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Mathematics teacher educators' knowledge sources in teacher education practices

Mehmet Fatih Ozmantar^{1,1} · Gulay Agac¹

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Abstract

This study aims to document and examine mathematics teacher educators' (MTEs) knowledge sources in teacher education practices. The study adopts a mixedmethod, embedded research design. Data were collected via a questionnaire with elicitations about MTEs' descriptive features as well as with open-ended items. Target population was all MTEs working in mathematics education departments across Turkish universities. The questionnaire was emailed to all MTEs, and 281 out of 522 returned with their answers. MTEs' written statements were examined via inductive thematic analysis. A post hoc quantitative analysis was also conducted to explore the relationships between knowledge sources and four variables: gender, self-identification, teaching experience and academic ranking. The analysis allowed the determination of a variety of knowledge sources in teacher education practices and established certain relationships. The findings are discussed with regard to MTEs' professional development, conceptions of knowing and learning and the contribution of different sources to the teacher education practices.

Keywords Knowledge and practice \cdot Knowledge sources \cdot Mathematics teacher educators \cdot Teacher education

Introduction: literature and the rationale of the study

In recent years, there has been a growing research attention on teacher educators (TEs) due to their apparent influence on the preparation of qualified teachers (Cochran-Smith & Zeichner, 2005). Teacher educators are generally defined as 'teachers of teachers' (Lanier & Little, 1986, p.528) with responsibilities for teacher qualifications (Howey & Zimpher, 1990). Teacher educators, broadly speaking, refer to induction tutors and mentors who work in schools with a responsibility to support on-the-job training and development including the

Mehmet Fatih Ozmantar ozmantar@gantep.edu.tr

¹ Department of Mathematics and Science Education, University of Gaziantep, Gaziantep, Turkey

practicum process as well as to academics who work in a higher education institution with a responsibility for teacher education and continuous professional development, research or subject studies and didactics (Dengerink et al., 2015). There has long been a serious research attention paid to mentoring and practicum process (Lawson et al., 2015 for a systematic review). However, until recently, studies on academics or university-based TEs in general and mathematics teacher educators (MTEs) in particular have remained limited (Superfine & Li, 2014). Since our research-based knowledge of MTEs is limited, this group is not wellknown to the research community (see Goodwin et al., 2014). Hence, this study focuses on MTEs actively involved in mathematics teacher education programs at higher education institutions.

The group of MTEs in higher education institutions is usually composed of both mathematicians and mathematics educators (NCTM, 2020). Hence, preservice mathematics teachers are prepared to the profession by MTEs with different, if not contradictory, approaches to mathematics and its teaching (Goldin, 2003). Bleiler (2015), noting such differences, states the importance of collaborations between these two groups in preparing preservice teachers. The research provides evidence that such collaborations have positive impact on the quality of MTEs' professional practices for teacher preparation. In a large-scale project, Goos and Bennison (2018), for example, have provided opportunities for mathematicians and mathematics teachers. The researchers reported that collaboration of this kind was 'generative of new practices—and, therefore, new learning' for MTEs and ultimately enhanced the quality of preservice teacher education program.

The work of TEs (Ellis et al., 2011), nature of their responsibilities (White et al., 2020) and relevant skills (Goodwin et al, 2014) were the focus of several research undertakings. The international research on 'Work of Teacher Educators' led by viv Ellis (but also occurred in Australia and New Zealand—see, Brennan & Zipin, 2016) provides important insights. In their research, Ellis and his colleagues examined the practical activities and material conditions of TEs in higher-education institutions. They worked with 13 TEs in England and Scotland. TEs in this study reported to spend much time on teaching related duties including course management, marking and relationship management with partner schools and individual preservice teachers. Research-related activities were also found to be a characteristic feature of TEs' work. However, contrary to common expectations, only six participants reported to have been involved in research activities. Interesting also is their finding that professional development efforts of the participants seemed to constitute just a small proportion of their work. White and his colleagues' study (White et al., 2020) gives also interesting insights into TEs' roles and responsibilities in an increasingly complex, multifaceted and digital world. The authors studied with 20 'accomplished' TEs with international reputations from across the USA, Canada, Europe, the UK, Asia and Australia. The authors asked participants to explain the nature of TE responsibilities and advices to neophytes. The participants overwhelmingly emphasised the importance of embracing professional identity, research activities including dissemination and working with multiple stakeholders including (preservice) teachers,

policy-makers and media. In addition to these, MTEs are also required to develop and demonstrate a specialisation in mathematics and mathematics-specific pedagogy (Jaworski, 2008). That is, MTEs should have a field-specific instructional perspective and content knowledge (Agaç, 2018). MTEs are also active members of mathematics education community participating in the development and advancement of practices, courses and mathematics teacher education programs and continuous professional development (see Hangul, 2018). Our considerations hitherto suggest that three areas of MTEs' work and responsibilities are prominent regarding teacher education practices: research engagement, teaching/ training-related duties and expertise in field-specific knowledge (i.e. content and pedagogy).

While the teachers trained by TEs are prepared to the profession through a predetermined program which is often based on an officially issued curriculum, it is an ambiguous issue how TEs are prepared for the profession. Probably with this in mind, van Veen (2013) claims that there are TEs who are responsible for preparing teachers, but there are no educators aiming to train or prepare TEs to the profession. van Veen argues that many become TEs (often based on expertise in subject matter pedagogy or teaching experience) when they are appointed as TEs. There are very few institutions around the world serving to the professional development of TEs such as MOFET in Israel (Korthagen, 2000) and NAFOL in Norway (Smith, 2020). MOFET aims to support and facilitate TEs' professional development and to broaden their field-specific knowledge. NAFOL was established to develop Norwegian TEs' research competencies. However, in many other countries, there does not appear any formal professional development program for TEs, and some explorative international studies have already provided evidence for the deficiency and necessity of it (Van der Klink et al., 2017). It is apparent that TEs' preparation to profession and professional developments are not well structured apart from very few examples. Therefore, TEs are not only expected to prepare highly qualified teachers but also hold the responsibility for their own professional development and/or learning. But how do TEs gain knowledge relevant to teacher education practices?

