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ORIGINAL ARTICLE

Teachers' responses to instances of student mathematical thinking with varied potential to support student learning

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Abstract
Teacher responses to student mathematical thinking (SMT) matter because the way in which teachers respond affects student learning. Although studies have provided important insights into the nature of teacher responses, little is known about the extent to which these responses take into account the potential of the instance of SMT to support learning. This study investigated teachers' responses to a common set of instances of SMT with varied potential to support students' mathematical learning, as well as the productivity of such responses. To examine variations in responses in relation to the mathematical potential of the SMT to which they are responding, we coded teacher responses to instances of SMT in a scenario-based interview. We did so using a scheme that analyzes who interacts with the thinking (Actor), what they are given the opportunity to do in those interactions (Action), and how the teacher response relates to the actions and ideas in the contributed SMT (Recognition). The study found that teachers tended to direct responses to the student who had shared the thinking, use a small subset of actions, and explicitly incorporate students' actions and ideas. To assess the productivity of teacher responses, we first theorized the alignment of different aspects of teacher responses with our vision of responsive teaching. We then used the data to analyze the extent to which specific aspects of teacher responses were more or less productive in particular circumstances. We discuss these circumstances and the implications of the findings for teachers, professional developers, and researchers.

Keywords Classroom discourse · Instructional activities and practices · Mathematics · Secondary · Student thinking · Teacher moves · Teacher responses

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Recommendations for effective mathematics teaching have consistently stressed the importance of engaging students in meaningful mathematical discourse (e.g., Australian Association of Mathematics Teachers [AAMT] 2006; National Council of Teachers of Mathematics [NCTM] 1991, 2000, 2014). Implicit in early recommendations and explicit in more recent ones is the critical role that student mathematical thinking (SMT) plays in such instruction. For example, NCTM (2014) described the goal of engaging students in discourse as "to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments" (p. 10). The teacher's role in meaningfully incorporating SMT into classroom discourse is critical (e.g., Hunter et al. 2016; NCTM 2014), but challenging (e.g., Hunter 2008, 2012; Peterson and Leatham 2009). Effective incorporation requires teachers to elicit and attend to, make sense of, and appropriately respond to SMT—actions widely recognized as the three components of *teacher noticing* (Jacobs et al. 2010). Here, we focus on the third component of noticing. Specifically, we examine how teachers respond to instances of SMT that are made public during classroom discourse. We do so by taking into consideration the productiveness of a teacher response in relation to the mathematical potential of the SMT that is being responded to—that is, the potential of the SMT to support students' mathematical learning.

Studies dating back over 40 years have recognized the importance of teachers' decisions—both long-term and in-the-moment—as an aspect of quality teaching (e.g., Bishop 1976; Borko et al. 2010). Recent research has shown that teacher responses to SMT that surfaces during a lesson matter because the way teachers respond to SMT affects student learning in their classrooms (e.g., Attard et al. 2018; Ing et al. 2015; Kazemi and Stipek 2001). For this reason, a growing body of research centers on better understanding teachers' responses to students' contributions. Some researchers have provided frameworks, coding schemes, and collections of moves that characterize how the responses support student talk (e.g., Chapin and O'Connor 2007), support student reasoning (e.g., Ellis et al. 2019), incorporate student thinking (e.g., Bishop et al. 2016), and focus on the mathematics in students' ideas (e.g., Selling 2016). Others have investigated variation among different teachers' responses (e.g., Schleppenbach et al. 2007) and among responses to different kinds of student thinking (e.g., Drageset 2014, 2015). Some studies have focused on teachers' responses to student contributions in their individual classrooms (e.g., Correnti et al. 2015), while others have focused on teachers' responses to a common set of SMT (e.g., Jacobs et al. 2011).

Studies such as these have provided important insights into the nature of teacher responses to SMT, yet much remains unknown. For example, although we know that not all SMT has the same potential to support student learning (e.g., Choy 2013; Leatham et al. 2015; Stockero and Van Zoest 2013), we do not know to what extent teachers' responses take that varying potential into account. This variation in potential is important, however, because the productiveness of a particular teacher response cannot be determined in general, but instead must be considered in relation to the nature of the SMT that prompted the response and the vision of teaching one is aspiring to achieve. To begin to better understand the relationship between the potential of SMT and teacher responses to that thinking, we investigated how teachers' responses vary depending on the mathematical potential of the SMT to which they are responding and in what ways this variation aligns with a particular vision of teaching.

Review of related literature

Our study builds on a long line of scholarship in mathematics education focused on teachers' instructional decisions (see, for example, Borko et al. 2010). In particular, we examine a *teacher's action during classroom instruction in response to a student contribution*—referred to henceforth as a *teacher response*. Because we investigated how teachers envisioned responding in the moment to SMT that might emerge during classroom discourse, we focus here on the body of literature that specifically examined teacher responses to instances of SMT, rather than other types of teacher actions related to SMT (e.g., anticipating or eliciting SMT). We organize our discussion around three areas of teacher response research related to different aspects of our research focus: teacher responses that support student learning, how different teachers respond to a common instance of SMT, and how teachers respond to different types of SMT.

Teacher responses that support student learning

Studies focused on teacher responses that support student learning provide insight into the characteristics of productive responses to instances of SMT. Some of these studies focus on teacher responses to specific types of student thinking. In their studies on teachers' responses to student errors, both Schleppenbach et al. (2007) and Son (2013) found that US teachers frequently responded to an error by evaluating the erroneous statement and telling students how to correct it. In contrast, Schleppenbach et al. (2007) found that Chinese teachers' responses to errors were more likely to be questions that prompted students to work through the error. They argued that the questioning about an error that was more prevalent in the Chinese classrooms was more likely to support student inquiry because it pushes students to engage in reasoning about and making sense of how to correct an error. Supporting this conclusion, Bray (2011) reported that teachers' responses to errors that engaged students in discussion about errors supported students' sense-making.

Studies that have made comparisons of teacher responses across varied instructional approaches also help us understand how teacher responses can support student learning. Correnti et al. (2015), for example, identified different distributions and patterns in the responses of two teachers who enacted different types of instruction—one aligned with traditional visions of teaching and another aligned with more *ambitious* mathematics teaching (e.g., Lampert et al. 2010) that is "deliberately responsive and discipline-connected" (p. 130). One key difference was how the teachers used *uptake* moves, teacher responses used to "extend, deepen, clarify, or elaborate the discussion" (p. 308). *Uptake* moves were used four times more often by the teacher whose instruction aligned with *ambitious* mathematics teaching compared to the teacher who engaged in traditional instruction, suggesting that such a move could be a key difference between such classrooms.

