitudes and pedagogy (Buckley et al., 2020; Maloney et al., 2013).

In addition to contributing to mathematics anxiety, negative beliefs about mathematical ability and potential also influence self-regulatory strategies used to manage learning, success and failure (Garcia & Pintrick, 1994). Attribution theory describes the way in which outcomes are attributed to particular causes (Weiner, 2010) and has been used to explain how beliefs and causal inferences about mathematics ability can be linked to negative regulatory behaviours in mathematical learning. For instance, learned helplessness is regarded as the result of continually attributing failure to a belief of inherent low ability, which then results in a perception that success is impossible and therefore persistence or effort is redundant (Middleton & Spanias, 1999). However, just as attribution theory can be used to explain negative behaviour in relation to mathematical learning, it can also be used to understand how adapting an individual's attributional style can lead to positive behavioural change. This is demonstrated by research supporting the positive impact of a growth mindset on mathematical learning and the popularity of mindset theory in the mathematics education community (Claro et al., 2016; Dweck, 1975; Yeager et al., 2016).

Mathematics anxiety, challenge and the 'problem' with uncertainty

Initiatives that address mathematics anxiety can target its causes by understanding, identifying and changing negative beliefs and patterns of thinking about mathematics ability and potential (Buckley et al., 2016). An obstacle to this process is an often reported culture of dislike among students for mathematical challenge (e.g. Sullivan et al., 2013). In the mathematics classroom, some teachers confront negativity about mathematics by encouraging students to embrace mathematical challenges and view effort and practice as fundamental to mathematical growth. This type of thinking directly addresses negative beliefs that underlie trait mathematics anxiety. It is also useful for helping indirectly to reduce state mathematics anxiety. For example, if students alter their beliefs to see practice as part of mathematical learning, they are less likely to feel anxious that the practise might result in errors and may appreciate that making errors is part of mathematical growth. However, time and experiential learning are required for belief structures to fully change, particularly when those beliefs are enduring and tied to family, peer and societal attitudes (Buckley et al., 2020; McLeod, 1992). The situations required to test out these new beliefs can involve mathematical challenge and dislike or fear of challenge in the mathematics classroom can lead to the experience of state mathematics anxiety. The physiological and cognitive symptoms of state mathematics anxiety can then reinforce old patterns of thinking and negativity and obstruct pathways to more positive beliefs.

We propose that underlying issues surrounding challenge in mathematics and its propensity to trigger symptoms of anxiety in students is a lowered tolerance or threshold for uncertainty. Uncertainty is considered to be 'the doubt that exists about whether or not a particular outcome will occur' (Rosen et al., 2014, p. 55). In the mathematics classroom, we take uncertainty to refer to the experience of students meeting questions or tasks which are unfamiliar and/or in which they are unsure of the solution type or solution method irrespective of whether they have received instruction on the task or subject previously. Two key components of our proposition for a lowered tolerance for uncertainty in mathematics classrooms are (1) uncertainty is linked to the experience of anxiety, and (2) intolerance of uncertainty seems common among some mathematics learners.

To elaborate on these points, we draw on Pekrun's (2006) control-value theory of achievement emotions and the theory's conceptualisation of anxiety. According to Pekrun and the control-value theory, underlying the experience of achievement emotions, or emotions felt in achievement settings, are perceptions of control and value. Anxiety is the emotion experienced when perceptions of low control combine with appraisals of value; that is, when students value mathematics in some way but feel they do not have control over their mathematical performance, they are likely to feel mathematics anxiety. On the other hand, perceptions of low control combined with appraisals of low value are likely to result in the experience of boredom (Pekrun et al., 2010). The control-value theory also conceptualises anxiety in achievement settings as the emotion experienced when uncertainty is associated with a learning situation, process or result and the focus of thought is on the anticipation and expectation of failure (Pekrun, 2006). Alternatively, the theory posits that uncertainty associated with a focus on the anticipation of success is related to the experience of hope.