Research on TEs provides some insights into the means of professional learning. Several researchers indirectly draw attention to TEs' knowledge sources. van Veen (2013) mentions teaching (to student teachers), research activities and interaction with colleagues as means of self-learning opportunities. While Çetinsaya (2014) points out the experiences gained as a faculty member in different universities, others (e.g. Hiebert et al., 2002; Kremer-Hayon & Zuzovsky, 1995) refer to teaching experience gained in schools as knowledge sources. Research projects also support TEs' professional learning (Anthony et al., 2018). van Velzen et al. (2010) state that the learning of TEs usually takes place informally in the workplace. Similarly, Eraut (2004) mentions that the workplace context adds new layers to, and hence deepens, TEs' understandings.

The qualifications sought for TEs' job assignment are also informative about professional learnings. In the Netherlands and Australia, for example, to find a position in higher education as a TE, one needs to have teaching experience or a postgraduate research degree (van Velzen et al., 2010). In the Netherlands (similar to England and Wales), teaching experience in the relevant field is required to become a TE (Harrison & McKeon, 2008), and having experience as a schoolteacher is considered a prerequisite for being a good TE (Swennen et al., 2010). Likewise, in the UK, almost all TEs have previously worked as teachers (Boyd, Harris & Murray, 2007). As in Portugal and Israel (Murray et al., 2009; Yogev & Yogev, 2006), TEs are described as academics in Finland, and they are expected to have a doctorate degree (Krokfors et al., 2011). In Turkey, TEs are also selected among Ph.D. holders from the relevant field of education. Hence, two widely accepted criteria to become a TE appear to be a postgraduate research degree and/or teaching experience.

Why are TEs' knowledge sources used for teacher education practices important? Research suggests two reasons. Firstly, researchers tend to forge a connection between the knowledge sources and the formation of professional identity. Williams and Ritter (2010), for instance, point out that identities of TEs develop through professional sharing among the members of the same community. Similar observations are also shared by others though in different contexts with different purposes (McKeon & Harrison, 2010; Poyas & Smith, 2007; Shagrir, 2010). Along with member interaction, Davey and Ham (2010) and Robinson and McMillan' (2006) indicate that literature readings play a role in the professional development and identity formation. From a broader perspective, researchers also point out that identity formation becomes particularly evident during the transitional stage to the profession (Margolin, 2011). Research on becoming TEs clearly suggests that identity formation is decisive for TEs to position themselves within the profession (Murray & Male, 2005; White et al, 2020).

Secondly, knowledge sources form the foundation of MTEs' field-specific expertise and play a decisive role in MTEs' prescriptions of effective mathematics teaching. In a survey study, Agaç (2018) reported that MTEs with teaching experience tend to focus more on student-teacher interaction and the assessment of learning outcomes. In a case study design, Hangül (2018) examined three MTE's evaluations about the quality of a video-recorded mathematics instruction. Hangül observed that MTEs highlighted different features as indicators of the quality, and they tended to justify their perspective with recourse to different knowledge sources such as personal research undertakings and theoretical frameworks introduced to them during postgraduate education.

Our consideration shared until now indicate that knowledge sources play a significant role in MTEs' self-regulated professional learning and positioning and hence their teacher education practices. Given this importance, we aim to document and examine MTEs' knowledge sources used for teacher education practices. We believe that this study would enable us, mathematics education research community, to gain insights into MTEs' management of somehow unplanned professional development.

Theoretical framework: relationships of knowledge and teacher education practice

In our literature review, it has become clear that MTEs' work and responsibilities centred around three main areas: research engagement, teaching related duties and expertise in field-specific knowledge base. These three areas provided a perspective for our efforts to document and examine MTEs knowledge sources. From this perspective, in making sense of, and establishing a relationship between MTEs' knowledge sources and their professional practices, we found useful the framework 'relationships of knowledge and practice' developed by Cochran-Smith and Lytle (1999).

Cochran-Smith and Lytle distinguish three prominent conceptions of teacher learning: knowledge-for-practice, knowledge-in-practice and knowledge-of-practice. In their account, knowledge-for-practice is related to a body of theoretical knowledge outside of the teacher, usually produced by researchers from universities in various disciplines. This conception of knowledge includes, though not limited to, the knowledge about content, pedagogy, theories of educational sciences, conceptual frameworks, teaching methods and strategies, human development and learning as well as teachers' professional practices and continuous development. Knowledgein-practice refers to experience-based knowledge developed over time and produced by teachers through their practices in actual settings with practical inquiries, narratives of (exemplary) practices and deliberate reflections. This knowledge is embedded in, and developed as a response to, the particularities of the schools/classrooms and manifested in teachers' actions, decisions and judgements. The third conception, knowledge-of-practice, refers to the production of knowledge from within the practice by taking a critical stance towards knowledge-for-practice produced by others. During such knowledge production, teachers make their classrooms sites for purposeful inquiry to relate their practices to larger schooling issues in the interest of a wider audience. This type of knowledge attempts to bridge knowledge-for-practice which is externally produced independent of teacher with the knowledge generated in situ with the realities of actual experiences (knowledge-in-practice).

Goodwin et al. (2014) translated these heuristics into teacher education discourse by arguing that these three conceptions have an explanatory power for the relationships of knowledge types and practices of TEs as well. The authors maintain that knowledge-for-practice is acquired during TEs' (post)graduate educations and the formal courses/programs. Knowledge-in-practice includes the insights gained through TEs' own experiences, experimentations on their practices and/or observations of peers/colleagues/mentors. Finally, knowledge-ofpractice could be construed as teacher education research that TEs engage as a member of the larger scholarly community.

Although Cochran-Smith and Lytle (1999) developed the framework with a focus on teacher learning, we agree with Goodwin et al. (2014) that it has also explanatory power in making connections between TEs' learning and knowledge types. We found these three conceptions of learning and knowing rather functional in explaining and examining MTEs' professional practices in relation to the type of knowledge acquired from different sources. This framework provides a lens through which we could gain insights into, and construe, the relationship between the knowledge sources used by MTEs and their professional practices of teacher preparation business.

Method

This study is a by-product of a research project examining MTEs' understandings and conceptualisations of effective mathematics teaching as well as the ways in which they put these understandings into practice while preparing preservice teachers. The ethics permission for the research was gained from the University of Gaziantep Ethics Committee in 2016 (protocol number 2016/21). Afterwards, the research project commenced in 2016 and was successfully completed in 2018. In this section, we give details about the research design, participants, data collection and data analysis.