Other studies have found that teacher responses that focus on the mathematics in a student contribution (Franke et al. 2011) and contribute to constructive interactions among students (e.g., Ellis et al. 2019; Kazemi and Stipek 2001) are particularly supportive of classroom instruction driven by SMT. Ing et al. (2015) focused specifically on characterizing teacher responses that encouraged students to engage with each other around SMT; these responses included asking students to explain each other's strategies, discuss differences among shared strategies, respond to or use another student's strategy, and make connections among ideas. These researchers found that such moves correlated with increased student participation and, as a result, with higher student achievement. These findings suggest that teacher responses that engage students with one another's mathematical ideas can be particularly effective in supporting student learning.

Collectively, these studies give some insight into how teacher responses may or may not support student learning. Because these studies took place in individual teacher's classrooms, however, the SMT to which the teachers were responding varied widely, making it difficult to study the effectiveness of the responses based on the nature of the SMT. To address this limitation, some research has begun to analyze different teachers' responses to the same instances of SMT.

Teacher responses to common instances

Analyses of teachers' responses to the same instance of SMT are found mainly in studies within the teacher professional noticing literature that investigate the *deciding how to respond* (Jacobs et al. 2010) aspect of this practice. In such studies, teachers are provided a video clip or written work depicting SMT and are asked what they would do next, either specifically in terms of posing a next problem (e.g., Jacobs et al. 2010; Schack et al. 2013) or in terms of providing a general response (e.g., Diamond et al. 2018; Jacobs et al. 2011). Although many such studies focus on comparing the same teachers' responses before and after an intervention, Jacobs et al.' (2010, 2011) work focuses specifically on comparing the responses of prospective and practicing teachers (with varying levels of experience with SMT) based on the extent to which the student's current thinking and understanding is taken into account in the response. This work found that when a teacher attends to students' strategies, this attention supports a response that is grounded in those strategies, but does not guarantee that this will be the case—an important insight into the relationship between teachers' attention to student strategies and the responses that follow. As of yet, however, research such as this does not provide insights into other aspects of the teacher response, such as what specific action the teacher would take, and does not analyze the responses in relation to different types of SMT.

Responses to different types of SMT

Research has begun to shed light on teachers' responses to different types of SMT. Some studies have documented no differences among teacher responses based on the nature of the SMT to which they are responding. Franke et al. (2009), for example, found that teachers frequently followed up on students' initial explanations by asking a variety of questions (e.g., specific, general, leading, and probing sequences), but found no relationship between these different types of questions and the nature of the initial explanation to which they were responding, including its clarity, correctness, or completeness. For instance, when teachers responded with single specific questions, the initial student explanations were evenly divided between those that were correct and complete, and those that were ambiguous, incomplete, or incorrect. Drageset (2014) found similar results in a study of teacher responses to three different types of student explanations (explaining action, explaining reason, and explaining concept), reporting no significant differences in how teachers responded based on explanation type. Most often, the teachers in these studies responded to student explanations with actions that focused students on rules and reasons. These results suggest that some teachers do not differentiate their responses based on the potential of the SMT to support student learning.

Other studies, however, have produced findings that suggest that some teachers do differentiate their responses. In their analysis of the classroom of a teacher characterized as *responsive*, Bishop et al. (2016) found that the teacher responded in different ways to contrasting types of student contributions. Contributions that were short, expected answers or routine calculations prompted teacher responses that did not incorporate the SMT; these responses typically dismissed or evaluated the student contribution. On the other hand, student contributions that shared strategies or reasoning prompted responses that typically engaged students in conversations with each other and focused on ideas in the contribution. Drageset (2015) found similar results in his study of different types of student contributions across multiple lessons in one teacher's classroom. The most frequent type of student contribution was a brief answer to a non-complex question (e.g., basic computation, fact recall), which was typically followed by a recall or procedural question that one would expect in a *funneling pattern* of teacher questioning (Bauersfeld 1980; Wood 1998). The second most frequent type of contribution, an unexplained answer, was typically followed by a different response—one in which the teacher explained or requested an elaboration or rationale. In contrast to the studies above in which there was no differentiation in teachers' responses, these studies found that the individual teachers in their studies did vary their responses based on the nature of the SMT. Thus, there is a need for additional research to better understand the extent to which teachers vary their responses based on the nature of the SMT.

Summary

Existing studies of teacher responses to SMT provide an important foundation for further investigations of teachers' responses during classroom instruction. Research focused on the nuances among responses to different types of SMT is still in its infancy, however, and is currently limited in at least two different ways. First, many studies to date have focused on a single teacher, which does not allow for comparison of the productivity of responses across teachers. Second, studies often focus on broad categories of SMT, which does not take into account that different types of student contributions provide different opportunities to support student learning (e.g., Choy 2014; Leatham et al. 2015; Stockero and Van Zoest 2013). Thus, studying teachers' responses to SMT in the absence of studying the nature of the SMT to which the teachers are responding provides insufficient information to make claims about the effectiveness of those responses. This study extends the existing literature by examining multiple teachers' responses to a common set of instances of SMT with varied potential to support student learning. Better understanding nuances of teacher responses in relation to the potential of the SMT to support student learning will help to illuminate when and how a range of responses can support *ambitious teaching* (Lampert et al. 2013).

Theoretical framework

Understanding how the variation of teachers' responses affects the alignment of the response to a particular vision of teaching requires an articulation of that vision. Our vision of instruction is compatible with that of NCTM (2014), as well as descriptions of ambitious (Lampert et al. 2013) and responsive (Robertson et al. 2016) teaching. Such teaching includes "foregrounding the substance of students' ideas" (Robertson et al. 2016, p. 1), "recognizing the disciplinary connections within students' ideas," and "taking up and pursuing the substance of student thinking" (p. 2). Incorporation of SMT in these ways is a central component of such instruction and the focus of our work. Specifically, our work focuses on understanding two critical aspects of teaching practice: (1) identifying instances of SMT that are important to *notice* (Jacobs et al. 2010) because of their potential to support student learning, and (2) responding to those instances in ways that align with our vision of instruction. In the following paragraphs, we describe the theoretical constructs that we used to operationalize these two critical aspects in our work.

Although much of the early noticing literature focused on noticing student thinking in general, without regard to the nature of that thinking (e.g., Sherin and van Es 2005; Stockero 2008), as in other more recent work (e.g., Choy 2014; Stockero et al. 2017a), we position some instances of SMT as more productive to notice than others. To identify instances of SMT that are particularly productive to notice because of their potential to support student learning, we use the MOST (Mathematically Significant Pedagogical Opportunities to Build on Student Thinking) Analytical Framework (Leatham et al. 2015). MOSTs are particularly important instances to notice since they are high-potential instances of SMT that, if made the object of consideration by the class, could help students better understand important mathematical ideas. As we have described elsewhere in greater detail (Leatham et al. 2015), MOSTs are instances of student thinking that are shared during whole-class interactions that simultaneously embody three critical characteristics: student mathematical thinking, mathematically significant, and pedagogical opportunity (Fig. 1). These characteristics emerged from literature on productive use of student thinking (e.g., Schoenfeld 2008; Thames and Ball 2013; Walshaw and Anthony 2008) and extensive analysis of classroom data. For each characteristic, two criteria determine whether an instance of student thinking embodies that characteristic.

