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ORIGINAL ARTICLE

# Evaluating the impact of a Spatial Reasoning Mathematics Program (SRMP) intervention in the primary school

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## Abstract
As part of the *Connecting Mathematics Learning through Spatial Reasoning* project, a Spatial Reasoning Mathematics Program (SRMP) intervention was implemented with one cohort of 30 students in grades 3 through 4. The SRMP embedded transformation skills in learning sequences comprising repeating and growing patterns, 2D and 3D relationships, structuring area and perimeter, directionality and perspective-taking. Analysis indicated a significantly better gain by the experimental group on the PASA-2 measure of awareness of pattern and structure and on the PASA-Sp assessment of spatial ability at the post-SRMP period. However, there were no significant differences found between groups on the PATMaths4 test of mathematics achievement. Qualitative analyses indicated that students demonstrated the development of complex spatial concepts well beyond curriculum expectations. The SRMP highlighted the important role of patterning and spatial structuring in the formation and representation of spatial concepts.

**Keywords** Mathematics education · Spatial reasoning · Intervention · Primary school · Assessment

## Introduction

The term spatial reasoning can be interpreted widely, depending on the disciplinary perspective and purpose of a study, but usually includes processes such

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as visualisation, mental rotation, symmetry, perspective-taking, locating, orienting, decomposing/recomposing shape and navigating. We define spatial reasoning as “the ability to recognize and (mentally) manipulate the spatial properties of objects and the spatial relations among objects” (Bruce et al. 2017, p. 147). Spatial reasoning plays a fundamental role within mathematics learning and beyond. We consider that mathematics learning is a complex, dynamic system of interconnected components that are fundamentally dependent on spatial reasoning, rather than one based on quantitative or numerical concepts as is often assumed (Davis and the Spatial Reasoning Study Group 2015; Mulligan et al. 2018). Indeed, the development of number concepts may be primarily spatial in origin (Mulligan and Woolcott 2015). Particular spatial skills have been found predictive of mathematics achievement, are malleable and can be developed from early childhood (Hawes et al. 2017; Uttal et al. 2013).

The recent surge in interest in Science, Technology, Engineering and Mathematics (STEM) has raised awareness of the role of spatial reasoning as a result of studies showing that success in STEM is related to spatial competencies and can predict entry to the STEM professions (Wai et al. 2009). However, evidence of causal connections between spatial reasoning and STEM competencies is sparse (Mix 2019).

“Spatialisation” of the curriculum has emerged across educational contexts as both critical and feasible for early childhood and primary education, given its role in mathematical development and STEM learning (Buckley et al. 2018; Davis and the Spatial Reasoning Study Group 2015; Newcombe 2017; Rittle-Johnson et al. 2019; Verdine et al. 2017). However, there has been no serious attempt to view spatial reasoning as core to the national curriculum and there have been few studies evaluating the effect of spatial reasoning programs on mathematical achievement and learning generally.

In keeping with the purpose of this special issue, we aim to contribute to the research on how spatial reasoning can be developed in the primary (elementary) school and its role in mathematics learning. Our focus is on establishing connections between mathematical patterns and structures and the development of spatial reasoning. Our project, *Connecting Mathematics Learning through Spatial Reasoning,* builds on prior research on mathematical pattern and structure (Mulligan and Mitchelmore 2013, 2018; Mulligan et al. 2020) and the work of the Spatial Reasoning Study Group (Bruce et al. 2017) and complements the work of Lowrie and colleagues (Lowrie et al. 2017; Lowrie et al. 2018; Ramful et al. 2015) and Hawes and colleagues (Hawes et al. 2017). Our ultimate goal is to develop a spatial reasoning mathematics intervention program, aligned with the *Australian F-10 Curriculum: Mathematics* (Australian Curriculum and Reporting Authority [ACARA] 2015) that can be implemented by classroom teachers in the regular primary classroom.

## Research perspectives

A number of studies have found that spatial reasoning is correlated with mathematical performance and can predict future achievement from pre-school years (e.g. Carr et al. 2018; Verdine et al. 2017). A pertinent study by Verdine et al. (2013) investigated 3-year olds' spatial assembly skills and found that spatial skills independently predicted a significant amount of the variability in concurrent mathematical performance. Another study focused on imaginary perspective-taking (IPT) with kindergartners and investigated whether children could imagine *what* is visible from a particular point of view and *how* an object or scene will look from a particular point of view (van den Heuvel-Panhuizen et al. 2014). The sample consisted of 4- and 5-year-old kindergartners in the Netherlands ($N = 334$) and in Cyprus ($N = 304$). In both countries, mathematics ability was significantly related to IPT performance. Most recently, Rittle-Johnson et al. (2019) assessed the mathematical knowledge of 73 preschoolers and found children's understanding of repeating patterns and spatial skills was predictive of concurrent and future mathematics achievement.

Such results suggest a possible causal relationship between spatial reasoning and mathematics achievement which can be confirmed using intervention studies. Recent reviews of research on spatial reasoning and spatial intervention studies in the primary school have provided an analytic synthesis of research and practice in this area (Woolcott et al. 2020a; Woolcott et al. 2020b). The review identifies the need to investigate core spatial knowledge and skills within the mathematics curriculum and for further research on interventions that provide sustainable, school-based spatial reasoning programs. We summarise a few of the most relevant studies below.

There have been a growing number of studies in the pre-school and primary school that indicate an effect of a spatial reasoning intervention on aspects of mathematics performance (Bruce et al. 2015; Cheng and Mix 2014; Kidd et al. 2014; Hawes et al. 2017; Lowrie et al. 2017; Lowrie et al. 2018; van den Heuvel-Panhuizen et al. 2014). Other studies have focused on specific spatial concepts or processes, linked to but not necessarily made explicit by mathematics curricula. For example, Casey et al. (2008) and Clements and Sarama (2007) focused on block construction and geometric concepts with young children. An intervention study with low-achieving first graders, Kidd et al. (2014) implemented an instructional program encompassing symmetrical, rotational and growing patterns. The study found that the program had significant effects on both reading and mathematics, with gains of 4 to 6 months. This study supported pedagogy that extended beyond simple repetitions to include more complex spatial patterns. In a small-scale intervention (Cheng and Mix 2014), spatial training in the form of mental rotation exercises was provided to an experimental group for 40 min. The intervention group significantly outperformed the control group on a mental rotation test but no differences were found on other spatial measures. The effect of the spatial training on mathematics achievement generally was limited to addition and subtraction tasks that required the completion of missing terms.