Theoretical conceptualisations of uncertainty in psychology propose that uncertainty intolerance is a key characteristic of anxiety and the component that leads to the experience of worry (Miceli & Castelfranchi, 2005), one of the defining features of state anxiety. While research has linked mathematics anxiety with intolerance of uncertainty (Williams, 2013), our proposition goes further to link these two phenomena with the culture of mathematics in schools. If the experience of anxiety is linked to perceptions of uncertainty, and mathematics anxiety is commonly reported by Australian adolescent students (e.g. see Thomson et al., 2013), then this suggests that secondary students experience a high degree of uncertainty in the mathematics classroom and react negatively to this uncertainty. However, uncertainty is not inherent to the study of mathematics and is present in other subject areas. This leads us to suggest that one aspect of the current culture of mathematics in many schools, particularly secondary schools, is a lowered threshold among students for tolerating uncertainty.

These arguments suggest that teaching students to be more open to uncertainty in the mathematics classroom could be beneficial to addressing the causes of mathematics anxiety. However, as stated before, being confronted with uncertainty means facing mathematical challenge, which commonly elicits symptoms of mathematics anxiety that in turn can negatively impact on learning. For students to succeed, they would ideally be taught how to regulate the negative emotion elicited when exposed to challenges. The challenges must also be presented and scaffolded by teachers in such a way that learning from mistakes is normalised and there are opportunities for students to experience mathematics in a way that helps to alter negative thinking and beliefs (e.g. rather than perceiving mistakes as an indicator of perpetual failure, they can be understood as productive in that they highlight pathways for improvement) (Boaler et al., 2018). In the next sections, we suggest two ways to reframe mathematics anxiety and uncertainty in order to benefit mathematical learning: firstly, through the use of emotion regulation; and secondly, through teacher actions in the classroom.

Reframing mathematics anxiety by improving emotion regulation skills

Pedagogical approaches that involve challenging negative beliefs and thinking about mathematics may not address both state and trait mathematics anxiety without the help of approaches that target anxiety directly. This is where the use of emotion regulation skills becomes important. Emotion regulation occurs when an individual modifies the impact of an emotion on their mental state by changing the type of emotion they are experiencing, its intensity or how it is expressed (Gross, 2002). A number of mathematics anxiety researchers have emphasised the importance of direct or psychological approaches that revolve around emotion regulation (Beilock et al., 2017; Buckley et al., 2016, 2020). In particular, these researchers have highlighted the effectiveness of reappraisal as an emotion regulation skill (Ramirez et al., 2018). Reappraising anxiety involves reframing the emotion so that it no longer has a negative impact. Research suggests that reappraisals may be more closely related to the experience of anxiety than attributions (Smith et al., 1993) and that reappraising or reframing mathematics anxiety can reduce and regulate its effects on performance and learning. For instance, studies have shown that students taught to reframe the symptoms of their mathematics anxiety as excitement — recognising that the physiological symptoms of anxiety and excitement are the same (e.g. increased heart rate and breathing rate) — perform better on a mathematics task than those instructed to remain calm (Brooks, 2014).

Another way of reappraising or reframing anxiety relates to acknowledging the positive qualities of the emotion (Jamieson et al., 2013). For instance, Jamieson et al. (2016) found that university students who were taught about the adaptive aspects of anxiety as an emotion that can improve performance showed reduced mathematics anxiety in subsequent testing situations and better performance than students who were instructed to ignore their anxiety. A third area of research to reaffirm the importance of reappraisal in the management of mathematics anxiety is neuroscience. In a study of university students, highly anxious students who showed greater activation in a region of the brain associated with emotion reappraisal before completing a mathematics task performed better than students who were also anxious but did not show this pattern of brain activation (Lyons & Beilock, 2012). These findings along with others reviewed in relation to reappraisal suggest that feeling anxious about mathematics is not the problem; the issue is how the emotion is managed once it is experienced.

Reappraising or reframing anxiety is one emotion regulation skill designed to increase a student's ability to regulate negative emotion and the impact that this emotion has on their behaviour and learning. Part of this approach is for teachers and students to recognise that regulating mathematics anxiety is not about eliminating the emotion from learning but reducing it to a point where it is not harmful to performance. In this section, we have detailed one emotion regulation skill or strategy that can be used to address the symptoms of mathematics anxiety. Other reviews describe the application of other strategies (e.g. Buckley et al, 2020; Park et al., 2014; Ramirez et al., 2018). In the classroom, teaching students to use emotion regulation skills to help regulate and reduce negative emotion is important as it has the potential to target anxiety directly by assisting students to understand and manage the anxiety they experience in the moment. This approach in combination with pedagogical techniques that teach students to embrace challenge is likely to help normalise uncertainty and reduce mathematics anxiety.