As shown in Fig. 1, the six criteria in the MOST Analytical Framework are considered linearly. An instance of SMT is classified according to the last criterion it satisfies (Student Mathematics, Mathematical Point, Mathematically Appropriate, Mathematically Central, Pedagogical Opening, or Pedagogical Timing). Those instances that

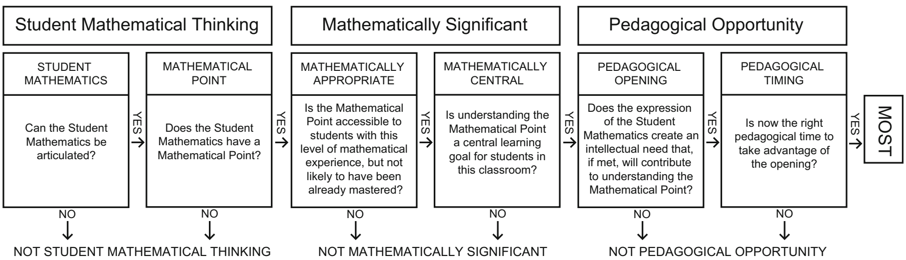


Fig. 1 The MOST Analytic Framework

appear mathematical, but for which the student mathematics cannot be inferred, are designated Cannot Infer. When an instance satisfies all six criteria, it embodies the three requisite characteristics and is an MOST.

The MOST framework was used in this study to classify the varying potential of instances of SMT to support student learning. For example, instances for which the Mathematical Point cannot be articulated have low potential to be used to support student learning since it would be unclear what mathematical understanding the teacher would use the instance to work towards. Instances of SMT that are classified as MOSTs, however, would by their nature have high potential to support student learning.

To interpret the alignment of teachers' responses to instances of varying potential with our vision of instruction, we consider four *Core Principles* underlying productive use of SMT that emerged from an examination of current research (e.g., Lampert et al. 2010; Robertson et al. 2016) and calls for reform (e.g., NCTM 2014) (see Fig. 2). The overarching principle is to position students as legitimate mathematical thinkers (Legitimacy Principle), which involves treating them as "capable of participating and achieving in mathematics" (NCTM 2014, p. 63) by engaging with mathematical ideas in a deep and meaningful way. Such positioning requires listening to students' contributions to discern what they are saying mathematically and then making decisions about the pedagogical potential of incorporating such contributions into the lesson. The other three principles contribute to this principle. Placing students' mathematics—"the mathematics in students' comments and actions" (NCTM 2014, p. 56)—at the forefront of decisions about whether and how to use SMT (Mathematics Principle) lays the groundwork for ambitious teaching. Engaging students in sense-making (Sense-making Principle) requires positioning them to "grapple with mathematical ideas and relationships" (NCTM 2014, p. 10). Working collaboratively (Collaboration Principle) means the students and teacher are working together as a classroom community in "collaborative experiences" (NCTM 2014, p. 7) towards a common mathematical goal. We use these Core Principles to capture the important aspects of our vision of instruction related to productively responding to SMT.

When taken in tandem, these frameworks allow us to theorize the productiveness of teacher responses to SMT. Thus, in addition to considering the alignment of a teacher response to the principles, interpreting the productiveness of a teacher response also requires considering the response in relation to the SMT that preceded it, since SMT classified at different points on the MOST Analytical Framework warrants different teacher responses. For example, SMT that is designated Cannot Infer would need clarification were it to be incorporated into instruction effectively, so teacher responses to such instances that seek clarification would be viewed as more productive than responses that did not do so. SMT designated Central would be aligned with important mathematical learning goals for the students and ready to be used by the teacher to work towards their goals in a variety of ways, but would not provide the same leverage as a MOST. As discussed in Van Zoest et al. (2017), MOSTs are instances of SMT

Legitimacy Principle: Students are positioned as legitimate mathematical thinkers.
Mathematics Principle: The mathematics of the instance of SMT is at the forefront.
Sense-making Principle: Students are engaged in sense making.
Collaboration Principle: Students are working collaboratively.

Fig. 2 Core Principles underlying productive use of SMT worth *building* on—that is, “student thinking worth making the object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea” (p. 36). We see building on a MOST as a particularly productive way for a teacher response to coordinate our Core Principles (Fig. 2). In general, to determine whether a teacher response to any instance of SMT aligns with our vision of instruction, we focus on the potential of the SMT, the extent to which the teacher response appropriately coordinates the Core Principles, and the relationship between the potential of the SMT and the teacher response.

Methodology

To understand the relationship between the potential of SMT and teacher responses to that thinking, this study investigated the following research questions: (a) To what extent do teachers’ responses vary depending on the mathematical potential of the SMT to which they are responding? and (b) To what extent are the variations in teacher responses in our data aligned with our Core Principles? Specifically, we analyzed 25 secondary school (ages 11–18) mathematics teachers’ responses to instances of SMT in the *Scenario Interview* (Stockero et al. 2015). The *Scenario Interview*, the resulting data, and the data analysis are described in the following sections.

The scenario interview

The *Scenario Interview* (Stockero et al. 2015) is a tool to investigate both how teachers respond to SMT during instruction and their reasoning underlying those responses. It was informed by standardized simulation interviews developed by others (e.g., Shaughnessy and Boerst 2017) to assess teachers’ actions during a teaching interaction. During the Scenario Interview, teachers are presented with instances of SMT from eight individual students—four from a geometry and four from an algebra context. The text portions of the instances were read by the interviewer and the student statements and relevant problems and diagrams from the context were provided in print form. The instances represent a range of contributions that satisfy different MOST Criteria. They also represent variation in the type of student thinking available, including ideas that are correct, incorrect, and incomplete. Four instances—two from each context—meet all the criteria of the MOST Analytic Framework and thus are MOSTs. Figure 3 provides the instances, their contexts, and their classifications according to the MOST Analytic Framework (see Fig. 1).

The Scenario Interview situates the interviewee as the teacher in their own mathematics classroom. They are provided information about what had already happened during the lesson and given the opportunity to ask for any contextual information (e.g., lesson and unit topics, grade level, student gender, what prompted the instance) that they wanted to know before responding. This contextual information was standardized in our interview protocol to ensure that all teachers received consistent information if it was requested. The interviewee is then asked to describe what they might do next were the instance to occur during whole-class discussion in their classroom and to explain why they would respond in that way. Each interviewee is then provided relevant contextual information to maintain the commonality of the instances to which they

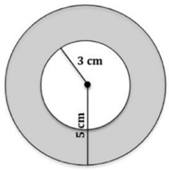
<table>
<tr><th>Scenario</th><th>Context</th><th>Instance</th><th>MOST Classification</th></tr>
<tr><td>G1</td><td rowspan="3">Students were sharing their solutions to the following task.
Given two concentric circles, radii 5cm and 3cm, what is the area of the band between the circles?