Bruce and colleagues of the Spatial Reasoning Study Group developed and evaluated a "spatialised" curriculum in grades 1 to 3 informed by laboratory-based research findings (Bruce et al. 2015). The program included static and dynamic symmetry, congruence and transformations and mental rotation that cultivated spatial reasoning in mathematically rigorous ways. The tasks extended beyond curriculum expectations and in particular encouraged dynamic aspects of spatial reasoning. Pre- and post-assessments revealed significant growth in students' spatial language, geometrical reasoning, mental rotation and numerical comparison. The researchers emphasised the need to reform curricula to integrate spatial reasoning activities.

Hawes et al. (2017) implemented a 32-week spatial reasoning intervention in grades K–2 employing six experimental and three control classes. Their program included five geometry and five spatial visualisation activities, implemented by classroom teachers. The study found significant differences in favour of the intervention group across measures of mental rotation, visual-spatial geometry and spatial language. In terms of performance in mathematics generally, relative gains were moderate and limited to numeral recognition.

Lowrie et al. (2017) developed and implemented a 10-week spatial training intervention program in grades 5 and 6 comprising 20 h centred on mental rotation, spatial orientation and spatial visualisation skills that replaced part of the regular mathematics program in ten intervention classes. Classroom teachers engaged in 5 days of intervention training that focused on an Experience-Language-Pictorial-Symbolic-Application (ELPSA) framework prior to their implementation of the program. On measures of mathematics performance and spatial skills, the study found that students in the intervention group significantly improved and outperformed those in the control group. However, on the test of general mathematics performance, the intervention group improved significantly on measurement and geometry but not on the number. Following this study, Lowrie and colleagues (Lowrie et al. 2018) implemented and evaluated the spatial training intervention program with students in grades 3 and 4 in order to extend the study to a wider age range. In total, the studies employed 337 students across 15 grade 3–6 classrooms. The effectiveness of the spatial program for the grade 3 and 4 students, measured on a test of spatial ability for the intervention group was significant, showing an effect size of $d = 0.83$, which was higher than previously reported in other studies (Uttal et al. 2013). The improvements are attributed to a focus on spatial concepts and the ELPSA pedagogical approach.

## The pattern and structure approach

The role of visual pattern recognition and analysis re-emerged in the 1990s and early 2000s as a focus of early mathematics education research (Papic et al. 2011; Warren and Cooper 2008; Woolcott et al. 2020b). Over two decades, the Australian Pattern and Structure Project investigated the development of patterning and structural relationships among children aged 4 to 8 years (Mulligan and Mitchelmore 2013). The focus has been on spatial structuring, defined as

> ... the mental operation of constructing an organization or form for an object or set of objects. It determines the object's nature, shape, or composition by identifying its spatial components, relating and combining these components,

> and establishing interrelationships between components and the new object (Battista 1999, p. 418).

Mulligan and Mitchelmore (2009) showed that children's ability to recognise mathematical pattern and structure in different mathematical contexts was highly correlated and could be regarded as fundamental to mathematics learning and understanding. They proposed the term Awareness of Mathematical Pattern and Structure (AMPS) to describe this general ability and developed an interview instrument, the Pattern and Structure Assessment (PASA), to measure AMPS. Three versions of PASA, suitable for kindergarten to grade 2, each contain 14–16 tasks involving repeating, growing and spatial patterns, rectangular and triangular arrays and grids, partitioning of lengths and regions, grouped counting, linear scales, and ten frames and hundred charts. Students' responses are scored according to the degree of AMPS they demonstrate. In a validation study of 618 students in grades K–2, it was shown that an overall AMPS score can be obtained from PASA responses which forms a Rasch scale across the three versions (Mulligan et al. 2015).

An analysis of the tasks included in the PASA showed that AMPS involves five substructures: Sequences, Shape and Alignment, Equal Spacing, Structured Counting and Partitioning (Mulligan et al. 2015). Two of these substructures (Shape and Alignment and Equal spacing) are explicitly spatial. But Sequences and Structured Counting also have spatial equivalents (e.g. an ABC repeating pattern is essentially spatial and readily links via grouping by 3s to multiplication by 3) and Partitioning can be purely spatial (e.g. dividing a rectangle into squares) or have an obvious spatial analogue (e.g. calculating fractions). PASA may, therefore, be regarded as to a large extent an assessment of spatial reasoning.

Mulligan and Mitchelmore (2016) also developed a Pattern and Structure Mathematics Awareness Program (PASMAP), a complete mathematics curriculum for grades K–2 designed to promote students' AMPS while learning the mathematics prescribed in the Australian F-10 Curriculum: Mathematics (ACARA 2015). In the PASMAP, key mathematical concepts and processes across number, measurement, geometry and statistics are encountered in several "pathways" through repeating and growing patterns, counting and grouping, grid structures, 2D and 3D shape, partitioning and sharing, base ten structure, additive and multiplicative structures, structuring measurement and data, and transformations. A 2-year longitudinal study of over 300 kindergarten students in Sydney and Brisbane (Mulligan and Mitchelmore 2018) showed conclusively that PASMAP did result in an improvement in AMPS, although little effect was shown on students' scores on the I Can Do Maths test (Doig and de Lemos 2000).