Research design

This study was designed as a mixed-method (embedded method) research. Mixed-methods research is formally defined here as the class of research where the researcher mixes or combines quantitative and qualitative research techniques, methods, approaches, concepts or language into a single study (Johnson & Onwuegbuzie, 2004, p.17). In this context, Creswell (2008) emphasizes that the blending of qualitative and quantitative research methods in mixed-method studies provides a better understanding of the research problem and research questions. The embedded design includes the collection of both quantitative and qualitative data in the same time, but one of the data types plays a predominant role within the overall design (Creswell, 2003). In this study, qualitative data were predominant in our design.

The qualitative part of this study was designed as qualitative descriptive research. According to Sandelowski (2000), qualitative descriptive research is a useful method to obtain straight descriptions of the phenomenon under investigation. In this study, as we aimed to obtain straight descriptions of MTEs' knowledge sources, this method was considered to be appropriate. We explored qualitative data by performing supplementary quantitative analyses to examine the relationships between the knowledge sources and certain variables, which we explained below.

Participants

Within the scope of our research, we targeted all MTEs working in higher education institutions in Turkey. We intended to reach all MTEs actively involved in mathematics teacher education/training programs at undergraduate levels. For this, a total of 81 education faculties located in Turkey were examined. In this process, the web pages of the mathematics teacher education programs were scrutinised, academic staff were identified and their e-mail addresses were obtained. The information accessed from the web pages has been confirmed on national academic database of the Higher Education Council (https://akademik.yok. gov.tr). Through our examinations, we determined a total of 522 academic staff actively taking part in mathematics teacher education programs as of 2017 academic year. All identified 522 academic staff were contacted via e-mail, informed about the research, and 281 (54% of the whole MTEs group in Turkey) MTEs participated in the study.

We examined the representativeness of the participants for the whole population over two variables which were the only available features from the national academic database: academic ranking and gender. Distribution of our participants over these two variables in comparison to the whole population is presented in Table 1.

As seen from Table 1, in terms of academic ranking, professors, associated professors and assistant professors were proportionally representative of the population. Though the proportion of research assistants was lower than this group's proportion in the population, they were considerably highly represented in our sample. We also examined the representativeness of the participants with a cross-tabular distribution in Table 2.

A cross-tabulated comparison also confirmed the proportional compatibility between the participants and the whole population over academic ranking and gender except for research assistant group which was again fairly represented in the participants.

Apart from gender and academic ranking, we collected information from the participants (see "Data collection tool and process" section below) about two more variables: teaching experience in schools and self-identification (as mathematician, mathematics educator or both). Cross tabular distribution of the participants with regard to four variables are presented in Table 3, and proportional distributions are presented in Table 4.

In summary, 13% of the participants were professors, 16% associate professors, 40% assistant professors and 31% other (research assistants and teaching staff such as lecturers). Sixty-two per cent of the MTEs had teaching experience before they were assigned to a position in universities. Fifty-seven per cent of the participants were male, and 43% were female. A high majority considered themselves as

	Participa	nts ($N = 281$)	Populatio	on ($N = 522$)
	\overline{f}	%	\overline{f}	%
Academic ranking				
Professor (with Ph.D.)	37	13.17	41	7.85
Associated professor (with Ph.D.)	44	15.66	79	15.13
Assistant professor (with Ph.D.)	112	39.86	207	39.66
Research assistant and teaching staff	88	31.32	195	37.36
Total	281	100.0	522	100.0
Gender				
Female	120	42.70	241	46.17
Male	161	57.30	281	53.83
Total	281	100.00	522	100.00

Table 1 Distribution of participants and population over academic ranking and gender

	Parti	cipants (1	V=281)			Popula	ation (N	(=522)		
	Male		Fema	ile	Total	Male		Fema	le	Total
	f	%	f	%		f	%	f	%	
Academic ranking										
Professor (with Ph.D.)	32	11.39	5	1.78	37	35	6.70	6	1.15	41
Associated professor (with Ph.D.)	31	11.03	13	4.63	44	56	10.73	23	4.41	79
Assistant professor (with Ph.D.)	65	23.13	47	16.73	112	117	22.41	90	17.24	207
Research assistant	33	11.74	55	19.57	88	73	13.98	122	23.37	195
Teaching staff										
Total	161	57.30	120	42.70	281	281	53.83	241	46.17	522

 Table 2
 Cross-tabular distribution of participants and population over gender and academic ranking

mathematics educators with 77%, mathematicians were 17% of the participants, and 6% identified themselves as both.

Data collection tool and process

A questionnaire consisting of two parts was formed as a data collection tool (see Appendix). The first part directed questions about MTEs' self-identification and teaching experience. The second part was composed of five open-ended questions prepared to elicit information about MTEs' views and practices for effective mathematics teaching. The theoretical framework of Cochran-Smith and Lytle (1999) guided the content of these five questions in relation to knowledge-for/in/of-practices. The initial two questions asked MTEs to express their views on effective mathematics teaching in theory and in practice. In the following two questions, MTEs were asked to share information about their curricular practices and extracurricular activities while preparing students as practitioners of effective mathematics teaching. In the last question, they were asked to state the sources of information that underpin their practices and approaches. We also requested MTEs to provide us with updated curriculum vitae. The questionnaire was sent out to all 522 MTEs via e-mail with information about the purpose of the study. E-mail was repeatedly sent three times with 2-week time intervals in order to increase the participation rate. As a result, 281 (54%) replied to our e-mails with a written response and hence constituted the participants of our study.

Data analysis

Participants' responses were analysed via content analysis technique with the help of MAXQDA 12 qualitative data analysis software. In analysing the data, we adopted an inductive thematic coding approach, following Strauss and Corbin's (1990) open and axial coding steps. These steps represent a grounded, data-driven

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Table 3 Distributio	Table 3 Distribution of participants over four variables	r four variables															
			Profe	ssor (Professor (n=37)		Associa (n=44)	ciated 4)	Associated professor $(n=44)$		Assis	stant pi	Assistant professor (<i>n</i> =112) Res. Ast. and Teac.Staf. (<i>n</i> =88)	=112)	Res. Ast. Teac.Staf. (n=88)	Ast. an Staf. 3)	p
		Teaching experience Yes No Missing Total Yes No Missing Total Yes No Missing Total Yes No Total	Yes	No	Missing	Total	Yes	No	Missing	Total	Yes	No	Missing	Total	Yes	No	Total
Self-identification Math. $(n=47)$	Math. (n=47)	Male (<i>n</i> =37)	6	5		14	1	3		4	11	9		17		5	2
		Female $(n=10)$	5			7		1		1		4		4	1	7	3
	Math.Edu. $(n=217)$ Male $(n=114)$	Male (<i>n</i> =114)	10	3	1	14	20	5		25	32	13		45	16	14	30
		Female $(n=103)$	3			3	×	1	1	10	28	12		40	22	28	50
	Both $(n=17)$	Male $(n=10)$	7	7		4	1	1		7	1	1	1	б	1		1
		Female $(n=7)$						7		7	3			з	7		2
TOTAL			26	26 10 1	1	37 30 13 1	30	13	1	44	75	75 36 1	1	112	112 42 46 88	46	88