</td><td>Chris shared his solution: "The radius of the big circle is 5 and the radius of the little circle is 3, so the gap is 2, so the area of the band is 4π cm^2."</td><td>MOST</td></tr>
<tr><td>G2</td><td>Before the teacher had a chance to respond to Chris, Pat says, "I also got 4π cm^2, but I did it a different way."</td><td>Student Mathematics</td></tr>
<tr><td>G3</td><td>Pat explained how he got the same answer as Chris (4π cm^2) a different way: "π times r^2 for the big circle is π times 5^2, which is 10π and π times 3^2 is 6π for the little circle. I minused (sic) them and got 4π as my answer."</td><td>MOST</td></tr>
<tr><td>G4</td><td>The teacher has just posed a problem parallel to the one above with circles of radii 4 cm and 1 cm.</td><td>Sam says, "The answer is 15π cm^2."</td><td>Opening</td></tr>
<tr><td>A1</td><td rowspan="2">Students had been discussing the following task and had come up with the equation y = 10x + 25.
Jenny received $25 for her birthday that she deposited into a savings account. She has a babysitting job that pays $10 per week, which she deposits into her account each week. Write an equation that she can use to predict how much she will have saved after any number of weeks.</td><td>Terry says, "If you deposit $20 per week instead of $10 per week, the number in front of the x in the equation would change, but the number that is added would stay the same."</td><td>Central</td></tr>
<tr><td>A2</td><td>Casey said, "You could also change the story so the number in front of the x is negative."</td><td>MOST</td></tr>
<tr><td>A3</td><td>The teacher asked, "How do we find the equation given any table?" and put this generic table of values [to the right] on the board for the students to use in their explanation.
<table><tr><th>X</th><th>Y</th></tr><tr><td>0</td><td>15</td></tr><tr><td>2</td><td>19</td></tr><tr><td>3</td><td>21</td></tr><tr><td>5</td><td>25</td></tr></table></td><td>Jamie said, "I found the number in front of the x by subtracting the y-values in the table, 21 - 19, so that number is 2."</td><td>MOST</td></tr>
<tr><td>A4</td><td>Following Jamie's response in the previous instance, the teacher asked, "Do others agree with Jamie?"</td><td>Jessie says, "It would have to be divided by x."</td><td>Cannot Infer</td></tr>
</table>

Fig. 3 Scenario Interview instances, their contexts, and their MOST classifications

are responding and given an opportunity to provide a revised response. If they did, their revised response was used in the analysis. By using a common set of instances of SMT and providing teachers a core of common knowledge of the context, the Scenario Interview allowed us to make direct comparisons among teacher responses to instances that had different MOST Classifications (see Fig. 1 and Fig. 3) and thus had varied potential to support student learning. This approach allowed us to explore whether teachers seem to vary their responses based on the potential of the instance of SMT to which they are responding.

The data

Data for this study consisted of video-recorded scenario interviews conducted with a purposeful sample of 25 secondary school (ages 11–18) mathematics teachers from across the USA who were identified by members of the research team as having at least some interest in having their students share their mathematical thinking in class. Although we have previously found that such interest manifests itself in a variety of ways (e.g., Leatham et al. 2014), this approach was used to guarantee that we would have uses of student thinking to analyze. This was important to the study because we would not have been able to answer our research questions if teachers were not attempting to use any student thinking in their instruction. We used video analysis software (SportsTec 1997-2015) to segment each interview into the eight instances of SMT and the teachers' associated responses. For the 8 instances of SMT and 25 teachers in this study, there were a total of 198[1] teacher responses.

We recognize that productive use of SMT often requires a coordinated collection of responses; however, to begin to understand teachers' responses to instances of SMT, we focused our analysis for this study on teachers' initial responses to the common set of SMT in the Scenario Interview. Thus, preparing data for coding required making inferences about the teachers' initial response to each instance. Three coders individually analyzed all data relevant to each instance for a participating teacher—the teacher's descriptions of the actions they would take immediately following an instance of SMT, including any elaboration they provided in response to additional interviewer questioning—and distilled the teacher's response to its essence, which we refer to here as the *teacher response*. As the coders wrote their version of the teacher response, they identified relevant dialog that provided supporting evidence. The coders then met to compare their evidence and arrive at a final agreed-upon teacher response for each of the 198 instances in the data set. If they were not able to agree, the evidence was brought to the larger research team for further discussion and resolution.

The analysis

Once the data (the teacher responses) were agreed upon, the three coders individually coded each of the resulting 198 teacher responses using the *Teacher Response Coding Scheme (TRC)*. As discussed in Peterson et al. (2017, 2018), this TRC is organized into three categories that capture *who* interacts with the SMT (Actor), *what* they do in those interactions (Action), and *how* the teacher response relates to the student thinking (Recognition-Student Actions and Recognition-Student Ideas). Figure 4 provides the TRC coding categories, codes, and code definitions.

The coders met to reconcile their TRC coding and discuss any disagreements until consensus was reached. Our approach was consistent with that described in Harry et al. (2005), in that "[w]e made no attempt to develop a numerical reliability rating, because our goal was consensus, with each point of difference being debated and clarified until the group agreed" (p. 6). Note that the TRC allows for multiple Actions to be present in a teacher response, each with their own set of TRC codes. To facilitate our analysis, when multiple actions were identified in a teacher response during the initial TRC coding, we later revisited the response to identify the *predominate codes* in the given response—*the codes that the students are most likely to experience as the instructional*

[1] One teacher did not respond to A3 because they were not able to envision posing the task in that way, and another teacher was not able to respond to G4 because their interview was cut short.