A major implication from our work on PASA and PASMAP is that AMPS plays a central role in mathematics learning and that AMPS can be developed by the study of pattern and structure in mainly spatial contexts. To investigate the development of AMPS in grades 3–5, it was decided to design an intervention that used the PASMAP approach but focused exclusively on spatial reasoning. It was predicted that AMPS developed in this way would generalise to improved mathematics performance overall. The resulting intervention was called the Spatial Reasoning Mathematics Program (SRMP).

## Design and development of the Spatial Reasoning Mathematics Program

The Spatial Reasoning Mathematics Program (SRMP) was designed not only to retain a focus on spatial patterning and structure but also include the three constructs of mental rotation, spatial orientation and spatial visualisation identified by Lowrie and colleagues (Lowrie et al. 2017). The spatial content was also informed by the work of Bruce, Hawes and colleagues (Bruce et al. 2015; Bruce and Hawes 2015; Hawes et al. 2017).

For this study, the SRMP lessons were designed to replace about 25% of the grade 3 and 4 mathematics learning program, replacing most of the regular mathematics syllabus content that focused on Patterns and Algebra and Geometry. The research team first identified patterning and spatial concepts and processes that would be challenging for grade 3 and 4 students, categorised and sequenced them into seven main components and organised them into approximately 40 lessons. The SRMP components were developed so that they incorporated the mandatory syllabus content but extended the scope and depth of patterning and geometric concepts to include growing patterns, transformations, interrelationships between 2D and 3D shapes, spatial visualisation and orientation including perspective-taking. The program also developed Working Mathematically processes in reasoning, problem-solving and communicating key ideas (Board of Studies NSW 2012) akin to the Proficiences developed in the *Australian F-10 Curriculum: Mathematics* (ACARA 2015). Because our prior studies had shown evidence that patterning was fundamental to spatial thinking, repetitions and growing patterns preceded the components on spatial orientation. Transformation processes of rotation, symmetry and reflection were integrated in the components, for example, in patterns, nets of solids, directionality and perspective-taking. Establishing connections between geometric models and number patterns was embedded in several components such as in triangular numbers and Fibonacci sequences, also aimed at encouraging pattern as generalisation. Using Bee-Bots as an iterable unit allowed students to use measurement skills and visualise, predict and symbolise pathways. Tasks on perimeter were loosely connected with the notion of a growing pattern of rectangles but connected with the work on directionality. Perspective-taking tasks were connected with transformation skills.

Table 1 shows the sequence of components and sub-components and some examples of key tasks.

In the next step, the seven components were broken down into about 40 lessons following the Modelling-Representing-Visualising-Generalising-Sustaining pedagogical model used in the PASMAP (Mulligan and Mitchelmore 2018). This model shares similarities with the ELPSA framework implemented by Lowrie and colleagues (Lowrie et al. 2017; Lowrie et al. 2018). Both approaches emphasise visualisation processes and representational (pictorial) and symbolic application of concepts with a focus on making predictions. However, in the PASMAP, the visualisation process is more focused on visual memory where students are required to represent mathematical models or task solutions from memory, with the development of their drawings, justifications and explanations being supported by teacher scaffolding and group discussion. Visualising and reproducing previously constructed representations allow the student to demonstrate the explicit structural features of their representations. The aim is to encourage students to look for patterns and similarities between models and

**Table 1** Components of the Spatial Reasoning Mathematics Program (SRMP)

| Component | Sub-component | Sample tasks |
|---|---|---|
| 1. Repeating patterns | Border patterns | Generate an ABB pattern to fit a given border. |
| | Symmetry | Generate ABB patterns in a 3 × 5 border and identify lines of symmetry within the pattern. |
| | Environmental patterns | Find repeating patterns in the environment. |
| | 3D patterns | Identify repeating patterns in a 3D structure. |
| 2. 2D and 3D relations | Representing solids on isometric paper | Create a large block using interlocking cubes and extend the pattern on isometric paper. |
| | Nets of solids | Given a rectangular prism made from polydron squares, find as many nets as possible. |
| 3. Growing patterns | Triangular numbers | Using multilink cubes, make the first five triangular numbers. How are the shapes the same? How are they different? How many cubes are needed to make the next biggest triangular number? How do they fit together? |
| | | Imagine stacking cans in a triangular shape so that each row has one more can than the row above it. Could you make a display with 55 cans? |
| 4. Directionality | Walked paths | Create and walk a simple path and draw the directions with arrows. Then represent the directions graphically. |
| | Bee-Bot paths | Create a path with 10 cm lengths and then predict the code needed to navigate the Bee-Bot path. |
| 5. Shape and perimeter | Bee-Bot steps | Program the Bee-Bot to create a rectangular path with a 10-step perimeter. |
| | Rectangles with a fixed perimeter | Create as many rectangles as possible that have a perimeter of 12 rods of the same length. |
| 6. Fibonacci sequences | Rectangular spirals | Create the Fibonacci sequence and then represent it as a spiralling path. |
| | Investigate areas within a Fibonacci spiral | Make squares whose sides form the Fibonacci sequence and fit them together to make a spiral |
| 7. Perspective-taking | Drawing different views | Draw a 2D representation of an L-shaped block from four different perspectives. |
| | Visualising different views | Complete the missing views if only the front and left views are given. |
| | Drawing top views | Draw the top view of the more complex models. |

representations in order to make simple generalisations about the spatial concepts and relationships they are investigating.

## Aims of the study

The aims of this study were to describe the feasibility of the SRMP in practice and to gather some preliminary data on its effectiveness. We address two key research questions:

1. What are the effects of the SRMP intervention on (i) spatial reasoning, (ii) Awareness of Mathematical Pattern and Structure (AMPS) and (iii) students' mathematical performance?