Reframing uncertainty in the mathematics classroom through teacher actions

There are several implications of the conceptualisation of anxiety and uncertainty presented in this article. If students approach mathematics in ways that embrace uncertainty and see success as a process (rather than a product), then it can be anticipated that they will be more likely to engage with learning opportunities and less likely to experience state mathematics anxiety. If, on the other hand, the inherent uncertainty in learning, especially learning mathematics, prompts fear of failure and a tendency to avoid effort (Elliot, 1999), opportunities may be missed in conditions that are also likely to foster trait mathematics anxiety.

There are two specific actions proposed for teachers that can have the effect of reducing anxiety around specific tasks and processes for learning mathematics as well as addressing deep-seated fears that some students experience consistently. The first relates to the nature of mathematics and how directed approaches to teaching mathematics emphasising micro skills, especially prevalent in secondary schools, can create anxiety. The second refers to ways that sequences of productively challenging tasks, appropriately differentiated, can normalise uncertainty. This is elaborated in the following section of the article. Some evidence is also presented that can give teachers confidence that these recommended approaches are effective.

Ways that the nature of mathematics may contribute to anxiety

It is possible there is a direct link between ways students experience uncertainty and the nature of the mathematics taught in some schools. Some approaches to teaching mathematics intend to minimise threat and risk and seek to reduce anxiety by avoiding challenging student thinking (e.g. see Clarke et al., 2014; Sullivan et al., 2013). This can involve teachers giving explanations of tasks to be performed after which graduated practice exercises are set. Such approaches are described in the High Impact Teaching Strategies (Department of Education and Training, 2019) as *Explicit Teaching* and also *Worked Examples* and are often promoted by systems and school leaders.

There are two compelling arguments that this approach has the opposite of the intended effect. The first is the mathematics to be learned by school students is a complex web of interconnected ideas that occur in varied contexts at unexpected times and in unanticipated ways. Consider the learning of fractions, decimals and percentages, which form the core of curriculum for early adolescents, the period at which anxiety is most evident (Meece & Wigfield, 1988) and arguably most debilitating. Even though these three forms are different ways of describing the same quantities and operations, they have unique representations, the processes for comparing and ordering fractions and decimals are far from obvious, and calculations involving each of them have their own idiosyncratic methods. These interconnections *cannot* optimally be learned by rules and rhymes. It is only by engaging with the nature of fractions, decimals and percentages that students can make meaning, the absence of which increases a sense of uncertainty. Since students naturally and intuitively search for meaning, in its absence, they can make inappropriate generalisations that lead to misconceptions. The experience of eventual exposure of misconceptions further contributes to anxiety.

The second argument relates to the grain size of the mathematics. If teachers see mathematics as sets of micro skills with efficiency the goal rather than understanding, they would be likely to present a single method for performing a particular procedure after which they pose questions arranged from simple to complex. This is a prevalent approach in secondary mathematics classes (Sullivan et al., 2014). The experience of many students would then be moving from knowing how to do simple questions to not knowing how to solve more complex ones. This is the exact opposite of learning and would no doubt create anxiety increasing lesson by lesson. A classroom climate that focuses on a single method for finding the answer is also likely to foster a culture of certainty or 'black-and-white' thinking and push students to see uncertainty as less acceptable.

Somewhat counter intuitively, uncertainty can be normalised if students frequently and consistently experience mathematics learning experiences and problems:

- For which the solutions and solution pathways are not immediately obvious;
- That emphasise connections;
- In which students have an active role in creating solutions and solution pathways; and
- Which can be readily differentiated based on student responses.

Each of these aspects represents a change from conventional approaches to teaching mathematics seen in some schools.

Ways of structuring mathematics teaching to reframe uncertainty and reduce anxiety

The second type of action teachers can take to reframe uncertainty relates to tasks they pose and associated pedagogies. As argued above, posing simple one-step tasks is not an effective way to minimise anxiety and is counterproductive in the longer term. This is partly due to the interconnectedness of mathematics ideas and the importance of building connections to foster understanding and make meaning. It is argued that students benefit when tasks are productively challenging supported by compatible pedagogies and effective sequencing. Christiansen and Walther (1986), for example, argued that non-routine tasks, because of the interplay between different aspects of learning, provide optimal conditions for cognitive development in which new knowledge is constructed relationally and items of earlier knowledge are recognised and evaluated. Lodge et al. (2018) proposed that:

> … difficulties and confusion are important in the process of learning, particularly when students are developing more sophisticated understandings of complex concepts. Work on desirable difficulties, impasse driven learning, productive failure, and pure discovery-based learning all provide clues as to how confusion could be beneficial for learning (p.8).