Code			Definition
			The teacher response...
Actor *(those who are publicly given the opportunity to consider the instance of SMT)*	Teacher		does not publicly invite or allow anyone other than the teacher to consider the instance.
	Same student(s)		invites or allows the contributor(s) to consider the instance.
	Other student(s)		invites or allows a subset of students other than the contributor(s) to consider the instance.
	Whole class		invites or allows the whole class to consider the instance.
			The teacher response...
Action *(what the actor is given the opportunity to do with respect to the instance of SMT)*	Adjourn		indicates (either explicitly or implicitly) that the instance will not be considered at that time, but suggests the instance may be considered later.
	Allow		creates an open space for interaction with the instance.
	Check-in		gives an opportunity for the actor to self-assess their reaction to or understanding of the instance.
	Clarify		gives the actor an opportunity to make the instance more precise.
	Collect		gives the actor an opportunity to contribute additional ideas, methods, or solutions.
	Connect		gives the actor an opportunity to make a connection between the instance and other ideas, representations, methods/strategies, or solutions.
	Correct		gives the actor an opportunity to rectify the student mathematics of the instance.
	Develop		gives the actor an opportunity to expand the instance beyond a simple clarification, but does not request justification.
	Dismiss		indicates (either explicitly or implicitly) that the instance will not be considered.
	Evaluate		gives the actor an opportunity to determine the correctness of the student mathematics of the instance.
	Justify		gives the actor an opportunity to contribute mathematical reasoning related to the instance.
	Literal		gives the actor an opportunity to provide brief factual information related to the instance.
	Repeat		gives the actor an opportunity to repeat (verbally or in writing) the instance (including minor rephrasing that does not change the meaning).
	Validate		gives the actor the opportunity to affirm the general value of the instance and/or encourage student participation (e.g., says "thank you," gives a thumbs up signal, asks for applause).
			The teacher response...
Student Recognition *(the extent to which the student who contributed the instance of SMT is likely to recognize their contribution in the teacher response)*	Student(s) Actions	Explicit	uses the student's specific actions (typically words).
		Implicit	uses indexical language (e.g., pronouns, synonyms) or stops short of using the student's specific actions because of conversational conventions.
		Not	does not explicitly or implicitly use the student's specific actions.
	Student(s) Ideas	Core	focuses on a main idea of the instance in a way that the student who contributed the instance would likely recognize the idea as their own.
		Peripheral	focuses on an idea that is related to, but not the main idea of the instance, or focuses on a main idea, but recognizing that would require a leap of logic that students are not likely to make.
		Other	focuses on an idea that does not seem to be related in any way to a main idea of the instance.
		Indeterminate	is worded such that it is not possible to infer its focus or the SMT cannot be inferred.

Fig. 4 Subset of the Teacher Response Coding Scheme (*TRC*) used in this paper

intent of the response. The results that we report use the predominate codes for each teacher response.

To illustrate our application of the TRC, Fig. 5 provides three teachers' responses to instance G1, a MOST. In the first response—given by Teacher 16 (T16)—Actor is coded *teacher* since the teacher is the only one who engages with Chris' thinking when deciding to do other things before discussing Chris' answer. T16 expresses the intent to come back to Chris' contribution, so the Action is to *adjourn* (rather than *dismiss*) the

Teacher Response	Actor	Action	Recognition	
			Student Actions	Student Ideas
Let’s table this answer for a while. Let’s ponder [this problem], add some more information, do another problem, get another answer and then make comparisons. (T16)	Teacher	Adjourn	Not	Indeterminate
Cool. How did you do that? (T7)	Same student	Develop	Implicit	Core
Who else has another [answer]? Did everybody get 4π? Give me some more answers. (T5)	Whole class	Collect	Explicit	Peripheral

Fig. 5 Coding for illustrative teacher responses to instance G1 of the Scenario Interview

thinking. Recognition-Student Actions is coded *not* because the response does not incorporate Chris’ contribution and Recognition-Student Ideas is *indeterminate* since there is no mathematical idea in T16’s response to compare to Chris’ idea. T7’s response invites the *same student* who contributed the thinking to continue to publicly consider the instance. It does so by asking Chris to *develop* his contribution by explaining how he arrived at his answer. The response *implicitly* refers to Chris’ words (“that”) and remains *core* to the Student Ideas because Chris is invited to say more about his idea. Finally, in T5’s response, Actor is *whole class* because all the students in the class are being asked to consider whether their thinking is different from Chris’. Action is *collect* as T5 requests that other students share their answers. Recognition-Student Actions is *explicit* because the teacher picks up the “4π” and Ideas is *peripheral* because asking for other solutions is related to Chris’ thinking, but not the same as pursuing that thinking.

Results and discussion

To answer our first research question about how teachers’ responses vary depending on the potential of an instance of SMT to support student mathematical learning, we begin by discussing the Actor and Action categories and their interactions, followed by the Recognition (Student Actions and Student Ideas) categories and their interactions. In discussing these results, we focus primarily on the categories that represented the largest number of teacher responses in the data (see Table 1)—those that illuminate typical teacher responses. To answer our second research question, we then discuss different MOST classifications of SMT to highlight how different response are more or less aligned with the Core Principles (Fig. 2).

Actor and action

Actor The majority (65%) of teacher responses to the 198 instances in our data had the *same student* as Actor, meaning that the proposed teacher response was directed back to the student who had contributed the original thinking (see Table 1). In 17% of instances, the *teacher* was the Actor, meaning that the teacher was the only one who had a direct opportunity to consider the SMT. In 16% of the instances, the *whole class* had the opportunity to consider the SMT.

Table 1 Actor and action (MOSTs, non-MOSTs, and all instances)

	Same student (S)			Whole class (W)			Teacher (T)			All instances (A)		
	M	NM	Total (% of S)	M	NM	Total (% of W)	M	NM	Total (% of T)	M (% of M)	NM (% of NM)	Total (% of A)
Adjourn	–	–	–	–	–	–	3	21	24 (71%)	3	21	24 (12%)
Clarify	5	23	28 (22%)	–	–	–	–	–	–	5	23	28 (14%)
Collect	2	–	2 (2%)	4	2	6 (19%)	–	1	1 (3%)	7	3	10 (5%)
Connect	1	1	2 (2%)	4	2	6 (19%)	–	–	–	6	3	9 (5%)
Develop	32	25	57 (44%)	5	–	5 (16%)	–	–	–	37	25	62 (31%)
Dismiss	–	–	–	–	–	–	1	7	8 (24%)	1	7	8 (4%)
Justify	16	10	26 (20%)	2	1	3 (9%)	–	–	–	18	11	29 (15%)
Total	65	63	128	26	6	32	4	30	34	99	99	198