2. What are the key aspects of the SRMP that support the development of spatial reasoning?

## Method

### Participants

As part of a larger study, an SRMP intervention was trialled in grades 3 through 4 at a government primary school in metropolitan Sydney, New South Wales (NSW), Australia. The school serves families from middle-income backgrounds and draws students from predominantly Asian backgrounds. Students whose first language was not English comprised 92% of the school population. The sample's academic achievement level on the National Assessment Plan for Literacy and Numeracy [NAPLAN] was ranked highly, with 86% of students achieving above the NSW State average in year 3 (ACARA 2019). Two grade 3 classes and their teachers were recruited to participate in the study. One class was randomly assigned as an experimental group and the other became a comparison group. The teacher of the experimental class agreed to implement the SRMP while the comparison class continued with their regular mathematics program for the duration of the intervention.

The resulting sample totalled 62 grade 3 students (32 males and 30 females), with 32 in the experimental group and 30 in the comparison group. The students were aged between 8 years 1 month and 9 years 7 months (mean 8 years 10 months) at the commencement of the study. Of these, 57 were retained in grade 4 (30 in the experimental group and 27 in the comparison group). The teacher of the grade 4 experimental class taught the same set of students as they had in grade 3.

### Data collection

To assess the impact of the SRMP, both quantitative and qualitative data were collected from the participating students. The quantitative assessments were as follows:

- Raven's Coloured Progressive Matrices [RCPM] (Raven 2004), a measure of general ability, was administered pre-intervention.
- The Progressive Achievement Tests of Mathematics, Version 4 [PATMaths4] (Stephanou and Lindsey 2013), a measure of mathematics achievement, was administered both pre- and post-intervention.
- The Pattern and Structure Assessment, Version 2 [PASA-2] (Mulligan et al. 2015), described above, was administered post-intervention.
- The Pattern and Structure Assessment-Spatial [PASA-Sp], an experimental measure of spatial reasoning, was also administered post-intervention.

Because the PASA-2 had a high ceiling in the previous validation study (Mulligan et al. 2015), it was expected that it would still provide a valid measure of AMPS among grade 3 and 4 students. The intercoder reliability on the PASA-2 in this study was 0.91.

The PASA-Sp employed a PASA-type interview using various spatial reasoning tasks judged to be suitable for grade 3 and 4 students. Four items—on rotation, partitioning of shapes, directionality and perspective-taking—were included. An earlier pilot study with a sample of 28 grade 4 students found that the four items taken together provided a valid and reliable measure of spatial reasoning that was sensitive enough to demonstrate the impact of the SRMP. Students' explanations and drawn responses to these four tasks were scored in the same way as the PASA-2 and the intercoder reliability was 0.87.

The qualitative data included observation and evaluation notes taken by the researchers and compiled following each learning session. These data were recorded and stored in a dedicated digital portfolio for each student, along with video recordings of the SRMP process in the classroom. Many student work samples, including students' written explanations and reflections, were also collected and analysed. In addition, the teacher implementing the SRMP was interviewed at the beginning, middle and end of each school year to ascertain her views about the impact of the program on student learning.

## The implementation of the SRMP

Prior to the commencement of the SRMP, the research team provided three professional learning sessions with the teacher on the aims and conceptual basis of the program and provided exemplars of how the PASMAP pedagogical model had been effective in prior classroom studies. She was then provided with explicit lesson guidelines that exemplified the spatial concepts and pedagogical sequence to be followed together with instructional pointers on what to notice and question about students' thinking and representations. The teacher was given no specific guidelines on how or what to teach in the remaining 75% of the time apart from teaching all the mandatory topics that had not been included in the SRMP.

A member of the research team who was an experienced mathematics teacher worked with the experimental teacher in implementing the SRMP. The teacher was given release from teaching time to debrief with the researcher, to discuss and reflect on the students' learning and to plan further lessons. In consultation with the researcher, the teacher reviewed each lesson prior to teaching and made any adjustments that were needed to accommodate students' range of abilities and to integrate the content into her regular program. The researcher acted as a mentor to the classroom teacher and collaborated in teaching the lessons using a team-teaching approach. Students mostly worked in mixed ability groups, followed by class discussion in which students shared their findings and representations and were encouraged to make simple generalisations. Following each lesson, a de-briefing meeting was conducted with the classroom researcher and a senior researcher to discuss the effectiveness of the pedagogical process and to review evidence of student learning. Student work samples and assessment tasks were reviewed in conjunction with planning for the next lesson. In a cyclic process of planning, teaching, review and refinement (Cobb et al. 2003), the SRMP implementation was characterised by systematic assessment and analysis of students' spatial reasoning for each sub-component. During the implementation of the SRMP, some lessons were also observed by other members of the research team and digital recordings of a representative sample of lessons were made for further analysis. This process ensured a high degree of fidelity in the program implementation.

Students in the grade 3 and grade 4 comparison classes completed the regular mathematics program. This program did not include any of the components or tasks included in

**Table 2** Mean scores on pre-intervention assessments and tests for differences

| Assessment | Group | Mean score | Mean rank | $U$ | $z$ | $p$ |
|---|---|---|---|---|---|---|
| RCPM | Experimental | 113.4 | 26.8 | 375 | − 0.867 | 0.386 |
| | Comparison | 109.6 | 30.4 | 489 | | |
| PATMaths4 | Experimental | 124.8 | 30.1 | 496 | − 0.225 | 0.821 |
| | Comparison | 125.0 | 30.4 | 464 | | |

the SRMP and it was observed that the teacher followed a teacher-directed style that focused on procedural skills. For algebra, the program was limited to extending some numerical repeating patterns, determining missing numbers in number sentences and recognising the properties of odd and even numbers. For geometry, the students named, described and sketched standard 3D objects and 2D shapes. They combined and split 2D shapes to create other shapes, compared angles using informal means and classified angles according to their size. For directionality, students used a grid-reference system to describe position and compass points to give and follow directions.

In grade 3, the SRMP was implemented over two successive school terms of 10 weeks' duration each, with lesson time averaging approximately 2 h per week. In grade 4, the SRMP extended over all four school terms, with an average lesson time of one and a half hours per week. The number of mathematics lessons, the mathematics lesson time slot and lesson duration were systematically recorded and found to be equivalent for the intervention and comparison classes in both grades 3 and 4.