Examples of two non-routine tasks that are productively challenging are as follows:

> The perimeter of a rectangle is 10 cm. Neither the length nor the width are whole numbers and both are longer than 2 cm. What are some possibilities for the length and the width?
> The perimeter of a rectangle is 50 cm. Neither the length nor the width are whole numbers but both are longer than 11 cm. What are some possibilities for the length and the width?

These tasks would be suitable for upper primary classes but may also be useful for junior secondary classes. There is challenge in that students must explore what is being asked for, it connects two domains of mathematics (perimeter and additive thinking with decimals), and they are open-ended in that there is a range of possible answers and students can choose their own responses and ways of communicating those responses. Within this approach to teaching, teachers would prepare an alternative task for students experiencing difficulty and also further tasks for those who finish quickly. The intent is that students engage in discussion of the first task and apply their new learning to the second task (see Sullivan et al., 2020, for a discussion of this). It is relevant here to note the connection between these tasks and the proficiencies of problem solving (meaning tasks that students do not already know how to do) and reasoning (meaning articulating and justifying thinking) in the Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority (ACARA, 2009). It is also important to note that these types of tasks can be used across the different stages of schooling. For instance, Russo and colleagues (Russo & Hopkins, 2019; Russo et al., 2019) found that early primary years' students engage willingly with challenging tasks even though teachers can sometimes be hesitant to use them with students in this age group.

A key step for teachers is sourcing tasks that allow productive challenge. Tasks that have only one solution or solution method are unlikely to prompt reflecting, thinking, learning from mistakes and so on. Tasks which are open-ended (Sullivan & Clarke, 1991), meaning those having more than one possible solution, are ideal for encouraging student engagement. Likewise, tasks which are open-middled, meaning those with more than one solution pathway, can also be more accessible for students. In both types of tasks, anxiety is reduced and uncertainty normalised because students make active choices thereby increasing their sense of control (Middleton, 1995).

Connected to this is the locus of thinking. If teachers insist that students solve problems as directed, anxiety can be the result of failing to reproduce taught methods. If, on the other hand, students use what they know to work on the task, the risk of failure is lessened. In this instance, rather than anxiety, students may experience feelings of hope. Of course, students must also identify aspects of the mathematics associated with the task that they do not know. The challenge is for them to find ways to come to know those unknowns. This connects to ways problems are sequenced. Another advantage of specifically planned sequences is that after struggling with a problem one day, it is possible that students will develop insights at a later stage, when their anxiety may also have diminished, and these insights can then be applied.

If students come to see learning as a journey where engagement with the present task can enhance chances of success with the next, the tolerance for uncertainty is increased. The argument is that if teachers plan sequences of tasks that are constructively varied, students can have confidence that the particular challenges they are experiencing currently are not the only opportunities to find solutions and come to understand the concepts.

An associated teacher action relates to ways that learning opportunities are differentiated. If, for example, students experience tasks which are readily accessible for all (sometimes described as 'low floor') but with potential for extension for others (categorised as 'high ceiling'), success is defined in new ways (Boaler et al., 2021). Likewise, if teachers create enabling prompts for students experiencing difficulty and extending prompts for those who are ready for more challenges (Sullivan et al., 2009), students can come to see learning mathematics not as right/wrong but a journey. It is stressed that students being given such prompts are *not* doing different work on different concepts. Indeed, such prompts are explicitly intended to avoid giving some students experiences that are different from the majority. The intention is that students requiring support are scaffolded into the mainstream tasks and those that finish quickly are encouraged to generalise or abstract the learning.

To illustrate these prompts, the following is an example of an enabling prompt that can be offered to students experiencing difficulty with the first of the task examples presented above:

> The perimeter of a rectangle is 10 cm. What are some possibilities for the length and the width?

This prompt removes the demand to use decimals while still engaging the student in thinking about perimeter. An extending prompt for students who finish that task quickly might be:

> Write a sentence that describes all the possible answers. What is the pattern?

This prompt is intended to encourage students to generalise their solutions.