Responses coded *same student* occurred at about the same frequency for the 99 responses to MOSTs (65%) and the 99 responses to non-MOSTs (63%). This even distribution did not occur, however, for instances with a *whole class* or *teacher* Actor. More MOSTs (26%) than non-MOSTs (6%) were coded whole class, while the opposite was true for instances coded teacher (4% of MOSTs; 30% of non-MOSTs). This result suggests that teachers may distinguish, at least to some extent, instances that have high potential to support student learning if discussed by the class from those that might be more appropriately dealt with, at least initially, by the teacher. This inference is further supported by analyzing the Actor data by specific instance. *Same student* was the primary Actor for every instance except for G2 and G4 (non-MOST instances classified, respectively, as Student Mathematics because the student's comment had no associated mathematical point and as an opening due to the poor timing of the student contribution). Together these two instances accounted for 82% of all instances with a *teacher* Actor. Three of the MOST instances (G3, A2, and A3) accounted for 77% of the instances with a *whole class* Actor. Thus, although the teacher response was most often directed to the student who contributed the idea for both MOSTs and non-MOSTs, there were specific cases where other Actors occurred relatively frequently. The fact that the instances with a *teacher* Actor were mainly non-MOSTs and those with a *whole class* Actor were mainly MOSTs suggests some discrimination in to whom the teachers directed their responses, potentially based on the nature of the SMT. This discrimination is important because MOSTs are instances of SMT that would be particularly advantageous to turn over to the whole class to consider and research suggests that teacher responses that engage students with one another's mathematical ideas can be particularly effective in supporting student learning (e.g., Ing et al. 2015). However, consistent with Schleppenbach et al. (2007), the findings also suggest that this group of teachers may have a tendency to return to the same student during their initial response to SMT, possibly even when it is neither necessary nor advantageous to do so (for example, when the instance is an MOSTs).

Action The data indicate that certain Actions may be the go-to moves for the teachers in our study. The *develop* Action occurred much more frequently than the others, accounting for nearly one-third (31%) of the data (see Table 1). Three other Actions—*adjourn*, *clarify*, and *justify*—occurred at relatively the same rate (12%, 14%, and 14.5%, respectively) and together accounted for 40.5% of the data. To help the reader make sense of these actions, examples of instances coded as *develop* and *adjourn* can be found in Fig. 5. An example of a *clarify* Action occurred in response to A4 where Jessie had made an ambiguous statement: "Tell me what you mean by divided by x, like what x?" (T3). An example of a *justify* Action was to ask Jamie in instance A3 why they used the numbers that they did: "Why did you do the 21 minus the 19? Why didn't you do the 19 minus the 15?" (T14).

Two of these dominant Actions, *develop* and *justify*, occurred more frequently in response to MOSTs than non-MOSTs, accounting for over half (55%) of responses to MOSTs. The other two dominant Actions, *adjourn* and *clarify*, occurred more frequently in response to non-MOSTs, accounting for nearly half (49%) of such responses. As with the Actor data, these findings suggest that the teachers may respond to SMT in different ways based on the nature of the SMT. These findings support and extend those of Bishop et al. (2016) and Drageset (2015) who documented that the individual teachers who were the focus of their respective studies differentiated their actions based the nature of the SMT being responded to. Our analysis of a larger set of teachers provides evidence that this finding may apply to teachers more generally.

Actor/action interactions We also considered the distribution of Action by Actor (see Table 1). Three Actions—*develop, clarify* and *justify*—accounted for over 85% of responses with a *same student* Actor (recall that this Actor was associated with 65% of the data), with *develop* Actions occurring about twice as often as the other two. (The *develop* example in Fig. 5 and the examples of *clarify* and *justify* provided in the Actions section illustrate these actions being directed to the *same student*.) In each case, the distribution of these Actions between MOSTs and non-MOSTs paralleled that for the data set overall, with *develop* and *justify* Actions occurring more frequently in response to MOSTs (particularly to G1, A2, and A3) and *clarifying* occurring more frequently in response to non-MOSTs (especially A4).

The predominant Actions directed to the *whole class* and to the *teacher* differed from those in the data overall, as well as from those with a *same student* Actor. In the case of *whole class*, *collect* and *connect* Actions occurred at the highest rate, each accounting for 19% of Actions; each occurred twice as often for MOSTs as for non-MOSTs (although the numbers are small). An example of a *whole class, collect* response comes from instance G1, where the teacher responded: "[W]ho else has another...did everybody get 4 pi? Give me some more answers." (T5); an example of a *whole class, connect* response from instance G3 is, "Ok, let's compare Pat's method and Chris' method. What are they doing that's similar? What are they doing that's different?" (T23).

In the case when the *teacher* was the Actor, *adjourn* was by far the most common Action at 71% (see Fig. 5 for an example), with *dismiss* Actions occurring 24% of the time. Dismiss responses were common in instance G4, such as this response given by T3: "I would just ignore. I wouldn't address Sam at all. I wouldn't draw attention to the fact that he's just blurted out an answer." These *adjourn* and *dismiss* Actions were primarily in response to non-MOSTs.

Together, the Actor/Action interaction data suggest that, although they predominantly engage the *same student* as the Actor, teachers do correlate different Actions with different Actors. We discuss the importance of this differentiation in the *Alignment with the Core Principles* section where we answer our second research question.

Recognition of student actions and ideas

The Recognition codes operationalize the extent to which the student who provided the SMT would recognize their thinking in the teacher's response. The majority of the 198 responses in the data set either *explicitly* (50%) or *implicitly* (26%) incorporated Student Actions—either their words (verbal) or gestures or work (non-verbal); there were more responses to MOSTs (86%) than to non-MOSTs (65%) classified in these categories. In terms of Student Ideas, the majority of responses (70%) remained *core* to the idea in the SMT, while 9.5% of responses were *peripheral* to the student ideas. As with the Student Action codes, more responses to MOSTs (89%) than non-MOSTs (69%) were classified as *core* or *peripheral.* It is noteworthy that a large percentage of the responses were both *explicit* and *core* (42% of all responses, 44% to MOSTs and 39% to non-MOSTs), meaning that the teachers in this study often honored the SMT by explicitly incorporating the student's actions while also staying focused on the student's core ideas. For example, T23's response to MOST G1, "Ok, Chris, you've given me some words. Come draw me a picture, [show] me what you're referring to as the gap," was *explicit* and *core* as it incorporated both the student's words (gap) and his ideas (having him illustrate his idea visually).

Alignment with the Core Principles based on the nature of the SMT

We now turn our attention to our second research question: To what extent are the variations in teacher responses in our data aligned with our Core Principles? Recall that we use the alignment of a response to the Core Principles (Fig. 2) as a means of interpreting the productiveness of a teacher response. Furthermore, as previously mentioned, the nature of the SMT influences what it means for a response to align with these principles. Thus, we organize this section around the four types of SMT included in our Scenario Interview: MOSTs, Openings, Central instances, and instances that have no mathematical point (Student Mathematics and Cannot Infer). In each section, we first use the Core Principles to theorize what productive responses would likely entail. We then describe the extent to which the teacher responses to those instances in our data adhere to this theoretical alignment.