## Results

### Pre-intervention assessments

Table 2 shows the group mean scores on the assessments administered before the intervention. Because the data were not normally distributed, 2-tailed Mann-Whitney $U$ tests were used to test for significant differences, but none were found. Incidentally, the results confirm that the entire sample was somewhat above average in ability.

## Descriptive analysis of the intervention: experimental group

A descriptive analysis of the SRMP intervention is provided below. We summarise students' responses to each sub-component and provide representative work samples drawn from the qualitative lesson data.

In the "repeating patterns" component, students were generally able to repeat the pattern in the border and identify the unit of repeat and the number of times it was replicated with ease. By visualising some of the border patterns as reflections or by folding them or drawing the lines of symmetry, they were able to distinguish between mirror symmetrical and asymmetrical patterns. However, a common error was to draw the lines of symmetry of the shape instead of the lines of symmetry of the pattern. Students found several repeating patterns in the environment, but patterns embedded in

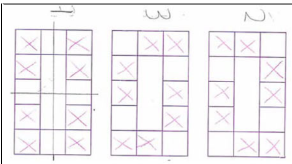
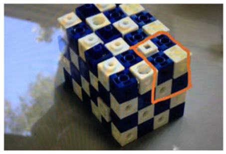
| | | |
|---|---|---|
| <br>"I made 3 different borders. Each one had 4 units of repeat. The #1 picture has 2 lines of mirror symmetry in the pattern. The others don't have any." | <br>"This one is a repeating pattern too… It's an ABCC pattern. Girl, boy, girl, girl. Girl, boy, girl, girl…I noticed it could also be girl, boy, short girl, tall girl." | <br>"I made this block and photographed it with my iPad. I then used Book Creator to show the 2 x 2 x 2 AB part that keeps repeating." |

**Fig. 1** Student work samples from the SRMP "repeating patterns" component

a cube were challenging for most of the students. Figure 1 shows three examples of students' work.

The focus of the "2D and 3D relations" component was on the interrelationship between 3D solids and their 2D representations. Drawing on isometric paper challenged students to analyse 3D relations more carefully, as did the representation of nets of solids on grid paper. This component also challenged students' transformation skills in recognising rotations or reflections of a net and built on their understanding of repeating patterns. Figure 2 shows three representative work samples.

The "growing patterns" component promoted the use of reasoning about the structure of shapes. Activities using multilink blocks opened up new ways of thinking about triangular numbers as students used their spatial skills to analyse the structure of the patterns. Physically manipulating the models provided a means of working mathematically in response to questions students posed, such as "What if I joined the 1st and the 2nd, and then the 2nd and the 3rd?" or "What would happen if I joined two of the same number?" Figure 3 shows some examples of students' work.

The "directionality" component focused on visualising, representing and constructing pathways. Students initially experienced considerable difficulties drawing arrows on a paper to denote the number of steps and direction taken, but programming a sequence of directions using a Bee-Bot helped them to analyse the structure of paths and enabled them to predict the commands that were needed to navigate their Bee-Bot through complex obstacle courses. Figure 4 shows some illustrative work samples.

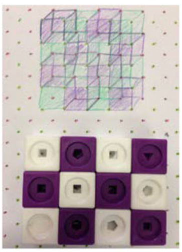
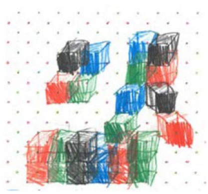
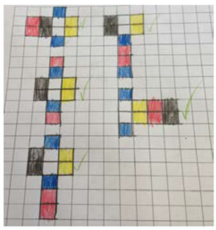
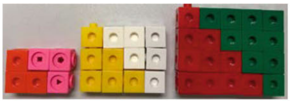
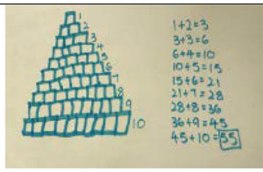
| | | |
|---|---|---|
| <br>"Drawing the bottom row was easy, but the other rows took longer to do. I found it easier to draw the cubes on top of each other but going back was hard" | <br>This student had considerable difficulty representing the pattern of cubes. At times the perspective changed completely. | <br>"I asked my friend to tick the nets that would make my rectangular prism. ... I made sure each net wasn't a rotation or reflection of one that I already drew" |

**Fig. 2** Student work samples from the "2D and 3D relations" component

“If you put the 1st and 2nd together, you get a square number. Hey, it works with the 2nd and 3rd together too”.



“If you put 2 of the 2nd together you get 2 x 3 = 6 and that was 2 high and 1 more across. If you put 2 of the 3rd together you get 3 x 4 = 12 and that was 3 high and 1 more across. So the one in the 10th spot would be …[thinking]… 10 high and 1 more across, so 10 x 11 = 110!”



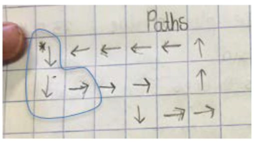

Some students modelled the problem with cups and then created a diagrammatic and numerical solution. Others could see the pattern developing without constructing the display.

**Fig. 3** Student work samples from the “growing patterns” component

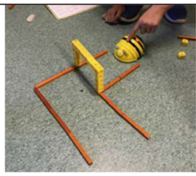
The aim of the “shape and perimeter” component was to help students become aware that the length of the perimeter of a rectangle does not determine its shape. Such an awareness was seen as a valuable precursor to the understanding that area and perimeter are different, independent properties. Most students recognised the advantage of creating a sequence of rectangles by systematically increasing their width by one unit, and all were able to observe and explain why the shapes eventually started repeating but in a reverse order and rotated by a quarter turn. Some student work samples are shown in Fig. 5.

The “Fibonacci sequences” component was designed to link the spatial reasoning skills of directionality, rotation and structural analysis using a novel example of a growing pattern. It was also an introduction to the idea of a recursive pattern. Students particularly enjoyed this experience and especially the opportunity to apply concepts they had encountered in earlier components.