TOT	

		f	Per cent
Experience as a schoolteacher $(n=281)$	Yes	173	62
	No	105	37
	Missing	3	1
Academic ranking $(n=281)$	Professor (with Ph.D.)	37	13
	Associated Professor (with Ph.D.)	44	16
	Assistant Professor (with Ph.D.)	112	40
	Research assistant and teaching staff	88	31
Gender $(n=281)$	Female	120	43
	Male	161	57
Self-identification $(n=281)$	Mathematics educator	217	77
	Mathematician	47	17
	Both	17	6

Table 4 Proportion of participants over four variables

approach to arranging information that ultimately leads to emergent codes and themes from the data.

In analysing the data, two researchers of this paper worked together. At the beginning, the written responses were repeatedly read by two coders separately to grasp the data. While reading, both coders took some notes about the knowledge sources mentioned by the participants. The coders then met to share their notes and observations, which were later compared and merged, if appropriate, and thus, an initial outline was created to analyse the written responses. A subsample of responses was examined in detail to clarify the codes and the emergent themes. During this clarification process, codes were defined, criteria for the inclusion were determined, and some illustrative quotations were selected. Following this, the analyses were extended to the full sample during which code definitions and emergent themes around the codes were refined. During the fullsample analysis, earlier observations and derived implications were revised as our analysis progressed. The two coders used a constant comparative method (Strauss & Corbin, 1990) across responses in order to achieve a deeper understanding of the knowledge sources mentioned by the participants. Hence, the development of the codes and the emergent themes went through an iterative, reflective and recursive process. Data analysis span over time during which the coders had periodical meetings to reflect on and critically evaluate the codes, emergent themes and illustrative examples. This process continued until they both reached an agreement. Both researchers also had discussions about the theoretical underpinnings of their observations in making sense of the data with a consideration of the relevant literature.

Inductive thematic analysis allowed us to produce descriptive codes and categories with the frequencies over a considerably large sample size. After the qualitative analysis of the written responses, a post hoc examination of the quantitative data was performed to explore the potential relationships between the reported knowledge sources and four variables: academic ranking, gender, teaching experience and self-identification. To this end, a chi-square test of independence was performed to examine the relation between the variables and the knowledge sources. Quantitative analysis was performed using SPSS version 25.0 software. The significant level was set at p = 0.05.

Findings

In this part of the study, the findings about MTEs' knowledge sources are presented in two sections. In the first, qualitative findings from the content analysis with the emerging codes and categories of knowledge sources are presented. Then, the relationships with significant chi-square values between the knowledge sources and variables were given.

Qualitative findings

Content analysis of MTEs' written statements revealed 31 different knowledge sources grouped under nine categories as presented in Table 5. Of these, three categories with 9 specific sources were concerned with knowledge-for-practice, five categories with 18 different sources were related to knowledge-in-practice, and one category with 4 particular sources was associated to knowledge-of-practice.

As seen in Table 3, our participants' knowledge sources in relation to knowledgefor-practice yielded three main categories: literature readings, educational backgrounds and official documents. The category of literature readings (78%) was by far the most cited knowledge source. Seventy-six per cent of MTEs stated that their understandings of and practices for effective mathematics teaching were influenced by academic publications (books and research papers). A small percentage (5%) of the MTEs pointed to the theories put forward in the field of (mathematics) education and expressed that these theories form the basis of their perspectives.

The second category of knowledge source under knowledge-for-practice is educational backgrounds which remained at 20%. Under this category, participants referred to their undergraduate (7%) and postgraduate (17%) educations as the knowledge sources. It is noteworthy that the majority of the participants (80%) did not mention their educational background as a knowledge source. Official documents were also regarded as knowledge sources shaping the participants' knowledge-forpractices, though by a small proportion with 10% of MTEs. It is worth noting that MTEs have mentioned the publications produced by either the Ministry of National Education or the institutions known with activities in mathematics education.

Our analysis suggested that there were five categories of sources related to knowledge-in-practice: experiences, observations, interaction with academics, interaction with preservice/in-service teachers and role models. Fifty-two per cent of MTEs referred to experiences among their knowledge source. Under this category, 22% of the participants referred to their 'personal experiences' without any explicit or specific explanations. Teaching experience, whether gained as a former schoolteacher

Table 5 Mathematics teacher educators' reported knowledge sources

	Category	f	Per cent Code	Code	Illustrative quotations	f	Per cent
Knowledge-for-practice Literature readings	Literature readings	219	78	Academic publications	 I read books and research articles published at national and interna- tional platforms 	214	76
			-	Theories in (mathematics) education	- Theories developed in (the field of) 15 mathematics education	15	5
			-	Teaching methods	- Mathematics teaching methods	5	1
	Educational backgrounds	56	20	Education at post-graduate levels	- The main source of these approaches is the education I received during my doctorate (program)	47	17
				Education at undergraduate levels	- The courses we attended during the undergraduate program	20	L
	Official documents	28	10	Curricular documents	- Official mathematics curricula documents	22	8
			-	Text books issued by the Ministry of National Education (MONE)	- Textbooks published by MONE	5	7
				NCTM documents	- NCTM standards and principles	9	2
			-	Teacher qualifications published by Ministry of National Education	-Teacher Competencies: General and Field specific competencies of teaching profession published by MONE in 2008	ŝ	-
Knowledge-in-practice	Experiences	147 52		Personal experiences (without further specification)	 My own personal experiences Experiences I gained from different environments 	61	22
				Experiences as an academic staff	- My experience as an academician	56	20
				Experiences as a former school teacher	 My teaching experience as a former school teacher is also an important source of information 	41	15