MOSTs We have theorized that the most productive response to an instance of SMT that is a MOST is to *build on* that SMT—to make the SMT an "object of consideration by the class in order to engage the class in making sense of that thinking to better understand an important mathematical idea" (Van Zoest et al. 2017, p. 36). We see building on a MOST as particularly productive because it aligns with all four Core Principles. The definition of building necessarily implies a *whole class* Actor; such a response would align with the Collaboration Principle since it would position all students to engage with the instance. The most productive Actions in response to MOSTs are those that engage the class in making sense of the SMT, specifically *develop* and *justify* actions, since such responses would align with the Sense-making

Principle. Given that the practice of building makes the MOST an object of consideration by the class, it would seem important with respect to alignment with the Mathematics Principle for the response to incorporate student actions in an *explicit* or *implicit* way and stay *core* to the student ideas to keep their mathematics at the forefront. When all of the aspects of building on a MOST are coordinated, we see them as collectively aligning with the Legitimacy Principle since the response positions students as capable of engaging in substantial ways with a peer's mathematical contribution.

Same student was the most common Actor for the teacher responses to all four of the MOSTs in our Scenario Interview. Responses with this Actor accounted for 84% of responses to G1, 68% of responses to A2, 63% of responses to A3, and 48% of responses to G3. Although the *same student* responses in our data often engaged the student who contributed the instance in sense-making through the use of *develop* and *justify* Actions, these responses did not directly engage the rest of the students and thus did not align with the Collaboration Principle. In fact, only 26% of the teacher responses to instances that were MOSTs had a *whole class* Actor, suggesting that in large part, the teachers in our study were not responding to the MOSTs in our data in the most productive way.

For MOSTs G1 and A2, the *develop* Action was the most common response in our data, accounting for 60% and 80% of responses to these instances, respectively. For MOST A3, the most common response was *justify*, which accounted for 54% of the teacher responses. For these three MOSTs, the initial Actions of the responses provided by the teachers in our study aligned well with the Core Principles, specifically the Sense-making Principle.

We did not see the same alignment with the Sense-making Principle for MOST G3, however, where Pat explained how they [incorrectly] arrived at their solution for the area between two concentric circles. For this instance, the Actions of the teacher responses were quite varied, with the most teachers using *literal* actions (24% of responses), followed by *allow* and *repeat* (16% each). Although it is possible that subsequent teacher Actions could have engaged students in sense-making, for this instance, the initial teacher responses did not do so, and thus, a potential opportunity to engage students in making sense of the SMT may have been lost.

For the three MOSTs where *develop* and *justify* moves were common (G1, A2, and A3), the teachers did in fact stay close to the students' actions and ideas, with 88% of responses to these three instances being either *explicit* or *implicit* and 77% being *core*. This indicates that these responses were aligned with the Mathematics Principle.

Opening Instances classified as Opening, such as G4, satisfy all of the characteristics of a MOST with the exception of timing. Thus, if the pedagogical timing of when the SMT was made public was better, the instance would in fact be a MOST. Because the timing is poor, however, we theorize that the most productive Actor and Action would be for the *teacher* to *adjourn* the SMT, particularly since we have found that bad timing is often the result of the SMT coming out before other students have had sufficient time to engage with a mathematical task. Adjourning the instance to allow all students the opportunity to engage with the mathematics before it is discussed aligns with the Legitimacy and Sense-making Principles since it positions all students (not just those who quickly arrive at a solution) as capable of making sense of mathematics. Additionally, we theorize that for Openings—where the most productive teacher Action is to *adjourn*—it is not necessary for the teacher to be *explicit* or *implicit* in their use of student actions, nor stay *core* to the ideas in the SMT. In fact, doing so would actually misalign with the Mathematics and Legitimacy Principles because, in essence, the teacher would be highlighting the student's words and ideas and then setting them aside.

For Opening instance G4, we found that 75% of teacher responses in our data had a *teacher* Actor, with most of these responses *adjourning* (39% of *teacher* Actor) or *dismissing* (33% of *teacher* Actor) the instance. Although these two Actions have a similar outcome—that the student idea is not taken up at the time it is made public—we see *adjourning* an Opening instance as more aligned with the Legitimacy Principle than *dismissing* the instance because it positions the SMT as an idea that will be taken up later in the discussion. In terms of the Recognition coding, we found that when the Action was *adjourn* or *dismiss,* the Recognition Student Actions was *not* 69% of the time, meaning that the response did not explicitly or implicitly use the student's actions. The Recognition Student Ideas for these responses was always *indeterminate*, meaning that it is not possible to infer a mathematical focus in the response. Thus, the data indicates that the teacher responses to our Opening instance were largely aligned with the Legitimacy, Mathematics, and Sense-making Principles.

Central We theorize that instances classified as Central, such as A1, can be responded to in a variety of ways. These instances contain mathematics that is aligned with learning goals for students, but do not create an intellectual need for students to engage with the instance. Although such instances do not provide a high-leverage opportunity to engage students in a focused discussion of an instance of SMT, they are often the very substance of such discussions. What differentiates a Central instance from a MOST is that it is not a lost opportunity if, rather than situating the instance as the focus of a whole-class discussion, the teacher simply allows the instance to be part of the ongoing class discussion. Thus, to align with the Core Principles, we theorize that a range of responses might be productive. In fact, the only type of response that we theorize might be unproductive is one that diminishes the student idea in some way, as such a response would misalign with the Legitimacy Principle by failing to acknowledge the important mathematics in the student's contribution.

The Actor of the responses to instance A1 in our data was *same student* 80% of the time, which was typical in our data overall. Importantly, we did not see any Actions in our data that would diminish the student idea. Although *justify* was the most common Action (40% of responses), we also saw responses that were coded as *clarify*, *connect*, *literal*, *develop*, *evaluate,* and *repeat*. Our data suggest that the teachers in our study did recognize the centrality of instance A1 since their response was *core* to the student ideas 80% of the time, indicating that they recognized that the SMT included important mathematical ideas. They *explicitly* or *implicitly* incorporated the student actions 76% of the time.

Instances with no Mathematical Point Student Mathematics and Cannot Infer instances of SMT are quite varied, yet share the characteristic that they have no associated mathematical point. This means that although the student may have something important to say, the teacher would not yet know what mathematics underlies their thinking. We have found it valuable to consider two general situations that influence the productivity of a teacher response to such instances: (a) those where the teacher thinks there may be something mathematical worth pursuing, and (b) those where they do not. We focus our discussion on the former situation, since the most productive response in the latter situation would be for the *teacher* to *dismiss* the instance. For the first situation, we theorize that the most productive response depends on whether the teacher intends to take up the SMT at that time; that is, whether the pedagogical timing seems good. If the teacher intends to take up the SMT at that time, a *same student* Actor seems most productive (aligned with the Mathematics and Legitimacy Principles), particularly paired with an Action of *clarify* or *develop* to make possible the articulation of the mathematics that underlies their thinking. Actions such as these would align best if they either *explicitly* or *implicitly* use the student's actions and stay *core* to the student's ideas, further aligning the response with the Mathematics Principle. If, on the other hand, the teacher did not intend to take up the SMT at that time, it would be more productive for the *teacher* to *adjourn* or *dismiss* the instance outright, since engaging with the instance while not intending to not take it up would misalign with the Mathematics and Legitimacy Principles (as argued in the Opening section above). As before, these responses do not require incorporation of the student actions or ideas. Regardless of the situation, it would seem unwise to turn Student Mathematics or Cannot Infer instances over to the *whole class* since the mathematics they were being asked to engage with would be unclear.