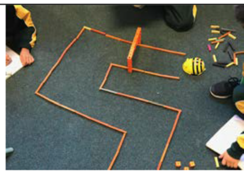
The final component, “perspective-taking”, like the “2D and 3D relations” component, focused on the representation of 3D shapes on 2D paper but with the added complication of requiring the student to place themselves in someone else’s position. Perspective-taking tasks (Fig. 7) were considered the most difficult of the seven components because mental rotation was involved and students were expected to have had little experience of using models to visualise different views. Some students found




Students were asked to take 1 step forward from the door and turn right and then to draw their path. Almost every student initially made the same error [the circled section].

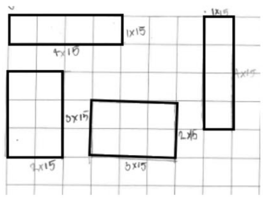


One group of students designed a path that required the Bee-Bot to travel under a bridge, turn left, and then exit the path. “At first, we forgot how long a Bee-Bot step was, so we changed it from F1, TL, F1 to the correct one [F2, TL, F2].”

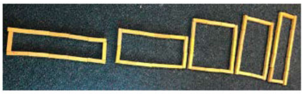


This group created a complex maze and correctly predicted the coding. They then challenged themselves to predict the code needed to return the Bee-Bot to its starting point without rotating through two consecutive right turns.

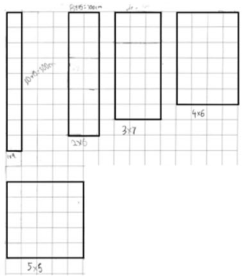
**Fig. 4** Student work samples from the “directionality” component



"I could only draw two rectangles [with a 10-step perimeter]. The other two were my first ones but just rotated."



"The skinniest rectangle is this first one [with a perimeter of 12 rods] because you can't have a side that's skinnier than one. And then each rectangle is one more wider. hmmm…. the square is a problem because it's not a rectangle but it fit with the pattern so we left it in. The first and last rectangles are the same, just rotated, just like the 2nd and 4th rectangles".



"The width is getting bigger and the length is getting smaller. I have drawn 5 rectangles. If I rotate these there would be 9. I think there are 5 unique rectangles with a perimeter of 20".

**Fig. 5** Student work samples from the "shape and perimeter" component

these tasks relatively easy and were able to take different perspectives without using models whereas others needed the concrete aid and still had difficulty.

Students' developing spatial reasoning exemplified in Figs. 1, 2, 3, 4, 5, 6 and 7 was reinforced by the teacher's (T) responses to the post-intervention interviewer (I). A pertinent excerpt follows:

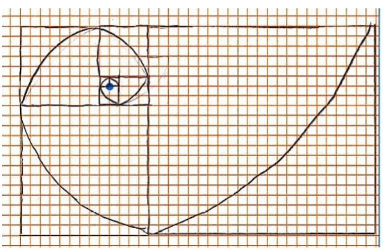
I: How do you think the student's spatial reasoning has developed?

T: They are much more aware of geometric things like rotations and reflections. They are able to see the shapes in their mind and describe these, and draw them without copying given shapes. This is beyond what we would have usually done in class.

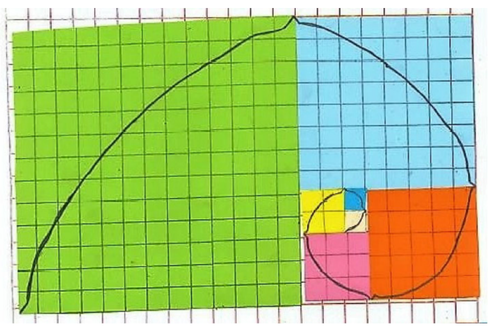
I: Are there any other things about their learning that you have noticed?

T: The tasks have helped these students develop their language and express these ideas. Before they were reticent to say much…I think they have learned to explain these geometric ideas.

I: How did explaining help them develop reasoning?



"The squares were hard to put together and the spiral wouldn't fit properly. It wouldn't go around the way that I wanted it to go.



"I tried it again with some help. The squares were okay but I liked my other spiral. It looked more like a spiral"

**Fig. 6** Student work samples for the "Fibonacci sequences" component

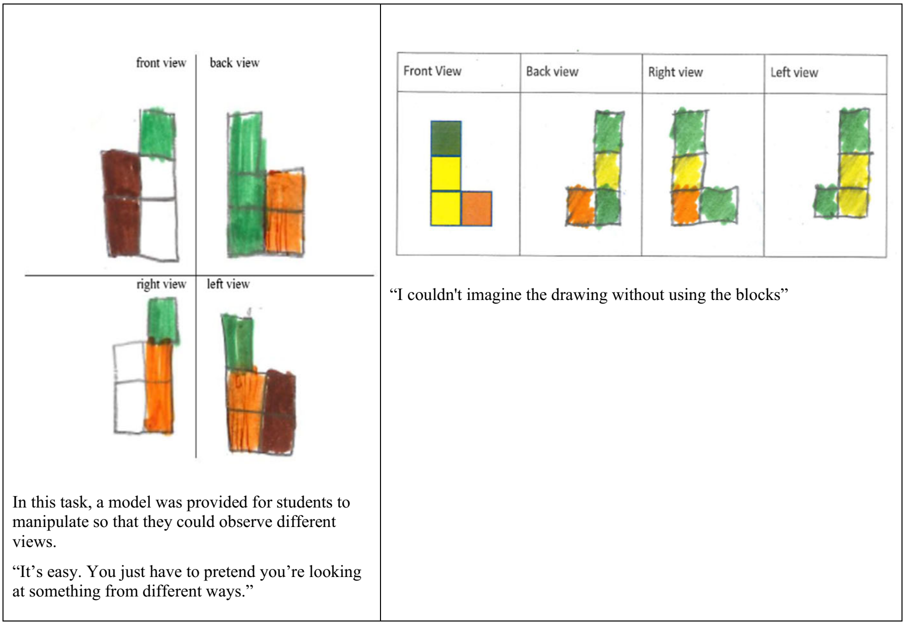



**Fig. 7** Student work samples from the "perspective-taking" component

T: I think the explaining and the representing helped them solve the challenging problem-solving tasks. They also had to produce their own independent solution rather than relying on others or waiting for my explanation. They had to think for themselves and argue (or reason) with others and respond to my questions.