Category	f	Per cent Code	Code	Illustrative quotations	Ŧ	Per cent
			Experience as a student	- My first source is my earlier expe- riences over the years as a (school) student	21	2
Observations	38	14	In-class observations	 My observations of student teach- ers' teaching practices during the practicum experience 	24	6
			Observations in different teaching environments	 Observations of maths classes 3-4 times in a week in a primary school where I am a mathematics education consultant 	14	S
				 Course observations at differ- ent universities 1 visited through Erasmus 		
			Observations of the academicians' lectures	- Making observations of academic staff in the courses I followed as a research assistant	9	7
			Observations on own learning	 Myobservations on my own learning during my education period as a student 	6	1
Interaction with academics	22	×	Share of knowledge/information	- I can also refer to the guidance made through our conversations with the academic staff as my main sources of information"	12	4
			Share of exemplary practices	- Good example-practice sharing with the colleagues in the field	10	4
Interaction with (preservice) teachers	23	8	Discussions with teachers	- Talks with and evaluation of teachers in various settings are also important sources of information	11	4

Table 5 (continued)							
	Category	f	Per cent Code	Code	Illustrative quotations	J.	Per cent
				Feedbacks from teacher candidates	 The feedback I received from the prospective teacherseven before their graduation, I receive their feedbacks when they go to schools for their practicum experience 	11	4
				Communications with in-service teachers during training activities	- Problems posed in the seminars (as 2 part of in-service trainings) I gave to teachers	0	1
				Coterie meetings with mathematics teachers	- Teacher experiences and sharing in coterie meetings	5	1
				Feedbacks from former students after induction	- Feedbacks from my students after their assignment to a school as teachers	6	-
	Role models	14	5	Mathematics teacher educators	- Mathematics educators lecturing me are in the first place	×	6
				Teachers	- My math teachers	5	2
				Educators	- Educators, who supported us in the pastsome were competent and others incompetent in my eye they have been role models that have helped me shape my own principles	3	_

Per	<i>f</i> Per cent Code	nal academic studies	Illustrative quotations f Per - Results I obtained from mv own 36 13	f 36	f Per cent 36 13
1			research	R :	Ĵ,
	Scie	Scientific meetings	- (Scientific) events I've attended such as conferences, seminars, etc.	. 18	9
	Supe	Supervisions to postgraduate theses	- MA theses and Ph.D. dissertations that I supervised	×	3
	Res	Research projects	 Projects I've submitted to our uni- versity, Turkish scientific research council, etc. 	9	2

(15%) or as an academician or TE (20%), seemed to be an important knowledge source for the participants. On the other hand, 7% of the participants stated that their experiences throughout their life as a student, without specifying a level, shaped their interpretations of effective mathematics teaching and their practices.

Another category of knowledge source related to knowledge-in-practice was MTEs' observations, referred by 14% of the participants. Some MTEs felt that they learnt much from their observations of the classroom environments (9%), their colleagues' practice (2%) and different teaching environments (5%). It is interesting to see that the ratio of those referring to in-class observations was limited to 9% among all participants.

Interactions with the academics and in-service/preservice teachers were two other categories of knowledge sources with regard to knowledge-in-practice. Through their interaction with the academics, MTEs appeared to mainly share information (4%) and best practices (4%). A mere 8% citation rate indicated that interaction with academics was not a common source among the MTEs, at least not as common as one might expect.

It was reported, though by a small number of participants (8%), that interactions with in-service and preservice teachers had an influence on their ideas and practices. In this regard, MTEs mentioned their discussions with teachers (%4), communications during coterie meetings (%1) and training events (%1) as knowledge sources. Feedbacks from preservice teachers during (4%) and after (1%) the preparation period were also considered as sources with an impact.

The final category related to knowledge-in-practice was role models. Some MTEs mentioned that academics (3%), teachers (2%) or educators (1%) that they had met were exemplary figures inspiring, in one way or another, their current efforts for teacher preparation. As seen, role models are the least referred one among all categories.

We identified only one category concerning knowledge-of-practice: academic activities. Among all participants, only 21% referred to this category. Under this category, MTEs stated that their personal academic studies (13%), scientific meetings (6%), supervised theses (3%) and the projects they participated in (2%) were among the important knowledge sources for them. It was interesting to observe that a large proportion of MTEs (79%) did not mention about any research-related activity that could be classified under this category.

Quantitative findings

Examination of four variables in relation to knowledge sources at code and category levels revealed certain correspondences. We extracted the relationships with significant chi-square values and presented the results as a crosstabs table in Table 6.

A chi-square test of independence suggested that there was a significant relationship between the self-identification and knowledge sources in three aspects. First, those identified themselves as mathematics educators tend to rely more on literature readings as knowledge sources than the mathematicians ($X^2 = 37.73$; p < 0.05). Secondly, mathematicians were also less likely to depend on teaching experience as a

				Knowle	dge for Pra	ctice					Know	ledge	in Practic	e
	Lite	rature readi	ngs	Educatio	onal backgr	ounds	Offic	ial docume	nts	1	Experiences		Ob	servations
	Count (%) Exp.C. (%)	$X^2(df)$	Count (%)	Exp.C. (%)	$X^2(df)$	Count (%)	Exp.C. (%)	$X^2(df)$	Count (%) Exp.C. (%)	$X^2(df)$	Count (%)	Exp.C. (%) X ² (d)
Self- identification	Lite	erature readi	ngs	At post	-graduate le	evels				As a for	mer school t	eacher		
M (n=47)	23 (11)	36.63 (16.73)		2 (4)	7.86 (16.72)		1			2 (5)	6.86 (16.73)			
ME (n=217)	187 (85)	169.12 (77.22)	37.73 (2)	44 (94)	36.3 (77.24)	8.65 (2)				32 (78)	31.66 (77.22)	13.67		
Both (n=17)	9 (4)	13.25 (6.05)	(2)	1(2)	2.84 (6.04)	(2)				7 (17)	2.48 (6.05)	(2)		
p (<.05)		.00			.013		1				.001			
Academic ranking	Literatu	re readings		At post	-graduate le	evels				As ar	academic s	taff		
Prof (n=37)	23 (11)	28.84 (13.16)		0 (0)	6.19 (13.17)		1			13 (23)	7.37 (13.16)			
Assoc.Prof. (n=44)	34 (15)	34.29 (15.65)	9.09	7 (15)	7.36 (15.66)	10.68				14 (25)	8.77 (15.66)	13.92		
Assis.Prof. (n=112)	86 (39)	87.29 (39.85)	(3)	19 (40)	18.73 (39.85)	(3)				19 (34)	22.32 (39.86)	(3)		
Res.A.&T.Staff (n=88)	76 (35)	68.58 (31.31)		21 (45)	14.71 (31.31)					10(18)	17.54 (31.32)	-		
p (<.05)		.028			.014		1				.003			
Gender				At post	-graduate le	evels								erent teaching ironments
Female (n=120)	-			30 (64)	20.07 (42.70)	10.29	1						2(14)	5.98 (42.71) 4.8
Male (n=161)	-			17 (36)	26.93 (57.30)	(1)							12 (86)	8.02 (57.29) (1)
p (<.05)	-				.002									.001
Experience as a school- teacher	_			At unde	ergraduate l	evels	Offic	ial docume	nts	Pers	onal experie	nces		
Yes (n=173)				6 (30)	12.45 (62.25)	9.52	10 (36)	17.42 (62.21)	9.31	47 (77)	37.96 (62.23)	7.30		
No (n=105)	-			14 (70)	7.55 (37.75)	(1)	18 (64)	10.58 (37.79)	(1)	14 (23)	23.04 (37.77)	(1)		
p (<.05)	_				.002			.003			.007			
	_									As a for	mer school t	eacher		
Yes (n=173)	_									40 (100)	24.89 (62.23)	28.36		
No (n=105)										0 (0)	15.11 (37.77)	(1)		
р (<.05)											.030			