Evidence of the effectiveness of these recommended approaches

In this article, we propose that changes to tasks posed and pedagogies used can normalise uncertainty and reduce anxiety, arguably improving access to and engagement with mathematics for students who are anxious. Of course, it is important that such approaches improve achievement for all students, even those who do not experience anxiety. Research suggests that the type of structured-inquiry approach proposed in this article to help reduce anxiety and increase students' tolerance to uncertainty in the mathematics classroom is also important for improving all students' mathematical learning in general. In a meta-analysis of 164 studies, Alfieri et al. (2011) distinguished between structured inquiry-based approaches and unstructured inquiry-based approaches. They found that unstructured inquiry was inferior to more explicit instructional approaches in terms of its impact on assessed student learning, whereas structured inquiry was superior to all other instructional approaches. They argued that 'participation in guided discovery is better for learners than being provided with an explanation or explicitly taught how to succeed on a task' (p. 11).

Some insights into the teacher actions proposed in this article can also be gained from the 2012 Programme in International Student Assessment (PISA) and its measurement of cognitive activation in relation to mathematics instruction (OECD, 2013). Cognitive activation was assessed by students' reports of how frequently their teachers exhibited certain behaviours or presented mathematics in specific ways that align with those proposed in this article as teacher actions that could help reframe uncertainty and reduce anxiety. In particular, these techniques represent pedagogies that prioritise student thinking and decision-making. For instance, the PISA 2012 measurement of cognitive activation asked students to report how frequently their teacher asked them to spend time considering a problem, to explain their thinking, to learn from their mistakes and to transfer their learning to different problems and how often they were presented with problems where the processes for solving the problem were not instantly apparent (OECD, 2013). Caro et al. (2015) analysed the 2012 PISA data to explore the relationships between mathematics performance and instructional approaches, while also taking into account socioeconomic context. Using data from students from 62 countries that participated in PISA, including Australia, their analysis showed that regardless of socioeconomic context, students who reported that their teachers used cognitively activating strategies in mathematics lessons more often were more likely to perform better on the PISA mathematics assessment. Students from higher socioeconomic contexts, however, benefitted more from cognitive activation strategies and less from teacher-directed instruction than students from lower socioeconomic contexts. Caro et al. noted, particularly for lower socioeconomic students, that.

> an integrated approach of teacher directed instruction and cognitively activating activities may be promoted which meets students' prior abilities but also challenges them sufficiently in order to develop new meta-cognitive skills such as self-directed learning and promote engagement and motivation. (p.18)

In other words, they argued that optimal mathematics pedagogies may be different for different cohorts of students but that cognitive activation strategies are important whatever the background of the students. Further analyses and study of the links between mathematics performance and learning, the use of cognitive activation strategies and the prevalence of mathematics anxiety for Australian students would lend support to the arguments made in this article.

Conclusion

In this article, we described the links between anxiety and uncertainty in mathematical learning and how this relates to the culture of mathematical classrooms and disengagement with challenge. Using educational and psychological perspectives, we proposed that improving Australian students' engagement with challenging mathematics tasks requires improving their tolerance of uncertainty in the mathematics classroom. We presented two approaches for reframing uncertainty and anxiety in mathematical learning: (1) improving students' emotion regulation skills using skills like reappraisal so that students are able to regulate any negative emotion elicited when they are exposed to challenges, and (2) using instructional techniques that normalise challenge to reframe uncertainty in the classroom. The latter approach is connected to the role adopted by teachers, the nature of the pedagogies used, the conceptualising of the mathematics and students being allowed time to think, reflect and learn.

This article drew on educational and psychological theory and constructs in our description of the relationships between anxiety, uncertainty, challenge and mathematical learning. Other theoretical perspectives, such as socio-cultural, epistemic and cultural-historical perspectives, could also add to an understanding of the relationships discussed in the article. However, by drawing on educational and psychological approaches and research evidence, our key argument is that rather than teachers seeking to remove or rename anxiety, students can be encouraged to manage anxiety and their reactions to uncertainty in the mathematics classroom promoting more positive mathematical learning and mathematical resilience.

Author contribution Both authors were involved in all stages and aspects of the article. The first author took the lead on the literature review and the discussion and interpretation of mathematics anxiety and emotion regulation, and the second author of summary and explanation of aspects related to mathematics and its pedagogy.

Availability of data and material Not applicable.

Code availability Not applicable.

Declarations

Conflict of interest The authors declare no competing interests.

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