I: Did you see any impact on the student's other general mathematical skills or development that wasn't covered by the SRMP?

T: Yes, what I said about the improvement in explaining and representing their own solutions. But also I saw changes for the better in the ability to notice detail and variation in mathematical patterns and connections…they looked for growing patterns and patterns in shapes more so than just for number as before. The work with the Bee-Bots was also great for their coding experience and I had not thought to link this to measurement before. It was advantageous to integrate like this.

## Post-intervention assessment

Table 3 shows the group mean scores on the assessments administered after the intervention. On this occasion, one-tailed Mann-Whitney *U* tests were used to test the hypothesis that the experimental group out-performed the comparison group. Although there was a very highly significant difference in favour of the experimental group on the PASA-2, the difference on the PASA-Sp was only just significant—possibly because of the small number of items on this experimental assessment. There was no evidence of any difference between the experimental and the comparison group on the PATMaths4.

**Table 3** Mean scores on post-intervention assessments and tests for differences

| Assessment | Group | Mean score | Mean rank | $U$ | $z$ | $p$ |
|---|---|---|---|---|---|---|
| PASA-2 | Experimental | 140.7 | 17.6 | 153 | −3.75 | 0.000 |
| | Comparison | 129.8 | 36.2 | 597 | | |
| PASA-Sp | Experimental | 17.5 | 19.0 | 272 | −1.740 | 0.041 |
| | Comparison | 15.2 | 32.0 | 496 | | |
| PATMaths4 | Experimental | 136.2 | 20.5 | 204 | −1.171 | 0.121 |
| | Comparison | 131.9 | 25.6 | 309 | | |

**Table 4** Mean scores on AMPS substructures at post-intervention assessment

| Substructure | Group | Mean score | Mean rank | $U$ | $z$ | $p$ |
|---|---|---|---|---|---|---|
| Sequences | Experimental | 137.8 | 14.9 | 211 | −2.132 | 0.016 |
| | Comparison | 128.2 | 28.4 | 437 | | |
| Shape and alignment | Experimental | 139.7 | 12.9 | 305 | −0.359 | 0.360 |
| | Comparison | 132.1 | 24.4 | 343 | | |
| Equal spacing | Experimental | 139.2 | 15.3 | 262 | −1.151 | 0.125 |
| | Comparison | 130.8 | 26.3 | 285 | | |
| Structured counting | Experimental | 140.6 | 10.4 | 170 | −2.906 | 0.002 |
| | Comparison | 128.4 | 30.5 | 478 | | |
| Partitioning | Experimental | 140.9 | 17.1 | 269 | −1.038 | 0.150 |
| | Comparison | 134.1 | 26.2 | 379 | | |

To get a better picture of the effects of SRMP on the students' PASA-2 scores, we obtained and analysed scale scores in the two groups for the five AMPS substructures measured by the PASA-2. The results are presented in Table 4.

Table 4 shows that the experimental group performed better than the comparison group on the Sequences and Structured Counting substructures, with no significant difference on the other scales. We shall discuss this apparently counter-intuitive result below.

## Summary

There was strong evidence from the qualitative and the quantitative analysis of development in students' AMPS and spatial reasoning. Students were able to seek and find patterns within each of the tasks and explain their reasoning and formulate simple generalisations. They demonstrated AMPS as they explored a variety of patterning and spatial concepts including transformations, interrelationships between 2D and 3D shapes, spatial visualisation and orientation and perspective-taking. Engagement in the spatial contexts also enabled students to connect spatial and numerical concepts when spatial challenges involved the construction of numerical patterns. The improvement in their AMPS was, however, confined to the Sequences and Structured Counting substructures.

## Discussion

We discuss the impact and feasibility of the SRMP in light of our two research questions:

1. What are the effects of the SRMP intervention on (i) spatial reasoning, (ii) Awareness of Mathematical Pattern and Structure (AMPS) and (iii) students' mathematical performance?
2. What are the key components of the SRMP that support the development of spatial concepts and processes?

**Research question 1(i)** Our analysis indicates a significantly better performance by the experimental group on the PASA-Sp post-intervention. Although the PASA-Sp was not administered pre-intervention and we acknowledge this as a limitation, the other pre-intervention assessments showed no evidence of any difference between the two groups. It therefore seems reasonable to conclude that the difference was a result of the SRMP intervention.

The four tasks in the PASA-Sp (rotation, partitioning shapes, directionality and perspective-taking) tap into the constructs of mental rotation, spatial orientation and spatial visualisation identified by Lowrie et al. (2017), so our finding could be seen as supporting their results. On the other hand, the four PASA-Sp items relate to four specific SRMP activities, so our result could also be seen as merely confirming that the experimental students had gained from their engagement with those four activities. A more definitive answer to this research question must await the development of a more general PASA-Sp, a project that is currently underway.

**Research question 1(ii)** Our analysis also indicates a significantly better performance by the experimental group on the Sequences and Structured Counting AMPS substructures. Again, although the PASA-2 was not administered pre-intervention, it seems reasonable to conclude that the differences were due to the SRMP intervention.

It is not difficult to see why SRMP should have led to gains on the Sequences structure. In the PASA-2, this structure is largely measured by a border pattern completion task and two spatial pattern continuation tasks, and there were many similar tasks included in the repeating patterns, growing patterns, shape and perimeter and Fibonacci sequences components. The gain on the Structured Counting substructure is much more difficult to explain, since there were no tasks involving Structured Counting in the entire SRMP. In particular, there was no exploration of base ten numeration, which provides two of the items that measure Structured Counting in the PASA-2. However, the essence of this structure is the awareness and application of equal grouping, and this technique was included in several components (including repeating patterns, 2D and 3D relations and growing patterns). It is feasible to conclude that transfer of understanding occurred from equal spatial groups to equal numerical groups and that even greater gains would have been made on this structure if SRMP had included a spatial treatment of the base ten system.