Table 6 Crosstabs analysis with significant Chi-squarevalues

knowledge source than the other two groups ($X^2 = 13.67$; p < 0.05). Thirdly, mathematics educators put more emphasis on their postgraduate education than the others ($X^2 = 8.65$; p < 0.05).

Regarding academic ranking, there were three significant chi-square values. To begin with, among the Ph.D. holders, as the academic ranking increased the reliance on the literature readings decreased (X^2 =9.09; p<0.05). Similarly, there was a decline in the consideration of postgraduate education as a knowledge source with the rise of academic ranking (X^2 =10.68; p<0.05). There was also significant association between the experience as an academician and seniority (X^2 =13.92; p<0.05). When the academic title increases, MTEs were more likely to depend on their academic experiences.

A chi-square test also showed a relationship between gender and two knowledge sources. Women differ from men in their reliance on postgraduate education $(X^2 = 10.29; p < 0.05)$. Men were more likely to benefit from observations in different teaching environments ($X^2 = 4.86; p < 0.05$).

Teaching experience was also associated to three sources with significant chisquare values. Those with teaching experience tended to consider their experience as knowledge source more than the ones without teaching experience ($X^2 = 6.78$; p < 0.05). Second, teaching experience was also related to consideration of graduate education as a knowledge source ($X^2 = 9.52$; p < 0.05). Lastly, those without teaching experience were more likely to draw on curricular documents ($X^2 = 8.08$; p < 0.05) than the ones with teaching experience.

Discussion and implications

The research describes MTEs' professional development for teacher education practices as a self-regulated learning process shaped under the influence of different knowledge sources (van Veen, 2013). The findings of this study revealed 31 such knowledge sources. These sources were relevant to three types of knowledge proposed by Cochran-Smith and Lytle (1999): knowledge-for-practice, knowledge-inpractice and knowledge-of-practice. We structure our discussion around these three types of knowledge and the related knowledge sources.

With regard to knowledge-for-practice, our participants' responses suggested 9 sources classified under three categories: literature readings, educational backgrounds and official documents. The literature readings were by far the most referred to category among the participants with 78%. It is clear that MTEs are highly dependent on reading academic publications (76%) that being an important means of their professional learning. It is not surprising to observe MTEs' heavy dependence on the literature readings, which might be regarded as a common sense (e.g., van Veen, 2013). Interesting, however, are the two relationships that chi-square test suggested: mathematicians and MTEs with higher academic ranking mentioned significantly less the literature readings as knowledge sources. The research has, for long, well established that mathematicians and mathematics educators had rather different views on and roles attributed to mathematics for teacher preparation. Goldin (2003), for instance, describes mathematicians as traditional conservatives with a 'rigid' approach to mathematics and its teaching. It is perhaps due to this rigidity that mathematicians were not as concerned with the literature readings. Regarding MTEs with higher academic ranking, it is not clear, to us, why with the increase of academic ranking, the references to the literature readings decreases. We wonder if this would be the case in different countries, the possible reasons and implications for the profession. We believe it would be an interesting research to delve more into this trend.

Educational backgrounds were also another category referred by 20% of the participants. While 17% of MTEs mentioned the courses during postgraduate education, 7% referred to undergraduate studies as knowledge sources. The researchers (e.g. Goodwin et al., 2014; van der Klink et al., 2017) repeatedly reported TEs' dissatisfaction with postgraduate studies in pedagogically preparing them for teacher education. Such results indicate that TEs are not well prepared to the profession due to the lack of knowledge-in-practice. This observation was also confirmed in our study as none of the MTEs referred to features of postgraduate studies regarding knowledge-in-practice. However, 17% of participants did refer to formal studies during postgraduate education as a knowledge source connected to knowledge-forpractice. Hence, we argue that contribution of postgraduate education to MTEs professional preparation might be better evaluated on the basis of different knowledge types. Otherwise, it would be an overgeneralisation to consider postgraduate studies as irrelevant to teacher education practice. When viewed from the perspectives of different conceptions of knowing and learning as employed in our study, we believe, the usefulness and functionality of MTEs' preparation to the profession regarding the postgraduate education could be evaluated more realistically.

The quantitative analysis provided an interesting relationship that women tended to consider postgraduate education as a knowledge source significantly more often than men. The reason for this is not readily apparent to us. However, this might be an indication of value attachment to a doctoral degree, different gender-related expectations imposed on the doctorate candidates or it might be related to cultural self-positioning of some kind. Considering that the number of women joining in the workforce, at least in Turkish context, overwhelmingly increasing (for example, much higher at research assistants, see Table 3), gender-related studies on MTEs might be a fruitful research agenda to understand the changes in and projections on the profession with possible consequences.