In our data, we found that all but one teacher did direct their response to Cannot Infer instance A4 to the *same student,* and 75% of these responses included a *clarify* Action, indicating that most of these teachers recognized that the situation called for returning to the student who contributed the SMT for additional information. Furthermore, 84% of the teacher responses were *explicit* and 92% were *core,* indicating that the teachers in our study used the student's words and ideas when specifying what aspect of the SMT required clarification—a response that aligns well with the Mathematics and Legitimacy Principles.

For the Student Mathematics instance G2 (where Pat claims to have arrived at the same answer as another student in a different way), there were two equally common and quite different sets of responses in our data. These sets of responses correspond with whether the teacher intended to take up the SMT at that time, as we theorized above. One subset of teachers gave a response that took up the SMT, providing responses similar to those in Cannot Infer instance A4. These teachers chose to return to the *same student* to *develop* Pat's thinking (48% of responses). These responses either *explicitly* or *implicitly* incorporated student actions 67% of the time and were *core* to their ideas 75% of the time. As with instance A4, such responses align with the Mathematics and Legitimacy Principles. The other subset of our teachers gave responses that did not take up the SMT at that time because they wanted to continue to pursue the idea that was already under discussion. The common response for this group was for the *teacher* to *adjourn* the SMT (40% of responses). As with the *adjourn* responses discussed in the Opening section, these responses were mainly coded *not* for Student Actions (80% of the time) and *indeterminate* for Student Ideas (100% of the time). These two different sets of common responses to this instance accounted for 88% of all responses in our data. Overall, the data indicated that these teachers responded to instances with no mathematical point (Student Mathematics and Cannot Infer) in ways that aligned with the Core Principles.

Conclusion and implications

This study investigated how teachers' responses to SMT vary depending on the potential of the SMT to support student mathematical learning, as well as the ways in which this variation aligns with our Core Principles. The findings contribute to the teacher response literature and have implications for teachers, professional developers, and researchers.

The findings of our study extend previous research that examined the extent to which teachers differentiate their responses based on type of SMT (e.g., Bishop et al. 2016; Drageset 2015). It does so by focusing not just on how teachers differentiate their Actions, but also on how they differentiate other aspects of their response—the Actor and Recognition by the student who contributed the SMT—as well as how that differentiation relates to the potential of an instance of SMT to support student learning. In terms of Actor, we found that teachers tended to direct their response to the *same student* who contributed the SMT the majority of time for both MOSTs and non-MOSTs. This suggests that teachers largely do not discriminate the actor of their response based on the potential of the SMT. We also found, however, that in the small number of instances where teachers did direct their response to the *whole class*, it was almost always in response to MOSTs, and when the *teacher* was the Actor, it was almost always in response to non-MOSTs. Thus, we see that although *same student* may be the go-to Actor for teachers, there is at least some indication that they have a sense of when it might be more productive to instead direct their response to the whole class or to address the SMT themselves.

In terms of Recognition, we found that teacher responses often incorporated the students' actions and stayed close to the ideas in the SMT, signaling that the teachers in our study proposed actions that did indeed value students' contributions. The only time this closeness was typically not the case was when the teacher was adjourning or dismissing a non-MOSTs, which we theorize was a productive time not to do so. This finding suggests that teachers were able to discriminate, based on variations in the mathematical potential of the instances, when incorporating the students' ideas and actions was productive and when it was not. Collectively, our findings provide more nuanced information about the ways that teachers do or do not differentiate their responses.

Our findings have implications for teacher professional development focused on helping teachers become more nuanced in the way they vary their responses based on the mathematical potential of the SMT (and as a consequence for teachers who are interested in developing this aspect of their practice). In many cases, it may be that small adjustments to teachers' responses could make a significant difference in the alignment of the responses to the Core Principles. For example, if the majority of teachers' responses align with the Legitimacy and Mathematics Principles, but engage only the student who contributed the instance (as was the case in this study), teachers (and professional development work) could focus specifically on exploring the potential value in directing a response to the whole class, and when it would and would not be appropriate to do so to better align the response with the Collaboration Principle. Specifically, this work could help teachers to develop and refine strategies for inviting the whole class to consider particular instances of SMT. Focused efforts such as this would allow teachers and professional developers to leverage teachers' strengths and thus develop teachers' practice more effectively, potentially accelerating teachers' abilities to engage students in meaningful mathematical discourse.

Our findings also have implications for further research on teachers' responses to student thinking. Although we found evidence of a certain degree of discrimination based on the nature of student thinking (and its potential for productive use), it is unclear to what extent teachers are aware of how they are making these decisions. It would be helpful for teachers and teacher educators to explore in what ways teachers make explicit this tacit understanding. Beyond expanding the literature base, such research would likely inform teachers and professional development providers, as the means for eliciting the data for research of this sort is also often an effective means for engaging teachers in coming to better understand and expand their own practice.

The study has two important limitations that also suggest directions for further research. First, our data were teachers' anticipated rather than actual responses. As has been documented in other research (e.g., Van Zoest et al. 2002), we recognize that there may be discrepancies in how teachers report they would respond to an instance of SMT in an interview setting and what they actually do in their classroom. Second, we can only infer based on the research of others (e.g., Ing et al. 2015) that teaching aligned with the Core Principles will improve student performance. Addressing these limitations requires classroom-based studies of how teachers respond to actual instances of SMT to understand how these responses relate to both what teachers report they would do and to student performance. Carrying out such research in the classroom has the potential to further advance our understanding of teacher responses that support student mathematical learning.

In summary, this study advances research on teachers' responses to SMT by considering teacher responses in relation to the potential of the SMT to which the teacher is responding. Examining teacher responses to a common set of instances of SMT, as we did in this study, allowed us not only to consider the extent to which teachers differentiated their responses, but also to highlight how variations in the Actor, Action, and Recognition aspects of teacher responses matter in determining the alignment of the response to a particular vision of teaching. This approach has the potential to advance our understanding of the related work of teacher noticing (e.g., Jacobs et al. 2010) as it illuminates an important connection between the interpreting and deciding aspects of this practice—the productivity of deciding depends on the interpretation of the potential of the SMT. In general, understanding the nuances of a teacher response in relation to the nature of the SMT will allow us to move beyond classifying particular responses as uniformly productive or unproductive, to illuminating when and how a range of teacher responses can support ambitious teaching.

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