At first sight, it is questionable why there were no gains on the other three AMPS substructures, especially the Shape and Alignment and Equal Spacing structures which seem to be the most explicitly spatial of the five. These two structures are measured in the PASA-2 by tasks involving the drawing of arrays, grids and scales. Students did not encounter arrays or scales in the SRMP, but grids were used or implied in every single component. However, there were very few cases where students actually drew their own grids (e.g. in directionality and perspective-taking) so there was no stimulus for them to deepen their understanding of these two structures. Similarly, there were no partitioning tasks in the SRMP so no gains could have been expected on this structure either.

**Research question 1(iii)** Stephanou and Lindsey (2013), after finding a correlation of 0.72 between AMPS scores and PATMaths scores in a large sample of grade 1 and grade 2 students and exploring the content of the PASA and the PATMaths in great detail, concluded that:

> ... the two constructs are aspects of the same variable. The PATMaths aspect focuses on *what* mathematics children can and cannot do, while the AMPS aspect of the construct seeks to find *how* children think about mathematical ideas underlying tasks (p. 22, emphasis in original).

In this study, we found significant differences between the experimental and comparison groups on the PASA-2 but not on the PATMaths4. These results suggest that the SRMP had a definite but limited effect on mathematics performance. It would appear that the intervention had an effect on the way the students thought about mathematics but that this change was not great enough to have any measurable effect on how they performed on standard mathematical tasks.

**Research question 2** Our classroom observations and interviews with the SRMP teacher allow us to identify a number of key aspects that may have been responsible for students' significant improvement in spatial reasoning.

Perhaps the most critical component was the pedagogy employed in SRMP. As we had found in our development of PASMAP (Mulligan and Mitchelmore 2018), the sequence Modelling-Representing-Visualising-Generalising tends to lead to deep internalisation of spatial concepts. Many such effects are discernible in the teacher's observations. The fact that students were "able to see the shapes in their mind ... and draw them without copying given shapes" is a direct result of the Representing-Visualising steps, where students gradually move from representing the structure they see in front of them to a mental representation of that structure. The continual challenge to look for similarities and differences and to seek generalisations helped students to "notice detail and variation", to "develop their language and express these ideas" and this helped them to "solve the challenging problem-solving tasks" and to "think for themselves and argue (or reason) with others"—and not only in spatial contexts.

Another key component of SRMP was the choice of tasks. As can be seen from the descriptions in Table 1, all the tasks were challenging and quite different from those advocated in the official curriculum and common textbooks. The attention to such unusual investigations 1 day a week may in itself have improved students' attitude to mathematics and inspired them to deeper thought. Although, 25% of the curriculum was not "covered", the experimental group still managed to out-perform the comparison group on the PASA-2 mathematics assessment as well as on the PASA-Sp assessment more clearly focussed on the intervention.

## Implications and further research

This study has shown that the SRMP can be integrated into a school's program and have a positive impact on students' mathematical and spatial development that more than satisfies the mandated curriculum requirements (ACARA 2015; Board of Studies NSW 2012). However, it was only conducted in one school with one teacher, in a class where most students were above average in mathematics achievement and with the accompaniment of a great deal of professional development. Thus, our findings do not permit immediate generalisation. Nevertheless, we have gained considerable insight into the program's potential effectiveness and can envisage further development of the SRMP.

In practical terms, the SRMP needs to be documented sufficiently well that teachers could use it with relatively little guidance and assistance from professional researchers. The work done and experience gained in the current study would provide the foundation for such a document, which might resemble the PASMAP professional guidelines developed for grades K–2 from our earlier pattern and structure studies (Mulligan and Mitchelmore 2016).

Since the 25% of mathematics class time allocated to the SRMP in this study has produced significant outcomes, we may expect that an increase in curriculum coverage would lead to even greater outcomes. At least the content should be extended to cover the entire spatial content of the curriculum, including measurement and data representation, and more tasks should be included where students construct their own grids and scales instead of merely using those supplied. Our experience with PASMAP suggests that it should be possible to approach the whole curriculum through an emphasis on spatial structure.

A further research priority is the continued development of the PASA into versions that are appropriate for grades 3 and 4. This study has shown that, as anticipated, the PASA-2 has such a high ceiling that it can be useful even for above-average students in grade 4. It should therefore be possible to arrive at suitable PASA-type assessments by selecting the PASA-2 items that performed well in this study, adapting others to perform better and incorporating the four items from PASA-Sp. In particular, care should be taken to include sufficient items that explicitly measure the Shape and Alignment, Equal Spacing and Partitioning substructures.

SRMP could then be trialled systematically in an experimental study utilising a larger and more representative sample of students and classrooms, as was done for PASMAP (Mulligan and Mitchelmore 2013). Studies should also be designed to explore some questions still remaining from this study, for example: How should spatial reasoning activities be designed to improve arithmetical competence? Which SRMP components contribute most to the spatial and non-spatial learning outcomes?

What role do the three constructs of mental rotation, spatial orientation and spatial visualisation (Lowrie et al. 2017) play in the development of AMPS? How do different students proceed through the Modelling-Representing-Visualising-Generalising pedagogical sequence? Which student characteristics most influence their progress? To what extent can we say that spatial skills are causal in the development of AMPS?

One outcome of studies such as ours and others reported in this paper and this special issue is to raise the profile of spatial reasoning within mathematics teaching and learning. There is a need for dedicated learning opportunities that explicitly connect mathematical concepts across strands and processes with spatial processes and concepts and for the development and support of corresponding curriculum and professional learning resources and strategies. It is hoped that such developments will indeed eventually lead to the "spatialisation" of our national curriculum.

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