The findings also suggested that MTEs relied on official documents as knowledge sources to develop their knowledge-for-practice. Under this category, MTEs mostly referred to the policy documents including the official mathematics curriculum scripts (8%). The researchers such as Smith (2005) and Zeichner (2005) emphasise the importance of bridging between policy and practice and consider this as one of TEs' responsibilities. The authors argue that TEs are expected to communicate to their student teachers about the policies, standards and implications for the prescribed teaching practices. A small proportion of our participants (10%) stated to have recourse to policy documents that shaped their views and practices. Hence, it appeared that some of our participants were concerned with bridging the policy and practice. Interesting, however, was that 90% did not mention official documents as knowledge sources. It is a matter of question if and how 90% of MTEs in our sample bridge the policy and practice of teacher preparation. The quantitative analysis shed some light on this question. A chi-square test of independence suggested a significant relationship that those without teaching experience were more likely to have recourse to policy documents than the ones with teaching experience. Our inference is that due to their previous teaching practices, which 62% of our participants had, MTEs had an accumulated knowledge of policy-related dimension of knowledgefor-practice gained through lived-in experiences. Therefore, MTEs with teaching experience may not feel the need to have recourse to policy documents. On the other hand, those having recourse to official documents, in so doing, may attempt to compensate the lack of teaching experience.

Regarding the knowledge-in-practice, our analysis led us to identify 18 sources under five main categories: experiences, observations, interactions with academics and with in-service/preservice teachers and role models. According to Cochran-Smith and Lytle (1999), knowledge-in-practice develops over time in responding to material conditions of the actual settings and through practical activities. Fifty-two per cent of our participants referred to their experiences as a category of knowledge sources. Quantitative analysis established that seniors and MTEs with teaching experience relied on their experiences more than the others. Cochran-Smith and Lytle (1999) state practical inquiry and deliberate reflections critical to the development of knowledge-in-practice. However, our data do not allow us to make decisive statements about if and in what ways MTEs' experiences involved reflections and inquiry. Nonetheless, MTEs referring to their observations (14%) as knowledge sources seemed to be involved in a purposeful pursuit of developing knowledge-in-practice.

Interesting to observe that among these five categories, there were only two with an explicit interactive feature: interaction with academics (8%) and with in-service/ preservice teachers (8%). The research provides convincing evidence that when MTEs work with teachers in close collaboration, they are likely to develop novel practice-based pedagogies which open up new possibilities in the advancement of profession (Anthony et al., 2018). Despite the reported benefits and potential for the development of knowledge-in-practice, however, MTEs referring to interaction with in-service/preservice teachers remained rather limited in our sample. This raises a question about the extent to which MTEs benefit from teachers' potential contributions in enriching their practices.

Goos and Bennison (2018) provide evidence for the important role of collaboration among the academics with different backgrounds to generate new practices relevant to mathematics teacher preparation. As a matter of fact, collaboration was considered as an indispensable part of TEs' work by many (Bleiler, 2015; Goodwin et al., 2014). In our participants' responses, the collaboration came to the fore in relation to sharing knowledge (4%) and exemplary practices (4%). The proportion of those referring to the interaction with colleagues or academics was limited to a mere 8%. According to Cochran-Smith and Lytle (1999), development of knowledge-in-practice depends also on collaborative efforts and interactions among the members of a community in which practitioners with different levels of experiences share accumulated wisdom and extended reflections with one another. However, the research at an international level on TEs proposes that such interaction is not often readily available in most teacher education institutions (van der Klink et al., 2017). In fact, some institutions established to support TEs' professional development in various ways such as MOFET in Israel (Korthagen, 2000) and NAFOL in Norway (Smith, 2020) aimed to ease opportunities for member interactions.

When the research results at an international level and our findings are considered together, we conclude that MTEs develop knowledge-in-practice largely in an 'iso-lated' manner (see also Lunenberg et al., 2007) without getting much peer support, and even missing the opportunities for learning, in the workplace context. According to Eraut (2004), the workplace context brings a new understanding to learning and argues that some key understandings, such as how people define the situations they face, what options are available for actions and which option should be preferred, emerge based on peer support and interaction in the workplace. In the workplace setting, hence, the importance of professional support provided by the more experienced colleagues to novice MTEs is all too apparent. In this regard, Mayer et al. (2011), for instance, argue that such support from experienced teacher educators is important not only for the adaptation of new comers to the institutional culture but also for achieving a balance between the teaching and research.

However, our findings showed that peer interaction and colleague support had turned into a source of knowledge for a small number of MTEs. From this perspective, it can be said that MTEs involved in mathematics teacher education could not find the necessary support in order to realise the desired outcomes both in organizing and maintaining their professional development and in the preparation of effective mathematics teachers. Lack of such support leads MTEs to individually regulate the development of knowledge-in-practice at their discretion, resulting in a perception that such knowledge is 'personal, likely idiosyncratic and contextually specific...sustaining the mythology of teaching/teacher educating as process without substance' (Goodwin et al, 2014, p.297). This situation creates at least two obvious difficulties for especially the beginning MTEs. First, beginning MTEs would miss workplace learning and development opportunities. Second, since the beginners of the profession are deprived of a certain culturalisation, their contribution to the process of mathematics teacher training may be transformed into an eclectic structure at the mercy of individual efforts and personal understandings detached from tradition and institutionalization. In order to overcome such problems, Kitchen and Parker (2009) suggest that MTEs would benefit from supportive learning communities formed by colleagues with meaningful collaboration opportunities.

The final category of knowledge sources related to knowledge-in-practice is the role models. Four per cent of our participants referred to educators or mathematics educators as role models while 2% stated that their mathematics teachers set an example. It is apparent that those who could act as role models for MTEs were scarce. The research indicates an ostensible incompatibility between what MTEs preach as features of an effective instruction and what they practice during their teaching (see Loughran, 2011). A question posed almost 20 years ago by Hoban (2004, p.29) still relevant to our discussion here: Do teacher educators acknowledge the complexity of teaching and practice what they preach or do they perceive themselves as specialist teachers of discipline knowledge? Considering our findings, the latter seems to be the case. Despite the fact that our participants have met and known a variety of MTEs with different levels of seniority, with different backgrounds and with different sorts of experiences, very few of participants considered certain figures as representing exemplary models. Without such role-models blending knowledge-for-practice with knowledge-in-practice, it would be a too challenging task for many MTEs to go beyond the reproduction of practices imposed on them in the past. Such reproduction in fact is considered among the reasons for teacher preparation practice remaining resistant to the change (Ladson-Billings, 2001).

Regarding the knowledge-of-practice, we identified only one category: academic activities, referred by 21% of the participants. MTEs are responsible for teaching as well as doing research and producing publications. When viewed from Cochran-Smith and Lytle's (1999) perspective, knowledge-of-practice plays a pivotal role in MTEs' professional development as it bridges the gap between knowledge-for-practice and knowledge-in-practice. Despite its importance, a relatively small number of MTEs referred to research activities in which they have been somehow involved (i.e. personal academic studies (13%); supervisions to theses (3%); and research projects (2%)). This finding is both understandable and interesting. This is understandable since MTEs cannot be expected to do research in each and every aspect that characterise their knowledge-in-practice. It is hence reasonable for MTEs to benefit from the research studies other than their own, i.e., knowledge-for-practice. On the other hand, this finding is also interesting in that MTEs appear to rely on somehow less-structured and less-systematic knowledge sources such as experiences gained from different settings (52%), educational backgrounds (20%) and observations (14%) made in diverse instructional environments more than their own research undertakings. That is, MTEs stated to have been benefiting from informally (e.g.

observations of student teachers' practicum activities), sporadically (e.g. course observations during Erasmus visits) and intuitively (e.g. selective readings of the relevant literature) selected knowledge sources more than their own research. When their research attempts are concerned with any dimension of instructional practices, then MTEs are likely to gain knowledge-in-practice along with knowledge-for-practice. Producing knowledge-of-practice requires, as Cochran-Smith and Lytle (1999) argue, MTEs to take a critical stance towards knowledge-for-practice and reconsider the knowledge-in-practice. MTEs' less reliance on knowledge-of-practice raises an important question as to the functionality of the knowledge gained from the less systematic sources (than research undertakings) in fostering an understanding useful to teacher preparation practices. For example, it is not clear how MTEs merge different perspectives gained from different knowledge sources, how the conflicting perspectives are negotiated or what kind of practices are appropriated under the influence of different sources. It is in fact through such negotiations and appropriations that MTEs develop understandings as to the nature of their responsibilities and duties as well as relevant activities and practices with regard to preparation of future teachers.

Three types of knowledge and their appropriations under the influence of various knowledge sources have implications for the issue of identity. In their research on work of TEs, White et al. (2020) conclude that an important dimension of becoming a teacher educator is related to embracing a new identity. Formation of TE identity is proven to involve serious hardships as the research on beginning TEs suggest. When transitioned to teacher education profession, beginning TEs have to deal with, for instance, adoption of new roles, confirming to the new rules, adjustment to the new cultures and institutional norms as well as adaptation to the research responsibilities. Shagrir (2010) considers such efforts as the works of becoming a member of a community of professionals and hence building the professional self or identity. Shagrir and many other researchers (Davey & Ham, 2010; McKeon & Harrison, 2010; Poyas & Smith, 2007) make connections between the development of TEs' professional identities and the knowledge sources such as literature readings and colleague interaction.

In the light of these studies, we could argue that knowledge sources also contribute to the formation of MTEs' professional identities. Each knowledge source devises certain ways to develop possible professional identities or selves. In other words, while professional identity, roughly speaking, refers to what MTEs could and would like to become (Markus & Nurius, 1986) with regard to teacher preparation business, knowledge sources act as a means of how this 'becoming' comes into life. The variety of knowledge sources determined in this study indicates that there are an infinite number of possible professional selves depending on the depth and extent of utilization and on any of their combinations. When we return to our findings, one can observe that MTEs, at least in Turkish context, tend to form their professional identities mostly individually (by reading academic publications—76%) and hence left to invent their practices. Further research in different teacher education cultures might help us understand the interrelationships between MTEs' professional identities and the utilised knowledge sources.

Based on our findings and considerations shared hitherto, we believe a fruitful research agenda worth noting would be: if, how and to what extent the professional growth of MTEs to produce meaningful, influential and exemplary teacher preparation practices can be achieved through the determined knowledge sources. A step towards this direction could be determination of professional qualifications or set of standards for MTEs, to the best of our knowledge, which has not yet been specified. However, there are attempts to establish certain standards for teacher educators in general. For instance, in the USA, the Association of Teacher Educators (ATE, 2008) identified some standards for teacher educators, including, but not limited to, cultural competence, scholarship, professional development and program development. We can observe similar, but not identical, standards determined in the Netherlands for teacher educators including competencies in content, pedagogy, communication and personal growth (Murray & Male, 2005). Research on MTEs has not fared so far as to determine necessary professional qualifications or set of standards for professional practices. However, we believe it would be a promising research agenda to focus on the qualifications that MTEs must possess and how these qualifications can be gained through different knowledge sources.

Our final words concern the limitations of our study. This study was conducted in Turkey with MTEs actively involved in teacher preparation in higher education institutions. Our study shed some light on the knowledge sources that MTEs use for teacher education. Considering that issues related to teacher education such as qualifications required for job assignment, MTEs' preparations to the profession and policies are all culture intensive (see also Leung, 2001), our findings should not be interpreted regardless of Turkish tradition of teacher education system. Studies on the MTEs' utilisation of knowledge sources in different educational cultures would be certainly useful to have a better grasp of MTEs' regulation of their professional development around the world. Further to this, our data were composed of written responses of the participants to several open-ended items. Explanations in written form often suffer from the lack of greater details and do not always allow for probing. There were some interesting trends in the data that we could not explore further due to this limitation. For instance, despite the fact that mathematicians were employed with the responsibilities of teacher preparation, their references to literature readings were considerably lower than mathematics educators. Our data did not allow us for a further exploration of this observation to unearth the reasons and results. Research designs on MTEs with in-depth data collection tools employed over a span of time would be more useful for future research attempts to delve into and follow up observed trends. We, like many others doing research on MTEs, relied on the accounts of our participants; however, we believe that research undertakings with a focus on MTEs' practices in situ are necessary to achieve a better understanding of this group.

Appendix 1. Questionnaire applied to mathematics teacher educators

Do you have teaching experience?	()No	
	()Yes	years at primary level
	Please indicate at what level	years at secondary level
	and the duration in years.	years at high school level
How would you identify yourself?	() Mathematician	() Mathematics Educator

- 1. How would you describe the notion of "effective mathematics teaching"?
- 2. What features do you focus on while evaluating the effectiveness (strengths and weaknesses) of a mathematics teaching in the classroom? Please explain in detail.
- 3. To what features do you pay particular attention during your lectures in order to set an example for your students to become effective teachers of mathematics in the future?
- 4. What kind of extracurricular activities do you design for your students towards their development to be able to teach mathematics effectively in the future?
- 5. What are your knowledge sources that constitute the basis for all your explanations until now about effective mathematics teaching?

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Declarations

Conflict of interests The authors declare no competing interests.

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